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Day-Ahead Bidding Strategies for Demand-Side Expected Cost Minimization

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Abstract—Day-ahead DSM techniques in the smart grid allow the supply-side to know in advance an estimation of the amount of energy to be provided to the demand-side during the upcoming day. However, a pure day-ahead optimization process cannot accommodate potential real-time deviations from the expected energy consumption by the demand-side users, as well as the randomness of their renewable sources. This paper proposes a day-ahead bidding system based on a pricing model that combines a price per unit of energy, depending on the day-ahead bid energy needs of the demand-side users, with a penalty system that limits the real-time fluctuations around their bid energy loads. In this day-ahead bidding process, demand-side users, possibly having energy production and storage capabilities, are interested in minimizing their expected cumulative expense. The resulting optimization problem is formulated as a noncooperative game and is solved by means of suitable distributed algorithms. Finally, the proposed procedure is tested in realistic situations.

I. INTRODUCTION

The electric energy network is undergoing a great technological evolution with the development of the smart grid concept, which allows more interaction between the supply-side and the demand-side of the network by taking advantage of advanced information and communication technology. These bases pose a plethora of new optimization and control issues related to stability, reliability, and pricing policies of the network.

Demand-side management (DSM) comprises the different initiatives intended to modify the time pattern and magnitude of the load demand. In particular, letting the consumers directly control and manage their individual consumption patterns, combined with time-dependent electricity prices, results in a grid that is more secure and efficient, easier to operate, and that facilitates the integration of renewable energy, distributed energy generation, and storage. A day-ahead demand-side optimization provides the supply-side with an estimation of the amount of energy to be delivered to the demand-side over the time-period of analysis [1]. Nonetheless, additional costs are incurred by the supply-side when the consumption schedule is not correctly predicted by the users, and are transferred to the demand-side in the form of penalty charges [2], [3].

The aim of this paper is to complement the model proposed in [4], [5], in which the participants were committed to follow strictly the bid consumption pattern, by taking into account the randomness of the users' energy consumption and non-dispatchable generation. For this purpose, we propose a two-stage pricing model: in the first stage, the price per unit of

energy is agreed during the day-ahead bidding process and explicitly depends on the aggregate bid energy load of all demand-side users, with the active users, i.e., those subscribers participating actively in the bidding process, bidding their energy needs according to individual consumption statistics; in the second stage, real-time penalties are applied to those users that deviate from their day-ahead bidding strategies, providing an incentive for better prediction.

By means of a fully distributed demand-side management method regulated by an independent central unit, active users are interested in accurately deriving the bidding strategies that minimize their expected individual monetary expense and simultaneously optimize eventual energy generation and storage strategies. Considered the selfish nature of the users, the resulting bidding optimization is formulated as a noncooperative game, for which we analyze the existence of Nash equilibria (NE). Besides, we propose two suitable distributed algorithms to calculate such solutions, deriving sufficient conditions for their convergence. In order to participate in the aforementioned demand-side bidding process, each active user is connected not only to the power distribution grid, but also to a communication infrastructure that enables two-way communication between his smart meter and the central unit.

The paper is structured as follows. Sections II and III introduce the overall smart grid framework, the adopted energy pricing model, and the demand-side bidding system. In Sections IV we consider demand-side users who can only adopt load bidding strategies, whereas in Section V we also include distributed generation and storage capabilities. Section VI illustrates the proposed methods and algorithms through experimental evaluations. Finally, we provide some conclusions in Section VII.

II. SMART GRID MODEL

The modern power grid is a complex network that can be conveniently divided into [6], [7]: (i) supply-side (energy producers and providers), (ii) central unit (regulation authority that coordinates the market-clearing process and the proposed demand-side bidding process), and (iii) demand-side (end users). In this paper, we focus our attention on the demand-side of the smart grid, which is introduced in Section II-A and further refined in Sections IV-A and V-A, whereas the supply-side and the central unit are modeled as plainly as possible.

A. Demand-Side Model

Demand-side users, whose associated set is denoted by \mathcal{D} , are characterized in the first instance by the random variable $l_n(h)$ representing the individual *per-slot energy load*, which gives the energy flow between user $n \in \mathcal{D}$ and the grid at time-slot h in the time period of analysis, which is conveniently divided in H time-slots. In particular, we have $l_n(h) > 0$ when user n purchases energy from the grid and $l_n(h) < 0$ when user n sells energy to the grid. Our demand-side model distinguishes between *passive* and *active* users. Passive users are basically energy consumers and resemble traditional demand-side users, whereas active users indicate those consumers participating in the demand-side bidding process, i.e., reacting to changes in the cost per unit of energy by modifying their demand.

For convenience, we group the P passive users in the set $\mathcal{P} \subset \mathcal{D}$ and the N active users in the set $\mathcal{N} \subset \mathcal{D} = \mathcal{D} \setminus \mathcal{P}$. We do not make any assumption on pdf of $l_n(h)$, but we suppose that active users can derive their individual load statistics.

B. Energy Cost and Pricing Model

This section describes the cost model on which depends the price of energy. Let us define the *cost per unit of energy* $C_h(\cdot)$ indicating the cost function at time-slot h set by the supply-side during the market-clearing process. Within the day-ahead bidding process, demand-side users agree the *per-slot aggregate bid energy load* $\hat{L}(h)$, and the price $C_h(\hat{L}(h))$ remains fixed during the time period of analysis, while real-time penalties for load deviations are subsequently applied. In this paper, we adopt the following cost function:

$$C_h(\hat{L}(h)) = K_h \hat{L}(h) \quad (1)$$

which corresponds to the non-normalized quadratic grid cost function widely used in the smart grid literature (e.g., in [4], [8], [9]). In general, the grid coefficients $K_h > 0$ are different at each time-slot h , since the energy production varies along the time period of analysis according to the energy demand and to the availability of intermittent energy sources.

For convenience, let us introduce $\hat{l}_n(h)$ as the *per-slot bid energy load* for each user $n \in \mathcal{N}$ at time-slot h , so that

$$\hat{L}(h) = \bar{L}^{(\mathcal{P})}(h) + \sum_{n \in \mathcal{N}} \hat{l}_n(h) \quad (2)$$

where $\bar{L}^{(\mathcal{P})}(h) = \sum_{n \in \mathcal{P}} \bar{l}_n(h)$ is the average per-slot aggregate energy consumption associated with the passive users connected to the grid, with $\bar{l}_n(h) = \mathbb{E}\{l_n(h)\}$. We suppose that the central unit can determine the average consumptions of every passive user $n \in \mathcal{P}$ referring to its available past statistics. For the sake of simplicity, we also assume that $\hat{L}(h) \geq L^{(\min)}(h)$ at each time-slot h , where $L^{(\min)}(h) > 0$ denotes the minimum per-slot aggregate energy load and can be also calculated by the central unit. Furthermore, once $\hat{L}(h)$ is fixed in the day-ahead bidding process, we assume that the real-time aggregate energy load needed by the demand-side users is always guaranteed by the supply-side.

In order to encourage participation of the demand-side in the bidding process, passive users pay an overprice on the

purchased energy given by the multiplicative constant $\gamma_h > 1$ with respect to the agreed cost per unit of energy $K_h \hat{L}(h)$:

$$f_n^{(\mathcal{P})} = \sum_{h=1}^H \gamma_h K_h \hat{L}(h) l_n(h), \quad n \in \mathcal{P} \quad (3)$$

where $l_n(h) > 0$ for users $n \in \mathcal{P}$, since we assume that only active users are allowed to sell energy to the grid.

On the other hand, each active user $n \in \mathcal{N}$ derives his *bid energy load vector* $\hat{\mathbf{l}}_n = (\hat{l}_n(h))_{h=1}^H$ in the day-ahead demand-side bidding process. Nonetheless, he can possibly deviate from such strategy in real time by buying/selling a different amount of energy $l_n(h)$, for which he pays/receives $K_h \hat{L}(h) l_n(h)$, while incurring in the penalties given by the following pricing system:

- (i) When $l_n(h) > \hat{l}_n(h)$, user n pays a penalty $K_h \hat{L}(h) \alpha_h (l_n(h) - \hat{l}_n(h))$, with $0 < \alpha_h \leq 1$ being the penalty parameter for discouraging user n from exceeding the agreed per-slot energy load $\hat{l}_n(h)$;
- (ii) When $l_n(h) < \hat{l}_n(h)$, user n pays a penalty $K_h \hat{L}(h) \beta_h (\hat{l}_n(h) - l_n(h))$, with $0 < \beta_h \leq 1$ being the penalty parameter for discouraging user n from falling behind the negotiated per-slot energy load $\hat{l}_n(h)$;

Accordingly, we introduce the *cumulative expense* over the time period of analysis for the active users, which represents the cumulative monetary expense incurred by user $n \in \mathcal{N}$ for obtaining the energy loads $\{l_n(h)\}_{h=1}^H$, including the penalties for deviating from the bid energy loads $\{\hat{l}_n(h)\}_{h=1}^H$:

$$f_n^{(\mathcal{N})} = f_n(\hat{\mathbf{l}}_n, \hat{\mathbf{l}}_{-n}) = \sum_{h=1}^H K_h (\hat{l}_{-n}(h) + \hat{l}_n(h)) (l_n(h) + \alpha_h (l_n(h) - \hat{l}_n(h))^+ + \beta_h (\hat{l}_n(h) - l_n(h))^+), \quad n \in \mathcal{N} \quad (4)$$

where $(x)^+ = \max(x, 0)$, $\hat{\mathbf{l}}_{-n} = (\hat{l}_{-n}(h))_{h=1}^H$ being the aggregate bid energy load of the other users $m \in \mathcal{N} \setminus \{n\}$, and

$$\hat{l}_{-n}(h) = \hat{L}(h) - \hat{l}_n(h) = \bar{L}^{(\mathcal{P})}(h) + \sum_{m \in \mathcal{N} \setminus \{n\}} \hat{l}_m(h). \quad (5)$$

Basically, when $l_n(h) \geq 0$, active user n pays a fraction β_h of the full price for the negotiated energy that he does not purchase, and is charged $1 + \alpha_h$ times for the energy that exceeds his day-ahead agreed amount. Likewise, when $l_n(h) < 0$, active users n receives a fraction β_h of the full price for the exceeding energy that he sells to the grid, while he is charged $1 + \alpha_h$ times the full price for the negotiated energy that he fails to sell.

The penalty parameters $\{\alpha_h, \beta_h\}_{h=1}^H$ are established in the day-ahead market-clearing process with the purpose of discouraging real-time deviations with respect to the bidding load either upwards or downwards. On the other hand, the parameter γ_h must be chosen to penalize passive users with respect to the active ones.

III. DEMAND-SIDE BIDDING SYSTEM

Once defined the overall model, in this section we present the proposed demand-side bidding process.

First, the grid energy prices for the time period of analysis, i.e., the grid coefficients $\{K_h\}_{h=1}^H$, and the penalty parameters $\{\alpha_h, \beta_h, \gamma_h\}_{h=1}^H$, are fixed in the day-ahead market-clearing process [6], [7]. Then, each active demand-side user reacts to the prices provided by the central unit through iteratively adjusting his per-slot bid energy loads, given by $\hat{\mathbf{l}}_n$, with the final objective of minimizing his expected cumulative expense throughout the time period of analysis, i.e.,

$$\bar{f}_n(\hat{\mathbf{l}}_n, \hat{\mathbf{l}}_{-n}) = \mathbb{E}\{f_n(\hat{\mathbf{l}}_n, \hat{\mathbf{l}}_{-n})\} \quad (6)$$

which is obtained in closed-form as in the following lemma.

Lemma 1 ([10, Sec. 1]). Given the pricing model in Section II-B, the expected cumulative expense of active user $n \in \mathcal{N}$, with per-slot bid energy loads $\hat{\mathbf{l}}_n$, is given by

$$\bar{f}_n(\hat{\mathbf{l}}_n, \hat{\mathbf{l}}_{-n}) = \sum_{h=1}^H K_h (\hat{l}_{-n}(h) + \hat{l}_n(h)) \phi_{l_n(h)}(\hat{l}_n(h)) \quad (7)$$

where

$$\begin{aligned} \phi_{l_n(h)}(x) &= \mathbb{E}\{l_n(h) + \alpha_h(l_n(h) - \hat{l}_n(h))^+ \\ &\quad + \beta_h(\hat{l}_n(h) - l_n(h))^+\} \\ &= (1 + \alpha_h)\bar{l}_n(h) - \alpha_h x \\ &\quad + (\alpha_h + \beta_h)(x F_{l_n(h)}(x) - G_{l_n(h)}(x)) \end{aligned} \quad (8)$$

$$G_{l_n(h)}(x) = \int_{-\infty}^x t f_{l_n(h)}(t) dt \quad (9)$$

with $F_{l_n(h)}(x)$ and $f_{l_n(h)}(x)$ denoting the cdf and pdf of $l_n(h)$.

Once the day-ahead bidding process finalizes, the prices per unit of energy $\{K_h \hat{L}(h)\}_{h=1}^H$ remain fixed. Then, passive and active users are charged in real-time as in (3)–(4), respectively.

IV. LOAD BIDDING STRATEGY FOR EXPECTED COST MINIMIZATION

In this section, we consider active users with load bidding capabilities, and we examine the bidding system introduced in Section III using a game theoretical approach.

A. Demand-Side Model with Energy Load Bidding

Let us introduce the individual *per-slot net energy consumption* $e_n(h)$, which indicates the energy needed by user $n \in \mathcal{D}$ to supply his appliances at time-slot h taking into consideration eventual non-dispatchable (renewable) energy resources that the user may adopt.¹ Since, in this section, $e_n(h)$ is the only contribution to the energy load $l_n(h)$, it thus follows that $l_n(h) = e_n(h)$ is the random net energy consumption and $\hat{l}_n(h) = x_n(h)$ is the optimization variable that represents the *per-slot bid net energy consumption* of user $n \in \mathcal{N}$.

Defining the *bidding strategy vector* as $\mathbf{x}_n = (x_n(h))_{h=1}^H$, the strategy set $\Omega_{\mathbf{x}_n}$ for active users can be expressed as

$$\Omega_{\mathbf{x}_n} = \{\mathbf{x}_n \in \mathbb{R}^H : \chi_n^{(\min)}(h) \leq x_n(h) \leq \chi_n^{(\max)}(h)\} \quad (10)$$

¹Non-dispatchable generators, having only fixed costs, imply no strategy regarding energy production, unlike dispatchable generators in Section V.

with $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ denoting the minimum and maximum per-slot bidding loads for user $n \in \mathcal{N}$, respectively.

B. Game Theoretical Formulation and Analysis of NE

We can now formally describe the demand-side bidding process described in Section III with the setup in Section IV-A as the game $\mathcal{G} = \langle \Omega_{\mathbf{x}}, \bar{\mathbf{f}} \rangle$, with $\Omega_{\mathbf{x}} = \prod_{n=1}^N \Omega_{\mathbf{x}_n}$ and $\bar{\mathbf{f}} = (\bar{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n}))_{n=1}^N$ given in (7), in which each player $n \in \mathcal{N}$ calculates his bidding strategy $\mathbf{x}_n \in \Omega_{\mathbf{x}_n}$ that minimizes his payoff function $\bar{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n})$, given the aggregate bid energy load vector of the other users $\hat{\mathbf{l}}_{-n}$:

$$\begin{aligned} \min_{\mathbf{x}_n} \quad & \bar{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n}) \\ \text{s.t.} \quad & \mathbf{x}_n \in \Omega_{\mathbf{x}_n} \end{aligned} \quad \forall n \in \mathcal{N}. \quad (11)$$

The solution of the game $\mathcal{G} = \langle \Omega_{\mathbf{x}}, \bar{\mathbf{f}} \rangle$ corresponds to the well-known concept of Nash equilibrium, which is a feasible strategy profile $\mathbf{x}^* = (\mathbf{x}_n^*)_{n=1}^N$ with the property that no single player n can profitably deviate from his strategy \mathbf{x}_n^* , if all other players act according to their optimal strategies [11]. The existence of such solutions is analyzed in next Theorem.

Theorem 1 (Existence of NE [10, Sec. 2]). *Given the game $\mathcal{G} = \langle \Omega_{\mathbf{x}}, \bar{\mathbf{f}} \rangle$ in (11), suppose that, $\forall n \in \mathcal{N}$, $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ are such that the pdf of the per-slot net energy consumption satisfies*

$$f_{e_n(h)}(x) \geq \left(\frac{1}{L^{(\min)}(h)} \right) \left(\frac{2\alpha_h}{\alpha_h + \beta_h} \right) \quad (12)$$

for $\chi_n^{(\min)}(h) \leq x \leq \chi_n^{(\max)}(h)$. Then, the game has a nonempty and compact solution set.

Remark 1.1. The condition in (12) limits the displacement of $x_n(h)$ around the mode of $e_n(h)$.

C. Computation of NE

Observe that, in the game $\mathcal{G} = \langle \Omega_{\mathbf{x}}, \bar{\mathbf{f}} \rangle$ in (11), the coupling between users lies at the level of the payoff functions $\bar{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n})$, whereas the individual feasible sets $\Omega_{\mathbf{x}_n}$ are decoupled. Hence, we consider the extremely flexible and easy-to-implement solution provided by a distributed algorithm based on the individual best-responses. In this scheme, each active user calculates, at each iteration, his optimal strategy given the aggregate bid energy loads for all time-slots in the time period of analysis, which obviously depend on the bidding strategies of the other active users.

We focus on the class of *totally asynchronous* algorithms, where some users may update their strategies more frequently than others and they may even use outdated information about the strategy profiles adopted by the other users. Let $\mathcal{T}_n \subseteq \mathcal{T} \subseteq \{0, 1, 2, \dots\}$ be the set of times at which user $n \in \mathcal{N}$ updates his own strategy \mathbf{x}_n , denoted by $\mathbf{x}_n^{(i)}$ at the i th iteration. We use $t_n(i)$ to denote the most recent time at which the strategy of user n is perceived by the central unit at the i th iteration. Besides, we assume that the standard conditions in asynchronous convergence theory given by (A1)–(A3) in [5, Sec IV], which are fulfilled in any practical implementation,

Algorithm 1 Best-Response Algorithm

Data : Set $i=0$. Given $\{K_h\}_{h=1}^H$ and any feasible starting point $\mathbf{x}^{(0)} = (\mathbf{x}_n^{(0)})_{n=1}^N$:

(S.1) : If a suitable termination criterion is satisfied: STOP.

(S.2) : For $n \in \mathcal{N}$, each user computes $\mathbf{x}_n^{(i+1)}$ as

$$\mathbf{x}_n^{(i+1)} = \begin{cases} \mathbf{x}_n^* \in \underset{\mathbf{x}_n \in \Omega_{\mathbf{x}_n}}{\operatorname{argmin}} \left\{ \bar{f}_n(\mathbf{x}_n, \hat{\mathbf{l}}_{-n}^{(t(i))}) \right\}, & \text{if } i \in \mathcal{T}_n \\ \mathbf{x}_n^{(i)}, & \text{otherwise} \end{cases}$$

End

(S.3) : $i \leftarrow i+1$; Go to (S.1).

are satisfied by the schedules \mathcal{T}_n and $t_n(i)$, $\forall n \in \mathcal{N}$ (see also [12, Ch. 6] for details). Hence, each individual user updates his strategy by minimizing his cumulative expense over the time period of analysis referring to the most recently available value of the per-slot aggregate energy load

$$\hat{L}^{(t(i))}(h) = \bar{L}^{(P)}(h) + \sum_{m \in \mathcal{N}} \hat{l}_m^{(t_m(i))}(h) \quad (13)$$

which considers the energy loads of the other users as perceived by the central unit, and from which user n obtains $\hat{l}_{-n}^{(t(i))}(h) = \hat{L}^{(t(i))}(h) - \hat{l}_n^{(t_n(i))}(h)$. This procedure is described in Alg. 1 (see [5, Alg. 1], [13, Alg. 4.1] for details).

Theorem 2 (Uniqueness of NE and Convergence of Algorithm 1 [10, Sec. 3]). *Given the game $\mathcal{G} = \langle \Omega_{\mathbf{x}}, \bar{\mathbf{f}} \rangle$ in (11), suppose that, $\forall n \in \mathcal{N}$, $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ are such that the pdf of the per-slot net energy consumption satisfies*

$$f_{l_n(h)}(x) > \frac{2 \max_h ((N-1) \max(\alpha_h, \beta_h) + \alpha_h)}{\min_h ((\alpha_h + \beta_h) L^{(\min)}(h))} \quad (14)$$

for $\chi_n^{(\min)}(h) \leq x \leq \chi_n^{(\max)}(h)$. Then, (i) the game has a unique solution, and (ii) any sequence $\{\mathbf{x}_n^{(i)}\}_{i=1}^{\infty}$ generated by Algorithm 1 converges to such unique Nash equilibrium.

Remark 2.1. The condition in (14) is more restrictive than that given in Th. 1, since it also implies the uniqueness of the NE. Besides, we can compute a solution even in the presence of multiple NE under more relaxed requirements (see Alg. 2 in next section). Furthermore, following [5, Rem. 3.1], we can also understand (14) as a limit on the number of active users with respect to the minimum per-slot aggregate load.

Note that the central unit cannot instantaneously control that each active user respects the condition in (14). Nonetheless, the former can apply further penalties to those users who persist in making inaccurate load bids [2] (this consideration holds equivalently for the convergence condition of Alg. 2 in (22)).

V. LOAD BIDDING, GENERATION, AND STORAGE STRATEGIES FOR EXPECTED COST MINIMIZATION

In this section, we analyze the bidding system formulated in Section III for more advanced demand-side users, who can possibly have energy generation and storage capabilities.

A. Demand-Side Model with Energy Generation and Storage

For convenience, let us use $\mathcal{G} \subseteq \mathcal{N}$ to denote the subset of users possessing dispatchable distributed energy generation (DG). For users $n \in \mathcal{G}$, $g_n(h) \geq 0$ represents the *per-slot energy production profile* at time-slot h . Introducing the *energy production scheduling vector* $\mathbf{g}_n = (g_n(h))_{h=1}^H$, we have that $\mathbf{g}_n \in \Omega_{\mathbf{g}_n}$, where $\Omega_{\mathbf{g}_n}$ is the strategy set for dispatchable energy producers $n \in \mathcal{G}$. Likewise, we use $\mathcal{S} \subseteq \mathcal{N}$ to denote the subset of users owning distributed energy storage (DS). Users $n \in \mathcal{S}$ are characterized by the *per-slot energy storage profile* $s_n(h)$ at time-slot h : $s_n(h) > 0$ when the storage device is to be charged (i.e., an additional energy consumption), $s_n(h) < 0$ when the storage device is to be discharged (i.e., a reduction of the energy consumption), and $s_n(h) = 0$ when the device is inactive. Introducing the *energy storage scheduling vector* $\mathbf{s}_n = (s_n(h))_{h=1}^H$, it holds that $\mathbf{s}_n \in \Omega_{\mathbf{s}_n}$, being $\Omega_{\mathbf{s}_n}$ the strategy set for energy storers $n \in \mathcal{S}$. Hence, for each user $n \in \mathcal{N}$, the per-slot energy load $l_n(h)$ and the per-slot bid energy load $\hat{l}_n(h)$ are now defined, respectively, as

$$l_n(h) = e_n(h) - g_n(h) + s_n(h) \quad (15)$$

$$\hat{l}_n(h) = x_n(h) - g_n(h) + s_n(h). \quad (16)$$

Observe that any generation and storage models resulting in compact and convex strategy sets validate the results in the next sections [13, Prop. 4.1] (see, e.g., those proposed in [5]).

Finally, let us define the per-slot strategy profile and the corresponding strategy vector of a generic user $n \in \mathcal{N}$ as

$$\mathbf{y}_n(h) = (x_n(h), g_n(h), s_n(h))^T, \quad \mathbf{y}_n = (\mathbf{y}_n(h))_{h=1}^H \quad (17)$$

For convenience, taking into account the strategy set $\Omega_{\mathbf{x}_n}$ defined as in (10), and the aforementioned sets $\Omega_{\mathbf{g}_n}$ and $\Omega_{\mathbf{s}_n}$, the corresponding strategy set for the generic user $n \in \mathcal{N}$ is

$$\Omega_{\mathbf{y}_n} = \{ \mathbf{y}_n \in \mathbb{R}^{3H} : \mathbf{x}_n \in \Omega_{\mathbf{x}_n}, \mathbf{g}_n \in \Omega_{\mathbf{g}_n}, \mathbf{s}_n \in \Omega_{\mathbf{s}_n} \} \quad (18)$$

with $\mathbf{g}_n = \mathbf{0}$ if $n \in \mathcal{N} \setminus \mathcal{G}$ and $\mathbf{s}_n = \mathbf{0}$ if $n \in \mathcal{N} \setminus \mathcal{S}$.

Lastly, for users $n \in \mathcal{G}$, the *production cost function* $W_n(g_n(h))$ gives the variable production costs for generating the amount of energy $g_n(h)$ at time-slot h , with $W_n(0) = 0$.

B. Game Theoretical Formulation and Analysis of NE

Let us formally describe the previous demand-side bidding process as the game $\mathcal{G} = \langle \Omega_{\mathbf{y}}, \tilde{\mathbf{f}} \rangle$, with $\Omega_{\mathbf{y}} = \prod_{n=1}^N \Omega_{\mathbf{y}_n}$ and $\tilde{\mathbf{f}} = (\tilde{f}_n(\mathbf{y}_n, \hat{\mathbf{l}}_{-n}))_{n=1}^N$, with

$$\tilde{f}_n(\mathbf{y}_n, \hat{\mathbf{l}}_{-n}) = \bar{f}_n((\boldsymbol{\delta} \otimes \mathbf{I}_H)^T \mathbf{y}_n, \hat{\mathbf{l}}_{-n}) + \sum_{h=1}^H W_n(\boldsymbol{\delta}_g^T \mathbf{y}_n(h)) \quad (19)$$

where $\boldsymbol{\delta} = (1, -1, 1)^T$ and $\boldsymbol{\delta}_g = (0, 1, 0)^T$. Here, each player $n \in \mathcal{N}$ calculates his bidding, production, and storage strategies $\mathbf{y}_n \in \Omega_{\mathbf{y}_n}$ that minimize his payoff function $\tilde{f}_n(\mathbf{y}_n, \hat{\mathbf{l}}_{-n})$, given the aggregate bid energy loads of the other users $\hat{\mathbf{l}}_{-n}$:

$$\begin{aligned} \min_{\mathbf{y}_n} \quad & \tilde{f}_n(\mathbf{y}_n, \hat{\mathbf{l}}_{-n}) \\ \text{s.t.} \quad & \mathbf{y}_n \in \Omega_{\mathbf{y}_n} \end{aligned} \quad \forall n \in \mathcal{N}. \quad (20)$$

Algorithm 2 Proximal Decomposition Algorithm

Data : Set $i = 0$ and the initial centroid $(\bar{\mathbf{y}}_n)_{n=1}^N = \mathbf{0}$.
 Given $\{K_h\}_{h=1}^H$, $\{\rho^{(i)}\}_{i=0}^\infty$, $\tau > 0$, and any
 feasible starting point $\mathbf{y}^{(0)} = (\mathbf{y}_n^{(0)})_{n=1}^N$:

(S.1) : If a suitable termination criterion is satisfied:
 STOP.

(S.2) : For $n \in \mathcal{N}$, each user computes $\mathbf{y}_n^{(i+1)}$ as

$$\mathbf{y}_n^{(i+1)} = \begin{cases} \mathbf{y}_n^* \in \operatorname{argmin}_{\mathbf{y}_n \in \Omega_{\mathbf{y}_n}} \left\{ \tilde{f}_n(\mathbf{y}_n, \hat{\mathbf{I}}_n^{(t(i))}) \right. \\ \left. + \frac{\tau}{2} \|\mathbf{y}_n - \bar{\mathbf{y}}_n\|^2 \right\}, & \text{if } i \in \mathcal{T}_n \\ \mathbf{y}_n^{(i)}, & \text{otherwise} \end{cases}$$

End

(S.3) : If the NE has been reached, then each user
 $n \in \mathcal{N}$ sets $\mathbf{y}_n^{(i+1)} \leftarrow (1 - \rho^{(i)})\bar{\mathbf{y}}_n + \rho^{(i)}\mathbf{y}_n^{(i+1)}$
 and updates his centroid: $\bar{\mathbf{y}}_n = \mathbf{y}_n^{(i+1)}$.

(S.4) : $i \leftarrow i + 1$; Go to (S.1).

Next theorem analyzes the existence of NE.

Theorem 3 (Existence of NE [10, Sec. 4]). *Given the game $\mathcal{G} = \langle \Omega_{\mathbf{y}}, \tilde{\mathbf{f}} \rangle$ in (20), suppose that, $\forall n \in \mathcal{G}$, the production cost function $W_n(x)$ is convex, and that, $\forall n \in \mathcal{N}$, $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ are such that the pdf of the per-slot net energy consumption satisfies*

$$f_{e_n(h)}(x) \geq \left(\frac{1}{2L^{(\min)}(h)} \right) \frac{(\alpha_h + 1)^2}{\alpha_h + \beta_h} \quad (21)$$

for $\chi_n^{(\min)}(h) \leq x \leq \chi_n^{(\max)}(h)$. Then, the game has a nonempty and compact solution set.

Remark 3.1. The condition in (21), although similar to that in (12), is more restrictive since the presence of DG and DS gives more degrees of freedom for the users' strategy (see [5, Sec. 4.1] for details).

C. Computation of NE

When energy generation and storage devices are included in the smart grid, the convergence conditions for Alg. 1 cannot be guaranteed (see [5, Sec. 4.1] for details). Therefore, bearing in mind the scheduling definitions provided in Section IV-C, we consider an alternative distributed algorithm based on the proximal decomposition algorithm [5, Alg. 2], [13, Alg. 4.2], which is guaranteed to converge in the presence of multiple NE and some additional constraints on the parameters of the algorithm that we provide next.

Theorem 4 (Convergence of Algorithm 2 [10, Sec. 5]). *Given the game $\mathcal{G} = \langle \Omega_{\mathbf{y}}, \tilde{\mathbf{f}} \rangle$ in (20), suppose that, $\forall n \in \mathcal{G}$, the production cost function $W_n(x)$ is convex, and that the following conditions hold:*

(a.1) $\forall n \in \mathcal{N}$, $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ are such that, the pdf

of the per-slot net energy consumption satisfies

$$f_{e_n(h)}(x) \geq \frac{1}{(\alpha_h + \beta_h)L^{(\min)}(h)} \left(\frac{1}{4}(\alpha_h + 1)^2 + N(\max(\alpha_h, \beta_h) + \alpha_h + \beta_h) \right) \quad (22)$$

for $\chi_n^{(\min)}(h) \leq x \leq \chi_n^{(\max)}(h)$;

- (a.2) The penalty parameters are such that $\alpha_h + \beta_h \leq 1$;
 (b.1) The parameter τ satisfies $\tau > 3(N - 1)(\max_h K_h)$;
 (b.2) $\{\rho^{(i)}\} \subset [R_m, R_M]$, with $0 < R_m < R_M < 2$.

Then, any sequence $\{\mathbf{y}_n^{(i)}\}_{i=1}^\infty$ generated by Algorithm 2 converges to a Nash equilibrium of the game.

VI. SIMULATION RESULTS

In this section, we illustrate numerically the performance of the proposed day-ahead bidding process.

We consider a smart grid of $N = 100$ active users and $P = 900$ passive users, considering a time period of analysis of $H = 24$ time-slots of one hour each. For the sake of simplicity, each demand-side user $n \in \mathcal{D}$ has the same energy consumption curve with daily average of $\sum_{h=1}^{24} \bar{e}_n(h) = 12$ kWh, occurring higher consumptions more likely during day-time hours, i.e., from 08:00 to 24:00, than during night-time hours, i.e., from 00:00 to 08:00, and reaching its peak between 17:00 and 23:00. The price per unit of energy is given by the grid cost function introduced in (1), with $\{K_h\}_{h=1}^8 = K_{\text{night}}$ and $\{K_h\}_{h=9}^{24} = K_{\text{day}}$, with $K_{\text{day}} = 1.5K_{\text{night}}$ as in [9], and whose values are chosen in order to obtain an initial price of 0.15 €/kWh, when neglecting the real-time penalties. Furthermore, we set $\alpha_h = 0.2$ for $h = 1, \dots, 8$ and $\alpha_h = 0.9$ for $h = 9, \dots, 24$, with $\beta_h = 1 - \alpha_h$.

We model $e_n(h)$ as a normal random variable with mean $\bar{e}_n(h)$ and standard deviation $\sigma_n(h)$, with $\chi_n^{(\min)}(h)$ and $\chi_n^{(\max)}(h)$ chosen such that the convergence of Alg. 1 and 2 is guaranteed. We use Alg. 1 to calculate the load bidding strategy for expected cost minimization, whereas we employ Alg. 2 when demand-side users have also dispatchable generation and storage capabilities. In particular, we suppose that the latter users follow the production and storage models proposed in [4, Sec. 2], with the same parameters used in [4, Sec. 5.2].

Fig. 1(a) illustrates the per-slot bid net consumptions $x_n(h)$ with respect to the average per-slot net consumptions $\bar{e}_n(h)$ for a generic active user, using three different standard deviations. Predictably, the user's bid is greater than his expected load when $\alpha_h > \beta_h$, since he is more likely to avoid severe penalties for surpassing the agreed load, and vice versa. In consequence, such displacement becomes greater as the standard deviation increases. Let us compare the cumulative expense obtained using Alg. 1 with respect to the case in which the user does not fully exploit his statistical knowledge and his bidding strategy is simply given by his expected consumption $\bar{e}_n(h)$. Each user obtains an expected save of 1.9% when $\sigma_n(h) = 0.25|\bar{e}_n(h)|$, of 3.7% when $\sigma_n(h) = 0.5|\bar{e}_n(h)|$, and of 5% when $\sigma_n(h) = 0.75|\bar{e}_n(h)|$.

In Fig. 1(b) we examine the expected expenses per time-slot using $\sigma_n(h) = 0.75|\bar{e}_n(h)|$, also considering the case in

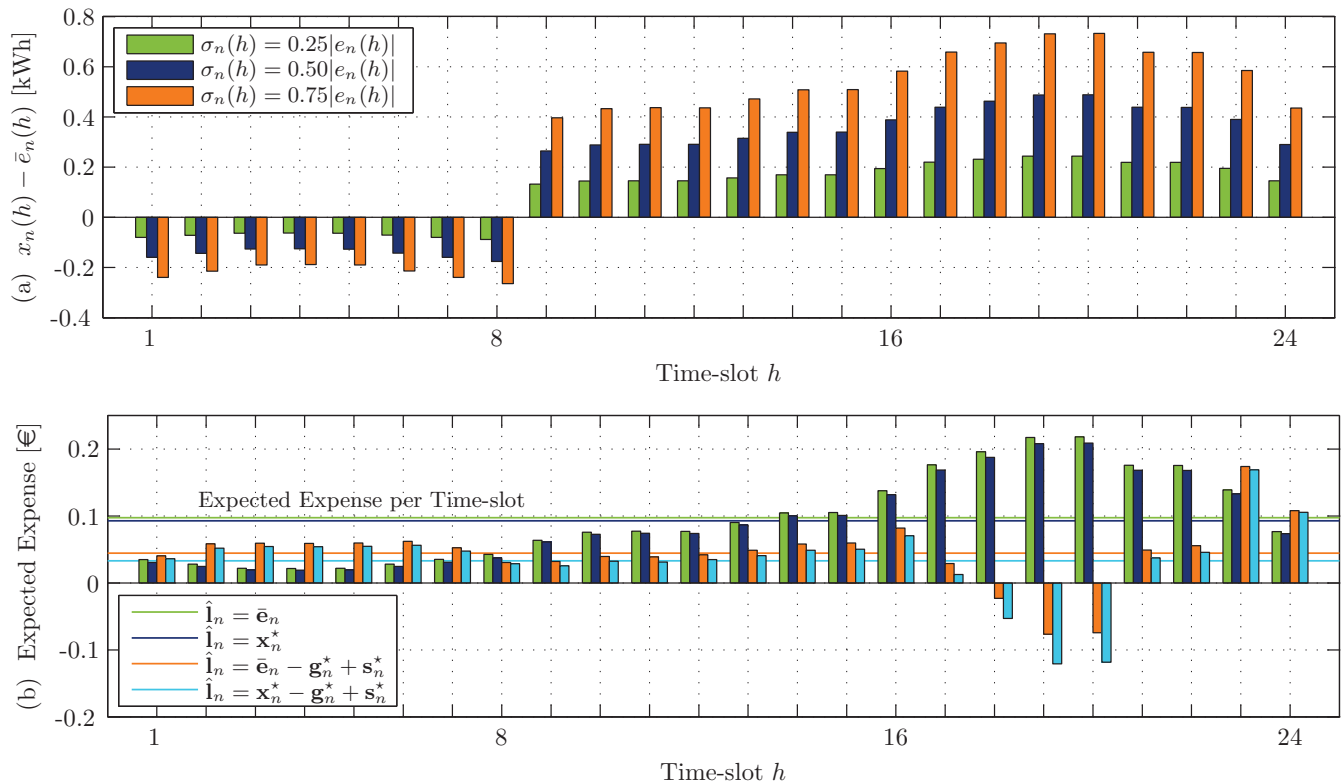


Fig. 1. (a) Difference between average consumption and bid consumption with three different $\sigma_n(h)$; (b) expected expenses per time-slot with bidding strategy based on \bar{e}_n , with optimum bidding strategy \mathbf{x}_n^* , and with optimum dispatchable generation and storage strategies \mathbf{g}_n^* and \mathbf{s}_n^* , using $\sigma_n(h) = 0.75|\bar{e}_n(h)|$.

which all users are dispatchable energy producers and storers. It is straightforward to see that, in both cases, the expenses are greater when the users adopt the bidding strategy resulting from Alg. 2, i.e., $\mathbf{x}_n = \mathbf{x}_n^*$. Moreover, when all users adopt DG and DS, the resulting cumulative expense is 25.3% less than when they bid their average energy loads.

VII. CONCLUSIONS

In this paper, we propose a day-ahead bidding process with real-time penalties for smart grid users, which also accommodates distributed energy production and storage. We provide two distributed and iterative algorithms that allow to compute the optimal bidding, production, and storage strategies of the users with minimum information exchange between the central unit and the demand-side of the grid. Simulations on realistic situations employing a practical cost function show that active users substantially reduce their expected expenses with respect to when their bidding strategy is based on their expected loads.

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