

Distributed Energy Efficient Channel Allocation

Oshri Naporstek, *Member, IEEE*, S. M. Zafaruddin, *Member, IEEE*, Amir Leshem, *Senior Member, IEEE*, Eduard Jorswieck, *Senior Member, IEEE*

Abstract—Design of energy efficient protocols for modern wireless systems has become an important area of research. In this paper, we propose a distributed optimization algorithm for the channel assignment problem for multiple interfering transceiver pairs that cannot communicate with each other. We first modify the auction algorithm for maximal energy efficiency and show that the problem can be solved without explicit message passing using the carrier sense multiple access (CSMA) protocol. We then develop a novel scheme by converting the channel assignment problem into perfect matchings on bipartite graphs. The proposed scheme improves the energy efficiency and does not require any explicit message passing or a shared memory between the users. We derive bounds on the convergence rate and show that the proposed algorithm converges faster than the distributed auction algorithm and achieves near-optimal performance under Rayleigh fading channels. We also present an asymptotic performance analysis of the fast matching algorithm for energy efficient resource allocation and prove the optimality for large enough number of users and number of channels. Finally, we provide numerical assessments that confirm the energy efficiency gains compared to the state of the art.

Index Terms—Auction algorithm, bipartite graph, channel assignment, energy efficiency (EE), linear programming, distributed protocol, multi-access channel, Rayleigh fading channel, resource management, wireless networks.

I. INTRODUCTION

Communication networks have been designed to optimize conventional performance measures such as bit-error-rate, latency, and data-rate in the past few decades. In the last few years, the issue of energy-efficient network design has gained more importance [1], [2], [3], [4]. Information and communication technologies (ICT) represent about 2% of the entire world's energy consumption, and the situation is likely to reach a point where ICT equipment in large cities will require more energy than is actually available [5]. For data networks, contrary to the intuition, more energy is consumed in access networks than in core networks. This happens because the number of devices in access networks (i.e. mobile terminals, base stations, and data modems installed on customers' premises) is much larger than the number of communication devices (routers, multiplexers, etc.) in the core network. This has sparked research in the field of wireless networks with a focus on the problem of optimizing the energy

efficiency (EE) at the physical layer by maximizing the ratio of a SINR-based function over the consumed energy.

Unlike cellular networks where the energy efficiency can be optimized in a centralized manner, in many cases centralized optimization is not possible. In this case distributed protocols are needed as a practical solution to maximize the energy efficiency of the network. Several approaches had recently been suggested for optimal distributed allocation. These approaches utilize an opportunistic version of carrier sensing to determine an orthogonal allocation which is optimal or near optimal in the sense of exploiting channel diversity. The first example is the use of the well known Gale-Shapley stable marriage theorem [6] to allocate spectra in a multichannel setup [7], [8]. In [8], Leshem et al. demonstrated how a stable channel allocation can be obtained without any explicit transmission using carrier sensing as a mechanism to prioritize channel. Analysis of this technique for Rayleigh fading channels appeared in [9]. In [10] and [11] a single user channel is considered and the optimization is carried out through transmit power control. In contrast, in [12]- [19] and [20], multiuser interference channels are considered and a competitive scenario in which users selfishly aim at individual EE maximization is addressed. Centralized and decentralized resource allocation in multi-hop networks for energy-efficiency maximization is studied in [21], [22].

Other approaches to the distributed channel allocation problem include game theoretic bargaining solutions [23], [24], [25] and distributed allocation using multichannel ALOHA [26], [27]. While the stable allocation which turns out to be the greedy assignment is almost optimal for Rayleigh fading channels, it is desirable to obtain the optimal allocation. It is well known that this allocation can be computed by solving a linear programming problem [28]. However, in order to compute the optimal distributed solution [29], [30] revised the auction technique of Bertsekas [31] which requires a shared memory or price exchange between the bidders and the auctioneer. Instead of knowledge of the highest price this technique only requires knowledge of local prices. Based on the local prices, an algorithm which can be implemented using multichannel opportunistic carrier sense multiple access (CSMA) is presented and its optimality is proved.

One of the disadvantages of the distributed auction algorithm is its convergence time which might be too long in practical scenarios. It was suggested in [29] to use a simplified version of the distributed auction algorithm and look for perfect matching instead of optimal matchings to optimize energy efficiency. It was proven that the matching algorithm is asymptotically optimal for sum rate maximization and simulated results showed fast convergence time. No analysis for the expected convergence time was given in [29]. In [32]

Oshri Naporstek was a Ph.D. student with Faculty of Engineering, Bar-Ilan university, Ramat-Gan, 52900, Israel (email: oshri8@gmail.com). S. M. Zafaruddin and Amir Leshem are with Faculty of Engineering, Bar-Ilan university, Ramat-Gan, 52900, Israel (email: smzafar@biu.ac.il, lesHEMA@biu.ac.il). Eduard Jorswieck is with the Faculty of Electrical and Computer Engineering, Communications Laboratory, TU Dresden, Dresden 01062, Germany (e-mail: eduard.jorswieck@tu-dresden.de)

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energy efficiency is optimized in a distributed manner for cellular scenario using fractional programming.

In this paper, we propose a distributed optimization algorithm for the channel assignment problem to increase the energy efficiency of multiple interfering transceiver pairs that cannot communicate with each other. We show that the problem can be solved without explicit message passing using a modified distributed auction algorithm. We then develop a novel scheme by converting the channel assignment problem of the distributed auction algorithm into finding perfect matchings on bipartite graphs. The proposed fast matching algorithm reduces the convergence time of the distributed auction algorithm and achieves near-optimal performance. We analyze the performance of the fast matching algorithm for energy efficiency maximization and show that the expected energy efficiency index achieved by the matching algorithm approaches the optimal energy efficiency index for large enough number of users and number of resources. We also prove that the expected number of iterations until convergence of the fast matching algorithm is $\mathcal{O}(N \log(N))$, where N denotes the number of users. We provide numerical assessments for various wireless channels that confirm the energy efficiency gains compared to the state of the art.

The paper is organized as follows: in Section II we define the maximal energy efficiency problem. Section III discusses the distributed auction algorithm. In Section IV we present a fast converging algorithm for a relaxation to the maximal energy efficiency problem and show that the algorithm for the relaxed problem terminates within $\mathcal{O}(N \log(N))$ iterations with high probability. The performance of the fast matching algorithm is studied in Section V. In Section VI we discuss simulated results for the proposed algorithm. Finally, Section VII concludes the paper.

II. PROBLEM FORMULATION

Before formulating the optimization problem for maximal energy efficiency, we present various definitions of energy efficiency for communication networks used in the literature.

A. Definitions of Energy Efficiency

The first and most widely used definition of EE is the ratio between the throughput and the transmit power (see [12]-[18], and references therein). Another proposed metric uses the goodput in place of the throughput [19]. In all of the above works, as far as the computation of the consumed power is concerned, only the transmit power is considered, whereas the power that is dissipated in the electronic circuitry of each terminal in order to keep the terminal active is neglected. This assumption was relaxed in [33], by defining the consumed power as the sum of the transmit power plus a constant term, independent of the transmit power, which models the circuit power needed to operate the terminal. Following [33], in [20]-[11] the consumed power is also defined as the sum of the transmitted power and the circuit power. Moreover, in these papers the throughput is replaced by the achievable rate in the definition of the energy efficiency.

Essentially, each transmitter n is not only interested in maximizing its own performance in terms of achieved SINR γ_n , but also in saving as much battery energy as possible. This trade-off is well modeled by defining the EE of a given terminal n , as the ratio between the so-called efficiency function which measures the SINR-based performance of user n and the power consumed to attain this performance level [20]-[11]:

$$EE_n = \frac{f(\gamma_n)}{p_n + P_{c,n}}. \quad (1)$$

In (1), $P_{c,n}$ is the power that is required by the transmitter electronic circuitry to operate the device, and which is dissipated even during non-transmission periods. For further details on the circuitry power term, we refer the reader to [34] and references therein, where several power consumption models for wireless networks are developed. As for $f(\gamma)$, in principle it can be a generic increasing function of the n -th user's SINR, with $f(0) = 0$ and such that (1) tends to zero for growing p_n . Two widely used efficiency functions are:

- 1) $f(\gamma_n) = R(1 - e^{-\gamma_n})$, where R is the communication rate and $(1 - e^{-\gamma_n})$ is an approximation of the probability of correct symbol reception. A similar approximation was used in [13], [14]. Thus, f is the number of bits that are correctly demodulated at receiver n per unit of time.
- 2) $f(\gamma_n) = W \log(1 + \gamma_n)$, where W is the communication bandwidth. For strictly static channels f represents the n -th user's achievable rate. For quasi-static channels, the use of f for resource allocation purposes is still well-motivated in view of the assumption that the channel coefficients remain constant for longer than the resource allocation phase.

Variations of option 1) are also available in the literature in the form of $f(\gamma_n) = R(1 - e^{-\gamma_n})^M$ and $f(\gamma_n) = R(1 - e^{-\gamma_n/2})^M$, and in this case the function $f(\cdot)$ is an approximation of the probability of error-free reception of a data packet of M symbols. An EE that considers both the case of $M > 1$ and the circuit power $P_{c,n}$ was considered in [33] for a single-hop system. There, it was shown that an equilibrium for the power allocation algorithm exists, but the convergence could not be proved. The techniques developed in this paper could be used to extend the results of [33] to the relay-assisted scenario, as well. However, in the following we choose to focus on the equally well-motivated case of $M = 1$. Thus, for any M , the resulting EE (1) is a measure of the number of bits that are correctly decoded at the receiver, per unit of time and per Joule of energy drained from the battery of the transmitter. Moreover, all the efficiency functions that we consider result in an EE (1) which is measured in bits per Joule, thus representing a natural measure of the efficiency with which each Joule of energy drained from the battery is being used.

Two pertinent social welfare performance metrics are the average EE and the system global EE (GEE), respectively defined by [14]-[18] as:

$$EE = \frac{1}{N} \sum_{n=1}^N \frac{f(\gamma_n)}{P_{c,n} + p_n} \quad (2)$$

and

$$\text{GEE} = \frac{\sum_{n=1}^N f(\gamma_n)}{\sum_{n=1}^N P_{c,n} + p_n}. \quad (3)$$

Customarily, the GEE in (3) is used to describe the energy efficiency of the overall system while EE in (2) focuses on energy efficiency of individual users.

B. Problem Formulation

We consider N transceiver pairs sharing a time slotted frequency band divided into K sub-bands. This can be seen as the open sharing model of the cognitive radio [35]. We assume $K \geq N$. This assumption can always be fulfilled: if $N > K$ than $N - K$ artificial channels with rate zero could be added and make $N = K$ [31]. Let \mathbf{P} be a matrix of transmission powers where each channel is used by a single user and $P_{n,k}$ is defined as the minimal power needed by the n -th user to achieve a preassigned target rate R_n in the k -th channel. We assume all the users have continuous sensing over all channels [8], [32]. This is a reasonable assumption since the sensing power is only a small percentage of the total power in wireless networks [36]. We also assume that only one user can transmit on each channel in each time slot and consider alien interference from other networks as the additive noise. Each user experiences frequency selective channel caused by both channel statistics and out of cell interference (since out of cell interference affects different users in a different way).

We propose a fully distributed method to maximize the energy efficiency of the system using different utility functions as described in the following:

1) *Average EE under rate constraint:* Under the expected rate constraints, each entry $P_{n,k}$ of the matrix \mathbf{P} is chosen to be the solution to the following ergodic rate equation

$$R_n = \mathbb{E} \left(\log_2 \left(1 + \frac{|H_{n,k}|^2 P_{n,k}}{\sigma_n^2} \right) \right), \quad (4)$$

where $H_{n,k}$ is the channel coefficient for the n -th user at the k -th frequency and σ_n^2 is the noise variance of the n -th receiver. By using Jensen's inequality, we get

$$R_n \leq \log_2 \left(1 + \frac{\mathbb{E}(|H_{n,k}|^2) P_{n,k}}{\sigma_n^2} \right). \quad (5)$$

The solution to the above equation gives us to minimize the energy

$$P_{n,k} \geq \frac{(2^{R_n} - 1) \sigma_n^2}{\mathbb{E}(|H_{n,k}|^2)}. \quad (6)$$

Using equations (6) and (2), we define the utility matrix \mathbf{U}^{av} such that $U_{n,k}^{\text{av}}$ is chosen to be the optimal individual EE for the n -th user in the k -th channel under rate constraint R_n

$$U_{n,k}^{\text{av}} = \frac{R_n}{\frac{\phi \sigma_n^2}{|H_{n,k}|^2} + P_n}, \quad (7)$$

where $\phi = (2^{R_n} - 1)$ is a function that makes the power $P_{n,k}$ used on the k -th channel by the n -th user to satisfy a QoS requirement. We denote P_n by the minimal power of the n -th device for its operation including the receiver power consumption and energy consumed during idle times.

2) *Average EE under goodput constraint:* Another problem we solve is the energy efficient channel assignment under a goodput requirement. The achievable goodput is defined as the rate of the successfully transmitted symbols. In this case $P_{n,k}$ is chosen to fulfill a goodput requirement:

$$T_n = R(1 - \text{SER}) = R \left(1 - e^{-\frac{|H_{n,k}|^2 P_{n,k}}{\sigma_n^2}} \right), \quad (8)$$

for a fixed $R > 0$ and T_n . The solution for $P_{n,k}$ is given by

$$P_{n,k} = \frac{\log \left(1 - \frac{T_n}{R} \right) \sigma_n^2}{|H_{n,k}|^2}, \quad (9)$$

and

$$U_{n,k}^{\text{good}} = \frac{R_n}{\frac{\phi \sigma_n^2}{|H_{n,k}|^2} + P_n}, \quad (10)$$

where $\phi = \log \left(1 - \frac{T_n}{R} \right)$.

3) *Global EE under rate constraint:* The third problem is the energy efficiency maximization with respect to the GEE criterion. Assuming preassigned target rates for the users, the problem simplifies into a power minimization problem under rate constraints. We assume that the instantaneous transmission power is limited per user by P_{\max} . For simplicity, we formulate the utility of the GEE as a maximization problem. We define the utility matrix \mathbf{U}^{GEE} for the GEE criterion as

$$U_{n,k}^{\text{GEE}} = \begin{cases} P_{\max} - P_{n,k}, & P_{n,k} \leq P_{\max} \\ 0, & P_{n,k} > P_{\max}. \end{cases} \quad (11)$$

Combining the utility functions in (7), (10), and (11), the maximum energy efficiency problem can be formulated as an integer programming problem in a general form as

$$\begin{aligned} \max_{\boldsymbol{\eta}} \quad & \sum_{n=1}^N \sum_{k=1}^K U_{n,k} \eta_{n,k} \\ \text{s.t.} \quad & \sum_k \eta_{n,k} = 1, \quad \forall n = 1, 2, \dots, N \\ & \sum_n \eta_{n,k} = 1, \quad \forall k = 1, 2, \dots, K \\ & \eta_{n,k} \in \{0, 1\}, \quad \forall n, k \end{aligned} \quad (12)$$

The constraint matrix of the problem in (12) is totally unimodular. Thus, the solution to the relaxed problem where we replace the integer constraint by $0 \leq \eta_{n,k} \leq 1$ is also the solution to the original problem. The relaxed problem is a linear programming (LP) problem and can be solved efficiently in a centralized manner by LP solutions methods such as the Hungarian method [28]. Although the original problem in (12) is relaxed, we prove that after the relaxation we can still achieve asymptotically optimal results in much lower time complexity than solving the original problem.

In the next section, we describe a distributed auction algorithm for the channel assignment problem in (12) to achieve maximal energy efficiency of the system. Subsequently, we describe a fast matching algorithm to reduce the convergence time of the distributed auction algorithm.

III. DISTRIBUTED AUCTION ALGORITHM

We propose the use of a fully distributed channel assignment algorithm for maximal energy efficiency that does not require any explicit message passing or a shared memory between the users. The algorithm relies on the auction algorithm [31] and the distributed algorithm suggested in [30] for sum-rate maximization. The distributed protocol consists of a bidding stage and an assignment stage. The description of the algorithm is as follows. The utility matrix \mathbf{U} is an $N \times K$ matrix of energy efficiency indexes and \mathbf{C} is an $N \times K$ cost matrix. The cost of a channel $C_{n,k}$ is a unitless number that merely represents how much user n wants channel k in comparison to the other users. We define the profit of user n from channel k as the reward (i.e. energy efficiency index) minus the price of the channel $U_{n,k} - C_{n,k}$. In the initialization stage each user sets the cost for all of the channels to be 0; i.e., $C_{n,k} = 0, \forall n, k$, select $\epsilon > 0$ and sets his state to unassigned. \tilde{k}_n is defined as the most profitable channel of the n -th user:

$$\tilde{k}_n = \arg \max_k U_{n,k} - C_{n,k}. \quad (13)$$

The distributed protocol proceeds in iterations. In each iteration two stages are sequentially performed, a bidding stage where users raise the price on their most profitable channel and an assignment stage where channels are assigned to the users who proposed the highest prices. In the bidding stage, each unassigned user n find his most profitable channel \tilde{k}_n and the profits from that channel γ_n and his second most profitable channel ω_n

$$\begin{aligned} \tilde{k}_n &= \arg \max_k (U_{n,k} - C_{n,k}) \\ \gamma_n &= U_{n,\tilde{k}_n} - C_{n,\tilde{k}_n} \\ \omega_n &= \max_{k \neq \tilde{k}_n} (U_{n,k} - C_{n,k}). \end{aligned} \quad (14)$$

Each unassigned user raises the price on his most profitable channel by

$$C_{n,\tilde{k}_n} = C_{n,\tilde{k}_n} + \gamma_n - \omega_n + \epsilon, \quad (15)$$

where ϵ is a predetermined positive constant that can be seen as the price of participating in the auction. After the unassigned users update their prices, all the users bid on their most profitable channels. If user n gets assigned to channel k he continues to bid on that channel without raising his bid. If a user n is unassigned he bids on \tilde{k}_n with the new bid C_{n,\tilde{k}_n} . In the assignment stage each channel is assigned to the highest bidding user. A channel without bids stays unassigned and users who were not assigned to channels become unassigned. The bidding and assignment stages proceed in iterations until all the users are assigned to channels. Once all of the users are assigned to channels, no one raises his bid and as a result the assignment becomes static. When all the users are assigned we say that the algorithm has *converged*. The distributed auction algorithm appears in Table I. It was proven in [30] that the distributed auction algorithm for the sum-rate problem converges in finite time to a solution within $N\epsilon$ from the optimal solution.

The distributed auction algorithm can be implemented using an opportunistic CSMA protocol without the use of explicit

TABLE I
DISTRIBUTED AUCTION ALGORITHM

Select $\epsilon > 0$, set all the users as unassigned and set $\mathbf{C}(n, k) = 0, \forall n, k$
Repeat
1. Each unassigned user n calculates his own maximum profit: $\gamma_n = \max_k (U_{n,k} - C_{n,k})$
2. Each unassigned user n calculates his second maximum profit: $\tilde{k}_n = \arg \max_k (U_{n,k} - C_{n,k})$ $\omega_n = \max_{k \neq \tilde{k}_n} (U_{n,k} - C_{n,k})$
3. Each unassigned user n updates the price of his best channel \tilde{k}_n to be $C_{n,\tilde{k}_n} = C_{n,\tilde{k}_n} + \gamma_n - \omega_n + \epsilon$
4. All the users bid. The unassigned users bid on their new best channel with the updated bid. The assigned users bid on the last channel they bid on and with the same price.
5. Assign channel to the highest bidder (channels with no bids stay unassigned)
Until all users are assigned

message passing among users. However, only coordination requirement between users lies with an auctioneer to decide which user made the highest bid. The opportunistic CSMA can be used as an auctioneer. We can define the reward that each user n gets from channel k to be the energy efficiency of that channel $U_{n,k}$. Using the opportunistic CSMA scheme, each user n tries to access his best profit channel as defined in (13) with a backoff time of $\tau_n = f(\tilde{k}_n)$ where $f(x)$ is a positive monotonically decreasing function. The price $C_{n,k}$ is determined and updated if necessary as described in Table I. The prices and their corresponding waiting times must converge in a finite number of iterations as in the distributed auction algorithm.

It was shown in [30] that the number of iterations needed until the convergence of the distributed auction algorithm is bounded by $\mathcal{O}(N^3)$. In the next sections, we suggest a relaxation to the maximal energy efficiency problem. We show that the suggested relaxation is asymptotically optimal and can be solved with $\mathcal{O}(N \log(N))$ expected number of iterations.

IV. FAST MATCHING ALGORITHM

In (12), we have formulated the maximum energy efficiency channel assignment problem for arbitrary values. In the previous section, we showed that this problem can be optimally solved in a fully distributed manner using the distributed auction algorithm. However, the expected convergence time for the near-optimal distributed auction algorithm might be too high for practical use. To speed up the convergence time, we develop a novel scheme by considering a relaxation to the channel assignment problem where each channel can either be "good" or "bad". Essentially, channel k is a good channel (corresponding to the channel gain) for the n -th user with respect to a properly chosen threshold. We represent a good channel by 1 and a bad channel by 0. Since every channel can be either good or bad, the utility matrix of the relaxed problem becomes $\{0, 1\}^{N \times K}$, which can be represented using bipartite graphs. Thus, the channel assignment problem for energy efficient transmissions is reduced finding perfect matchings on bipartite graphs.

TABLE II
ALGORITHM FOR MAXIMAL CARDINALITY MATCHING

- 1) Initialize $h_v = 0, \forall v \in V, U_{\text{free}} = \{1, 2, \dots, N\}$ and set $M = \emptyset$
- 2) While $|M| < N$ do
 - a) Choose $u \in U_{\text{free}}$
 - b) $j = \arg \min_{v \in n_u} h_v$
 - c) $M = M \cup (u, j)$
 - d) $u_{\text{old}} = \{u \in U : (u, j) \in M\}$
 - e) $M = M \setminus (u_{\text{old}}, j)$
 - f) $U_{\text{free}} = U_{\text{free}} \cup u_{\text{old}} \setminus u$
 - g) $h_j = h_j + 1$
- 3) Return

A. Maximum Cardinality Matching on Bipartite Graphs

To formulate the maximum energy efficiency assignment problem as a matching problem on bipartite graph and for theoretical analysis, we need various definitions and relevant results on bipartite graph. These have been presented in the Appendix A. Using these definitions, we define the maximum cardinality matching problem as follows: Let $G = (U, V, E)$ be bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . Find a matching M such that $|M|$ is maximal. Here, U, V in the bipartite graph represent the users and the channels, respectively, and the edges represent the energy efficiency of each user in each channel.

The maximum cardinality matching (MCM) problem can also be formulated as the max-energy efficiency problem (12) where the reward matrix is a binary matrix with 0, 1 values. We now present an algorithm that finds a maximum cardinality matching on bipartite graphs which can be implemented in a fully distributed manner. This iterative algorithm assigns an unassigned user to a channel according to the following scheme: Each channel $k \in K$ is assigned a value h_k that represents how many times the channel was reassigned to different users. Let $\mathbf{h}^{(i)} = [h_1, h_2, \dots, h_K]$ be the vector of the values of the channels on the i -th iteration. At the beginning of the algorithm all the values of the channels are initialized to 0; i.e.,

$$h_k^{(0)} = 0, \quad \forall k = 1, 2, \dots, K.$$

Let U_{free} be the set of all free users. In each iteration, an unassigned user $u \in U_{\text{free}}$ is chosen and assigned to the channel with a minimal value he can access and raises its value by 1. The MCM algorithm is summarized in Table II.

B. Expected Number of Iterations of Fast Matching Algorithm

In this section, we analyze the expected number of iterations until the algorithm converges for random bipartite graphs. $G = (U, V, E)$ is called a *bipartite random graph* if G is a bipartite graph and the edges in E are independently chosen with probability p ; i.e.,

$$\Pr((u, v) \in E) = p, \quad \forall u \in U, \quad \forall v \in V. \quad (16)$$

Denote the set of all random bipartite graphs with vertex sets $|U| = |V| = N$ and probability p for an edge by $B(N, p)$. The following known result on perfect matching in random bipartite graphs was proven by Erdős and Rényi in [37] and Motwani in [38]:

Theorem (Erdős and Rényi [37]). *Let $p = \frac{(1+\epsilon)\log(N)}{N}$ and $G \in B(N, p)$ then*

$$\lim_{N \rightarrow \infty} \Pr(G \text{ contains a perfect matching}) - e^{-2N^{-\epsilon}} = 0. \quad (17)$$

Theorem (Motwani [38]). *Let $G \in B(N, p)$ where $p \geq \frac{(1+\epsilon)\log(N)}{N}$ then for every $\gamma > 0$ there exists N_γ such that for every $N \geq N_\gamma$*

$$\Pr(G \in B(N, p)) \geq 1 - N^{-\gamma}. \quad (18)$$

The next theorem proven in [39] shows that for random bipartite graphs with $p \geq \frac{(1+\epsilon)\log(N)}{N}$ the number of iterations until the convergence of the algorithm is less than $\frac{cN \log(N)}{\log(Np)}$ with high probability, where $c > 0$ is a constant.

Theorem (Naparstek and Leshem [39]). *Let $G = (U, V, E)$ be a random bipartite graph with $|U| = |V| = N$ and $p \geq \frac{(1+\epsilon)\log(N)}{N}$. Let T be the number of iterations until the algorithm converges then:*

$$\lim_{N \rightarrow \infty} \Pr\left(T \leq \frac{cN \log(N)}{\log(Np)}\right) = 1. \quad (19)$$

Above theorem ensures that the fast matching finds a perfect matching with a probability that approaches 1 in $\mathcal{O}(N \log(N))$ iterations.

In what follows, we analyze the fast matching algorithm in wireless channels and show that the proposed scheme can be implemented in practical systems to find asymptotically optimal solution to the energy efficiency maximization problem with a small number of iterations.

V. PERFORMANCE OF FAST MATCHING ALGORITHM

In this section, we analyze the expected number of iterations required by the fast matching algorithm in a Rayleigh fading channel, and show that the proposed algorithm is asymptotically optimal for the maximally energy-efficient channel assignment problem.

As described in the previous section, the main idea of the fast matching algorithm is to transform \mathbf{U} into a binary 0, 1 matrix $\tilde{\mathbf{U}}$, and then apply the matching algorithm on $\tilde{\mathbf{U}}$. The transformation from \mathbf{U} to $\tilde{\mathbf{U}}$ is done by applying a judiciously chosen threshold $a_n^{\text{thresh}} \geq 0$ for each row:

$$\tilde{U}_{n,k} = \begin{cases} 1, & U_{n,k} \geq a_n^{\text{thresh}} \\ 0, & U_{n,k} < a_n^{\text{thresh}}. \end{cases} \quad (20)$$

To ensure asymptotically optimal solutions to the max-energy efficiency problem, a_n^{thresh} must satisfy the following requirements:

- 1) Only the best channels (corresponding to the channel gain) of each user should be above a_n^{thresh} .
- 2) $\tilde{\mathbf{U}}$ should contain a perfect matching with high probability.

The first condition ensures that the solution to $\tilde{\mathbf{U}}$ can provide a good solution to the max-sum problem. The second condition ensures that with high probability all of the users will get assigned by the algorithm. We define a parameter m such that $m \log(N)$ best channels of each user are above the threshold.

Assume that each row n of \mathbf{U} consists of i.i.d random variables and assume that the cumulative distribution function (CDF) of each entry of $U_{n,k}$ is given by F_n . A proper choice of a_n^{thresh} would be

$$a_n^{\text{thresh}} = F_n^{-1} \left(1 - \frac{m \log(N)}{N} \right), \quad (21)$$

where $m > 2$ [39]. The choice of the threshold satisfies the first condition since only $m \log(N)$ best channels of each user are above the threshold. Theorem by Erdős and Rényi in [37] ensures that with high probability \mathbf{U} contains a perfect matching if $a_n^{\text{thresh}} \geq 0$ for all $n = 1, 2, \dots, N$. Hence, the proposed algorithm can converge faster with high probability when target rate of each user be chosen independently such that with high probability $a_n^{\text{thresh}} \geq 0$ for all $n = 1, 2, \dots, N$.

A. Target Rates for Rayleigh Fading Channels

We model the channels of each user as Rayleigh fading channels; i.e., the channel attenuation $|H_{n,k}|^2$ is an exponential random variable given by

$$|H_{n,k}|^2 = G_n \cdot F_n \cdot \frac{1}{r_n^\alpha}, \quad (22)$$

where G_n is a global normalizing factor, F_n is an exponentially distributed gain (due to the Rayleigh fading channel with a multi-path effect), r_n is the distance between the n -th transmitter from its receiver and α is the path loss exponent. Hence, the PDF of $|H_{n,k}|^2$ is:

$$f_{|H_{n,k}|^2}(x) = \lambda_n e^{-\lambda_n x}, \quad (23)$$

where

$$\lambda_n = \frac{r_n^\alpha}{G_n} = \frac{1}{\mathbb{E}(|H_{n,k}|^2)}. \quad (24)$$

The instantaneous rate of user n in channel k is given by [40]:

$$R_{n,k} = \log_2 \left(1 + \frac{|H_{n,k}|^2 P_{n,k}}{\sigma_n^2} \right). \quad (25)$$

We now present sufficient conditions on R_n such that the fast matching algorithm converges within $\mathcal{O}(N \log(N))$ expected number of iterations.

Theorem 1. *The fast matching algorithm converges within $\mathcal{O}(N \log(N))$ expected number of iterations in a Rayleigh fading channel with SNR $\bar{\gamma} = \frac{P_{\max} \mathbb{E}(|H_{n,k}|^2)}{\sigma_n^2}$ if the following requirement is satisfied for all n :*

$$R_n \leq \log_2 \left(1 + \bar{\gamma} \log \left(\frac{N}{m \log(N)} \right) \right) \quad (26)$$

for the rate requirement, and

$$\frac{T_n}{R} \geq 1 - \left(1 + \frac{N}{m \log(N)} \right)^{\bar{\gamma}} \quad (27)$$

for the goodput requirement.

Proof: The proof is presented in Appendix B. ■

Theorem 1 provides conditions on the target rate of each user under which the proposed algorithm has a faster convergence rate. Examining (26) reveals that a target rate equal to the capacity of the channel satisfy the rate requirement for

TABLE III
FAST MATCHING ALGORITHM USING OPPORTUNISTIC CARRIER SENSING
FOR THE n -TH USER

```

Initialize  $h_k = 0, \forall k \in K$ , set assigned=false
set  $b_n$  to be the indices of  $m \log(N)$  best
channels of the  $n$ -th user, set  $N_{\text{iter}} = 0$ 
Repeat
1. If assigned=false then
  1.1  $N_{\text{iter}} = N_{\text{iter}} + 1$ 
  1.2 Find the channel with minimal value  $\hat{j} = \arg \min_{i \in b_n} h_i$ 
  1.3 Choose random backoff time  $\tau_n$ 
  1.4 If a busy tone was transmitted on channel  $i$  before  $\tau_n$  then
    1.4.1 No transmission attempt by the  $n$ -th user in the current
    time slot
    1.4.2  $h_i = h_i + 1$ 
  1.5 Else
    1.5.1 Transmit a busy tone on the  $\hat{j}$ -th channel.
    1.5.2  $h_{\hat{j}} = h_{\hat{j}} + 1$ 
    1.5.3 Set assigned=true
  1.6 End if
2. Else
  2.1 If a busy tone was transmitted on channel  $i$  before  $\tau^{\max}$  then
    set  $h_i = h_i + 1$ 
  2.2  $N_{\text{iter}} = N_{\text{iter}} + 1$ 
  2.3 If  $i = \hat{j}$ 
    2.3.1 No transmission attempt by the  $n$ -th user in the current
    time slot
    2.3.2 Set assigned=false
  2.4 Else transmit a busy tone on the  $j$ -th channel.
3. End If
Until all users are assigned or  $N_{\text{iter}} = (N - 1)^2$ 
If  $N_{\text{iter}} = (N - 1)^2$ 
  run the distributed auction algorithm from [30].
End If

```

faster convergence. Note that users do not require external knowledge or message passing among users since the conditions can be verified by each user independently. Hence each user can choose a target rate and verify the convergence conditions without relying on other users.

B. Asymptotic Optimality

We now show that the fast channel assignment is asymptotically optimal for Rayleigh fading with properly chosen target rates. For the asymptotic analysis, we use some known results from order statistics [41], and presents in Appendix C.

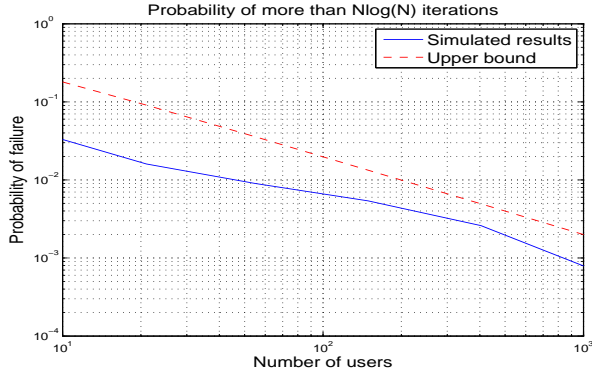
Theorem 2. *Let $A_{\text{OPT}}^{\text{GEE}}$ be the optimal solution to the max-energy efficiency problem for the global energy efficiency defined in (3) and let $A_{\text{FMA}}^{\text{GEE}}$ be the solution obtained by the fast matching algorithm. If the rates satisfy (20) and a perfect matching exists, then for Rayleigh fading channels:*

$$\lim_{N \rightarrow \infty} \frac{\mathbb{E}\{A_{\text{FMA}}^{\text{GEE}}\}}{\mathbb{E}\{A_{\text{OPT}}^{\text{GEE}}\}} = 1 \quad (28)$$

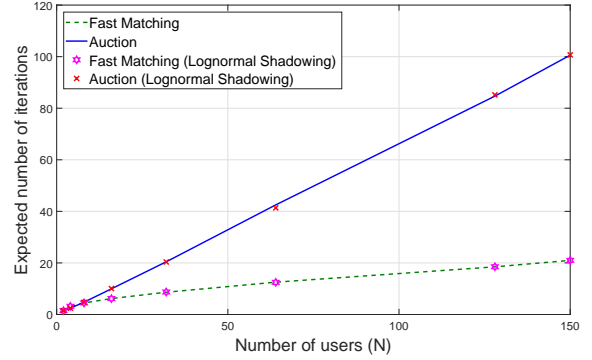
Proof: The proof is presented in Appendix D. ■

Theorem 3. *Let $A_{\text{OPT}}^{\text{EE}}$ be the optimal solution to the max-energy efficiency problem for the individual energy efficiency defined in (2) and let $A_{\text{FMA}}^{\text{EE}}$ be the solution obtained by the fast matching algorithm. If the rates satisfy (20) and a perfect matching exists, then for Rayleigh fading channels:*

$$\lim_{N \rightarrow \infty} \frac{\mathbb{E}\{A_{\text{FMA}}^{\text{EE}}\}}{\mathbb{E}\{A_{\text{OPT}}^{\text{EE}}\}} = 1 \quad (29)$$



(a) Probability that the algorithm terminates after more than $N \log(N)$ iterations..



(b) Expected number of iterations.

Fig. 1. Expected number of iterations required by algorithms for convergence.

Proof: The proof is presented in Appendix E. ■

It is noted that the asymptotic results of Theorem 2 and Theorem 3 have exactly the same meaning as any other asymptotic analysis. That is, the performance of the fast matching algorithm approaches the optimal solutions for large enough users and resources. Also note that in most types of asymptotic analysis, the important question: "How close is the asymptotic performance to the optimal solution?" is primarily dealt with through simulations, as discussed in the next section.

C. Distributed Implementation of Fast Matching Algorithm

We can implement the fast matching algorithm without the use of explicit message passing using opportunistic CSMA. Opportunistic CSMA [42] is a distributed transmission protocol suggested for wireless sensor networks. Opportunistic CSMA is composed of carrier sensing and a waiting strategy. Since continuous sensing of all channels by all users is assumed, each user in the network calculates a fitness measure ψ_n and maps it into a waiting time τ_n based on a predetermined common decreasing function $f(\psi_n)$. Here, each user waits until the waiting time ends and if no one transmitted on its most wanted channel then it is allowed to transmit. Hence, the user with the highest ψ_n transmits in the channel. This can be seen as a distributed winner determination algorithm where the winner gets the channel. Note that since instantaneous sensing is assumed it implies that there are no collisions. This is because the probability of two users having the same random listening time is zero.

The fully distributed fast matching algorithm for maximal energy efficiency using prioritized CSMA proceeds as follows: At the beginning of each time slot each unassigned user finds the channel with the lowest value among the good channels (defined in) i.e. one of the best $m \log(N)$ channels. Next, each unassigned user waits a random amount of time and if no one transmitted on any of the channels before the waiting time then that user transmits a busy signal on the channel with lowest value and raises the value of the channels by 1. Each

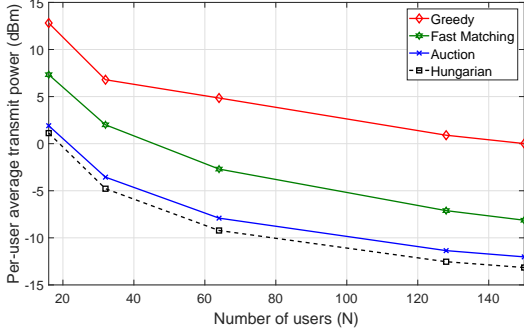
assigned user waits for the maximum time allowed τ^{\max} . if no user transmits on the assigned channel until τ^{\max} , then the assigned user transmits on the channel. When a user senses that a busy signal was transmitted on a channel before τ^{\max} , that user raises the value of that channel by 1. The schematic description of algorithm is depicted in Table III.

VI. NUMERICAL ANALYSIS

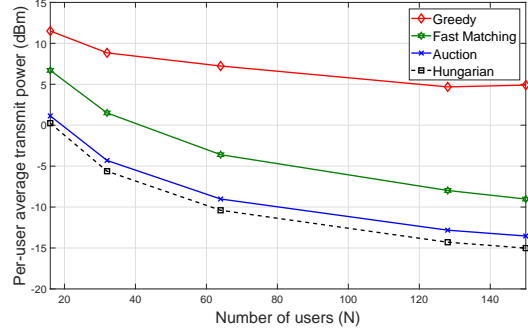
In this section, we demonstrate the performance of the fast matching algorithm using computer simulations. We compared the proposed algorithm with the greedy, distributed auction, and centralized Hungarian methods under multi-path fading and longterm shadowing effect.

We considered a network of N users distributed uniformly in a radius of 50 m to 500 m with a BS in the center. The carrier frequency was 2 GHz with a per-user transmission bandwidth of 200 KHz. The channel was divided into N sub-channels. The channel of each user pair was generated by the extended pedestrian A model (EPA) of the LTE standard with 9 random taps. The path loss exponent was taken as $\alpha = 3$. The users were assumed to be moving at a speed of 3 km/h. We also considered channel between users to the BS to be log-normal distributed with a spreading factor of 4 dB. We assumed maximum spectral efficiency of 8 bits/sec/Hz for each user. We compared the performance of the fast matching algorithm with other algorithms by computing the average power and global energy efficiency of the network. We fix the parameter $m = 2.5$ to get $m \log(N)$ channels above the threshold with $m > 2$. The target rates were chosen using (45) and the threshold was computed using $\alpha_n^{\text{thresh}} = F^{-1}(1 - \frac{m \log(N)}{N})$. The simulations were averaged over 5000 iterations.

We investigated the expected number of iterations achieved by the fast matching algorithm compared to the distributed auction algorithm. First, we verify that the number of iterations required by the fast matching algorithm exceeds $cN \log(N)$ with a probability of less than $\frac{1}{N}$. In Fig. 1a, we plotted the empirical probability that the fast algorithm exceeds $N \log(N)$ iterations against the theoretical bound of $\frac{1}{N}$ for $N = 10 \dots 10^3$.



(a) EPA channel model without log-normal shadowing.



(b) EPA channel model with log-normal shadowing.

Fig. 2. Average per-user transmit power requirement to achieve a spectral efficiency of 8 bits/sec/Hz.

The figure shows that the simulations support theoretical result on the expected number of iterations for convergence. Next, we compared the expected number of iterations of the fast matching algorithm with the distributed auction algorithm until convergence. The comparison is shown in Fig. 1b. As predicted, the expected number of iterations needed by the fast auction algorithm is smaller than the expected number of iterations by the distributed auction algorithm.

In Fig. 2, we demonstrate the average transmit power requirement to achieve a desired target rate (i.e. spectral efficiency taken 8 bits/sec/Hz) by various algorithms under two channel scenarios. It can be seen that the fast matching algorithms performs better than the greedy method. However, the proposed algorithm requires higher transmit power than the auction method but requires significantly less number of iterations in convergence, as shown in Fig. 1b. As expected, the centralized scheme using Hungarian requires the minimum average transmit power. Moreover, the optimal distributed scheme, the auction method, performs very close to the Hungarian method.

We also compared the network energy efficiency of various algorithms in Fig. 3. Although the distributed auction algorithm achieves the maximum energy efficiency (as depicted in Fig. 3a), the proposed fast matching algorithm provide greater gain per iteration (as depicted in Fig. 3b) due to less number of iterations required for convergence.

Finally, we demonstrate the asymptotic optimality of the proposed algorithm by simulating over large network in Fig. 4. The figure shows that the fast matching algorithm approaches the optimal auction algorithm as network size increases as proved through analysis in Theorem 2. However, it requires much larger system sizes to achieve the optimal solution since the increase in the reward becomes slower as number of users increases (i.e., when $N > 200$). It can also be seen that the greedy algorithm is not asymptotic optimal and performs poorly compared with the fast matching algorithm.

VII. CONCLUSION

We presented a fully distributed protocol for resource allocation to optimize the energy efficiency of a wireless network.

We converted the channel assignment problem into finding perfect matchings on bipartite graph which was shown converge within $\mathcal{O}(N \log(N))$ expected number of iterations with high probability. The algorithm was based on a version of the auction algorithm which solves a matching problem. We also showed that under mild assumptions on the fading distribution, the fast matching algorithm produces asymptotically optimal solutions to the energy efficiency problem. The proposed algorithm was shown to perform better than the greedy method and achieves near-optimal performance with lesser number of iterations than the auction algorithm. The fast matching algorithm can be implemented using carrier sensing protocol to compute the channel assignments.

ACKNOWLEDGMENT

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APPENDIX A: BIPARTITE GRAPH

Definition 1. Let $G = (V, E)$ be a graph with a vertex set V and an edge set E . The neighbor set of vertex $v \in V$ is given by

$$n_v = \{u \in V : (u, v) \in E\}. \quad (30)$$

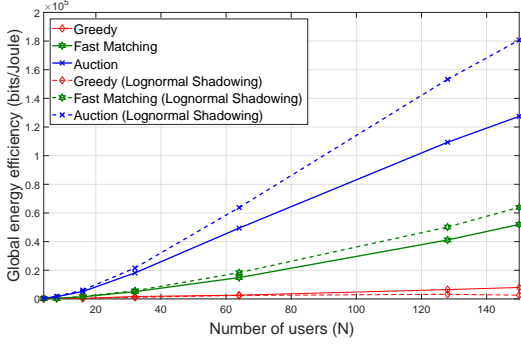
Definition 2. Let $G = (\hat{V}, E)$ be a graph with a vertex set \hat{V} and an edge set E . If \hat{V} can be divided into two subsets U, V such that

$$\begin{aligned} n_u \cap U &= \emptyset, \quad \forall u \in U \\ n_v \cap V &= \emptyset, \quad \forall v \in V. \end{aligned}$$

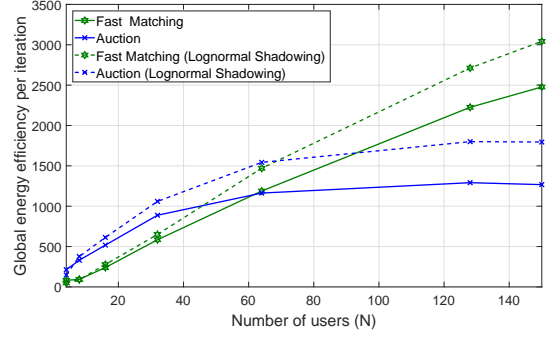
we say that G is a bipartite graph and we denote it by $G(U, V, E)$.

Definition 3. Let $G = (U, V, E)$ be a bipartite graph with a vertex sets $|U| = |V| = N$ and an edge set E . Let $M \subseteq E$ and let $\tilde{G} = (\tilde{U}, \tilde{V}, M)$ be a bipartite subgraph of G with vertex sets $|\tilde{U}|, |\tilde{V}| = N$ and an edge set M . M is a matching on G if

$$\max_{v \in U \cup V} |n_v| = 1. \quad (31)$$



(a) Overall GEE.



(b) GEE per iteration.

Fig. 3. Global energy efficiency of network for the EPA channel model with and without log-normal shadowing.

Definition 4. Let $G = (U, V, E)$ be a bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . M is a perfect matching if M is a matching and $|M| = N$.

Definition 5. Let $G = (U, V, E)$ be a bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . Let $M \subseteq E$ and let $\tilde{G} = (U, V, M)$ be a bipartite subgraph of G with vertex sets $|U|, |V| = N$ and an edge set M . A vertex $v \in U \cup V$ is free if $|n_v| = 0$ otherwise we say it is not free.

Definition 6. Let $G = (U, V, E)$ be bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . If G is a random graph where each edge occurs with probability p we say that $G \in B(N, p)$.

Definition 7. if $G \in B(N, p)$ and for any non-maximal matching M there exists an augmenting path for M of length at most $2L+1$ where $L = \frac{c \log(N)}{\log(Np)}$ and $c > 0$ is some constant.

Lemma 1. Let T be the number of iterations until the algorithm terminates and let h_v be the value of vertex v at termination, then

$$T = \sum_{v=1}^N h_v \quad (32)$$

Lemma 2. Let $G = (U, V, E)$ be a bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . Let $M(i) \subseteq E$ be a non maximal matching obtained by the algorithm in the i -th iteration. Let $h_v(i)$ be the value of vertex v in the i -th iteration of the algorithm. Let $D_l(i)$ be a subset of V defined by:

$$D_l(i) = \{v \in V : h_v(i) \geq l\}. \quad (33)$$

If $v_0 \in D_l(i)$ and $(u, v_0) \in M(i)$ then

$$n_u \subseteq D_{l-1}(i) \quad (34)$$

Lemma 3. Let $G = (U, V, E)$ be bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . Let $M(i)$ be a non maximal matching obtained by the algorithm in the i -th iteration. Let $u_0 \in U$ be a free vertex such that in the i -th iteration of the algorithm $n_{u_0} \subseteq D_l(i)$. Let $u_1 \in U$ be the

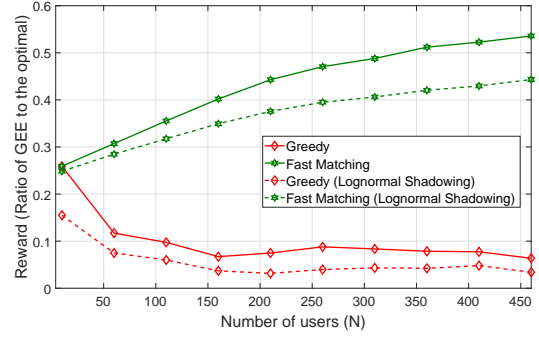


Fig. 4. Asymptotic convergence of the fast matching algorithm to the optimal solution.

end point of an alternating path P with $|P| = 2$ starting from u_0 then

$$v \notin D_{l-2}(i) \setminus D_{l-1}(i) \quad \forall v \in n_{u_1} \quad (35)$$

Lemma 4. Let $G = (U, V, E)$ be bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . Let $M(i)$ be a non maximal matching obtained by the algorithm in the i -th iteration and let $u_0 \in U$ be a free vertex such that in the i -th iteration of the algorithm $n_{u_0} \subseteq D_l(i)$; then every augmenting path of G on $M(i)$ starting from u_0 is at least of length $2l+1$.

Theorem (Berge [43]). Let $G = (U, V, E)$ be a bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . If G contains a perfect matching there exists an augmenting path in G for any partial matching M .

Using the above theorem and Lemma 4, we have the following results:

Lemma 5. Let $G = (U, V, E)$ be a bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . If G contains a perfect matching and for any non-perfect matching $M \subseteq E$ there exists an augmenting path of length at most $2l+1$, then for every $v \in V$ at each iteration of the algorithm until the algorithm terminates

$$h_v(i) \leq l+1, \forall i, v \in V \quad (36)$$

Lemma 6. Let $G = (U, V, E)$ be a bipartite graph with vertex sets $|U| = |V| = N$ and an edge set E . If G contains a perfect matching and T be the number of iterations until the algorithm terminates then

$$T \leq N(N-1) \quad (37)$$

Lemma 7. Let $G \in B(N, p)$ and let and let T be the number of iterations until the algorithm terminates then

$$T \leq N(L+1). \quad (38)$$

where $L = \frac{c \log(N)}{\log(Np)}$.

APPENDIX B: PROOF OF THEOREM 1

Using (6), minimal power needed to achieve rate R_n is given by:

$$P_{n,k} = \frac{(2^{R_n} - 1) \sigma_n^2}{|H_{n,k}|^2}.$$

For Rayleigh fading channels, $|H_{n,k}|^2$ is exponentially distributed with the CDF:

$$F_{|H_{n,k}|^2}(x) = 1 - e^{-\lambda_n x}. \quad (39)$$

Hence, the CDF of $P_{n,k}$ is given by

$$F_{P_{n,k}}(x) = e^{-\frac{\lambda_n \sigma_n^2 (2^{R_n} - 1)}{x}}. \quad (40)$$

The expected number of iterations will be $\mathcal{O}(N \log(N))$ only if there exists a perfect matching in the graph with a probability of at least $1 - \frac{2}{N^{\alpha-1}}$. From Theorem of Erdős and Rényi in [37], there exists a perfect matching with a probability of at least $1 - \frac{2}{N^{m-1}}$ only if the expected number of edges connected to each vertex is at least $m \log(N)$ for $m > 2$. Hence, to fulfill this requirement, the expected number of channels in which each user is able to transmit without violating his power constraint is at least $m \log(N)$. A user can transmit on a channel only if the power needed on the channel to achieve the target rate is less than P_{\max} . Thus,

$$F_{P_{n,k}}(P_{\max}) \geq \frac{m \log(N)}{N} \quad (41)$$

Hence, the target rates for each user must satisfy:

$$F_{P_{n,k}}^{-1} \left(\frac{m \log(N)}{N} \right) \leq P_{\max}. \quad (42)$$

The inverse CDF of $F_{P(n,k)}(x)$ is given by

$$F_{P(n,k)}^{-1}(x) = \frac{\lambda_n \sigma_n^2 (2^{R_n} - 1)}{\log(\frac{1}{x})}. \quad (43)$$

Using (42) and (43), the target rates must satisfy:

$$F_{P_{n,k}}^{-1} \left(\frac{m \log(N)}{N} \right) = \frac{\lambda_n \sigma_n^2 (2^{R_n} - 1)}{\log \left(\frac{N}{m \log(N)} \right)} \leq P_{\max} \quad (44)$$

Simplifying (44), we get:

$$R_n \leq \log_2 \left(1 + \frac{P_{\max} \log \left(\frac{N}{m \log(N)} \right)}{\lambda_n \sigma_n^2} \right). \quad (45)$$

Thus, the fast matching algorithm converges with an expected $\mathcal{O}(N \log(N))$ number of iterations.

Following the same steps for the goodput requirement, we get

$$\frac{T_n}{R} \geq 1 - \left(1 + \frac{N}{m \log(N)} \right)^{\frac{P_{\max}}{\lambda_n \sigma_n^2}}. \quad (46)$$

Using $\lambda_n = \frac{1}{\mathbb{E}(|H_{n,k}|^2)}$, $\bar{\gamma} = \frac{P_{\max} \mathbb{E}(|H_{n,k}|^2)}{\sigma_n^2}$ in (45) and (46), we prove the Theorem 1.

APPENDIX C: ORDER STATISTICS

Definition 8. Let A be a random variable with CDF $F_A(r)$ and let $A_{1:N} < A_{2:N} < \dots < A_{N:N}$ be random variables obtained by taking N samples from A and ordering the samples in an increasing order. $A_{k:N}$ is called the k -th order statistics of A with N samples.

Definition 9. Let k_N be a function of N such that $k_N \rightarrow \infty$ as $N \rightarrow \infty$ and $\lim_{N \rightarrow \infty} \frac{k_N}{N} = 0$ then $A_{N-k_N+1:N}$ and $A_{k_N:N}$ are called intermediate order statistics.

Definition 10. Let $F(x)$ be a differentiable, absolutely continuous distribution function. If

$$\lim_{x \rightarrow F^{-1}(1)} \frac{d}{dx} \left(\frac{1 - F(x)}{f(x)} \right) = 0 \quad (47)$$

then the third Von Mises condition is satisfied.

Theorem (Falk [44]). Let F be an absolutely continuous CDF satisfying one of the Von Mises conditions. Suppose $k_N \rightarrow \infty$ as $N \rightarrow \infty$ and $\lim_{N \rightarrow \infty} \frac{k_N}{N} = 0$. Then there exist norming constants α_N and $\beta_N > 0$ such that

$$\frac{A_{N-k_N+1} - \alpha_N}{\beta_N} \xrightarrow{d} N(0, 1). \quad (48)$$

where $\alpha_N = F^{-1} \left(1 - \frac{k_N}{N} \right)$ and $\beta_N = \frac{\sqrt{k_N}}{N f(\alpha_N)}$.

APPENDIX D: PROOF OF THEOREM 2 (GEE ASYMPTOTIC OPTIMALITY)

We first derive probability distribution function and quantile function of $\mathbf{U}(n, k)$ given that the power is lower than P_{\max} :

$$F_{U_{n,k}^{\text{GEE}}}(x) = 1 - e^{-\frac{a_n}{P_{\max}}} e^{-\frac{a_n}{P_{\max}-x}}, \quad (49)$$

where $a_n = \lambda_n \sigma_n^2 (2^{R_n} - 1)$ and

$$F_{U_{n,k}^{\text{GEE}}}^{-1}(\rho) = P_{\max} + \frac{a_n}{\log(1-\rho) - \frac{a_n}{P_{\max}}}. \quad (50)$$

We now observe that the probability distribution in (50) satisfies the third Von Mises condition resulting

$$\begin{aligned} \lim_{N \rightarrow \infty} \mathbb{E} \left(U_{N-m \log(N)+1:N}^{\text{GEE}} \right) &= F^{-1} \left(1 - \frac{m \log(N)}{N} \right) \\ &= P_{\max} + \frac{a_n}{\log(\log(N)) - \log(N) + \frac{a_n}{P_{\max}}} \end{aligned} \quad (51)$$

We now obtain simple bounds on $\mathbb{E}(A_{\text{OPT}}^{\text{GEE}})$ and $\mathbb{E}(A_{\text{FMA}}^{\text{GEE}})$:

$$\begin{aligned} \mathbb{E}(A_{\text{OPT}}^{\text{GEE}}) &\leq \sum_{n=1}^N \mathbb{E}(A_{N:N}) \\ &= N P_{\max} - \sum_{n=1}^N \frac{a_n}{\log(N) - \frac{a_n}{P_{\max}}} \end{aligned} \quad (52)$$

and

$$\begin{aligned} \mathbb{E}(A_{\text{FMA}}^{\text{GEE}}) &\geq \sum_{n=1}^N \mathbb{E}\left(A_{N(1-\frac{m \log(N)}{N}):N}\right) \\ &= NP_{\max} - \sum_{n=1}^N \frac{a_n}{\log(N) - \log(m \log(N)) - \frac{a_n}{P_{\max}}} \end{aligned} \quad (53)$$

It is now easy to see that

$$\frac{NP_{\max} - \sum_{n=1}^N \frac{a_n}{\log(N) - \log(m \log(N)) - \frac{a_n}{P_{\max}}}}{NP_{\max} - \sum_{n=1}^N \frac{a_n}{\log(N) - \frac{a_n}{P_{\max}}}} \leq \frac{\mathbb{E}(A_{\text{FMA}}^{\text{GEE}})}{\mathbb{E}(A_{\text{OPT}})} \leq 1. \quad (54)$$

It can be seen that

$$\lim_{N \rightarrow \infty} \frac{NP_{\max} - \sum_{n=1}^N \frac{a_n}{\log(N) - \log(m \log(N)) - \frac{a_n}{P_{\max}}}}{NP_{\max} - \sum_{n=1}^N \frac{a_n}{\log(N) - \frac{a_n}{P_{\max}}}} = 1. \quad (55)$$

Finally, we use (54) and (55) to get (28) of Theorem 2.

APPENDIX E: PROOF OF THEOREM 3 (EE ASYMPTOTIC OPTIMALITY)

The proof is identical to the proof of Theorem 2. The only difference is that the CDF $F_{U_{n,k}^{\text{av}}}(x)$ is given by

$$F_{U_{n,k}^{\text{av}}}(x) = 1 - e^{-\frac{\lambda_n(2^{R_n} - 1)\sigma_n^2 x}{R_n - c(n)x}}. \quad (56)$$

It is easy to verify that all arguments applied to the CDF in Theorem 2 can also be readily applied to the CDF in (56).

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