

Power Allocation Strategies across N Orthogonal Channels at Both Source and Relay

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Abstract

We consider a wireless relay network with one source, one relay and one destination, where communications between nodes are performed via N orthogonal channels. This, for example, is the case when orthogonal frequency division multiplexing is employed for data communications. Since the power available at the source and relay is limited, we study optimal power allocation strategies at the source and relay in order to maximize the overall source-destination capacity under individual power constraints at the source and/or the relay. Depending on the availability of the channel state information at the source and relay, optimal power allocation strategies are performed at both the source and relay or only at the relay. Considering different setups for the problem, various optimization problems are formulated and solved. Some properties of the optimal solution are also proved.

Index Terms

Optimal power allocation strategy, amplify-and-forward relay networks, greedy algorithm, orthogonal channels, sum capacity.

I. INTRODUCTION

Driven by the ever increasing demand for high-speed services within limited spectrum resources, cooperative relay transmission, as a promising spectrally efficient technique, has received significant interests in recent years [1]–[4]. Cooperative relaying promises numerous gains for wireless networks such as improved reliability [2] and increased network capacity [3]. The

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benefits of relay transmission can also be reaped by orthogonal frequency division multiple (OFDM) access systems to support broadband services, e.g., the IEEE 802.16j [4].

Power limitation is a common problem in wireless networks. Therefore, allocating the limited power resources to the network nodes (i.e., the source and relay nodes) is a design consideration which has received much attention [5]–[21]. For example, relay selection schemes, where only a subset of relay nodes is considered for cooperation, are studied in [5]–[9] as a simple power allocation technique. It has been shown in [10] that the optimal power allocation between the source and relay nodes can improve the network performance. Moreover, considering a multi-hop network in [11], the optimal allocation of power among the hops is shown to significantly improve the performance. Also, in [12]–[15], assuming that a common message is sent from the source node to multiple amplify-and-forward (AF) or decode-and-forward (DF) relay nodes over orthogonal channels, optimal power allocation strategies among relay nodes are studied.

Recently, considering relay networks with multiple source-destination pairs (multicast), relevant power allocation strategies have been addressed [16]–[21]. The set up of [16] is that one source sends independent messages to a number of destinations. A single OFDM channel is used between the source and a single relay node, where each private message is assigned to one frequency tone and then amplified and forwarded on different frequency tones to various users. Thus, power allocation at the relay is studied to maximize the minimum SNR among all destinations. In [17]–[19], various power allocation schemes at both the source and relay nodes are developed for the general multi-source, -relay, and -destination set up. Transmissions are done over orthogonal channels, and the corresponding power allocation schemes are based on maximizing the minimum SNR among all users, minimizing the maximum transmit power over all sources, as well as maximizing the network throughput, the minimum rate among all users, and the weighted-sum of rates. Power allocation at the relay nodes only in the multi-relay multi-destination set up is considered also in [20], and is based on maximizing the sum capacity of the network. In [21], power allocation at both the source and relay nodes in a set up similar to [18], [19] is studied for maximizing the sum network capacity under power constraints for

orthogonal subchannels.

In this paper, we consider a two-hop AF relay network, which consists of a single source node, a single destination, and a single relay nodes. We assume that the communications between nodes occur across N orthogonal subchannels, where each subchannel is assigned independent information. Thus, the network capacity is the sum of the individual capacities of subchannels. This setup may correspond to the case, for example, when OFDM signalling is employed between the nodes. Our motivation for considering this setup is the widespread integration of OFDM into various wireless standards. Therefore, power allocation among the orthogonal channels (across frequency tones) is an important issue.

Taking into account individual power constraints at the source and relay nodes, we study strategies for optimal allocation of the limited power among the N orthogonal subchannels. Depending on the availability of channel state information (CSI) at the source and/or the relay, the power allocation is done at both the source node and the relay node or only at the relay node. The goal is to maximize the overall data rate of the network.

We consider two cases for data forwarding at the relay. In one case, we assume the information received on the i -th source-relay subchannel is amplified and forwarded on the i -th relay-destination subchannel. In the other case, similar to [22], we allow the relay to switch the source message received on one source-relay subchannel to another relay-destination subchannel. With optimal power allocation, this strategy significantly improves the overall achievable rate.

Power allocation for a similar OFDM-based relay network has been studied in [23]. The main differences of our work are as follows. (i) With global CSI available at both the source and relay, we first prove that the optimal solution should satisfy a certain symmetry. This symmetry, in turn, allows us to directly find the optimal power allocation for both the source and relay. In contrast, in [23], an iterative optimization approach is studied. An iterative solution may not find the global optima in a reasonable number of iterations. Our approach, however, finds the optimal solution in one shot. (ii) We also consider the situation when only the relay has global CSI. In other words, the source only knows the source-relay channels. This is a more practical setup

as the source need not know the relay-destination channels. For this case, we propose a simple greedy algorithm to maximize the overall network achievable rate. It is also worth noting here that the optimization method of [23] needs global CSI and cannot be used for the latter case.

Note that availability of global CSI (and therefore optimal power allocation) only at the relay has been considered in, for example, [20]. However, the existing optimal power allocation schemes at the relay mostly focus on maximizing the sum capacity and it is not clear how the source (who lacks global CSI) should allocate rates across subchannels. Instead, our approach maximizes the achievable sum rate, while considering the outage. For this, we study how the source chooses its actual data rates across subchannels to avoid outage. Interestingly, at high SNR, the achievable rate is very close to that obtained in the case when global CSI is available to both source and relay.

The rest of this paper is organized as follows. In Section II, we outline the system model. Our optimization problems for maximizing the achievable sum rate are also defined. In Section III, we propose optimal power allocation strategies at both the source and relay or only at the relay for different cases with respect to the availability of CSI. Some of the properties of the optimal solution are also studied. Section IV provides asymptotic analysis for optimal power allocation at high SNR. In Section V, numerical results are provided to demonstrate the benefits of the optimal power allocation. The paper is concluded in Section VI.

II. SYSTEM MODEL AND PROBLEM DEFINITIONS

A. System and channel models

Consider a two-hop relay network which consists of one source node (S), one destination node (D), and one relay node (R) with no line of sight between S and D . Thus, R is employed in order to assist the communications between S and D . Moreover, it is supposed that communication between nodes is done across N orthogonal links.

Two cases are considered for data forwarding at R . In the first case, the message received on the i -th S to R subchannel (SR_i) is simply amplified and forwarded by the relay on the i -th R to D subchannel (RD_i). This case is referred to as AF relaying. In the second case, as in [22], we

assume that the relay sorts S to R subchannels and R to D subchannels based on their quality and assigns data on the i -th best S to R subchannel to the i -th best R to D subchannel. This amplify–sort–and–forward setup is referred to hereafter as ASF. For both AF and ASF cases, complete transmissions from S to D are composed of two phases: (i) S sends the messages while R listens; (ii) R forwards the message to D .

Considering that each node is equipped with a single antenna, the received signal at R through SR_i subchannel can be expressed as

$$y_i = g_i \sqrt{P_{S_i}} m_i + n_i \quad (1)$$

where m_i is the source message with a unit energy transmitted by S on the link SR_i , P_{S_i} is the transmit power on the link SR_i , g_i is the associated flat Rayleigh fading channel coefficient for this link, which is a complex Gaussian random variable with zero mean and variance $\sigma_{g_i}^2$, i.e., $g_i \sim \mathcal{CN}(0, \sigma_{g_i}^2)$, and n_i stands for a complex valued additive white Gaussian noise (AWGN) such that $n_i \sim \mathcal{CN}(0, 1)$.

For AF relaying, at the end of the second phase, the received signal at D via subchannel RD_i can be written as

$$x_i = h_i \sqrt{P_{R_i}} \frac{y_i}{\sqrt{E\{|y_i|^2\}}} + n'_i \quad (2)$$

where $E\{\cdot\}$ denotes the expected operation, P_{R_i} is the transmit power assigned to the link RD_i , $h_i \sim \mathcal{CN}(0, \sigma_{h_i}^2)$ is the channel coefficient for this link, and $n'_i \sim \mathcal{CN}(0, 1)$ is the AWGN.

Moreover, for the ASF relay network, the received signal at D from R via the i -th ordered link (denoted as $RD_{(i)}$) can be represented as

$$x_{(i)} = h_{(i)} \sqrt{P_{R_{(i)}}} \frac{y_{(i)}}{\sqrt{E\{|y_{(i)}|^2\}}} + n'_{(i)} \quad (3)$$

where the subscript (i) stands for the i -th ordered link when sorting is based on $|h_i|$ in the decreasing order such that $|h_{(1)}| \geq \dots \geq |h_{(N)}|$. Also, $P_{R_{(i)}}$ is the transmit power on the link $RD_{(i)}$, and $n'_{(i)} \sim \mathcal{CN}(0, 1)$ is the AWGN. Note that $y_{(i)}$ in (3) can be found from (1) when all SR_i links are ordered to $SR_{(i)}$ and again sorted based on $|g_i|$ in the decreasing order such that $|g_{(1)}| \geq \dots \geq |g_{(N)}|$.

This system can be viewed as N subsystems operating on separate orthogonal suchchannels. For the i -th subsystem, we can define a source, a relay, and a destination denoted by S_i , R_i , and D_i respectively, where the communication from S_i to D_i is done over SR_i-RD_i . This point of view reduces our system to N parallel conventional single-relay systems each operating on a single channel, which helps our optimization approach later. Using existing results for the SNR of conventional relay systems [24], the overall SNR of the i -th subsystem in the AF case is

$$\rho_i = \frac{P_{S_i} P_{R_i} |g_i|^2 |h_i|^2}{P_{S_i} |g_i|^2 + P_{R_i} |h_i|^2 + 1}. \quad (4)$$

Similarly, in the ASF case, the corresponding SNR is

$$\rho^{(i)} = \frac{P_{S^{(i)}} P_{R^{(i)}} |g^{(i)}|^2 |h^{(i)}|^2}{P_{S^{(i)}} |g^{(i)}|^2 + P_{R^{(i)}} |h^{(i)}|^2 + 1}. \quad (5)$$

B. Problem formulations

Depending on the availability of the CSI at the relay and source nodes two different setups are considered: (i) when global CSI is available at both the source node and relay node, i.e., both S and R have knowledge of both g_i and $h_i, \forall i$. (ii) only local CSI is available at the source node, i.e., S knows only $g_i, \forall i$ while R , knows g_i and $h_i, \forall i$. Thus, in both cases, the transmitter has the knowledge of the channel. It is worth mentioning that for water-filling protocols, it is typical in standards deploying OFDM to provide the CSI to the transmitter [4].

As mentioned earlier, we consider two different types of relaying: AF and ASF. Thus, a total of four different cases with respect to type of relaying and availability of global CSI can be considered. For these four cases, we study optimal power allocation strategies across N orthogonal links at both S and R . For given finite energy budgets at each node, our goal is to maximize the achievable rate of the network. As can be seen from (4), the SNR of the link i , i.e., SR_i-RD_i , and, thus the capacity of this link depend on both P_{S_i} and $P_{R_i}, \forall i$. Therefore, we seek the optimal power levels P_{S_i} and $P_{R_i}, \forall i$ that maximize the sum rate of the network. Finding the optimal solution for the ASF relay network can also be motivated in a similar way from (5).

The optimal power allocation problems in these four cases can be stated as follows.

1) *Case I: AF relaying, global CSI at both S and R* : The goal is to optimally allocate power in both S and R so that the sum capacity of the network is maximized under the individual power constraints at S and R . Thus, we have

$$\max_{P_{S_i}, P_{R_i}, \forall i} \sum_{i=1}^N C_i \quad (6)$$

$$\text{s.t. } \sum_{i=1}^N P_{S_i} \leq P_S, \quad \sum_{i=1}^N P_{R_i} \leq P_R \quad (7)$$

where C_i denotes the capacity of the link $SR_i - RD_i$, P_S and P_R are the individual power budgets at S and R , respectively.

2) *Case II: AF relying, global CSI only at R* : In this case, we study power allocation strategy only at R . This is because, in this case, S knows only $g_i, \forall i$ and, thus, the associated optimal power allocation at S is independent of $h_i, \forall i$. This means that the optimal power allocation at S is the traditional water-filling rule. Thus, we focus only on optimal power allocation at R .

In the first phase, S decides the allocation of its limited power P_S to N orthogonal links using water-filling. For the obtained power allocation, then, S knows the link capacity for $SR_i, \forall i$. Based on the known link capacity, S decides a data rate γ_i for each link $SR_i, \forall i$.

In the second phase, within its limited power P_R , the relay amplifies and forwards the source data received via N links. Particularly, the data transmitted on SR_i should be amplified (and forwarded on RD_i) by the relay so that the overall capacity of this link $SR_i - RD_i$ is equal to or greater than γ_i . Otherwise, channel outage occurs. It is important to notice that the relay, due to its limited power, may not be able to forward all source's data. In such a case, no power should be assigned to the links whose data is not forwarded. In addition, on the links RD_i that the relay decides to forward the source data, it is supposed to put just enough power in order to avoid the channel outage. The goal is, therefore, to forward as much source data as possible.

To achieve this goal, we can formulate the following optimization problem

$$\max_{P_{R_i}, \forall i} \sum_{i=1}^N C_i \quad (8)$$

$$\text{s.t.} \quad \sum_{i=1}^N P_{R_i} \leq P_R \quad (9)$$

$$C_i \in \{\gamma_i, 0\} \quad (10)$$

where γ_i is the data rate decided by S for the link SR_i , and C_i denotes the capacity of the link SR_i-RD_i controlled by P_{R_i} .

The constraint (10), as discussed earlier, is taken into consideration in order to avoid outage on links whose data is forwarded as well as to avoid power wastage on links whose data is not forwarded. Also, it is worth stressing that there is no point in allocating extra power on a link to increase C_i beyond γ_i . The choice of γ_i will be discussed in Section III, where the optimization problem is solved.

3) *Case III: ASF relaying, global CSI at both S and R :* Similar to Case I, we aim at finding the optimal solution for the power allocation at both S and R . However, the difference from Case I is that the data transmitted on the i -th ordered link $SR_{(i)}$ is amplified and forwarded on the i -th ordered link $RD_{(i)}$. Therefore, we can pose the optimization problem as

$$\max_{P_{S_{(i)}}, P_{R_{(i)}}, \forall i} \sum_{i=1}^N C_{(i)} \quad (11)$$

$$\text{s.t.} \quad \sum_{i=1}^N P_{S_{(i)}} \leq P_S, \quad \sum_{i=1}^N P_{R_{(i)}} \leq P_R \quad (12)$$

where $C_{(i)}$ denotes the capacity of the link $SR_{(i)} - RD_{(i)}$.

4) *Case IV: ASF relying, global CSI only at R :* In this case, similar to Case II, the optimization problem can be formulated as

$$\max_{P_{R_{(i)}}, \forall i} \sum_{i=1}^N C_{(i)} \quad (13)$$

$$\text{s.t.} \quad \sum_{i=1}^N P_{R_{(i)}} \leq P_R, \quad C_{(i)} \in \{\gamma_{(i)}, 0\} \quad (14)$$

where $\gamma_{(i)}$ is the data rate decided by S for the link $SR_{(i)}$, and $C_{(i)}$ denotes the capacity of the link $SR_{(i)}-RD_{(i)}$ controlled by $P_{R_{(i)}}$.

III. OPTIMAL RATE SOLUTION

A. Case I: AF relaying, global CSI at both S and R

In this case, it is assumed that both S and R know g_i and $h_i, \forall i$. Thus, using (4), the individual capacity of link i is $C_i = \log(1 + \rho_i)$. Then, the sum capacity of the network is

$$C \triangleq \sum_{i=1}^N C_i = \sum_{i=1}^N \log \left(1 + \frac{P_{S_i} P_{R_i} |g_i|^2 |h_i|^2}{P_{S_i} |g_i|^2 + P_{R_i} |h_i|^2 + 1} \right). \quad (15)$$

Based on (15), the optimal power allocation problem (6)-(7) can be reformulated as

$$\max_{P_{S_i}, P_{R_i}, \forall i} \sum_{i=1}^N \log \left(1 + \frac{\alpha_i \beta_i P_S P_R |g_i|^2 |h_i|^2}{\alpha_i P_S |g_i|^2 + \beta_i P_R |h_i|^2 + 1} \right) \quad (16)$$

$$\text{s.t.} \quad \sum_{i=1}^N \alpha_i \leq 1, \quad \sum_{i=1}^N \beta_i \leq 1 \quad (17)$$

where α_i denotes the ratio of P_{S_i} to P_S , i.e., $\alpha_i = P_{S_i}/P_S$, and similarly $\beta_i = P_{R_i}/P_R$.

Because of constraints on α and β , solving (16)-(17) directly can be difficult. Therefore, we first find a necessary condition on α_i and β_i that significantly helps solving (16)-(17). For this, we notice that the individual power constraints in (17) lead to the following necessary condition on the sum of the given individual powers at S and R

$$\sum_{i=1}^N P_{S_i} + P_{R_i} \leq P_S + P_R \quad \text{or} \quad \sum_{i=1}^N \alpha_i + \sum_{i=1}^N \beta_i \tau \leq 1 + \tau \quad (18)$$

where τ is the ratio of P_R to P_S , i.e., $\tau = P_R/P_S$. While this condition is not sufficient, any conclusion received from enforcing (18) is necessary for the optimal solution of (16)-(17). In the next theorem, we derive a relationship between optimal values of α_i and β_i .

Theorem 1: When g_i and $h_i, \forall i$ are known to both S and R , the optimal power allocation at S and R satisfies

$$\frac{\beta_i^*}{\alpha_i^*} \tau = \frac{\alpha_i^* P_S |g_i|^2 + 1}{\beta_i^* P_R |h_i|^2 + 1} \quad (19)$$

where α_i^* and β_i^* denote the optimal values of α_i and β_i , respectively.

Proof: Let $\mathbf{P}_i = [P_{S_i} \ P_{R_i}]$ denote a vector with elements of transmit powers used on subchannel i . Then, it can be shown from (15) that for a given \mathbf{P}_i , the sum capacity of the network is a concave function of \mathbf{P}_i since $\partial^2 C / \partial^2 \mathbf{P}_i \leq 0$ for all values of $\mathbf{P}_i \in \mathcal{R}^2, \forall i$. Therefore, (16)-(17) is a convex optimization problem with respect to $\mathbf{P}_i, \forall i$.

We now consider the objective (16) along with the constraint (18). The objective (16) can be optimized by using a Lagrangian multiplier method. The associated Lagrange function can be written as

$$\mathcal{O} = \sum_{i=1}^N \log \left(1 + \frac{\alpha_i \beta_i P_S P_R |g_i|^2 |h_i|^2}{\alpha_i P_S |g_i|^2 + \beta_i P_R |h_i|^2 + 1} \right) + \lambda \left(1 + \tau - \sum_{i=1}^N \alpha_i - \sum_{i=1}^N \beta_i \tau \right) \quad (20)$$

where λ stands for the Lagrange multiplier.

Taking derivatives of (20) with respect to α_i and $\beta_i, \forall i$, and equating them to zero, we obtain that the optimal values of α_i and $\beta_i, \forall i$ can be expressed as:

$$\alpha_i^* = -\frac{2 + \beta_i^* P_R |h_i|^2}{2 P_S |g_i|^2} + \frac{1}{2} \sqrt{\beta_i^* \tau \frac{|h_i|^2}{|g_i|^2} \left(\beta_i^* \tau \frac{|h_i|^2}{|g_i|^2} + \frac{4}{\lambda} \right)} \quad (21)$$

$$\beta_i^* = -\frac{2 + \alpha_i^* P_S |g_i|^2}{2 P_R |h_i|^2} + \frac{1}{2} \sqrt{\frac{\alpha_i^* |g_i|^2}{\tau |h_i|^2} \left(\alpha_i^* \tau^{-1} \frac{|g_i|^2}{|h_i|^2} + \frac{4}{\lambda} \right)}. \quad (22)$$

Inserting (22) into (21), it is revealed that

$$\alpha_i^* (\alpha_i^* P_S |g_i|^2 + 1) = \tau \beta_i^* (\beta_i^* P_R |h_i|^2 + 1) \quad (23)$$

and the claim of Theorem 1 follows straightforwardly. ■

Theorem 1 reveals an interesting symmetry for balancing power among individual links. Based on this result, if the optimal α_i^* is known, then the optimal β_i^* can be immediately found.

Noticing that (19) is a necessary condition under which (16) is optimized under the constraint (17), we can provide a modified optimization problem only based on α_i . So, the complexity of solving the optimization problem is greatly reduced. Then, using (15) and (19), the sum capacity of the network can be reformulated with respect to only α_i as

$$C = \sum_{i=1}^N \log \left(1 + \frac{\alpha_i P_S |g_i|^2 (-1 + A(\alpha_i)) / 2}{\alpha_i P_S |g_i|^2 + (1 + A(\alpha_i)) / 2} \right) \quad (24)$$

where $A(\alpha_i) = \sqrt{1 + 4\alpha_i P_S |h_i|^2 (\alpha_i P_S |g_i|^2 + 1)}$. Then the problem (16)-(17) can be reformulated as

$$\max_{\alpha_i, \forall i} \sum_{i=1}^N \log \left(1 + \frac{\alpha_i P_S |g_i|^2 (-1 + A(\alpha_i)) / 2}{\alpha_i P_S |g_i|^2 + (1 + A(\alpha_i)) / 2} \right) \quad (25)$$

$$\text{s.t.} \quad \sum_{i=1}^N \alpha_i \leq 1 \quad (26)$$

$$\sum_{i=1}^N \frac{-1 + A(\alpha_i)}{2P_R |h_i|^2} \leq 1 \quad (27)$$

where constraint (27) is equivalent to $\sum_{i=1}^N \beta_i \leq 1$. Notice that in this optimization problem, the objective function and the constraints are all convex functions of α_i . Therefore, traditional efficient numerical convex optimization techniques can be applied in order to solve it.

B. Case II: AF relying, global CSI only at R

In this case, as addressed earlier, we assume that in the first phase, S decides a power level P_{S_i} and a data rate γ_i for the link SR_i . The goal of the relay in the second phase is to amplify and forward as much source data received over N source-relay links as possible, subject to avoiding outage on subchannels that R decides to forward through. This means that the data received over some links may not be forwarded by the relay, due to its limited available power. The optimization problem in this case is the problem (8)–(10).

For given ρ_{SR_i} , we can represent the individual achievable rate of the link SR_i – RD_i as

$$C_i = \log \left(1 + \frac{\rho_{SR_i} \beta_i P_R |h_i|^2}{\rho_{SR_i} + \beta_i P_R |h_i|^2 + 1} \right). \quad (28)$$

The sum rate of the network can therefore be expressed as

$$C \triangleq \sum_{i=1}^N C_i = \sum_{i=1}^N \log \left(1 + \frac{\rho_{SR_i} \beta_i P_R |h_i|^2}{\rho_{SR_i} + \beta_i P_R |h_i|^2 + 1} \right) \quad (29)$$

where β_i is zero for links RD_i that are not chosen to be amplified.

Based on (8) and (29), subject to avoiding outage on links with $\beta_i \neq 0$, the optimization problem can be reformulated as

$$\max_{\beta_i, \forall i} \sum_{i=1}^N \log \left(1 + \frac{\rho_{SR_i} \beta_i P_R |h_i|^2}{\rho_{SR_i} + \beta_i P_R |h_i|^2 + 1} \right) \quad (30)$$

$$\text{s.t.} \quad \sum_{i=1}^N \beta_i \leq 1 \quad (31)$$

$$C_i \in \{\gamma_i, 0\}, \forall i. \quad (32)$$

Due to the discrete constraint (32), convex optimization techniques cannot be applied in order to solve (30)–(32). When seeking the optimal solution, it may be needed to consider all possible power allocations. The number of different power allocations, however, increases exponentially with N . Therefore, we propose a greedy algorithm which finds near optimal solutions (and in many cases the optimal solution). The idea is to allocate the limited available power in the most efficient way. Particularly, notice that on the relay–destination link RD_i , there is a minimum β_i guaranteeing successful source–destination communication. This value of β_i , denoted by $\tilde{\beta}_i$, is considered as the cost of communication on this link. This is because $\tilde{\beta}_i$ is proportional to the amount of power spent on this link, if the link is chosen to be amplified and forwarded. Solving

$$\log \left(1 + \frac{\rho_{SR_i} \tilde{\beta}_i P_R |h_i|^2}{\rho_{SR_i} + \tilde{\beta}_i P_R |h_i|^2 + 1} \right) = \gamma_i,$$

the value of $\tilde{\beta}_i$ can be found as

$$\tilde{\beta}_i = \frac{(2^{\gamma_i} - 1) 2^{\gamma_i/\delta_i}}{(2^{\gamma_i/\delta_i} - 2^{\gamma_i}) P_R |h_i|^2} \quad (33)$$

where δ_i denotes the ratio of γ_i to the capacity of the link SR_i . While various subchannels can have different δ_i , in this work we assume $\delta_i = \delta, \forall i$ that is the worst–case. In Section V, we observe that even with $\delta_i = \delta, \forall i$, the achievable sum rate converges at high SNR to the case of Global CSI at both S and R .

Consuming the cost $\tilde{\beta}_i$ on link i , in return, the data rate γ_i from the source to the destination is obtained. Then, we can define $\eta_i = \gamma_i/\tilde{\beta}_i$ as the efficiency of allocating power to link i . Notice that the larger the value of η_i , the better the link i . This is because for the same cost in terms of

the power spending, larger data rate γ_i is achievable on links that have larger values of η_i . So, all links are sorted in the decreasing order of η_i . We assign power to the links that have better η_i until we run out of power (before $\sum_i \beta_i$ becomes greater than one.)

The greedy algorithm is designed such that the limited power is spent on the best links in terms of the achievable rate. However, this solution may not be optimal because by assigning power according to this greedy algorithm, we may end up with some positive leftover power. In such cases, other strategies that result in zero leftover power, may turn out to achieve a slightly higher rate. Nonetheless, the greedy algorithm is optimal in the sense of using the limited power in the most efficient way and in most cases provides the optimal solution for sum rate maximization problem. In particular, as N grows large (i.e., the effects of the leftover power is negligible), it almost always gives the optimal solution.

Now, we modify this greedy algorithm to avoid any leftover power. The idea is to allow $\delta \in [0, 1]$ to be a function of channel statistics. We notice that δ remains constant as long as channel statistics are constant. To remove the leftover power, we consider the following max – max sum rate optimization problem

$$\max_{\delta \in [0,1]} E \left\{ \max_{\tilde{\beta}_i, \forall i} \sum_{i=1}^N \log \left(1 + \frac{\rho_{SR_i} \tilde{\beta}_i P_R |h_i|^2}{\rho_{SR_i} + \tilde{\beta}_i P_R |h_i|^2 + 1} \right) \right\} \quad (34)$$

$$\text{s.t. } \sum_{i=1}^N \tilde{\beta}_i \leq 1 \quad (35)$$

$$C_i \in \{\gamma_i, 0\}, \forall i. \quad (36)$$

To solve this optimization problem, for any given value of $\delta \in [0, 1]$, we use the aforementioned greedy algorithm to find the maximum sum rate. Then, we find such a δ^* that provides the maximum sum rate. Please notice that in practice channel statistics do not change fast, so for given channel statistics, the optimal values of δ can be found off–line and tabulated for later use. As soon as δ^* is chosen by the network, a single run of our greedy algorithm is needed to find the optimal power allocation per channel realization.

C. Case III: ASF relaying, global CSI at both S and R

The only difference from Case I is channel ordering. In other words, data transmitted on $SR_{(i)}$ (the i -th ordered link) is amplified and forwarded on $RD_{(i)}$. The ordering is in the decreasing order of the channel gains $|g_i|^2$ and $|h_i|^2$, respectively. Therefore, using (5), the individual capacity of link (i) can be written as

$$C_{(i)} = \log(1 + \rho_{(i)}). \quad (37)$$

where $\rho_{(i)}$ denotes the SNR received via link $SR_{(i)}-RD_{(i)}$. The overall capacity of the network can be defined as

$$C \triangleq \sum_{i=1}^N C_{(i)} = \sum_{i=1}^N \log \left(1 + \frac{\alpha_{(i)}\beta_{(i)}P_S P_R |g_{(i)}|^2 |h_{(i)}|^2}{\alpha_{(i)}P_S |g_{(i)}|^2 + \beta_{(i)}P_R |h_{(i)}|^2 + 1} \right) \quad (38)$$

Using (38), the power allocation optimization problem (11)–(12) can be represented by the following problem

$$\max_{\alpha_{(i)}, \beta_{(i)}, \forall i} \sum_{i=1}^N \log \left(1 + \frac{\alpha_{(i)}\beta_{(i)}P_S P_R |g_{(i)}|^2 |h_{(i)}|^2}{\alpha_{(i)}P_S |g_{(i)}|^2 + \beta_{(i)}P_R |h_{(i)}|^2 + 1} \right) \quad (39)$$

$$\text{s.t.} \quad \sum_{i=1}^N \alpha_{(i)} \leq 1, \quad \sum_{i=1}^N \beta_{(i)} \leq 1. \quad (40)$$

Notice that the optimization problem (39)–(40) is mathematically equivalent to the problem (16)–(17) in Case I. The only difference from (16)–(17) is that the indices of optimal powers $\alpha_{(i)}^*$ and $\beta_{(i)}^*$ in (39)–(40) correspond to the ordered links. Thus, in order to find the optimal solution to (39)–(40), we follow the same approach as in Case I by replacing g_i, h_i, α_i and β_i with $g_{(i)}, h_{(i)}, \alpha_{(i)}$ and $\beta_{(i)}$, respectively.

D. Case IV: ASF relaying, global CSI only at R

Again, the optimization problem in this case is mathematically identical to the one in Case II. The only modification needed is to replace g_i, h_i, β_i and γ_i with $g_{(i)}, h_{(i)}, \beta_{(i)}$ and $\gamma_{(i)}$, respectively.

IV. ASYMPTOTIC ANALYSIS WITH GLOBAL CSI AT BOTH S AND R AT HIGH SNR

In this section, we study the optimization problem of Case I at high SNR, i.e., when P_S and P_R tend to infinity. Again, we assume $P_R = \tau P_S$. Thus, for a given τ , we study the case that P_S tends to infinity. Through this analysis, we discuss the behavior of the optimal solution at high SNR regime. Some interesting observations are discussed.

Using (4) and (24), when P_S increases, the overall SNR on link i can be given by

$$\begin{aligned} \lim_{P_S \rightarrow \infty} \rho_i &= \lim_{P_S \rightarrow \infty} \frac{\alpha_i P_S |g_i|^2 (-1 + 2\alpha_i P_S |h_i| |g_i|) / 2}{\alpha_i P_S |g_i|^2 + (1 + 2\alpha_i P_S |h_i| |g_i|) / 2} \\ &= \alpha_i \varphi_i P_S \end{aligned} \quad (41)$$

where $\varphi_i \triangleq |h_i| |g_i| / (1 + |h_i| |g_i|)$. Then, it can be obtained asymptotically from (41) that the sum capacity of the network is expressed only in terms of α_i by

$$\lim_{P_S \rightarrow \infty} C = \lim_{P_S \rightarrow \infty} \sum_{i=1}^N C_i = \sum_{i=1}^N \log(\alpha_i \varphi_i) + N \log(P_S). \quad (42)$$

Based on (25) and (42), the optimization problem in Case I can be rewritten at high SNR as

$$\max_{\alpha_i, \forall i} \sum_{i=1}^N \log(\alpha_i \varphi_i) + N \log\left(\frac{P}{1 + \tau}\right) \quad (43)$$

$$\text{s.t. } \sum_{i=1}^N \alpha_i \leq 1 \quad (44)$$

$$\frac{1}{\tau} \sum_{i=1}^N \alpha_i \frac{|g_i|}{|h_i|} \leq 1. \quad (45)$$

It is worth noticing that the objective function (43) is a monotonic function of $\alpha_i, \forall i$. Using the Lagrange multiplier method, optimal solution to (43)–(45) can be obtained. Then the Lagrange function can be formulated as

$$\mathcal{O} = \sum_{i=1}^N \log(\alpha_i \varphi_i) + N \log\left(\frac{P}{1 + \tau}\right) + \lambda_1 \left(1 - \sum_{i=1}^N \alpha_i\right) + \lambda_2 \left(1 - \frac{1}{\tau} \sum_{i=1}^N \alpha_i \frac{|g_i|}{|h_i|}\right) \quad (46)$$

where λ_1 and λ_2 are Lagrange multipliers associated with the constraints. Differentiating (46) with respect to $\alpha_i, \forall i$ and equating the results to zero, we obtain the following closed-form expression for the optimal value of α_i

$$\alpha_i^* = \max \left\{ 0, \left(\lambda_1 + \frac{\lambda_2 |g_i|}{\tau |h_i|} \right)^{-1} \right\}. \quad (47)$$

In (47), the Lagrange multipliers λ_1 and λ_2 are found once for a given set of values $g_i, \forall i$ and $h_i, \forall i$. Therefore, it can be observed that when $\lambda_2 > 0$, for two subchannels with $h_i = h_j$, the optimal power allocation solutions put more source power on the weaker subchannel. In other words, if $g_i < g_j$ then $\alpha_i^* > \alpha_j^*$, and vice versa. This is the inverse of a more traditional water-filling scheme, where only one power constraint exists and the optimal solution puts more power on the stronger subchannel. Similarly, rewriting the above equations in terms of β_i^* , we have

$$\beta_i^* = \max \left\{ 0, \left(\lambda_1' + \lambda_2' \tau \frac{|h_i|}{|g_i|} \right)^{-1} \right\}. \quad (48)$$

which means that the behavior of the optimal β_i^* is the inverse of water-filling for $\lambda_2' > 0$.

Furthermore, from Theorem 1, at high SNR, it is obtained that the optimal solution to the problem (43)–(45) must satisfy

$$\lim_{P_S \rightarrow \infty} \alpha_i^* = \tau \beta_i^* \sqrt{\frac{|h_i|^2}{|g_i|^2}} \Rightarrow \lim_{P_S \rightarrow \infty} P_S \alpha_i^* = P_R \beta_i^* \sqrt{\frac{|h_i|^2}{|g_i|^2}}. \quad (49)$$

Notice that $P_S \alpha_i^*$ and $P_R \beta_i^*$ represent the allocated power to links SR_i and RD_i , respectively. It can be concluded then that when $|g_i| > |h_i|$ on link i , the optimal assigned power on link SR_i must be less than the optimal assigned power on link RD_i .

V. SIMULATIONS

Consider a two-hop single relay network where nodes communicate through $N \in \{4, 20\}$ orthogonal channels. The source-relay and relay-destination channels are assumed to be independent Rayleigh flat fading with variance σ^2 . The total sum power assigned to these $2N$ channels at both the source and relay is $P = P_S + P_R$. The noise is assumed to be additive zero-mean white Gaussian with unit variance. Thus, the total transmit SNR, which is used in our figures, is defined as $\text{SNR} = P\sigma^2$. Notice that transmit SNRs at S and R can individually be expressed as $P_S\sigma^2$ and $P_R\sigma^2$, correspondingly.

A. AF relaying

Assuming that the AF strategy is employed at the relay node, Fig. 2 shows the achieved sum capacity versus the total transmit SNR for Case I when the global CSI is available at both the

source and relay nodes and the proposed optimal power allocation strategy is used. Here $\tau = 1$ (i.e., $P_S = P_R = P/2$). It can be seen from the figure that the sum capacity increases for larger N . Intuitively, this is because for limited P_S and P_R , larger N provides more chances of obtaining better source-destination subchannels on which the power allocation can result in a better sum capacity.

Fig. 3 depicts the results for Case II, when the global CSI is available only at the relay and, thus, the power allocation can only be made at the relay. The sum rate is shown versus the total transmit SNR for $\tau = 1$ and $N = 20$. Three scenarios for the greedy algorithm are considered. In the first one, $\delta = 0.5$ is fixed for all SNRs, while in the second and third scenarios, the optimized δ , i.e., δ^* , is used. The difference between the second and third scenarios is that water-filling at the source is assumed in one, while equal power allocation across all subchannels at the source is used in the other. The proposed greedy algorithm is used in order to allocate P_R to N orthogonal subchannels. It can be seen from the figure that, as expected, higher data rates are achieved for optimal δ^* . Interestingly, the equal power allocation at the source does not suffer from significant rate loss. Thus, in further examples for greedy-based power allocation, we consider only equal power allocation at the source.

In Fig. 4, the results of the optimal power allocation in Cases I and II are compared to each other in terms of the sum rate plotted versus the total transmit SNR when $\tau = 1$, $N = 20$, and optimal δ^* is used. In agreement with our analysis, the global optimization in Case I always outperforms the other case. Particularly, at sum rate of 2 bits/s/Hz, Case I is shown to obtain 4.5 dB power gain compared to Case II. Here, it is worth mentioning that this significant power gain is achieved at a modest value of N .

The results of the optimal power allocation in Cases I and II are also compared to each other in Fig. 5 in terms of the sum rates when $\tau = 0.5$ (i.e., $P_S = 2P_R$), $N = 20$, and $\delta = \delta^*$. It can be seen from the figure that again, Case I outperforms Case II for all values of the total transmit SNR. However, the performance gap between Cases I and II decreases as compared to the set up examined in Fig. 4. This is because for $\tau \ll 1$ all source-relay subchannels are

likely to have much better conditions than the relay-destination subchannels, while the sum rate is mainly affected by the power allocation across the relay-destination subchannels. In addition, in both figures (Figs. 4 and Fig. 5), one can see that Case II converges to Case I at high SNRs. This is because all subchannels become almost deterministic at high SNR, and therefore, the global optimization of the power allocation is less beneficial, while optimization of the power allocation only at the relay used in Case II tends to be almost sufficient. This is because Case II takes into consideration as many best relay-destination subchannels as possible within a limited power budget.

Let us now consider $\tau = 2$ (i.e., $2P_S = P_R$). The corresponding sum rates is shown versus the total transmit SNR for both Cases I and II in Fig. 6 for $N = 20$ and $\delta = \delta^*$. It can be seen from this figure that Case I is better than Case II for all values of SNR. Interestingly, comparing all cases tested, i.e., $\tau = 0.5$ (Fig. 5), $\tau = 1$ (Fig. 4), and $\tau = 2$ (Fig. 6), the global optimal power allocation is most beneficial when $\tau = 1$.

B. ASF relaying

Fig. 7 depicts the sum rate versus the total transmit SNR for Case III when the global CSI is available at both the source and relay nodes and the optimal power allocation is performed for ASF relaying. For this figure, $N \in \{4, 20\}$ and $\tau = 1$. Case I is also depicted for comparison purposes. It can be observed from this figure that Case III outperforms Case I, as expected.

Optimizing δ and the power allocation only at the relay in Case IV, the sum rate is shown in Fig. 8 versus the total transmit SNR for $N = 20$ and $\tau = 1$. The sum rate curves for Cases I, II, and III are also depicted for comparison. It is confirmed in Fig. 8 that Case III always outperforms all other cases including Case IV for the same reasons that have been explained while comparing Cases I and II. Interestingly, it can also be observed in this figure that Case IV is superior to the global optimization of Case I for moderate and high SNRs.

VI. CONCLUSIONS

Two-hop AF and ASF relay networks, consisting of single source, destination, and relay nodes with limited individual power constraints at the source and relay are considered. Optimal power allocation strategies across N orthogonal subchannels between nodes are studied, depending on the availability of CSI at both the source and relay or only at the relay.

When source has global CSI knowledge, through a symmetry property proved for the optimal power allocation, the optimization problem is solved. Via an asymptotic analysis, it is found that the global power optimization assigns more power on the weaker subchannels, which is the inverse of the traditional water-filling. When only the relay has global CSI knowledge, a greedy algorithm maximizing the achievable sum rate is proposed. For this, the actual data rate at the source is optimized and the minimum powers on subchannels are found at the relay in order to guarantee successful source-destination communication. It is also shown numerically that the optimal power allocation performed only at the relay can outperform the global power optimization scheme at moderate and large SNRs if simple subchannel sorting capabilities are added at the relay.

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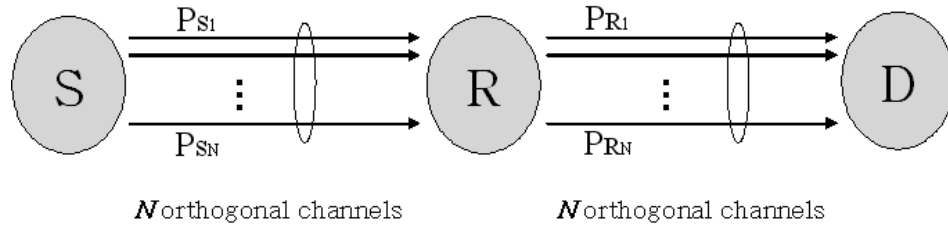


Fig. 1. Block diagram of a two-hop single relay wireless network with N source-relay and N relay-destination orthogonal channels.

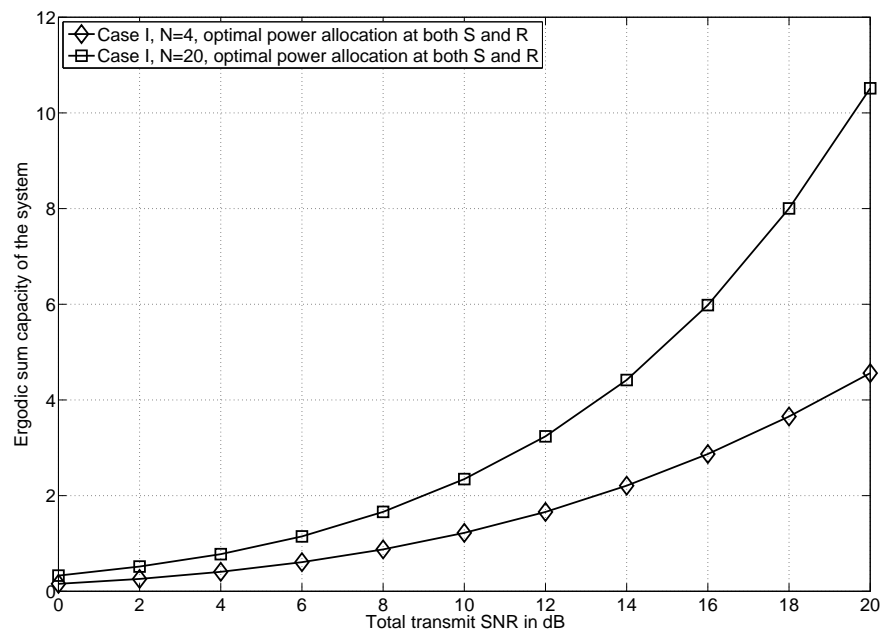


Fig. 2. The sum capacity versus the total transmit SNR for Case I: the optimal power allocation at both the source and the relay for $N \in \{4, 20\}$ and $\tau = 1$.

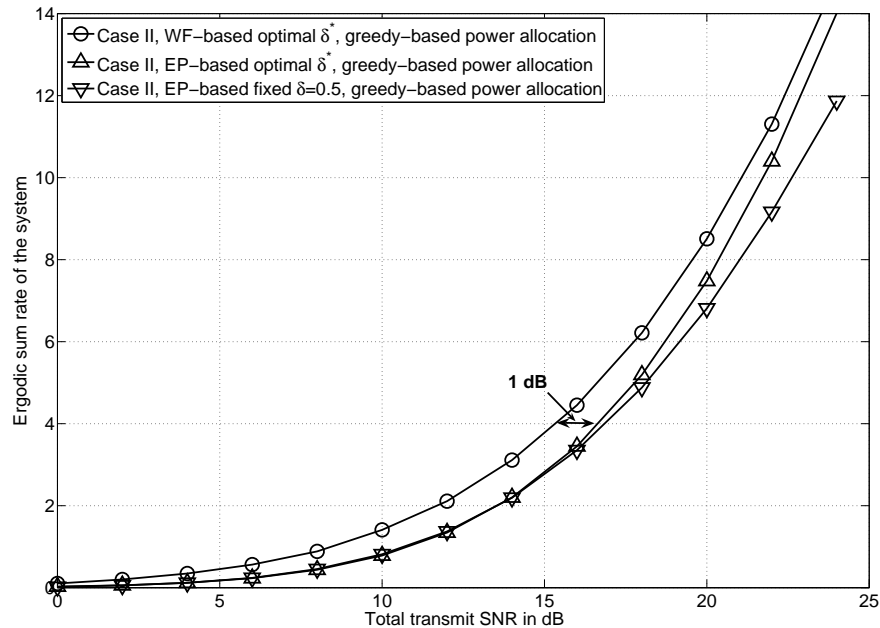


Fig. 3. The sum rate versus the total transmit SNR using the proposed greedy algorithm for $N = 20$ and $\tau = 1$. Three cases of δ are examined: (i) optimal δ^* with the water-filling at the source (WF-based optimal δ^*); (ii) optimal δ^* with the equal power allocation across N orthogonal channels at the source (EP-based optimal δ^*); (iii) fixed $\delta = 0.5$ with the equal power allocation across N orthogonal channels at the source (EP-based fixed $\delta = 0.5$).

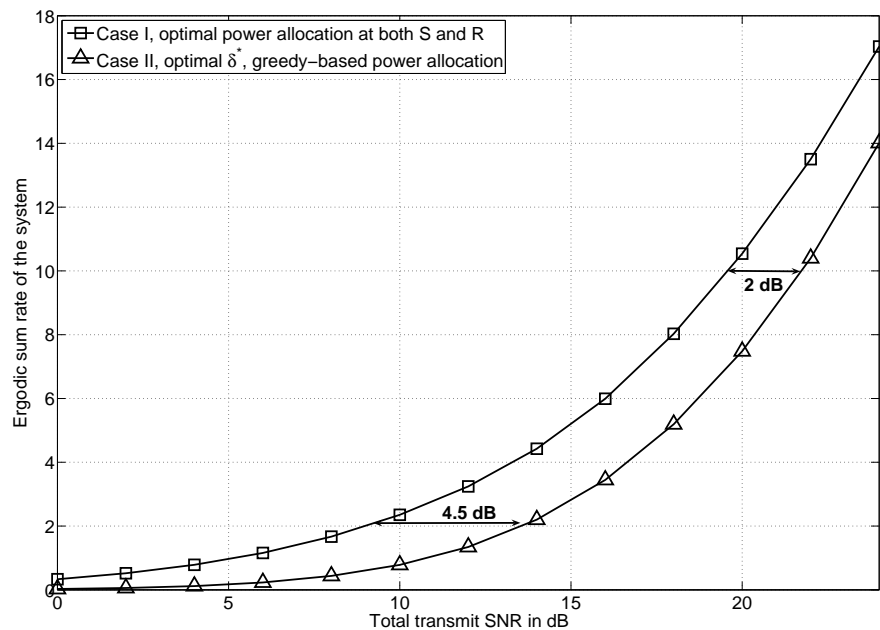


Fig. 4. Comparison of the sum rates for Cases I and II: $N = 20$, $\tau = 1$, and $\delta = \delta^*$ for equal power allocation across N orthogonal channels at the source.

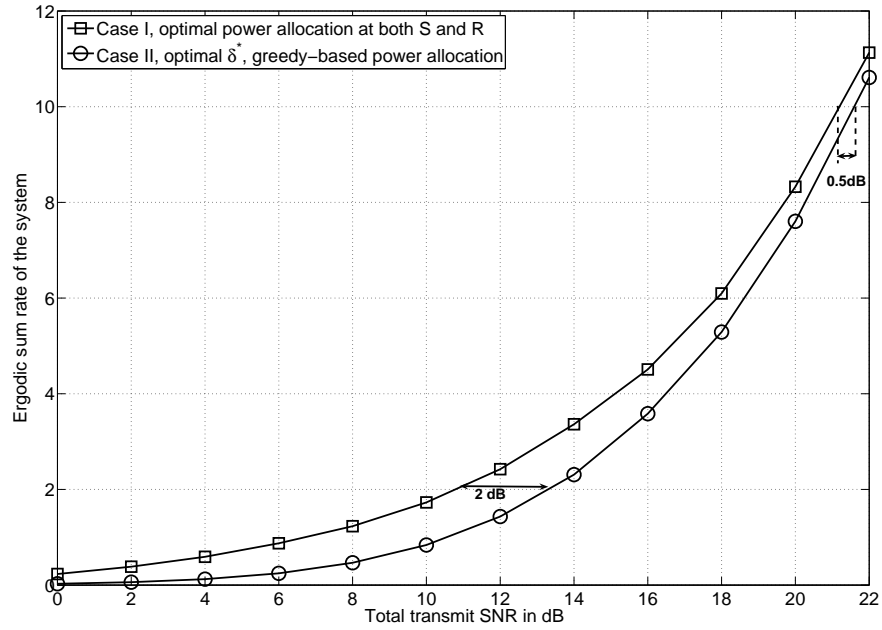


Fig. 5. Comparison of the sum rates for Cases I and II: $N = 20$, $\tau = 0.5$, and $\delta = \delta^*$ for equal power allocation across N orthogonal channels at the source.

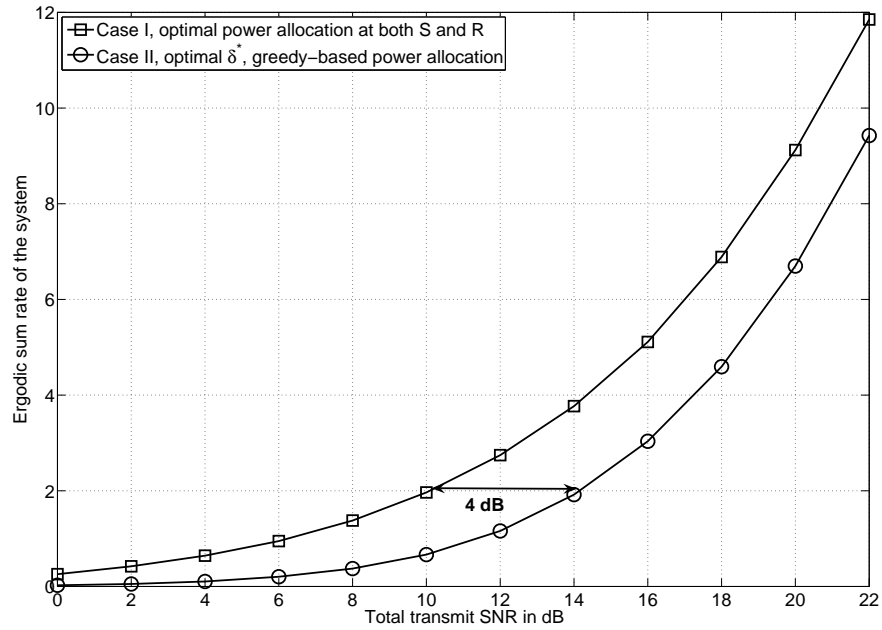


Fig. 6. Comparison of the sum rates for Cases I and II: $N = 20$, $\tau = 2$, and $\delta = \delta^*$ for equal power allocation across N orthogonal channels at the source.

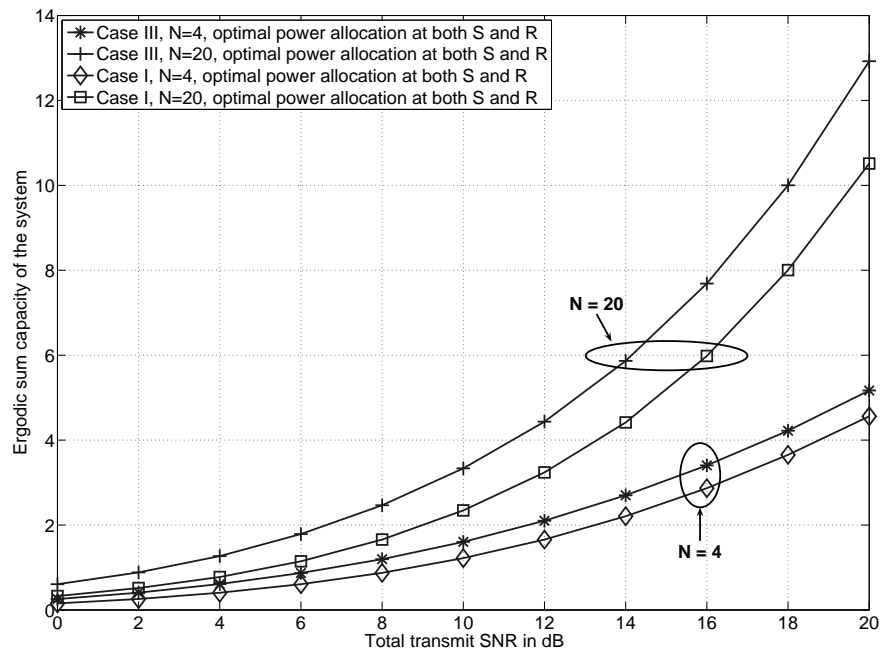


Fig. 7. The sum capacity versus the total transmit SNR: Case III of the optimal power allocation at both the source and the relay for $N \in \{4, 20\}$, and $\tau = 1$. Case I is depicted for comparison.

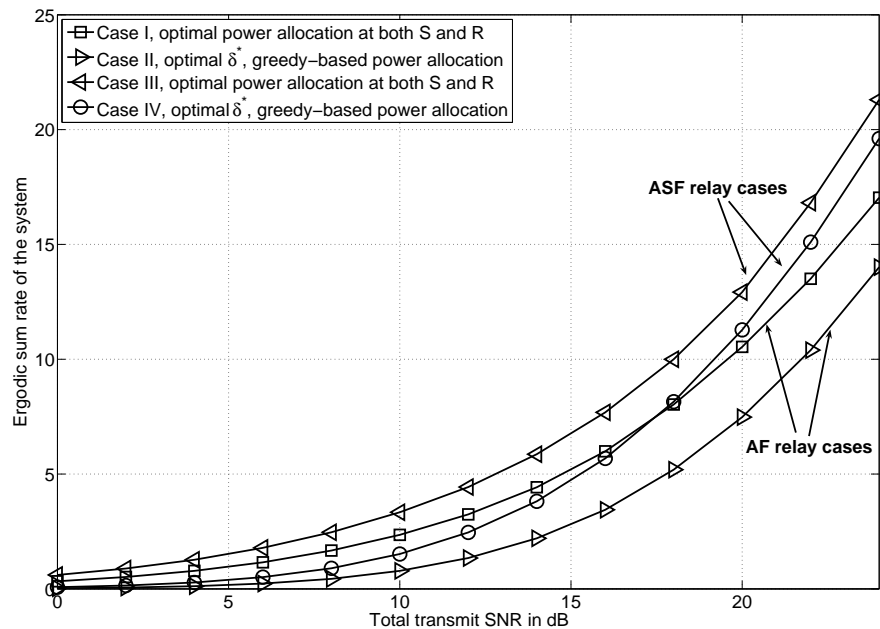


Fig. 8. The sum rate versus the total transmit SNR using the proposed greedy algorithm in Case IV for $N = 20$, $\tau = 1$, $\delta = \delta^*$ for equal power allocation across N orthogonal channels at the source. Cases I, II, and III are depicted for comparison.