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Optimized Metric Clipping Decoder Design for Impulsive Noise Channels at High Signal-to-Noise Ratios

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***Abstract*—In many practical communication systems, the channel is corrupted by non-Gaussian impulsive noise (IN). It introduces decoding metric mismatch for the traditional Euclidean metric decoders and limits system performance. The situation is worsen by the practical difficulty in accurately estimating the IN statistics. Recently, some metric clipping based decoders with a properly chosen clipping threshold has been shown to be very effective in mitigating the effect of IN, even without a precise knowledge of its statistics. However, we observe that such a clipping threshold is derived based on some assumptions which lead to an error floor in the bit error probability curve at high signal-to-noise ratio (SNR). In this work, a clipping threshold is derived by an optimization approach without exploiting the IN statistics. It is demonstrated by simulation that with our proposed clipping threshold, the optimzed metric clipping decoder is able to perform close to the maximum likelihood decoding performance at high SNR under the Bernoulli Gaussian noise model with various parameters.**

I. Introduction

In most communication channel models [1], the noise is assumed to be Gaussian distributed. However, the assumption is violated by the impulsive noise with random arrivals and high spikes in some real-world applications, e.g., power line communications (PLC) [3], digital subscriber line (DSL) loops [4], ultra wide-band (UWB) communication system [5], cognitive radio systems [6].

In the literature, there are several well-known statistical models for the impulsive noise. The Bernoulli Gaussian (BG) noise model [7] is a simplistic and representative one. It models the impulsive noise at the k-th time slot with two components, i.e.,

$$n_k = w_k + b_k \cdot g_k \tag{1}$$

where w_k represents the white background Gaussian noise, g_k denotes the additive Gaussian distributed impulsive noise with a larger noise power, the power ratio between the impulsive and Gaussian components is defined as the impulsive to Gaussian noise power ratio (IGR) Γ, and $b_k \in \{0, 1\}$ is a Bernoulli random variable with impulse arrival probability p for b_k being 1. A more practical but complicated model is the Middleton Class-A (MCA) noise model [8] which models the impulsive component as a weighted linear combination of an infinite number of Gaussian distributions with different variances.

To tackle the impulsive noise, better decoding schemes have been developed by exploiting the knowledge of the noise statistics. In Song *et al.*'s work [5], a detection rule for the m-ary signal at a low signal to noise ratio (SNR) was proposed by exploiting the noise variance statistics, e.g., the noise dispersion parameter in Cauchy distribution, the degree of freedom parameter in Student's t-distribution. A locally optimal detector was proposed in [9] by preserving only one impulsive noise term in the MCA model and estimating the noise state (e.g., impulsive or Gaussian) based on the noise variance. However, estimating the impulsive noise statistic is a non-trivial task since the number of impulsive noise samples is much less than that of the background Gaussian noise, and more importantly, the accurate impulsive noise model is unknown in many practical applications. Recently, it was discovered that it is possible to derive a decoding metric without knowing the impulsive noise statistics such that the resultant decoding performance is close to the optimal maximum likehood decoder (MLD) performance under certain conditions. This was first demonstrated by the so-called joint erasure marking and Viterbi algorithm (JEVA) proposed by Li *et al.* [6], [7] for the convolutionally coded data by extending the Viterbi decoding algorithm to exploiting the code structure to mark erasures. However, the computational cost for JEVA is high and becomes unmanageable for long codelengths. Fertonani and Colavolpe [10] made a similar conclusion for low-density parity-check codes with iterative decoding based on a robust (against impusive noise) soft decoding metric, which can be obtained by simply clipping the Gaussian noise metric. However, a systematic way to choose the clipping threshold is unavailabe in [10]. In [11], by interpreting the Viterbi algorithm with a clipped Euclidean metric as a special form of JEVA that can mark a varying number of erasures, the so-called metric erasure Viterbi algorithm (MEVA) was proposed and a systematic way to choose the clipping threshold was derived by assuming the impulsive noise probability is close to 1. The aforementioned metric clipping based decoders are easy to implement as practical decoders always adopt some form of metric clipping due to the finite bit precision in digital circuit implementation. However, the assumption made in [11] (and implicitly in [10]) about the noise statistics for deriving the clipping threshold are violated at high SNR which leads to an error floor in bit error probability curve, as shown in the Experimentsl Results Section.

In this work, we focus on the high SNR behavior of the decoding performance of a coded system under an impulsive noise channel. An optimized metric clipping decoder (OMCD) is proposed. The selection of the metric clipping threshold is formulated as an optimization problem without estimating the impulsive noise statistics. Compared with MEVA [11], our optimization problem is analytically formulated using the exact pairwise error provability (PEP), rather than choosing the parameter with the plots of Chernoff bounds. To evaluate the performance, simulations are conducted under the Bernoulli Gaussian model over a wide range of parameters. Experimental results demonstrate that the proposed clipping method is able to approach the MLD performance at high SNR.

II. Proposed Approach

In this work, we consider the decoding problem in the presence of impulsive noise. The decoding decision is based on choosing $\mathbf{x}$ that maximizes the decoding metric $m(\mathrm{t}, \mathbf{r}, \mathbf{x})$, where t is the clipping threshold of the Euclidean metric [11] which is to be optimized, $\mathbf{x}$ is the transmitted signal sequence, i.e., a codeword, $\mathbf{r}$ is the received signal samples at the output of the communication channel. Denoting by $\hat{\mathbf{x}}$ the competing codeword, the metric difference of the two codewords can be expressed as

$$\Delta_m(\mathrm{t},\mathbf{r},\mathbf{x},\hat{\mathbf{x}}) = m(\mathrm{t},\mathbf{r},\mathbf{x}) - m(\mathrm{t},\mathbf{r},\hat{\mathbf{x}}), \tag{2}$$

where the path metric for a codeword of length N is given by the sum of the bit metrics, i.e., $m(\mathrm{t},\mathbf{r},\mathbf{x}) = \sum_{i=1}^{N} m(\mathrm{t},r_i,x_i)$, and the pairwise error probability (PEP) is given by

$$P\{\mathbf{x}\to\hat{\mathbf{x}}\} = P\{\Delta_m(\mathrm{t},\mathbf{r},\mathbf{x},\hat{\mathbf{x}}) < 0\}. \tag{3}$$

In this section, we aim at optimizing the clipping threshold by minimizing the PEP as a function of t. According to the unified PEP calculation formula in [12], the exact PEP can be expressed as

$$\begin{aligned} P\{\mathbf{x}\to\hat{\mathbf{x}}\} &= \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}\Phi(s,\mathrm{t})\frac{ds}{s} \\ &= \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} E(e^{-s\cdot\Delta_m(\mathrm{t},r_i,x_i,\hat{x}_i)})^d\frac{ds}{s} \end{aligned} \tag{4}$$

where $\Phi(s,\mathrm{t})$ is the moment generating function (MGF) of the metric difference $\Delta_m(\mathrm{t},\mathbf{r},\mathbf{x},\hat{\mathbf{x}})$, d is the pairwise distance of the pair of codewords, and each dimension is assumed to be independent and identically distributed. Therefore, the metric difference $\Delta_m(\mathrm{t},\mathbf{r},\mathbf{x},\hat{\mathbf{x}}) = \sum_{i=1}^{d}\Delta_m(\mathrm{t},r_i,x_i,\hat{x}_i)$. According to the numerical calculation of the Gauss quadrature rule [12], the optimal parameters which lead to the minimum PEP can be found by minimizing the Chernoff bound, i.e., $\min_{s,\mathrm{t}}\Phi(s,\mathrm{t})$. Therefore, the optimal choice of clipping threshold t can be obtained by setting the first order partial derivatives of $\Phi(s,\mathrm{t})$ to zeros. That is,

$$\frac{\partial\Phi(s,\mathrm{t})}{\partial\mathrm{t}} = 0 \quad\text{and}\quad \frac{\partial\Phi(s,\mathrm{t})}{\partial s} = 0. \tag{5}$$

Assuming the all-zero codeword is transmitted (i.e., a linear code is used), we have $r_i = \sqrt{E}+n_i$, where n_i is the additive channel noise. Thus, $\Delta_m(\mathrm{t},r_i,x_i,\hat{x}_i)$ can be expressed in terms of t and n_i and for notational convenience, we define $\Delta(\mathrm{t},n_i,x_i,\hat{x}_i) = \Delta_m(\mathrm{t},\sqrt{E}+n_i,x_i,\hat{x}_i)$. The metric difference for each bit $\Delta(\mathrm{t},n_i,x_i,\hat{x}_i)$ under $\mathrm{t}\geq\sqrt{E}$ is given in [11] as,

$$\Delta(\mathrm{t},n_i,x_i,\hat{x}_i) = \begin{cases} 4\sqrt{E}(\sqrt{E}+n_i), & -\mathrm{t} < n_i < \mathrm{t}-2\sqrt{E} \\ \mathrm{t}^2 - n_i^2, & \mathrm{t}-2\sqrt{E} \leq n_i < \mathrm{t} \\ (2\sqrt{E}+n_i)^2 - \mathrm{t}^2, & -\mathrm{t}-2\sqrt{E} \leq n_i \leq -\mathrm{t} \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

It should be noted that, different from [11], we are only interested in the case of $\mathrm{t}\geq\sqrt{E}$, since $\mathrm{t} < \sqrt{E}$ leads to a disconnected decision region and severe degradation of system performance. Eq. (5) can be expanded as

$$\begin{aligned} &\frac{\partial\Phi(s,\mathrm{t})}{\partial\mathrm{t}} = \frac{\partial}{\partial\mathrm{t}}\int_{-\infty}^{\infty} e^{-s\cdot\Delta(\mathrm{t},n_i,x_i,\hat{x}_i)}f(n_i)dn_i = 0 \\ \Rightarrow &\int_{\mathrm{t}}^{\mathrm{t}+2\sqrt{E}} e^{-s[\mathrm{t}^2-(n_i-2\sqrt{E})^2]}f(n_i-2\sqrt{E}) \\ &\qquad - e^{-s[(n_i-2\sqrt{E})^2-\mathrm{t}^2]}f(n_i)dn_i = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} &\frac{\partial\Phi(s,\mathrm{t})}{\partial s} = \frac{\partial}{\partial s}\int_{-\infty}^{\infty} e^{-s\cdot\Delta(\mathrm{t},n_i,x_i,\hat{x}_i)}f(n_i)dn_i = 0 \\ &\Rightarrow \int_{\sqrt{E}}^{\mathrm{t}} 4\sqrt{E}(n_i-\sqrt{E})\Big[e^{-4\sqrt{E}s(n_i-\sqrt{E})}f(n_i-\sqrt{E}) \\ &- e^{4\sqrt{E}s(n_i-\sqrt{E})}f(n_i)\Big]dn_i + \int_{\mathrm{t}}^{\mathrm{t}+2\sqrt{E}}[\mathrm{t}^2-(n_i-2\sqrt{E})^2]\times \\ &\Big[e^{-s[\mathrm{t}^2-(n_i-2\sqrt{E})^2]}f(n_i-2\sqrt{E}) - e^{s[\mathrm{t}^2-(n_i-2\sqrt{E})^2]}f(n_i)\Big]dn_i = 0 \end{aligned} \tag{8}$$

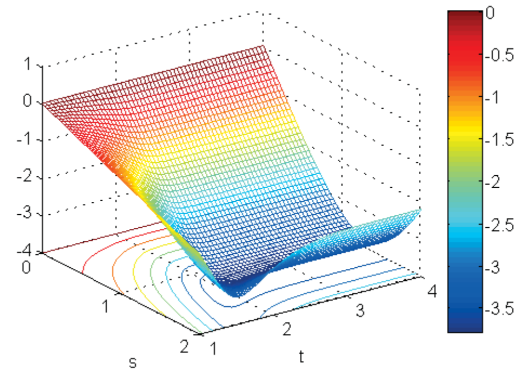

Fig. 1. A 3D plot of the Chernoff bound $\Phi(s,\mathrm{t})$versus the clipping threshold t and the Chernoff parameter s under the Bernoulli Gaussian noise model with $\Gamma = 15, p = 0.1$ and SNR = 15 dB.

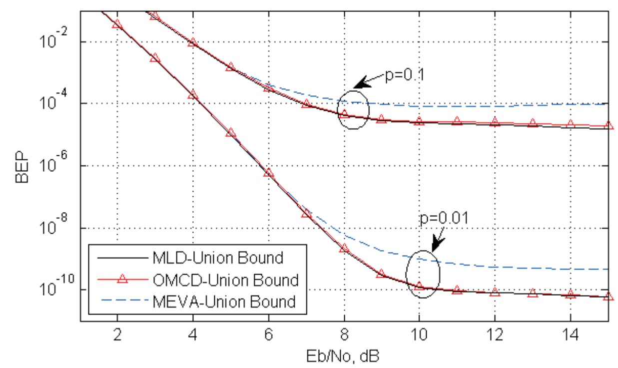

Fig. 2. The union bound on the Bit Error Probability (BEP) of MLD, MEVA and the proposed OMCD for a convolutional coded system under the BG noise model with $\Gamma = 400$ and $p = \{0.01, 0.1\}$.

It can be seen that Eq. (7) and (8) consist of integrals of products of an amplifying factor and the noise probability. The clipping threshold t controls the exponential amplifying factor and the integral limits while s affects the amplifying factors. It is conjectured that the clipping threshold should be chosen as $\mathrm{t} = 2\sqrt{E}$.

Intuitively, the reasons are two folds. First, although both t and s are able to control the amplifying factors, s is more efficient since the factors vary monotonically as s increases while t is not. Secondly, only at $\mathrm{t} = 2\sqrt{E}$, the amplifying factor and the noise probability are monotonically decreasing/increasing in the integral interval, $[t, t+2\sqrt{E}]$, which leads to a stable choice of s for an exponential amplifying factor. The argument is strengthened in the high SNR case where the probability distribution is more concentrated.

The conjecture can be verified with a 3D plot of $\Phi(s,\mathrm{t})$ which shows that the global minimum is near $\mathrm{t} = 2\sqrt{E}$. Similar observations have been found for a wide range of parameters.

III. Experimental Results

In this section, simulation results under the Bernoulli Gaussian (BG) noise model will be presented. The BG model is chosen for two reasons. First, it's an intuitive model for controlling the thickness of the tail probability with two parameters, i.e., impulse arrival probability p and impluse to Gaussian ratio Γ [7]. Second, as is shown in the literature that a locally optimal detector can be derived by reducing the infinite impulsive noise terms to only one dominant term [9] which is the same as the BG model. To illustrate

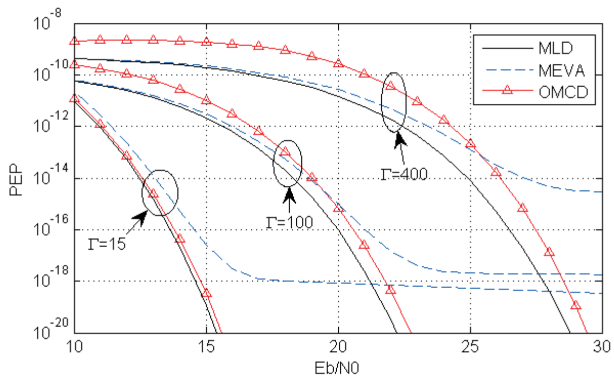

Fig. 3. The pairwise error probability of MLD, MEVA and the proposed method under the BG noise model with $\Gamma \in \{15, 100, 400\}$ and $p = 0.02$.

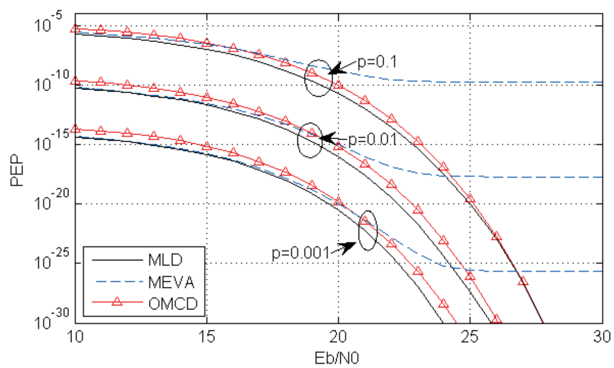

Fig. 4. The pairwise error probability of MLD, MEVA and the proposed method under the BG noise model with $\Gamma = 100$ and $p \in \{0.001, 0.01, 0.1\}$.

the performance of the proposed clipping threshold, experiments are conducted to evaluate the pairwise error probability under a wide range of IGR, $\Gamma \in \{15, 100, 400\}$, impulsive noise probability $p \in \{0.001, 0.01, 0.1\}$. The IGR and impulse arrival probability parameters characterize the "heaviness" of the tail distribution. It should be noted that the impulsive noise parameters are chosen similarly to those of the experiments in [7] and [11].

The PEP is the major comparisons metric in our experiment since the exact PEP curves can be plotted efficiently according to [12]. However, it should be noted that the union bound on the overall bit error probability (BEP) can be expressed as a weight sum of PEPs of the dominating codeword pairs [1]. In the experiment, we adopt $(2, 1, 6)$ CC with generator 75,53 [13]. It can be seen from the union bounds of different approaches in Figure 2 that the range of error floors can be as high as 10^{-4} which depends on the noise parameters in the BG model. In the following, we will consider the pairwise error probability which dominates the bit error probability, i.e., codeword pairs with $d = 8$ [13].

To compare the performances of different approaches, we first fix the impulse arrival probability at $p = 0.02$ and varies the IGR in $\Gamma \in \{15, 100, 400\}$. As can be seen in Fig. 3, the PEPs of the proposed OMCD are able to approach those of the MLD closely for $\Gamma = 15$ while error floors are observed for MEVA. This is due to the approximation errors of the decoding metric in MEVA are not negligible at high SNR where the impulse arrival probability $p \ll 1$. It should be noted that, as Γ increases, the error floor occurs at higher SNR (from 15 to 25 dB) and higher probability (from 10^{-18} to 10^{-14}). The gap between PEPs of the proposed method and MLD are enlarged under larger IGR, $\Gamma = 400$, while it should be noted that the gap is bounded and not diverging. On the other hand, the gap between MEVA and MLD are reduced under the median SNR condition (around 15 dB) as the Γ increases, the error floor cannot be eliminated in the high SNR cases. Secondly, the experiments are conducted by fixing IGR at $\Gamma = 100$ and varying the impulse arrival probability in $p \in \{0.001, 0.01, 0.1\}$. Similar asymptotic behavior for the proposed OMCD method can be observed in Fig. 4, while the error floors occur in the high SNR region for MEVA.

IV. Conclusion

Some recently proposed metric clipping based decoders were demonstrated to achieve near optimal performance without estimating the impulsive noise statistics under some conditions. We pointed out that these existing metric clipping decoders suffer from the presence of error floors. In this work, we formulate the clipping threshold design problem as a pairwise error probability optimization problem so that the resultant optimized metric clipping decoder can eliminate the error floor. It is demonstrated by simulation that the resultant optimized metric clipping decoder can approach the MLD performance at high SNR under BG noise model with a wide range of parameters. One of our future works are to provide a comprehensive proof on the proposed clipping threshold method and evaluate its effectiveness under different impulsive noise models, e.g., the Middleton Class A model and the symmetric Alpha stable model.

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