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On the Control of Robots with Visco-Elastic Joints

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***Abstract*— Feedback linearization is a viable nonlinear control technique for solving trajectory tracking problems in robots with (and without) elastic joints. However, the additional presence of dissipative effects due to joint viscosity destroys full state feedback linearizability. For robots with visco-elastic joints, the use of a static state feedback can achieve at most input-output linearization and decoupling, since an internal nonlinear dynamics is left in the closed-loop system. Although the stability properties of this unobservable dynamics still guarantee perfect output tracking in nominal conditions, control design based on static feedback becomes ill-conditioned as joint viscosity decreases. Instead, resorting to a nonlinear dynamic state feedback leads to the same closed-loop properties, but with a regularized control effort for any level of joint viscosity and elasticity. Static and dynamic nonlinear feedback control designs are presented for a reduced and a complete dynamic model of visco-elastic joint robots. A numerical comparison on a simple case study illustrates the benefits of the dynamic input-output linearization approach.**

***Index Terms*— Joint elasticity, dissipative effects, robot tracking control, feedback linearization.**

I. Introduction

When robots are required to execute trajectory tracking tasks with high speed and precision, complete and accurate dynamic models are necessary for a successful control design.

Inclusion of a joint elasticity model has been found relevant for *industrial robots* that use harmonic drives (where the teflon teeth of the flexspline introduce small angular displacement between the motor and the driven link), belts (typically, in Scara-type arms), or long shafts (e.g., for the last 3 dof of Puma robots) as reduction/transmission elements [1].

More recently, special attention has been devoted to the use of *service robots* for close human cooperation [2], [3]. The need to increase mechanical compliance and to reduce apparent inertia for safety purposes has lead to different elastic actuation/transmission arrangements in the robot design, which include: relocation of actuators close to the robot base and transmission of motion through steel cables and pulleys, like in the 8R *SM-Dexter* arm [4], or through tendons, like in the dextrous *UBHand* [5]; combination of harmonic drives and lightweight link design, as in the 7R *DLR III* arm [6]; use of parallel and distributed macro-mini actuation with elastic couplings, pioneered in the *DECMMA* project of Stanford University [7]; introduction of actuators with intrinsic variable stiffness (double McKibben muscles), as in the 3R *SoftArm* built at the University of Pisa [8].

In all the above cases, compliant phenomena are reasonably captured by assuming a concentrated elasticity at the robot joints. Dynamic modeling of elastic joint (EJ) robots has been considered in [9], [10], [11]. In a Lagrangian formulation, a doubling of generalized coordinates is needed (for each joint, one for the motor and one for the link positions) and a linear spring is introduced at each joint with associated potential energy. However, the common observation that relative motor/link internal oscillations damp out quickly over time suggests to consider additional dissipative effects. These are due to friction (of any type), separately acting on the motor and on the link side of the transmissions, and to the intrinsic viscosity associated to elastic joints. Although possibly small, these terms may affect static precision in positioning or dynamic accuracy in trajectory tracking tasks.

From a control point of view, robots with joint elasticity have challenged researchers for a long time. For regulation tasks, Lyapunov arguments have been used to prove global asymptotic stability of a desired equilibrium configuration when using a PD control action with constant [10] or on-line [12], [13] gravity compensation. These controllers use only motor measurements. For trajectory tracking tasks, exact linearization by static state feedback [9] and passivity-based adaptive control [14] have been proposed for a reduced dynamic model of EJ robots. When joint stiffness is sufficiently large, a singular perturbation control approach can also be followed [15]. When inertial cross-couplings between motors and links are included in the dynamic model of EJ robots, full linearization of the robot equations can still be obtained through a dynamic state feedback [16]. This holds true also in the case of robots with mixed sequences of rigid and elastic joints [17], [18].

The additional presence of viscous friction and/or joint viscosity does not affect the terminal behavior of regulation controllers. In fact, such dissipative effects typically improve the oscillatory transients of the control laws presented in [10], [12], [13], which need thus no modifications. Moreover, it can be shown that viscous friction acting on the motor and/or on the link side of an elastic joint can be easily included in the design of nonlinear feedback laws intended for trajectory tracking. Therefore, our main interest here is to evaluate the impact of visco-elasticity at the robot joints on the design of tracking controllers based on feedback linearization [9], [16]. Although this class of nonlinear feedback laws is not the only available for solving trajectory tracking problems (see, e.g., the backstepping approach in [19]), it is certainly the best performing one in nominal conditions.

After presenting the dynamic model of visco-elastic joint (VEJ) robots (Sect. II), we detail first the synthesis of static versus dynamic nonlinear feedback control laws under the same simplifying modeling assumptions used in [9] (Sect. III). Then, we consider the control design for the more complete dynamic model including cross-inertial terms (Sect. IV). Finally, simulation results are presented in Sect. V to illustrate the theoretical findings and to assess quantitatively the improved control behavior obtained with the dynamic linearization approach.

II. Dynamic modeling of VEJ robots

Consider an open kinematic chain of N rigid links, interconnected by N joints undergoing visco-elastic deformation. The robot is actuated by N electrical drives, the i-th being located at the i-th joint or mounted on a previous link of index $j < i$. Let $q \in I\!R^N$ be the link position coordinates and $\theta \in I\!R^N$ be the motor (i.e., rotor) position coordinates, as reflected through the gear ratios. We assume: *i)* small joint deformations, in the linear elastic domain; *ii)* balanced motors, i.e., rotors are uniform bodies with center of mass on their rotation axes. As a result, inertia and gravity terms will not depend on θ.

Using a Lagrangian approach, the kinetic energy is

$$T = \frac{1}{2} \begin{bmatrix} \dot{q}^{\mathrm{T}} & \dot{\theta}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} M(q) & S(q) \\ S^{\mathrm{T}}(q) & J \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{\theta} \end{bmatrix},$$

where the $N \times N$ blocks in the overall inertia matrix are the link inertia matrix $M(q) > 0$, the motor inertia diagonal matrix $J > 0$, and the strictly upper triangular matrix $S(q)$ which takes into account the motor/link inertial couplings [10]. The potential (gravitational plus elastic) energy is

$$U = U_g(q) + \frac{1}{2}(q - \theta)^{\mathrm{T}} K (q - \theta),$$

with the joint stiffness diagonal matrix $K > 0$.

From the Euler-Lagrange equations for $L = T - U$, the dynamic model of VEJ robots is expressed by a set of $2N$ second-order differential equations

$$\begin{bmatrix} M(q) & S(q) \\ S^{\mathrm{T}}(q) & J \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} C(q,\dot{q})\dot{q} + C_1(q,\dot{q})\dot{\theta} \\ C_2(q,\dot{q})\dot{q} \end{bmatrix} + \begin{bmatrix} g(q) + K(q-\theta) \\ K(\theta - q) \end{bmatrix} = \begin{bmatrix} 0 \\ \tau \end{bmatrix} - \begin{bmatrix} \tau_{V,q} \\ \tau_{V,\theta} \end{bmatrix}, \tag{1}$$

where $C\dot{q}$, $C_1\dot{\theta}$, and $C_2\dot{q}$ are centrifugal and Coriolis terms, $g = (\partial U_g/\partial q)^{\mathrm{T}}$ are gravity terms, $\tau \in I\!R^N$ are the torques supplied by the motors, and the viscous dissipative terms are

$$\begin{bmatrix} \tau_{V,q} \\ \tau_{V,\theta} \end{bmatrix} = \begin{bmatrix} D(\dot{q} - \dot{\theta}) + F_q\dot{q} \\ D(\dot{\theta} - \dot{q}) + F_\theta\dot{\theta} \end{bmatrix}, \tag{2}$$

with the joint viscosity diagonal matrix $D > 0$ and, respectively, the link and motor viscous friction diagonal matrices $F_q > 0$ and $F_\theta > 0$. We refer to the first N equations in (1) as the link equations, and to the last N as the motor equations.

The simplifying assumption that angular kinetic energy of the motors is due only to their own spinning [9] implies $S \equiv 0$ in eq. (1), from which $C_1 = C_2 = 0$ also follow. Using eq. (2), the reduced model of VEJ robots becomes

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + D(\dot{q} - \dot{\theta}) + K(q - \theta) + F_q\dot{q} = 0 \tag{3}$$

$$J\ddot{\theta} + D(\dot{\theta} - \dot{q}) + K(\theta - q) + F_\theta\dot{\theta} = \tau. \tag{4}$$

III. Input-output linearization of VEJ robots

For a VEJ robot, we consider the problem of tracking a sufficiently smooth reference (output) trajectory $q = q_d(t)$ defined in terms of link coordinates. Note that such a desired link motion algebraically determines a unique cartesian trajectory of the robot end-effector via the standard direct kinematics. On the other hand, the motor trajectory $\theta_d(t)$ associated to $q_d(t)$ has to be determined dynamically.

For the sake of simplicity, ideal operative conditions are assumed, i.e., measures of the full state and an accurate dynamic model are available for feedback control design. Therefore, we suppose to have a sensor for each of the four variables q, θ (link and motor position) and $\dot{q}$, $\dot{\theta}$ (link and motor velocity) and to have identified the 'rigid' robot dynamic coefficients as well as the friction and visco-elastic joint parameters.

If joint viscosity were not present ($D \equiv 0$), the reduced model (3–4) of EJ robots would be exactly linearizable via nonlinear static state feedback [9], whereas the same result would hold for the complete model (1) by resorting to a nonlinear dynamic state feedback [16]. The trajectory tracking problem can be easily solved by completing the stabilizing control design on the linear and decoupled side of the obtained system, similarly to the well-known computed torque method for rigid robots [20].

Unfortunately, the above full state linearization properties are lost for VEJ robots. Therefore, we shall pursue the approach of designing a nonlinear control law that maximizes the dimension of the linear part of the closed-loop dynamics, either by static or by dynamic state feedback. This linear part will be described in terms of the link output q and its derivatives. We shall first consider the reduced model (3–4), handling then the complete one in Sect. IV. Preliminarily, note that by neglecting the (presumably small) joint viscosity in the control design, exact tracking capabilities would be destroyed even in the assumed ideal conditions.

A. Static state feedback design

Rewrite the link equation (3) as

$$M(q)\ddot{q} + \alpha(q,\dot{q}) - D\dot{\theta} - K\theta = 0, \tag{5}$$

where $\alpha = C(q,\dot{q})\dot{q} + g(q) + (D + F_q)\dot{q} + Kq$. Differentiating once, we obtain[1]

$$M(q)q^{[3]} + \beta(q,\dot{q},\ddot{q}) - D\ddot{\theta} - K\dot{\theta} = 0,$$

with $\beta = \dot{\alpha}(q,\dot{q}) + \dot{M}(q)\ddot{q}$. Substituting $\ddot{\theta}$ from motor equation (4) and imposing $q^{[3]} = v$, where $v \in I\!R^N$ is the new control input, leads to the static state feedback

$$\tau = JD^{-1}\Big[M(q)v + \beta(q,\dot{q},\ddot{q}) - K\dot{\theta}\Big] + \gamma(q,\theta,\dot{q},\dot{\theta}), \quad (6)$$

with the linear function $\gamma = D(\dot{\theta} - \dot{q}) + K(\theta - q) + F_\theta\dot{\theta}$. Note that $z = (q,\dot{q},\ddot{q},\dot{\theta}) = T_1(q,\dot{q},\theta,\dot{\theta})$ defines a global change of coordinates, by virtue of the invertibility of $M(q)$ in eq. (5). Therefore, the implementation of the control law (6) does *not* require acceleration measurements: link acceleration $\ddot{q}$ is computed from eq. (5) using the model and the available measures of the robot state. On the other hand, wishing to avoid the associated inversion of the link inertia matrix $M(q)$, link acceleration in eq. (6) could be also estimated by numerical differentiation of link velocity.

The resulting closed-loop system is described by

$$\begin{aligned} q^{[3]} &= v \\ \ddot{\theta} &= D^{-1}\Big[M(q)v + \beta(q,\dot{q},\ddot{q}) - K\dot{\theta}\Big]. \end{aligned} \quad (7)$$

The input-output behavior between v and q is linear and decoupled, while an additional internal nonlinear dynamics has appeared in eq. (7). In order to complete the control design, the new control input v is synthetized as a linear exponentially stabilizer (e.g., by pole placement) of the tracking error $e = q_d - q$. The stability of the resulting overall closed-loop system has to be verified. For this, it is sufficient that the zero dynamics associated to the output $y = q - q_0 \equiv 0$ is asymptotically stable [21]. It is easy to see that the zero dynamics for system (7) is

$$\dot{z}_4 = -D^{-1}Kz_4,$$

i.e., linear, first order, asympotically stable, and arbitrarily fast for $D \to 0$.

Although the obtained closed-loop characteristics are certainly satisfactory, we note that the static control law (6) for τ is ill-conditioned when D approaches zero. As a matter of fact, input-output linearization has been achieved by relying on (and inverting) a weak dissipative term. This results in a low-authority control action that will be highly sensitive to perturbations or disturbances.

B. Dynamic state feedback design

Define first a nonlinear feedback law for τ, in terms of a new variable $u \in I\!R^N$:

$$\tau = Ju + \gamma(q,\theta,\dot{q},\dot{\theta}).$$

The robot dynamics (3–4) becomes

$$\begin{aligned} M(q)\ddot{q} + \alpha(q,\dot{q}) - D\dot{\theta} - K\theta &= 0 \quad (8)\\ \ddot{\theta} &= u. \quad (9) \end{aligned}$$

[1]The compact differential notation $x^{[i]} = d^ix/dt^i$ is used throughout the paper.

By differentiating twice eq. (8), we obtain

$$\begin{aligned} M(q)q^{[3]} + \beta(q,\dot{q},\ddot{q}) - Du - K\dot{\theta} &= 0 \quad (10)\\ M(q)q^{[4]} + \delta(q,\dot{q},\ddot{q},q^{[3]}) - D\dot{u} - Ku &= 0, \quad (11) \end{aligned}$$

where $\delta = \dot{\beta}(q,\dot{q},\ddot{q}) + \dot{M}(q)q^{[3]}$. Imposing now $q^{[4]} = v$ and solving for $\dot{u}$ leads to the dynamic state feedback

$$\begin{aligned} \tau &= Ju + \gamma(q,\theta,\dot{q},\dot{\theta}) \\ \dot{u} &= D^{-1}\left[M(q)v + \delta(q,\dot{q},\ddot{q},q^{[3]}) - Ku\right], \end{aligned} \quad (12)$$

where $u \in I\!R^N$ is the state of the obtained dynamic compensator. The change of coordinates $z = (q,\dot{q},\ddot{q},q^{[3]},u) = T_2(q,\dot{q},\theta,\dot{\theta},u)$ is globally defined, thanks to the invertibility of $M(q)$ in eqs. (8) and (10). Once again, the implementation of the control law (12) does *not* require acceleration or jerk measurements: link acceleration $\ddot{q}$ and link jerk $q^{[3]}$ are computed, respectively, from eq. (8) and eq. (10), using the model and the available measures of the robot state.

The resulting closed-loop system is

$$\begin{aligned} q^{[4]} &= v \\ \dot{u} &= D^{-1}\left[M(q)v + \delta(q,\dot{q},\ddot{q},q^{[3]}) - Ku\right]. \end{aligned} \quad (13)$$

The structure of eqs. (13) is similar to that of eqs. (7), although the input-output relation is now of fourth-order. Control design for v is successfully completed as in Sect. III-A (now for the decoupled chains of four input-output integrators), since the zero dynamics associated to system (13),

$$\dot{z}_5 = -D^{-1}Kz_5,$$

is the same as before.

The basic difference, however, stands in the fact that the dynamic control law (12) is now well-defined even for vanishing D. Stated differently, the ill-conditioning of the static feedback law (6) has been transformed in a singularly perturbed dynamic compensator. The notable consequence is that, in the limit case of $D \to 0$, the compensator dynamics can be always considered to have already reached its steady-state. By setting then $\dot{u} = 0$ in eq. (12) and solving for u, yields the following control torque

$$\tau = JK^{-1}\Big[M(q)v + \delta(q,\dot{q},\ddot{q},q^{[3]})\Big] + \gamma(q,\theta,\dot{q},\dot{\theta}),$$

i.e., the same expression of the static feedback linearizing law for EJ robots derived in [9]. This shows the numerical stability of the dynamic feedback design for any value of the joint viscosity D, as opposed to the static controller (6).

IV. Including complete inertial couplings

Consider now the complete model (1) of VEJ robots and assume, for the sake of a simpler presentation, that the matrix S is constant (though non-zero). This happens, e.g., in planar robotic structures and implies that the velocity matrices C_1 and C_2 vanish in the model. We recall that EJ robots are not static feedback linearizable as soon as $S \neq 0$ [16], and this is indeed true also for VEJ robots. Therefore, the only feasible choice in this case is to consider a dynamic state feedback design. The purpose of this section is to show that the constructive three-step algorithm that achieves full state linearization via dynamic feedback for EJ robots [16] can be suitably modified so as to provide input-output linearization of the complete model of VEJ robots.

At **step 1**, the motor equations in (1) are feedback linearized by means of the following globally defined static control law

$$\tau = \left[J - S^T M^{-1}(q) S\right] u + \tau_{V,\theta} + K(\theta - q) \\ -S^T M^{-1}(q) \left[\tau_{V,q} + K(q-\theta) + g(q) + C(q,\dot{q})\dot{q}\right],$$

where $u \in I\!R^N$ is a new input variable. The resulting robot equations are conveniently rewritten as

$$f(q,\dot{q},\ddot{q}) + S\ddot{\theta} - D\dot{\theta} - K\theta = 0 \quad (14)$$

$$\ddot{\theta} = u, \quad (15)$$

with $f = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) + (D + F_q)\dot{q} + Kq$. Note that eq. (14) differs from eq. (8) by the presence of matrix S, which brings the motor acceleration $\ddot{\theta}$ into the picture. Moreover, the structural singularity of S (a strictly upper triangular matrix) is the reason for the failure of static feedback as an input-output decoupling scheme.

At **step 2**, input-output decoupling is performed w.r.t. the ficticious output f. For this, differentiate twice eq. (14)

$$\ddot{f} + S\ddot{u} - D\dot{u} - Ku = 0$$

and add a dynamic extension of $2(i-1)$ integrators (with state ϕ_{ij}) on the i-th input channel, for $i = 1, \ldots, N$, so as to avoid input differentiation (because now $S \neq 0$, as well as $D \neq 0$).

For illustration, consider a VEJ robot with $N = 2$ joints. The dynamic extension is

$$\begin{aligned} u_1 &= \bar{w}_1 \\ u_2 &= \phi_{21} \qquad \dot{\phi}_{21} = \phi_{22} \qquad \dot{\phi}_{22} = \bar{w}_2, \end{aligned} \quad (16)$$

where $\bar{w}_i$, $i = 1, 2$, are the temporary inputs to the integrator chains. Since S is strictly upper triangular, this leads to

$$\ddot{f}_1 + s_{12}\bar{w}_2 - d_1\dot{\bar{w}}_1 - k_1\bar{w}_1 = 0 \quad (17)$$

$$\ddot{f}_2 - d_2\phi_{22} - k_2\phi_{21} = 0. \quad (18)$$

In order to avoid ill-conditioned operations for a vanishing joint viscosity coefficient d_2, differentiate twice eq. (18)

$$f_2^{[4]} - d_2\dot{\bar{w}}_2 - k_2\bar{w}_2 = 0,$$

and apply the additional dynamic feedback

$$\begin{aligned} \dot{\bar{w}}_1 &= \frac{1}{d_1}\left[-k_1\bar{w}_1 + s_{12}\bar{w}_2 + w_1\right] \\ \dot{\bar{w}}_2 &= \frac{1}{d_2}\left[-k_2\bar{w}_2 + w_2\right]. \end{aligned} \quad (19)$$

The combination of eqs. (16) and (19) leads to the same input-output linear and decoupled result (for step 2) of the case of no joint viscosity, namely

$$\ddot{f}_1 = w_1 \qquad f_2^{[4]} = w_2.$$

Note that an internal unobservable dynamics of dimension $N = 2$ has been left. The associated zero-dynamics (i.e., constraining the output $f \equiv 0$) is

$$\begin{aligned} \dot{\bar{w}}_1 &= -\frac{k_1}{d_1}\bar{w}_1 + \frac{s_{12}}{d_1}\bar{w}_2 \\ \dot{\bar{w}}_2 &= -\frac{k_2}{d_2}\bar{w}_2 \end{aligned}$$

i.e., linear, first order, asympotically stable, and arbitrarily fast for vanishing d_1 and d_2. Furthermore, following an analysis similar to the one at the end of Sect. III, for $d_1, d_2 \to 0$ the dynamics (19) smoothly reduces to a static transformation (as in EJ robots).

More in general, for VEJ robots with N dofs, N additional compensator states (compare with eq. (19)) are introduced in step 2 of the algorithm, beside the original $N(N-1)$ states of the dynamic extension needed for EJ robots (compare with eq. (16)).

At **step 3**, the algorithm resumes without changes with respect to [16]. Linearization and decoupling of the link position output q is obtained using a further dynamic extension of dimension $N(N-1)$.

For robots with N visco-elastic joints, the obtained result is summarized in the following

Proposition: A dynamic controller of dimension $N(2N-1)$ achieves input-output linearization of a general VEJ robot. The resulting input-output system is composed of N decoupled chains of $2(N+1)$ integrators each. The closed-loop system contains also an exponentially stable zero dynamics of dimension N. ■

V. Simulation Results

In order to compare the performance of the static and dynamic controllers of Sect. III, we have simulated a trajectory tracking task for the simplest VEJ robot, consisting of a single link ($N = 1$) moving in the vertical plane without friction. In this case, $S = 0$ and the (scalar) parameters in eqs. (3–4) have been chosen as:

$$M = 0.66 \quad J = 0.10 \quad mg_0 l_c = 9.81 \quad K = 100.$$

To show the effects of a small joint viscosity, its coefficient is fixed at $D = 0.01$.

The reference trajectory $q_d(t)$ for the link is a 9th-order rest-to-rest polynomial of $T = 1$ s (with zero boundary conditions up to the fourth derivative) from the downward equilibrium ($q = 0$) to the horizontal position. For the trajectory error stabilization, closed-loop poles have been set to $(-10, -10, -100)$ in association to the controller (6), with an additional pole in -100 for the dynamic feedback design.

Figures 1–2 refer to the exact output tracking situation, i.e., for matched initial conditions, with the robot at rest in the initial configuration and without joint deformation. The two torque profiles are practically identical, while the dynamic compensator state shows a bounded rest-to-rest behavior.

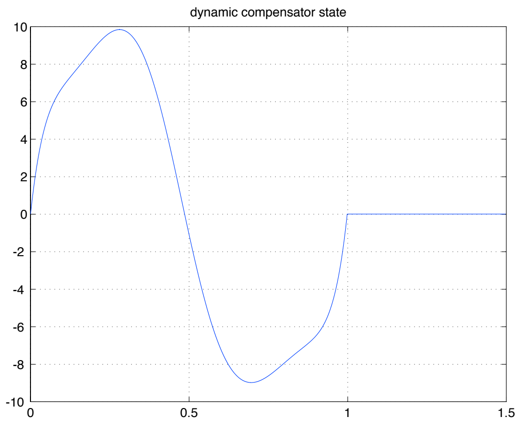

Fig. 1. Exact tracking – evolution of the dynamic compensator state u of eq. (12)

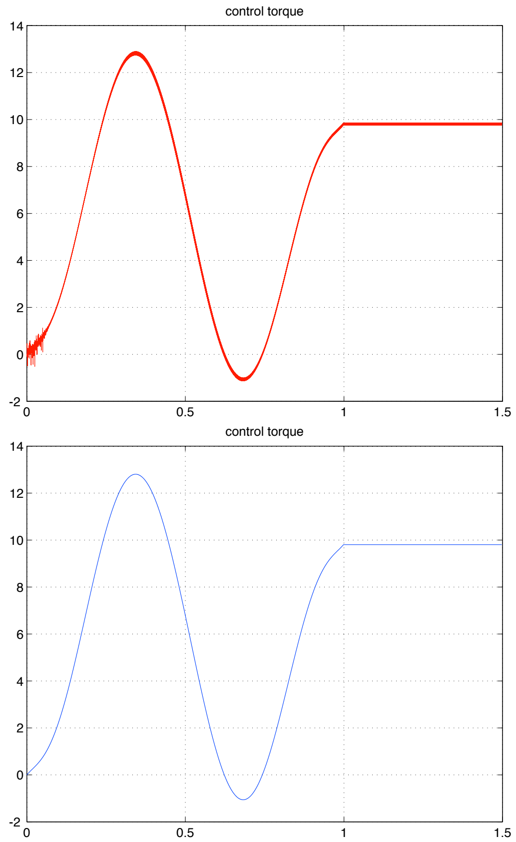

Fig. 2. Exact tracking – static (top, red) and dynamic (bottom, blue) control torques [Nm]

Asymptotic tracking results for an initial off-trajectory configuration ($q(0) = 5°$) are presented in Figs. 3–4. Actuator saturation has been also introduced as a typical non-ideal condition. The initial error is recovered within ≈ 0.3 s and the output performance is indistinguishable with the two controllers. However, a more jerky behavior has been observed in the deformation of the robot joint in the case of static feedback (not reported here). A large difference is instead present in the control effort, with a much smoother torque profile obtained in the dynamic case.

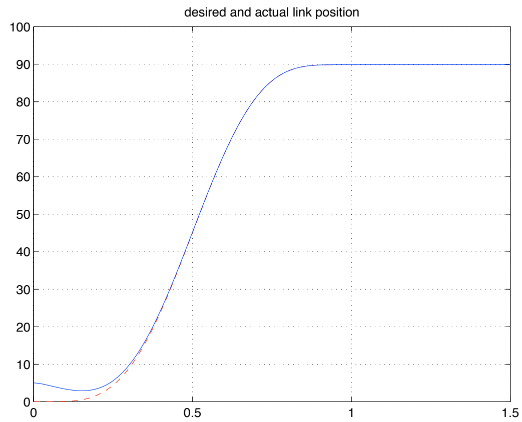

Fig. 3. Asymptotic tracking – link position (solid, blue) and its reference (dashed, red) with dynamic control law (similar with static law) [deg]

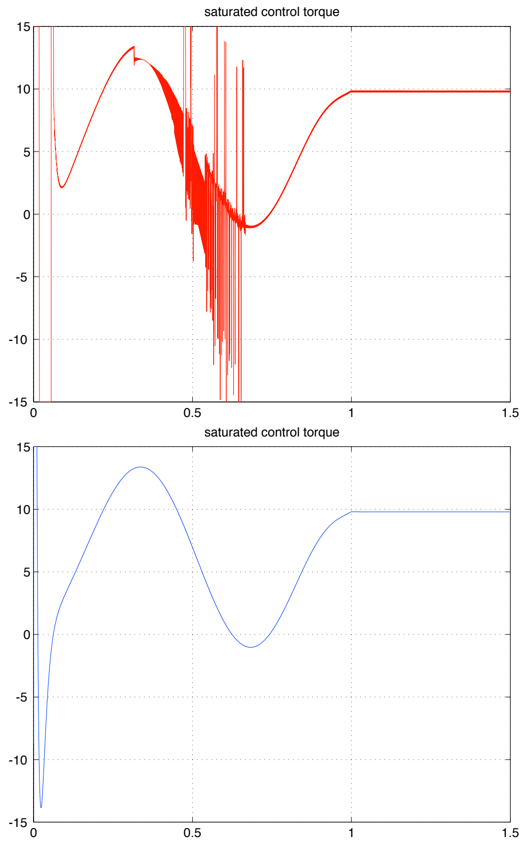

Fig. 4. Asymptotic tracking – static (top, red) and dynamic (bottom, blue) control torques [Nm]

VI. Conclusions

We have considered modeling and control issues for robots with visco-elastic joints. Accurate trajectory control can be obtained by input-output model-based linearization techniques, using either static or dynamic state feedback. In case of common applicability of the static and dynamic designs, output performance is identical in nominal conditions but control sensitivity to tracking errors and unmodeled dynamics is quite different. In fact, dynamic feedback 'regularizes' the problem for vanishing joint viscosity, generating smoother control torques and recovering in the limit case the same control laws that are valid in the absence of viscosity, namely dynamic or, respectively, static feedback linearization of the full robot state, depending on the presence or absence of cross-coupling inertial terms in the model. In all cases, no link acceleration or higher-order derivatives are needed for the computation of the control laws.

We have considered in this paper only issues related to tracking accuracy and control effort for motion tasks in free space. When interacting with the environment, e.g., with a human user, the presented control designs can be modified accordingly, or simpler cartesian impedance schemes [4], [22] may be used for the purpose of safe operation. There is indeed a natural trade-off between the accuracy achievable during free motion and a safe/compliant behavior during interaction.

Finally, we mention that in the case of EJ or VEJ robots having variable stiffness transmissions, dynamic feedback is instrumental for obtaining simultaneous and decoupled control of the robot motion (in terms of the link position output q) and of its compliance (in terms of the joint stiffness output K). This feature, which is relevant for minimizing expected injuries for accidental collisions between robots and humans, is the subject of current research.

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