Towards the Optimal Amplify-and-Forward Cooperative Diversity Scheme

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Abstract

In a slow fading channel, how to find a cooperative diversity scheme that achieves the transmit diversity bound is still an open problem. In fact, all previously proposed amplify-and-forward (AF) and decode-and-forward (DF) schemes do not improve with the number of relays in terms of the diversity-multiplexing tradeoff (DMT) for multiplexing gains r higher than 0.5. In this work, we study the class of slotted amplify-and-forward (SAF) schemes. We first establish an upper bound on the DMT for any SAF scheme with an arbitrary number of relays N and number of slots M. Then, we propose a sequential SAF scheme that can exploit the potential diversity gain in the high multiplexing gain regime. More precisely, in certain conditions, the sequential SAF scheme achieves the proposed DMT upper bound which tends to the transmit diversity bound when M goes to infinity. In particular, for the two-relay case, the three-slot sequential SAF scheme achieves the proposed upper bound and outperforms the two-relay non-orthorgonal amplify-and-forward (NAF) scheme of Azarian *et al.* for multiplexing gains $r \leq 2/3$. Numerical results reveal a significant gain of our scheme over the previously proposed AF schemes, especially in high spectral efficiency and large network size regime.

Index Terms

Cooperative diversity, diversity-multiplexing tradeoff (DMT), relay, relay scheduling, slotted amplify-and-forward (SAF).

I. Introduction and Problem Description

As a new way to exploit spatial diversity in a wireless network, cooperative diversity techniques have recently drawn more and more attention. Since the work of Sendonaris *et al.* [1], [2], a flood

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of works has appeared on this subject and many cooperative protocols have been proposed (see, for example, [3]–[8]). A fundamental performance measure to evaluate different cooperative schemes is the diversity-multiplexing tradeoff (DMT) which was introduced by Zheng and Tse [9] for the MIMO Rayleigh channel. It is well known that the DMT of any N-relay cooperative diversity scheme is upper-bounded (referred to as the *transmit diversity bound* in [4]) by the DMT of a MISO system with $N+1$ antennas,

$$d(r) = (N+1)(1-r)^{+}. \tag{1}$$

This bound is actually proved achievable by the cooperative multiple access scheme [6], using a Gaussian code with an infinite cooperation frame length.

However, how to achieve (1) in a single-user setting (*i.e.*, half-duplex relay channel) in the general case is still an open problem, even with an infinite cooperation frame length. In the single-relay case, the best known cooperative scheme, in the class of amplify-and-forward strategies, is the Non-orthogonal Amplify-and-Forward (NAF) scheme and the Dynamic Decode-and-Forward (DDF) scheme in the class of decode-and-forward strategies. The NAF scheme was proposed by Nabar *et al.* [5] and has been proved to be the optimal amplify-and-forward scheme for a half-duplex single-relay channel by Azarian *et al.* [6]. It is therefore impossible to achieve (1) by only amplifying-and-forwarding with one relay. The DDF scheme was proposed independently in [6], [10], [11] in different contexts. In [6], it is shown that the DDF scheme does achieve (1) in the low multiplexing gain regime ($r < 0.5$) but it fails in the high multiplexing gain regime, which is due to the *causality* of the decode-and-forward scheme. Intuitively, to achieve the MISO bound with a multiplexing gain r, the source and the relay need to cooperate during at least r-portion of the time. However, before this might possibly happen, the relay also needs at least r-portion of the time to decode the source signal (even with a Gaussian source-relay link). Therefore, it is impossible for the DF schemes to achieve the MISO bound for $2r > 1$.

Being optimal in the single-relay case, the generalization of the NAF and the DDF schemes proposed in [6], also the best known in each class, fails to exploit the potential spatial diversity gain in the high multiplexing gain regime ($r > 0.5$) with the growth of the network size. The suboptimality of these two schemes becomes very significant for a large number of relays, as shown in Fig. 1. Our goal is therefore to find a practical scheme that can possibly fill the gap between the two schemes and the MISO bound. In this work, we focus on the class of slotted

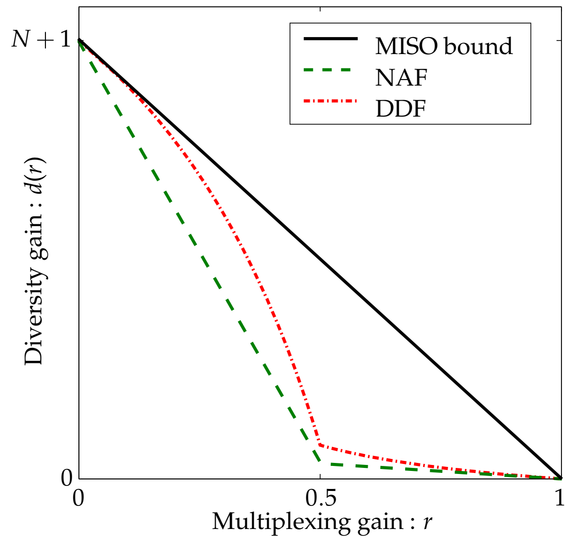


Fig. 1. Diversity-multiplexing tradeoff of an N-relay channel : NAF, DDF *vs.* MISO bound.

amplify-and-forward (SAF) schemes because of the following attractive properties :

1) Low relaying complexity. The relays only need to scale the received signal and retransmit it.
2) Existence of optimal codes with finite framelength. We will show that any SAF scheme is equivalent to a linear fading channel, whose DMT is achieved by perfect [12] $M \times M$ codes. The code length for an M-slot SAF scheme is therefore at most M^2.
3) Flexibility. The source does not have to know the number of relays or the relaying procedure. The coding scheme only depends on the number of slots M and is always optimal in terms of DMT.

A natural question is raised : *Is it possible for a half-duplex SAF scheme to achieve the MISO bound (1)? And how to achieve it if it is possible?* This question is partially answered in this work. The main contributions of this work are as follows :

- For a general N-relay M-slot SAF scheme, we establish a new upper bound :
$$d^*(r) = (1-r)^+ + N\left(1 - \frac{M}{M-1}r\right)^+, \tag{2}$$
from which we conclude that it is impossible to achieve the MISO bound with a finite length, even without the half-duplex constraint. This bound is however tending to the MISO bound

when M goes to infinity. Then, we argue that the suboptimality of the N-relay NAF scheme is due to the fact that only half of the source signal is *protected* by the relays.

- Inspired by the upperbound (2), we propose a half-duplex sequential SAF scheme. The basic idea is to let as many slots as possible (*i.e.*, $M-1$) be forwarded by the relays in the simplest way. For $M=2$ and an arbitrary N, the proposed scheme corresponds to the single-relay NAF scheme combined with the relay selection scheme [7] and the DMT upper bound is achieved. For arbitrary (N, M), we show that the sequential SAF achieves the DMT upper bound in the extreme case where all relays are isolated from each other, *i.e.*, there is no physical link between the relays. Nevertheless, even without the relays isolation assumption, simulation results show that a significant power gain over the NAF scheme is obtained by the sequential SAF scheme.
- In particular, we show explicitly that the two-relay three-slot sequential SAF scheme dominates the two-relay NAF scheme for multiplexing gains $r \leq 2/3$. It is therefore the best known two-relay amplify-and-forward scheme.

In this paper, we use boldface lower case letters $\boldsymbol{v}$ to denote vectors, boldface capital letters $\boldsymbol{M}$ to denote matrices. $\mathcal{CN}$ represents the complex Gaussian random variable. $[\cdot]^{\mathsf{T}}, [\cdot]^{\dagger}$ respectively denote the matrix transposition and conjugated transposition operations. $\|\cdot\|$ is the vector norm and $\|\cdot\|_{\mathrm{F}}$ is the Frobenius matrix norm. $(x)^{+}$ means $\max(0, x)$. The dot equal operator $\doteq$ denotes asymptotic equality in the high SNR regime, *i.e.*,

$$p_1 \doteq p_2 \quad \text{means} \quad \lim_{\mathsf{SNR}\to\infty} \frac{\log p_1}{\log \mathsf{SNR}} = \lim_{\mathsf{SNR}\to\infty} \frac{\log p_2}{\log \mathsf{SNR}},$$

and $\dot{\leq}$, $\dot{\geq}$ are similarly defined.

The rest of the paper is organized as follows. Section II introduces the system model and the class of SAF schemes. In Section III, we establish an upper bound on the DMT of any SAF schemes, using a genie-aided model. Then, Section IV proposes a sequential SAF scheme that achieves the previously provided DMT upper bound in certain conditions, when using two scheduling schemes. To show the performance of the proposed scheme, numerical results with the sequential SAF scheme are presented in Section V, compared to the NAF scheme and the non-cooperative scheme. Finally, we provide some concluding remarks in Section VI. For continuity of demonstration, all detailed proofs are left in the Appendix.

II. System Model

A. Basic Assumptions

The considered system model consists of one source s, one destination d and N relays (cooperative terminals) $\mathsf{r}_1, \ldots, \mathsf{r}_N$. The physical links between terminals are slowly faded and are modeled as independent quasi-static Rayleigh channels, *i.e.*, the channel gains do not change during the transmission of a cooperation frame, which is defined according to different schemes (protocols). The gain of the channel connecting s and d is denoted by g_0. Similarly, g_i and h_i respectively denote the channel gains between r_i and d and the ones between s and r_i. γ_{ij} is used to denote the channel gain between r_i and r_j. Channel quality between terminals is parameterized by the variance of the channel gains. Unless otherwise indicated, the relays work in half-duplex mode, that is, they cannot transmit and receive at the same time.

B. Slotted Amplify-and-Forward

1) Definition: In the paper, we study a particular class of amplify-and-forward schemes that we call slotted amplify-and-forward (SAF). More precisely, an N-relay M-slot scheme is specified by the following requirements :

- a cooperation frame is composed of M slots of l symbols, denoted by $\boldsymbol{x}_i \in \mathbb{C}^{l\times 1},\ i = 1, \ldots, M$;
- during the i^{th} slot, the source s transmits $\boldsymbol{x}_i$ and the j^{th} relay r_j, $j = 1, \ldots, N$ transmits $\boldsymbol{x}_{r_j,i} \in \mathbb{C}^{l\times 1}$;
- the received symbols at the j^{th} relay and the destination are respectively denoted by $\boldsymbol{y}_{r_j,i}, \boldsymbol{y}_i \in \mathbb{C}^{l\times 1}$, with

$$\begin{cases} \boldsymbol{y}_i = g_0\,\boldsymbol{x}_i + \sum_{j=1}^{N} g_j\,\boldsymbol{x}_{r_j,i} + \boldsymbol{z}_{d,i} \\ \boldsymbol{y}_{r_j,i} = h_j\,\boldsymbol{x}_i + \sum_{k=1,k\neq j}^{N} \gamma_{k,j}\,\boldsymbol{x}_{r_k,i} + \boldsymbol{z}_{r_j,i} \end{cases} \tag{3}$$

  where $\boldsymbol{z}_{d,i}, \boldsymbol{z}_{r_j,i} \in \mathbb{C}^{l\times 1}$ are i.i.d. AWGN with unit variance;
- according to the AF constraint, $\boldsymbol{x}_{r_j,i}$ can only be linear combination of the vectors $\boldsymbol{y}_{r_j,1}, \ldots, \boldsymbol{y}_{r_j,i-1}$ that it receives in previous slots, *i.e.*,

$$\boldsymbol{x}_{r_j,i} = \sum_{k=1}^{i-1} p_{i,k}^{(j)} \boldsymbol{y}_{r_j,k} \tag{4}$$

where $p_{i,k}^{(j)}$ depends on the AF protocol and the scheduling;

- the transmitted signal $\boldsymbol{x}_i$ and $\boldsymbol{x}_{r_j,i}$ are subject to the short-term[1] power constraint

$$\mathbb{E}\left(\|\boldsymbol{x}_i\|^2 + \sum_{j=1}^{N} \left\|\boldsymbol{x}_{r_j,i}\right\|^2\right) \leq l \cdot \mathsf{SNR}, \quad \forall i. \tag{5}$$

For example, the NAF scheme [6] is an N-relay $(2N)$-slot scheme and the non-orthogonal relay selection scheme [7] is an N-relay two-slot scheme. Furthermore, any AF scheme with cooperation frame length L can also be regarded as an L-slot SAF scheme with slot length constraint $l = 1$.

In the SAF model, the knowledge of channel state information (CSI) is not specified. We assume that the cooperations between terminals are coordinated by a scheduler (that exists physically or logically). Depending on how much CSI the scheduler has, the coordination (scheduling) can be static (no CSI, *e.g.*, NAF) or dynamic (based on global CSI, *e.g.*, relay selection). Therefore, for each relay, the coefficients $\left\{p_{k,i}^{(j)}\right\}$'s in (4) are decided basing on its own CSI and the scheduling information it receives from the scheduler. To be realistic, we assume in our work that all terminals have receiver CSI only, and that depending on applications the scheduler may have global CSI but can only send order information to the relays, in order to minimize the signaling overhead.

2) Equivalent channel: Note that in the considered scheme, there is only one source signal stream $[\boldsymbol{x}_1 \cdots \boldsymbol{x}_M]$ and all relayed signal $\boldsymbol{x}_{r_j,i}$ can be eventually expressed as a noisy linear combination of $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M$, as shown by (3) and (4). Therefore, without going to the details, we can verify that the transmission of a cooperation frame with any SAF scheme described above can be written in the following compact form

$$\underbrace{[\boldsymbol{y}_1 \cdots \boldsymbol{y}_M]^{\mathsf{T}}}_{\boldsymbol{Y}} = \sqrt{\mathsf{SNR}}\, \boldsymbol{H} \underbrace{[\boldsymbol{x}_1 \cdots \boldsymbol{x}_M]^{\mathsf{T}}}_{\boldsymbol{X}} + \underbrace{\boldsymbol{Z}_d + \boldsymbol{Z}_e}_{\boldsymbol{Z}} \tag{6}$$

where $\boldsymbol{X} \in \mathbb{C}^{M \times l}$ is the normalized[2] (by $\sqrt{\mathsf{SNR}}$) codeword matrix; $\boldsymbol{H} \in \mathbb{C}^{M \times M}$ is the equivalent channel matrix consisting of functions of the channel coefficients and the $\left\{p_{i,k}^{(j)}\right\}$'s in (4); $\boldsymbol{Z}_d \in \mathbb{C}^{M \times l} \sim \mathcal{CN}(0, \mathbf{I})$ is the AWGN at the destination and $\boldsymbol{Z}_e \in \mathbb{C}^{M \times l} \sim \mathcal{CN}(0, \boldsymbol{\Sigma}_e)$ is the effective

[1]We do not consider power control in our work.

[2]For simplicity, we keep the same notation $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_M$ to denote the normalized codeword.

accumulated noise[3] caused by the AF operations at each relay during the whole transmission; the total noise is thus $\boldsymbol{Z} = \boldsymbol{Z}_d + \boldsymbol{Z}_e \sim \mathcal{CN}(0, \boldsymbol{\Sigma})$ with $\boldsymbol{\Sigma} = \mathbf{I} + \boldsymbol{\Sigma}_e$.

C. Diversity-Multiplexing Tradeoff and Achievability

Let us recall the definition of the multiplexing and diversity gains.

Definition 1 (Multiplexing and diversity gain [9]): A coding scheme $\{\mathcal{C}(\mathsf{SNR})\}$ is said to achieve *multiplexing gain* r and *diversity gain* d if

$$\lim_{\mathsf{SNR}\to\infty} \frac{R(\mathsf{SNR})}{\log \mathsf{SNR}} = r \quad \text{and} \quad \lim_{\mathsf{SNR}\to\infty} \frac{\log P_{\mathrm{e}}(\mathsf{SNR})}{\log \mathsf{SNR}} = -d$$

where $R(\mathsf{SNR})$ is the data rate measured by bits per channel use (PCU) and $P_{\mathrm{e}}(\mathsf{SNR})$ is the average error probability using the maximum likelihood (ML) decoder.

Theorem 1: The DMT of any SAF scheme with equivalent channel model (6) is

$$d(r) = d_{\boldsymbol{H}}(Mr), \tag{7}$$

with $d_{\boldsymbol{H}}(r)$ being the DMT of the linear channel (6). Furthermore, by vectorizing a full rate $M \times M$ space-time code with non-vanishing determinant (NVD), we get a code that achieves the tradeoff $d(r)$ for the SAF scheme. The code construction only depends on the slot number M.

Proof: The equality (7) is obvious, since M is the normalization factor of the channel use. The achievability is immediate from the results in [13], [14], stating that the DMT of a fading channel with any fading statistics can be achieved by a full rate NVD code. ∎

Since the optimal code construction is independent of the fading statistics of the channel, the only information that the source needs for coding is the number of slots M. In practice, M is decided by the scheduler, based on the channel coherence time, decoding complexity, etc. The relaying strategies are between the destination and the relays and can be completely ignored by the source. When no relay is helping, the equivalent channel matrix is diagonal. In this case, even if the source is not aware of the non-relay situation, the destination can decode the signal with linear complexity. All these properties make SAF schemes very flexible and suitable for wireless networks, especially for *ad hoc* networks where the network topology changes frequently.

[3]The l columns of $\boldsymbol{Z}_e$ are mutually independent and each column has the same covariance matrix $\boldsymbol{\Sigma}_e$.

III. Genie-Aided SAF and Upper Bound of the DMT

From (3) and (4), it is clear that an SAF scheme is actually defined by $\left\{p_{k,i}^{(j)}\right\}$. Therefore, it is impossible to get the DMT of an SAF without precising $\left\{p_{i,k}^{(j)}\right\}$. However, we can establish an upper bound on the DMT of any SAF scheme, which is independent of the choice of $\left\{p_{i,k}^{(j)}\right\}$. To this end, we will first introduce the genie-aided SAF model.

A. The Genie-Aided Model

We consider the following genie-aided model. We assume that before the transmission of the i^{th} slot, the relays know exactly the coded signal $\boldsymbol{x}_j$ for any $j < i$, via the genie. However, the relays are not allowed to decode the message embedded in the signal, due to the AF constraint. The half-duplex constraint is also relaxed. Therefore, in the i^{th} slot, the relays can transmit *any* linear combinations of the vectors $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{i-1}$, *i.e.*,

$$\boldsymbol{x}_{r_j,i} = \sum_{k=1}^{i-1} l_{i,k}^{(j)} \boldsymbol{x}_k \tag{8}$$

where $l_{i,k}^{(j)}$ can be set arbitrarily as long as the power constraint (5) is satisfied. Obviously, the genie-aided SAF provides better performance than the original SAF does, since unlike in (4) where we can only choose the coefficients of $\boldsymbol{y}_{r_j,i}$, we are now free to choose the coefficients of $\boldsymbol{x}_k$. Moreover, there is no accumulated noise in the genie-aided model.

The equivalent channel model for the genie-aided SAF is still in the form of (6), except that $\boldsymbol{Z}_e = \mathbf{0}$ and that $\boldsymbol{H}$ can be specified as

$$\boldsymbol{H} = g_0 \, \mathbf{I} + \sum_{j=1}^{N} g_j \, \boldsymbol{L}_j \tag{9}$$

where each matrix $\boldsymbol{L}_j \in \mathbb{C}^{M \times M}$ is strictly lower-triangular with $\boldsymbol{L}_j(i,k) = l_{i,k}^{(j)}$.

B. Upper Bound on the DMT

Theorem 2: The optimal DMT of an N-relay M-slot genie-aided SAF scheme is

$$d^*(r) = (1-r)^+ + N\left(1 - \frac{M}{M-1} r\right)^+, \tag{10}$$

for any $M > 1$. It is achievable by using uniquely the relay with largest relay-destination gain to send $\boldsymbol{x}_i$ in the $(i+1)^{\text{th}}$ slot, $i = 1, \ldots, M-1$.

Proof: See Appendix B. ■

Corollary 1: The DMT of any N-relay M-slot SAF scheme is upper-bounded by (10) for any $M > 1$.

In this theorem, we exclude the case $M = 1$ for the obvious reason that the single-slot SAF scheme corresponds to the non-cooperative case. In the two-slot case ($M = 2$), this upper bound is actually achievable by previously proposed half-duplex schemes : 1) with single-relay ($N = 1$), the NAF is shown in [6] to achieve (10); 2) for $N > 1$, the upper bound is achievable by the relay selection NAF scheme [7] if the scheduler have global CSI or by beamforming if the relays could have transmitter CSI. Intuitively, the upper bound is tight in the two-slot case since the half-duplex constraint is implicitly imposed by the SAF model.

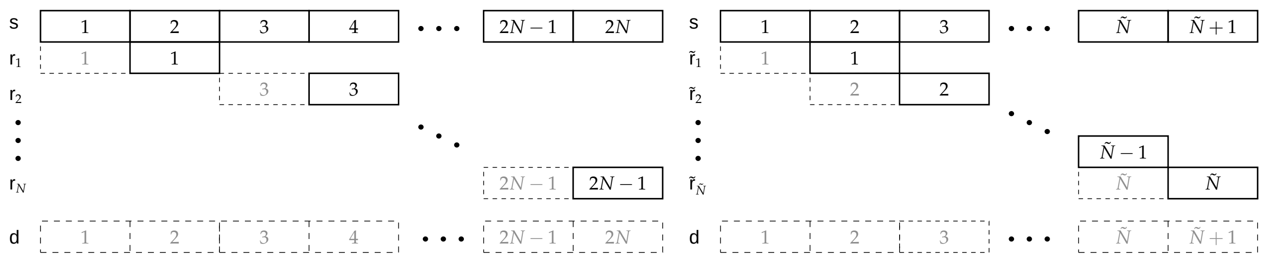
On the other hand, in the single-relay case ($N = 1$), the upper bound is not tight for $M > 2$: it is shown in [6] that the NAF scheme is the best single-relay half-duplex AF scheme in the DMT sense. The looseness of the bound in the single-relay case is due to the fact that the upper bound is obtained by relaxing the half-duplex constraint which is too strong in the single-relay case.

C. Implications

From the upper bound (10), two observations can be made : 1) SAF schemes can never achieve the MISO bound with a finite number of slots, even without the half-duplex constraint, and 2) SAF schemes can never beat the non-cooperative scheme for $r > \frac{M-1}{M}$. In fact, the first observation can be seen as a necessary condition of the second one, and it applies to all AF schemes as they can be seen as L-slot SAF schemes.

Intuitively, even in the genie-aided model, the last slot is not protected by any relay. This is due to the causality of the relay channel, not to the half-duplex constraint. Therefore, at most $M - 1$ slots out of M slots can be protected, which explains the suboptimality for $r > \frac{M-1}{M}$. In the same way, since only N slots out of $2N$ slots are protected by one relay in the NAF scheme, the NAF scheme is not better than the non-cooperative scheme for $r > 0.5$.

As stated in [6], an important guideline for cooperative diversity is to let the source keep transmitting all the time so that the maximum multiplexing gain is achieved. Here, we provide another guideline : *let most of the source signal be protected by extra paths*. Based on this



(a) NAF

(b) Sequential SAF

Fig. 2. Frame structure and relaying procedure of NAF and sequential SAF, solid box for transmitted signal and dashed box for received signal.

guideline, we propose, in next section, a sequential SAF scheme and we show that this scheme actually achieves the upper bound (10) in some particular cases.

IV. The Sequential SAF Scheme

As previously stated, the NAF scheme is optimal in the single-relay case, due to the half-duplex constraint. We consider the multiple-relay case in the rest of the paper.

Let us consider the following sequential SAF scheme. First of all, in order to achieve the full multiplexing gain, the source must transmit during all the M slots. Then, from the beginning of the second slot, in each slot, there is one and only one relay forwarding a scaled version of what it received in the previous slot. In such a way, $M-1$ slots out of M slots of the source signal are forwarded by at least one relay. Here, we can see that this is only possible when we have more than one relay, where different relays can alternatively help the source to alleviate the half-duplex constraint. Thus, we have $\tilde{N} \triangleq M-1$ effective relays $\tilde{\mathsf{r}}_1, \ldots, \tilde{\mathsf{r}}_{\tilde{N}}$ during the transmission of a specific source. The mapping between the real relays and the effective relays is accomplished by relays scheduling that will be discussed later on. The frame structure and the relaying procedure are illustrated in Fig. 2, compared to the NAF scheme.

A. Equivalent Linear Fading Channel

In SAF schemes, there is no difference in data processing for different symbols within the same slot. Thus, we can consider one symbol from a slot, without loss of generality. With the previous description of the sequential SAF scheme, we have the following signal model :

$$\begin{cases} y_{d,i} = \sqrt{\pi_i \,\mathsf{SNR}}\, g_0\, x_i + \sqrt{\bar{\pi}_i \,\mathsf{SNR}}\, \tilde{g}_{i-1}\, \tilde{b}_{i-1}\, y_{r,i-1} + z_{d,i} \\ y_{r,i} = \sqrt{\pi_i \,\mathsf{SNR}}\, \tilde{h}_i\, x_i + \sqrt{\bar{\pi}_i \,\mathsf{SNR}}\, \tilde{\gamma}_{i-1,i}\, \tilde{b}_{i-1}\, y_{r,i-1} + z_{r,i} \end{cases} \tag{11}$$

where x_i is the transmitted symbol from the source in the i^{th} slot; $y_{r,i}$ and $y_{d,i}$ are the received symbols at the i^{th} effective relay and at the destination, respectively, in the i^{th} slot; $\boldsymbol{z}_{d,i}$'s and $\boldsymbol{z}_{r,i}$'s are independent AWGN with unit variance; $\tilde{h}_i$ and $\tilde{g}_i$, $i = 1, \ldots, \tilde{N}$, are the channel gains from the source to the i^{th} effective relay and from the i^{th} effective relay to the destination, respectively; $\tilde{\gamma}_{i-1,i}$ is the channel gain between the $(i-1)^{\text{th}}$ and the i^{th} effective relay; $\tilde{b}_i$ is the processing gain at the i^{th} effective relay subject to the power constraint $\mathbb{E}\Big(|\tilde{b}_i\, y_{r,i}|^2\Big) \leq 1$. The power allocation factors π_i, $\bar{\pi}_i$, $i = 1, \ldots, M$ satisfy $\sum_{i=1}^{M}(\pi_i + \bar{\pi}_i) = M$. Finally, we set $\bar{\pi}_1 = 0$ and $\tilde{b}_0 = 0$.

We can express the signal model (11) of M slots in the following vector form

$$\begin{cases} \boldsymbol{y}_d = \sqrt{\mathsf{SNR}}\, g_0 \operatorname{diag}(\boldsymbol{a})\, \boldsymbol{x} + \boldsymbol{U_c}\, \boldsymbol{y}_r + \boldsymbol{z}_d \\ \boldsymbol{y}_r = \sqrt{\mathsf{SNR}} \operatorname{diag}(\tilde{\boldsymbol{h}}) \operatorname{diag}(\boldsymbol{a}) \boldsymbol{x} + \boldsymbol{U_d}\, \boldsymbol{y}_r + \boldsymbol{z}_r \end{cases} \tag{12}$$

where $\boldsymbol{T} \triangleq \boldsymbol{U_c}(\mathbf{I} - \boldsymbol{U_d})^{-1}$, $\boldsymbol{a} \in \mathbb{R}_+^{M \times 1}$ with $a_i \triangleq \sqrt{\pi_i}$, and $\boldsymbol{U_c}$, $\boldsymbol{U_d}$ are $M \times M$ matrices defined as

$$\boldsymbol{U_c} \triangleq \begin{bmatrix} \mathbf{0}^{\mathsf{T}} & 0 \\ \operatorname{diag}(\boldsymbol{c}) & \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{U_d} \triangleq \begin{bmatrix} \mathbf{0}^{\mathsf{T}} & 0 \\ \operatorname{diag}(\boldsymbol{d}) & \mathbf{0} \end{bmatrix}$$

with $\boldsymbol{c}, \boldsymbol{d} \in \mathbb{C}^{\tilde{N} \times 1}$ whose components are defined by $c_i \triangleq \sqrt{\bar{\pi}_{i+1}\, \mathsf{SNR}}\, \tilde{g}_i\, \tilde{b}_i$ and $d_i \triangleq \sqrt{\bar{\pi}_{i+1}\, \mathsf{SNR}}\, \tilde{\gamma}_{i,i+1}\, \tilde{b}_i$ for $i = 1, \ldots, \tilde{N}$. Both $\boldsymbol{U_c}$ and $\boldsymbol{U_d}$ are forward-shift like matrices.

From (12), we finally get the equivalent vector channel

$$\boldsymbol{y}_d = \sqrt{\mathsf{SNR}}\, \boldsymbol{H} \boldsymbol{x} + \boldsymbol{z}.$$

where the equivalent channel matrix and noise are in the following form :

$$\boldsymbol{H} = \Big(g_0 \mathbf{I} + \boldsymbol{T} \operatorname{diag}(\tilde{\boldsymbol{h}})\Big) \operatorname{diag}(\boldsymbol{a}) \tag{13}$$

$$\boldsymbol{z} = \boldsymbol{z}_d + \boldsymbol{T}\, \boldsymbol{z}_r, \tag{14}$$

From (14), the covariance matrix of the noise is $\boldsymbol{\Sigma_z} = \mathbf{I} + \boldsymbol{T}\boldsymbol{T}^{\dagger}$. We can show that the largest and smallest eigenvalues of $\boldsymbol{\Sigma_z}$ satisfy $\lambda_{\max}(\boldsymbol{\Sigma_z}) \doteq \lambda_{\min}(\boldsymbol{\Sigma_z}) \doteq \mathsf{SNR}^0$, which implies that the DMT of the proposed scheme depends only on $\boldsymbol{H}$ and not on $\boldsymbol{\Sigma_z}$.

Now, let us take a closer look at the equivalent channel matrix $\boldsymbol{H}$, which is lower-triangular. For simplicity, we ignore the term $\mathrm{diag}(\boldsymbol{a})$ in our analysis since it does not impact the DMT. The main diagonal of the equivalent channel is $g_0\,\mathbf{I}$, representing the direct (source-destination) link. The off-diagonal entries are defined by $\boldsymbol{T}\,\mathrm{diag}(\tilde{\boldsymbol{h}})$, where the i^{th} sub-diagonal[4] is $\boldsymbol{U_c}\cdot\boldsymbol{U_d}^{i-1}\cdot\mathrm{diag}(\tilde{\boldsymbol{h}})$, representing the source-relays-destination i-hop link. Since the off-diagonal entries are independent of the main diagonal entries, extra protection to the source signal is provided and therefore the diversity gain is obtained.

B. Isolated Relays

Calculating the DMT of the sequential SAF being prohibitive in general, we search for an approximation. Intuitively speaking, the source signal degrades with the number of hops, since the channel in each hop is faded and that each normalization at the relays weakens the signal power. Therefore, one possible approximation is to ignore the i-hop links for $i > 1$, which is equivalent to the special scenario where relay r_j is isolated with r_{j-1} for $j = 2, \ldots, \tilde{N}$. In this case, the DMT can be obtained explicitly.

Proposition 1: When the relays are isolated from each other, ***i.e.***, $\tilde{\gamma}_{i,i+1} = 0,\ \forall i$, the DMT (10) is achievable with the sequential SAF scheme.

This proposition is proved in the following paragraphs. With the assumption of relay isolation, we have $\boldsymbol{T} = \boldsymbol{U_c}$ and $\boldsymbol{H}$ is therefore a bidiagonal matrix. The special form of $\boldsymbol{H}$ allows us get the following lemma that is crucial to the proof.

Lemma 1:

$$\max_{\tilde{\boldsymbol{b}},\boldsymbol{\pi},\bar{\boldsymbol{\pi}}} \det\left(\mathbf{I} + \mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^{\dagger}\right) \ \dot{\geq}\ \left(1 + \mathsf{SNR}\,|g_0|^2\right)^M + \prod_{i=1}^{\tilde{N}}\left(1 + \mathsf{SNR}\left|\tilde{g}_i\tilde{h}_i\right|^2\right) \tag{15}$$

Proof: Using the bidiagonal property of $\boldsymbol{H}$ (See Appendix A for details), we have

$$\det\left(\mathbf{I} + \mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^{\dagger}\right) \geq \left(\mathsf{SNR}\,|g_0|^2\right)^M + \prod_{i=1}^{\tilde{N}}\left(1 + \mathsf{SNR}\left|\tilde{g}_i\tilde{h}_i\right|^2\bar{\pi}_{i+1}\mathsf{SNR}\left|\tilde{b}_i\right|^2\right).$$

[4] $\boldsymbol{T} = \boldsymbol{U_c}\left(\mathbf{I} - \boldsymbol{U_d}\right)^{-1} = \boldsymbol{U_c}\ \left(\mathbf{I} + \boldsymbol{U_d} + \boldsymbol{U_d}^2 + \cdots\right)$.

Since we can always find $\boldsymbol{\pi}$,$\bar{\boldsymbol{\pi}}$ and $\tilde{\boldsymbol{b}}$ that satisfy simutaneously $\bar{\pi}_{i+1}\mathsf{SNR}|\tilde{b}_i|^2 \doteq \mathsf{SNR}^0$ and the power constraint (5), the lemma is proved. ■

We can now introduce the scheduling strategies that permit the sequential SAF to achieve the DMT upper bound :

1) *Dumb scheduling*: For $\tilde{N} = kN$ with k being any integer, the relays help the source in a round-robin manner, *i.e.*, $\tilde{\mathsf{r}}_i = \mathsf{r}_{(i-1)_N+1}$. For $\tilde{N} = kN + m$ with $m \in [1, N-1]$, we first order the relays $\mathsf{r}_1, \ldots, \mathsf{r}_N$ in such a way that

$$\min\{C_1, \ldots, C_m\} \geq \max\{C_{m+1}, \ldots, C_N\}.$$

where C_i are the cost function defined by

$$C_i \triangleq \frac{\mathsf{SNR}^2 \, |b_i \, g_i \, h_i|^2}{1 + \mathsf{SNR} \, |b_i \, g_i|^2}. \tag{16}$$

Then, we apply the round-robin scheduling.

2) *Smart scheduling*: First, select the two “best” relays in the sense that they have largest cost function C_i defined by (16). Then, we apply the dumb scheduling on these two relays, as if we were in the two-relay M-slot case.

These two scheduling strategies maximize *statistically* the RHS of (15) in the high SNR regime, so that upper bound (10) is achieved. The detailed proof is provided in Appendix C. Even though both schemes achieve DMT (10) under the relay isolation assumption, the smart scheme outperforms the dumb scheme in a general case, without relay isolation. Since the cost function C_i is the effective SNR of the relayed signal at the destination if the i^{th} relay is used, the basic idea of the smart scheduling is to avoid using the “bad” relays, where the noise level is higher than the other relays in average. Therefore, in M slots, noise amplification is less significant with the smart scheduling than with the dumb scheduling. The impact is investigated in the next section, with the simulation results. Note that which scheduling scheme to be used depends strongly on the available CSI at the scheduler. If the scheduler has no CSI at all, dumb scheduling is used and we set $\tilde{N} = kN$ (or $M = kN + 1$).

As an example, Fig. 3 shows the DMT of different cooperative schemes for a three-relay channel, with relay isolation assumption. For $M = 2$, the DMT of the proposed scheme coincides with that of the NAF scheme. With increasing M, the proposed scheme is approaching the MISO bound, which makes it asymptotically optimal.

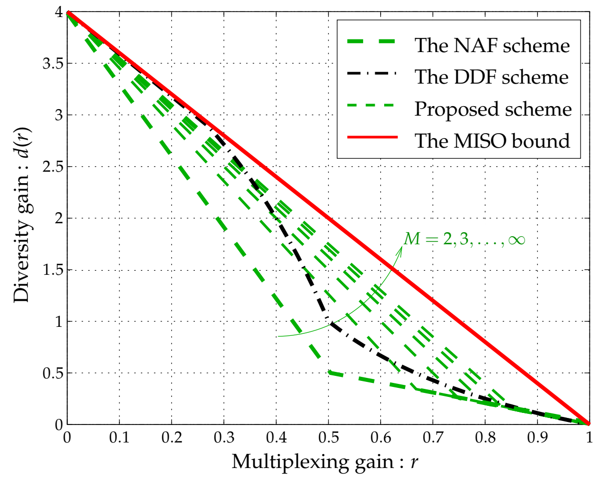


Fig. 3. Diversity-multiplexing tradeoff of different three-relay schemes with isolated relays.

C. Non-Isolated Relays

With interconnected relays, the DMT of the sequential SAF is generally unknown, except for the following two cases.

1) Two-Slot with Arbitrary Number of Relays: Note that for the particular cases $M = 2$, i.e., $k = 0$ *and* $m = 1$, *the above analysis is valid whether the relays are isolated from each other or not.* This is because the maximum number of hops in the channel is 1. Therefore, the DMT (10) for $M = 2$ and arbitrary N is achieved by the sequential SAF with scheduler CSI, where the scheduler selects the relay with largest C_i. It also corresponds to the relay selection NAF scheme [7].

2) Two-Relay and Three-Slot:

Proposition 2: The two-relay three-slot sequential SAF scheme achieves the DMTs of Fig. 4, where the relay ordering is such that $|h_2|^2 \geq |h_1|^2$, *i.e.*, the relay with worse source-relay link transmits first.

Proof: The DMTs are obtained with the same method as previously, by expressing explicitly the determinant $\det\left(\mathbf{I} + \mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^{\dagger}\right)$. See Appendix D for details. ■

As shown in Appendix D, even though we have the closed-form determinant expression, we can only have a lower-bound on the DMT because of the complex determinant form. Unfortunately, the lower-bound we get does not coincide with the upper bound (10) for $r < 0.5$. By adding a

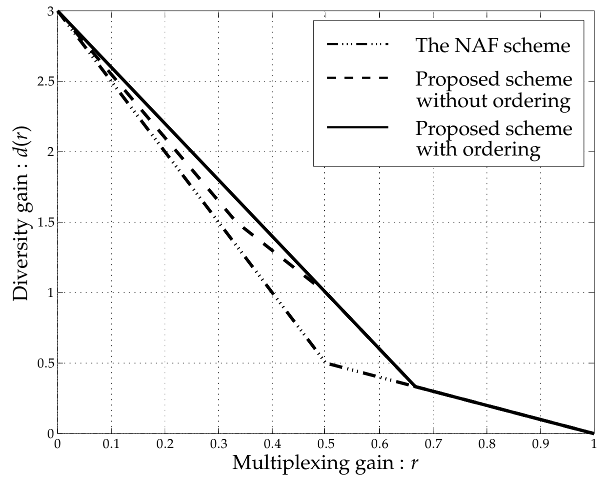


Fig. 4. Diversity-multiplexing tradeoff of the two-relay schemes.

relay ordering procedure ($|h_2|^2 \geq |h_1|^2$), we finally get a lower-bound equal to the upper bound. However, this does not necessarily mean that the relay ordering improves the performance, as we will show in the next section.

As shown in Fig. 4, the sequential SAF scheme (with or without relay ordering) outperforms the two-relay NAF scheme. Since with the three-slot structure we protect $\frac{2}{3}$ of the source signal, we can beat the non-cooperative scheme for $0 \leq r \leq \frac{2}{3}$. It is therefore the best AF scheme known for the two-relay case. To further improve the DMT, we should increase the number of slots.

D. Discussions

1) Artificial Relay-Isolation: Although it is hard to tell if the multi-hop links are harmful, proposition 1 shows that the relay-isolation condition is sufficient to achieve the DMT (10). If the scheduler has global CSI, it can order the relays in such a way that consecutive relays are separated as far as possible to approximate the relay-isolation condition. An example scheme is shown in Fig. 5.

2) Practical Considerations: In practice, an individual scheduler might not exist physically in the network. In this case, we can integrate the scheduler's role into the destination receiver. To implement the relay ordering, which is essential for the smart scheduling and the $\tilde{N} \neq kN$

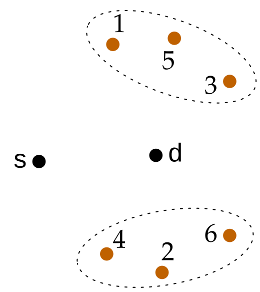


Fig. 5. A scheme to create weak inter-relay connections, in order to approximate the relay-isolation condition. The order of the relays are indicated by the numbers.

case of the dumb scheduling, an intelligent way is similar to the RTS/CTS scheme proposed in [7] described as follows :

- If we have the reciprocity for the forward and the backward relay-destination links, *i.e.*, the channel gains are the same (g_i) for the forward and backward links, an intelligent way to implement the relay ordering is similar to the RTS/CTS scheme proposed in [7]. First, the relays measure the source-relay channel quality $|h_i|$ by the reception of the *RTS* (Ready-to-Send) frame from the source. Then, the destination broadcasts a *relay-probing* frame, from which the relays can estimate the relay-destination channel $|g_i|$. Each relay calculates the cost function C_i and reacts by sending an *availability* frame after t_i time which is proportional to C_i. Therefore, the relay with the largest cost function is identified as relay 1, and so on. Finally, based on the order, the destination decides a scheduling strategy and broadcasts the parameters (*e.g.*, the relay ordering for the relays and number of slots M for the source, etc...) in the *CTS* (Clear-to-Send) frame.
- When there is no reciprocity for the relay-destination links, we modify the last three steps as follows. Each relay quantizes the source-relay gain and sends it in the *availability* frame to the destination using its own signature. Then, the destination can estimate the relay-destination links quality $|g_i|$ and also gets the estimates $|h_i|$ by decoding the signal. Finally, the destination decides the order based on the cost functions and broadcasts the *CTS* frame.

Since we only consider slow fading channels, the ordering would not be so frequent and the signaling overhead is negligible in both cases (the overhead issue is mentioned in [7]). In the worst case where the above signaling is impossible, a cooperation order for the relays should be predefined and we apply the dumb scheduling with a slot number M such that $M-1=kN$. In this case, the same DMT is achieved.

V. Numerical Results

In this section, we investigate the numerical results obtained by Monte-Carlo simulations. By default, we consider a symmetric network, where all the channel coefficients are i.i.d. Rayleigh distributed with unit variance. There is therefore no *a priori* advantage of the source-relay links over the source-destination link. The power allocation factors are $\pi_i=\bar{\pi}_i=0.5$ for $i=2,\ldots,M$ and $\pi_1=1$. Information rate is measured in bits per channel use (BPCU). We compare the proposed sequential SAF scheme to the NAF scheme and the non-cooperative scheme in both small network scenarios (2 relays) and large network scenarios (12 relays).

A. Two-Relay Scenario

1) Three-Slot Case: Fig. 6 shows the performance of the proposed two-relay three-slot scheme for different spectral efficiencies. Note that with a low spectral efficiency (2 BPCU), the proposed schemes have almost the same performance as the NAF scheme. However, when increasing the spectral efficiency, the gain of our schemes compared to the NAF strategy increases. For 10 BPCU, the NAF scheme barely beats the non-cooperative scheme. Also note that in all cases, the scheme with relay ordering proposed in Sec. IV-C.2 is not better than the one without relay ordering. Based on that observation, we conjecture that we can achieve the DMT (2) even without relay ordering in the two-relay three-slot case.

Then, we consider the error rate performance of NVD codes (*i.e.*, achieving the DMT) under ML decoding. For the two-relay NAF scheme, we use the optimal code $\mathcal{C}_{2,1}$ (QAM) proposed in [14]. For the sequential SAF scheme, we use the perfect 3×3 code construction proposed in [15], based on QAM constellations, the best known 3×3 real rotation [16] and the “non-norm” element $\gamma=\frac{1+2i}{2+i}$. The vectorized code (frame) lengths are 8 and 9 QAM symbols for the NAF and the sequential SAF, respectively. 4-QAM and 64-QAM uncoded constellations are used, corresponding to the 2 BPCU and 6 BPCU counterparts in the outage performance. The frame error rate (FER) is shown in Fig. 7(a). It is surprising to see such a similarity between code performance and outage performance: for a given probability (error or outage respectively), all SNR differences between the compared schemes are almost the same. We have a power gain of

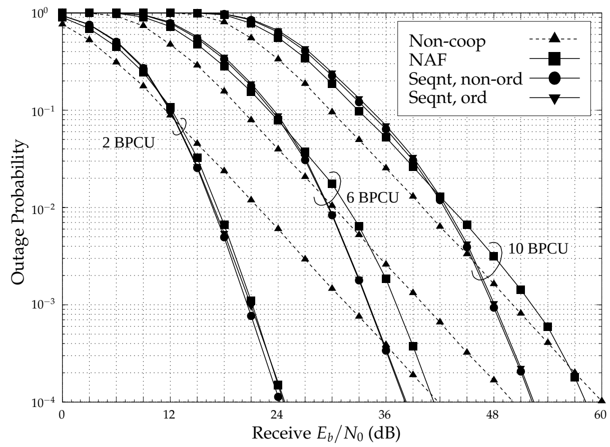
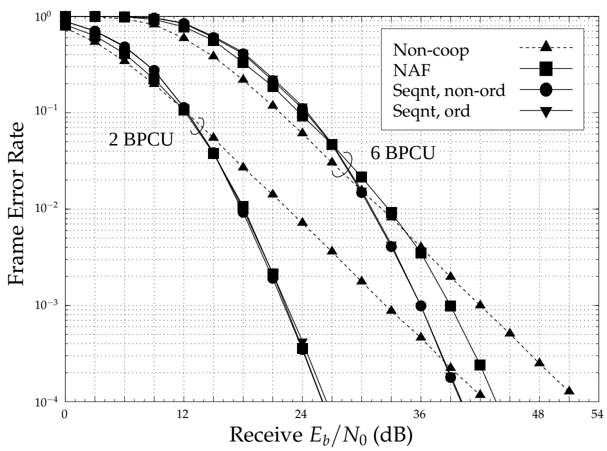
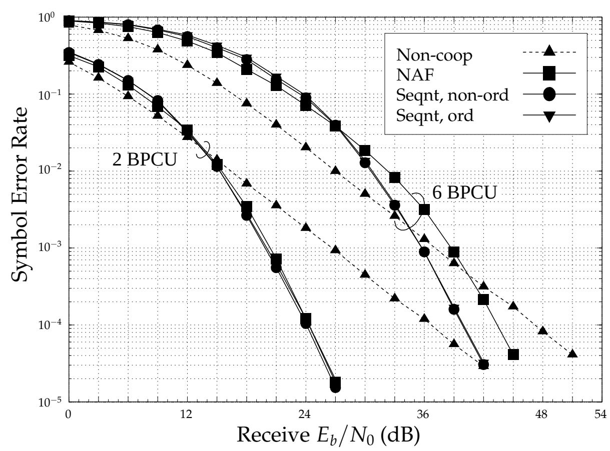


Fig. 6. Outage probabilities for the non-cooperative, NAF and sequential SAF scheme with three slots. Two-relay symmetric network. Considered information rates: 2, 6 and 10 BPCU.



(a) Frame Error Rate



(b) Symbol Error Rate

Fig. 7. Error rate performance: sequential SAF *vs.* NAF scheme. Two-relay symmetric network, perfect 3×3 code for the three-slot SAF scheme and $\mathcal{C}_{2,1}$ for the NAF scheme for the NAF. 4- and 64-QAM for 2 and 6 BPCU, respectively.

more than 3 dB for FER lower than 10^{-3} with 64-QAM. For fairness of comparison between different frame length, we also show the symbol error rate performance in Fig. 7(b).

As stated in theorem 1, we can always construct optimal codes for a given SAF scheme. To focus on the cooperative scheme itself, we only consider the outage probability hereafter.

2) Impact of the Number of Slots: Fig. 8 shows the outage performance with different numbers of slots. For 2 BPCU, the difference is minor (within 1 dB). However, for 6 BPCU, the power gain

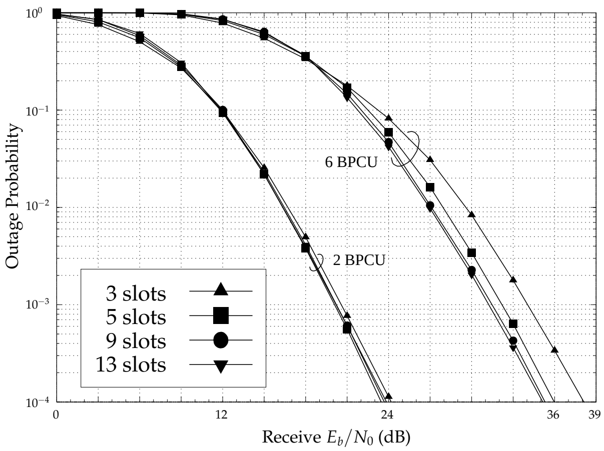


Fig. 8. Outage probability of the sequential SAF scheme with 3, 5, 9 and 13 slots. Two-relay symmetric network.

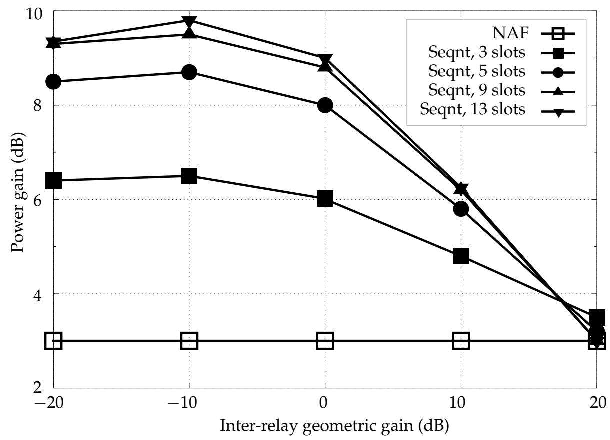


Fig. 9. Power gain to the non-cooperative scheme : impact of the inter-relay geometric gain. Two-relay network. Target information rate : 6 BPCU. Target outage probability : 10^{-3}.

compared to the three-slot scheme increases to 2 and 3 dB for 5 slots and 13 slots, respectively. The increasing SNR gain shows the superiority of the schemes with a larger number of slots in terms of DMT, even without the relay isolation assumption.

3) Inter-Relay Geometric Gain: In Fig. 9, we show the impact of the inter-relay geometric gain (defined as $\mathbb{E}\left|\gamma_{ij}\right|^2/\mathbb{E}\left|h_j\right|^2$) on the outage performance. In this scenario, all paths have the same average channel gain (0 dB), except for the inter-relay channels whose channel gains vary form -20 dB (weak interconnection) to 20 dB (strong interconnection). The y-axis represents the power gain to the non-cooperative scheme with 6 BPCU and outage probability of 10^{-3}. The x-axis represents the inter-relay geometric gain. As shown in Fig. 9, the NAF scheme is independent of the geometric gain since there is no inter-relay communication at all in the NAF scheme. In the weak interconnection regime (< 0 dB), the sequential SAF scheme is not sensitive to the geometric gain and we always have a better performance by increasing the slot number. However, in the strong interconnection regime (> 0 dB), the performance degrades dramatically with the increase of inter-relay gain and the increase of the number of slots. Intuitively, the task of the i^{th} effective relay is to protect the source signal $\boldsymbol{x}_i$, transmitted in the i^{th} slot. A strong interconnection between the $(i-1)^{\text{th}}$ relay and the i^{th} relay makes $\boldsymbol{x}_i$ drowned in the combined signal of $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{i-1}$ from the $(i-1)^{\text{th}}$ relay.

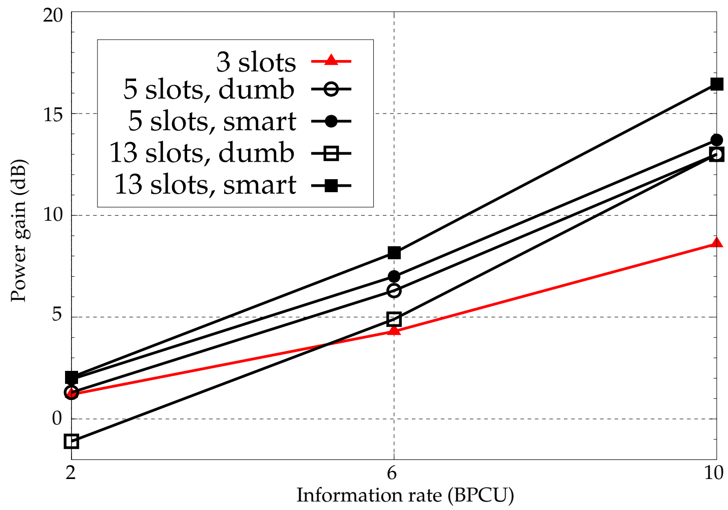


Fig. 10. Power gain to the NAF scheme with selection : dumb *vs.* smart scheduling. Symmetric network with 12 relays. Target outage probability : 10^{-3}.

B. Large Network : Dumb vs. *Smart Scheduling*

Now, we consider a large symmetric network with 12 available relays. We compare the proposed scheme to the NAF scheme. To ensure fairness, the considered NAF is combined with the relay selection scheme, *i.e.*, the source is only helped by the best relay (with largest C_i). For the sequential SAF scheme, both the dumb and the smart schedulings are considered. Obviously, with 3 slots, the dumb scheduling is the same as the smart scheduling. As shown in Fig. 10, the power gain increases with spectral efficiency, showing the superiority of our scheme in terms of DMT. The increase is more significant with a larger slot number. With the same slot number, the curve of the dumb scheduling is parallel to that of the smart scheduling, meaning the same DMT for the same slot number. The power gain is up to 8 and 16 dB for 6 BPCU and 10 BPCU, respectively. For 2 BPCU, the 13-slot dumb scheduling scheme is worse than the NAF, since the noise amplification is significant. As we see, the smart scheduling is always better than the dumb scheduling. In the considered cases, the 5-slot smart scheduling outperforms the 13-slot dumb scheduling. Since the optimal codes are respectively of length 5^2 and 13^2 for the 5 slot and the 13 slot cases, the use of smart scheduling can dramatically reduce the decoding complexity.

VI. Conclusion and Future Work

In this paper, we considered the class of slotted amplify-and-forward schemes. We first derived, for the SAF schemes, an upper bound of the DMT which asymptotically (when the framelength grows to infinity) achieves the MISO bound. Then, we proposed and analyzed a sequential SAF scheme for which the DMT upper bound is achieved in some special cases. In particular, the two-relay three-slot sequential SAF is optimal within the $N = 2, M = 3$ class and therefore outperforms all previously proposed two-relay AF schemes.

The superiority of the sequential SAF scheme over the previously proposed AF schemes lies in the fact that it exploits the potential diversity gain in the high multiplexing gain regime ($r > 0.5$), whereas all previously proposed AF schemes do not beat the non-cooperative scheme for $r > 0.5$. An important guideline for the design of AF schemes was then proposed : let most of the source signal be protected by extra paths. We also showed that, by using a smart relay scheduling, the complexity of decoding can be dramatically reduced. Numerical results on both the outage and error rate performance reveal a significant gain of our scheme compared to previously proposed AF schemes. Since we can always find optimal codes of finite length for any SAF scheme and the code construction is independent of the number of relays, the proposed scheme is a combination of efficiency and flexibility.

Even though we showed that the sequential SAF scheme is asymptotically optimal in some particular cases, the DMT for the general case is unknown. It would also be interesting to find a new SAF scheme, more sophisticated than the sequential one in order to improve the statistical properties of the equivalent channel matrix.

Appendix

A. Preliminaries

For any linear fading Gaussian channel

$$\boldsymbol{y} = \sqrt{\mathsf{SNR}}\, \widetilde{\boldsymbol{H}}\, \boldsymbol{x} + \boldsymbol{z}$$

where $\boldsymbol{z}$ is an AWGN with $\mathbb{E}\{\boldsymbol{z}\boldsymbol{z}^\dagger\} = \mathbf{I}$ and $\boldsymbol{x}$ is subject to the input power constraint $\mathrm{Tr}\left\{\mathbb{E}\left[\boldsymbol{x}\boldsymbol{x}^\dagger\right]\right\} \leq 1$, the DMT $d_{\boldsymbol{H}}(r)$ can be found as the exponent of the outage probability in the high SNR regime, *i.e.*,

$$\begin{aligned} P_{\text{out}}(r\log\mathsf{SNR}) &\doteq \text{Prob}\Big\{\log\det\left(\mathbf{I}+\mathsf{SNR}\,\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\dagger}\right)\le r\log\mathsf{SNR}\Big\}\\ &= \text{Prob}\Big\{\det\left(\mathbf{I}+\mathsf{SNR}\,\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\dagger}\right)\le \mathsf{SNR}^{r}\Big\}\\ &\doteq \mathsf{SNR}^{-d_{\boldsymbol{H}}(r)}. \end{aligned} \tag{17}$$

Lemma 2 (Calculation of diversity-multiplexing tradeoff): Consider a linear fading Gaussian channel defined by $\boldsymbol{H}$ for which $\det\left(\mathbf{I}+\mathsf{SNR}\,\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\dagger}\right))$ is a function of $\boldsymbol{\lambda}$, a vector of positive random variables. Then, the DMT $d_{\boldsymbol{H}}(r)$ of this channel can be calculated as

$$d_{\boldsymbol{H}}(r) = \inf_{\mathcal{O}(\boldsymbol{\alpha},r)} \varepsilon(\boldsymbol{\alpha})$$

where $\boldsymbol{\alpha}_i \triangleq -\log v_i/\log\mathsf{SNR}$ is the exponent of v_i, $\mathcal{O}(\boldsymbol{\alpha},r)$ is the outage event set in terms of $\boldsymbol{\alpha}$ and r in the high SNR regime, and $\varepsilon(\boldsymbol{\alpha})$ is the exponential order of the pdf $p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha})$ of $\boldsymbol{\alpha}$, *i.e.*,

$$p_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) \doteq \mathsf{SNR}^{-\varepsilon(\boldsymbol{\alpha})}.$$

Proof: This lemma can be justified by (17) using Laplace's method, as shown in [9]. ■

Lemma 3: Let X be a χ^2-distributed random variable with $2t$ degrees of freedom and Y be a uniformly distributed random variable in an interval including 0. Define $\xi \triangleq -\frac{\log X}{\log\mathsf{SNR}}$ and $\eta \triangleq -\frac{\log|Y|^2}{\log\mathsf{SNR}}$, then we have

$$p_{\xi} \doteq \begin{cases} \mathsf{SNR}^{-\infty} & \text{for } \xi<0,\\ \mathsf{SNR}^{-t\xi} & \text{for } \xi\ge 0; \end{cases}$$

and

$$p_{\eta} \doteq \begin{cases} \mathsf{SNR}^{-\infty} & \text{for } \eta<0,\\ \mathsf{SNR}^{-\eta/2} & \text{for } \eta\ge 0. \end{cases}$$

Lemma 4: Let $\boldsymbol{G}$ be $(k+1)\times(k+1)$ bidiagonal matrix defined by

$$\boldsymbol{G} \triangleq x_0\,\mathbf{I} + \begin{bmatrix} \mathbf{0}^{\mathsf{T}} & 0\\ \text{diag}(\boldsymbol{x}) & \mathbf{0} \end{bmatrix}.$$

Then,

$$\det\left(\mathbf{I}+\boldsymbol{G}\boldsymbol{G}^{\dagger}\right) \ge |x_0|^{2(k+1)} + \prod_{i=1}^{k}\left(1+|x_i|^2\right).$$

Proof: Define $\boldsymbol{M}_{k+1} \triangleq \mathbf{I} + \boldsymbol{G}\boldsymbol{G}^{\dagger}$ which is tridiagonal in the following form

$$\begin{bmatrix} 1+|x_0|^2 & x_0 x_1^* & \cdots & 0 \\ x_0^* x_1 & 1+|x_0|^2+|x_1|^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & x_0 x_k^* \\ 0 & \cdots & x_0^* x_k & 1+|x_0|^2+|x_k|^2 \end{bmatrix}.$$

For simplicity, let $X_i \triangleq |x_i|^2$ for $i = 0, \ldots, k$, $D_k \triangleq \det(\boldsymbol{M}_k)$ and use the formula for the calculation of the determinant of a tridiagonal matrix [17], we have

$$\begin{aligned} D_{k+1} &= (1 + X_0 + X_k)D_k - X_0 X_k D_{k-1} \\ &= (1 + X_0)D_k + X_k(D_k - X_0 D_{k-1}). \end{aligned} \tag{18}$$

Let us rewrite the last equation as

$$D_{k+1} - X_0 D_k = X_k(D_k - X_0 D_{k-1}) + D_k \tag{19}$$

and define $B_k \triangleq D_k - X_0 D_{k-1}$, from (18) and (19), we get

$$\begin{bmatrix} D_{k+1} \\ B_{k+1} \end{bmatrix} = \begin{bmatrix} 1+X_0 & X_k \\ 1 & X_k \end{bmatrix} \begin{bmatrix} D_k \\ B_k \end{bmatrix}. \tag{20}$$

First, it is easy to show that $D_2 = X_0^2 + 2X_0 + (X_1 + 1)$ and $B_2 = X_0 + X_1 + 1$. Then, from (20), it is obvious that, as a polynomial of $(X_0, \ldots, X_k)$, D_{k+1} has nonnegative coefficients for any k. Finally, as a polynomial of X_0, D_{k+1}'s coefficients can be found recursively using (18) and we have

$$D_{k+1}(X_0) = X_0^{k+1} + \prod_{i=1}^{k}(1 + X_i) + P(X_0).$$

where $P(X_0) \geq 0$ is a polynomial of X_0 and is always nonnegative. Thus, we have

$$D_{k+1} \geq X_0^{k+1} + \prod_{i=1}^{k}(1 + X_i).$$

■

B. Proof of Theorem 2

The DMT of the genie-aided model can be obtained by considering the equivalent channel matrix defined by (9). First, it is upper-bounded, as shown in the following lemma.

Lemma 5: For the genie-aided model (9), let us define $|g_{\max}|^2 \triangleq \max_{i=0\ldots N} |g_i|^2$, then we have

$$\begin{aligned}\det\left(\mathbf{I} + \mathsf{SNR}\boldsymbol{H}\boldsymbol{H}^\dagger\right) \;\dot{\leq}\; &\left(1+\mathsf{SNR}\,|g_0|^2\right)^M \\ &+ \left(1+\mathsf{SNR}\,|g_{\max}|^2\right)^{M-1}.\end{aligned} \tag{21}$$

Proof: We can prove it in a recursive manner. First, any $(n+1)\times(n+1)$ lower-triangular matrix $\boldsymbol{H}_{n+1}$ can be written as

$$\boldsymbol{H}_{n+1} = \begin{bmatrix} \boldsymbol{H}_n & \mathbf{0} \\ \boldsymbol{v}_n^\dagger & g \end{bmatrix}$$

Let us define $D_{n+1} \triangleq \det\left(\mathbf{I} + \mathsf{SNR}\boldsymbol{H}_{n+1}\boldsymbol{H}_{n+1}^\dagger\right)$ and $C \triangleq 1 + \mathsf{SNR}\,|g|^2$. Then, we have

$$\begin{aligned}D_{n+1} &= C\det\left(\mathbf{I} + \frac{\mathsf{SNR}}{C}\boldsymbol{v}_n\boldsymbol{v}_n^\dagger + \mathsf{SNR}\boldsymbol{H}_n^\dagger\boldsymbol{H}_n\right) \\ &\overset{(a)}{\leq} C\left(1+\mathsf{SNR}\lambda_1 + \frac{\mathsf{SNR}}{C}\|\boldsymbol{v}_n\|^2\right)\prod_{i=2}^{n}(1+\mathsf{SNR}\lambda_i) \\ &= C\,D_n + \mathsf{SNR}\|\boldsymbol{v}_n\|^2\prod_{i=2}^{n}(1+\mathsf{SNR}\lambda_i) \\ &\leq C\,D_n + \left(1+\mathsf{SNR}\|\boldsymbol{v}_n\|^2\right)\left(1+\mathsf{SNR}\,\|\boldsymbol{H}_n\|_{\mathrm{F}}^2\right)^{n-1} \\ &\leq C\,D_n + \left(1+\mathsf{SNR}\,\|\boldsymbol{H}_{n+1}\|_{\mathrm{F}}^2\right)^{n}\end{aligned}$$

with λ_i the i^{th} smallest eigenvalue of $\boldsymbol{H}_n\boldsymbol{H}_n^\dagger$. The inequality (a) comes from the fact that $\boldsymbol{v}_n\boldsymbol{v}_n^\dagger$ has only one nonzero eigenvalue and that for any nonnegative matrix $\boldsymbol{A}$ and $\boldsymbol{B}$, $\det(\boldsymbol{A}+\boldsymbol{B})$ is maximized when they are simultaneously diagonalizable and have eigenvalues in reverse order. By setting $\boldsymbol{H}_{n+1} = \boldsymbol{H}$ of the genie-aided model, we have

$$D_{n+1} \;\dot{\leq}\; C\,D_n + \left(1+\mathsf{SNR}\,|g_{\max}|^2\right)^n \tag{22}$$

since $\|\boldsymbol{H}_{n+1}\|_{\mathrm{F}}^2 \leq M\,|g_0|^2 + \sum_{j=1}^{N}|g_j|^2\,\|\boldsymbol{L}_j\|_{\mathrm{F}}^2 \;\dot{\leq}\; |g_{\max}|^2$ where we use the fact that $\|\boldsymbol{L}_j\|_{\mathrm{F}}^2 \;\dot{\leq}\; \mathsf{SNR}^0$ to meet the power constraint (5). The inequality (22) leads directly to (21) in a recursive manner.

■

Then, the upper bound (21) is achievable by setting $\boldsymbol{H}$ bidiagonal with

$$\boldsymbol{H} = g_0\,\mathbf{I} + \begin{bmatrix} \mathbf{0}^{\mathsf{T}} & 0 \\ g_{\max}\,\mathbf{I} & \mathbf{0} \end{bmatrix},$$

which can be justified by Lemma 4. This setting is equivalent to using uniquely the relay with the best relay-destination channel gain to send $\boldsymbol{x}_{i-1}$ during the i^{th} slot.

Now, define $\boldsymbol{\alpha} \triangleq [\alpha_{g_0} \ldots \alpha_{g_N}]$, where α_{g_i} is such that $|g_i|^2 \doteq \mathsf{SNR}^{-\alpha_{g_i}}$. By applying Lemma 2 on the right hand side (RHS) of (21), we get the DMT of the genie-aided SAF

$$\bar{d}_{\boldsymbol{H}}(r) = \inf_{\mathcal{O}(\boldsymbol{\alpha},r)} \sum_{i=0}^{N} \alpha_{g_i}$$

with

$$\mathcal{O}(\boldsymbol{\alpha}, r) = \left\{ \begin{array}{c} M(1-\alpha_{g_0})^+ < r; \\ (M-1)(1-\alpha_{g_i})^+ < r, \quad \text{for } i = 1, \ldots, N \end{array} \right\}.$$

Due to the symmetry of α_{g_i} for $i = 1, \ldots, N$, we can solve the linear programming problem by adding the constraint $\alpha_{g_1} = \ldots = \alpha_{g_N}$. Applying Theorem 1, we can get the closed-form DMT (10).

C. Lower-bound on the DMT with Isolated Relays

1) Dumb scheduling: In the $\tilde{N} = k\,N$ case with any integer k, a round-robin scheme is optimal since the $\tilde{N}$ slots are equally protected by all the relays. The RHS of (15) becomes

$$\left(1 + \mathsf{SNR}\,|g_0|^2\right)^M + \prod_{i=1}^{N} \left(1 + \mathsf{SNR}\,|g_i\,h_i|^2\right)^k. \tag{23}$$

We carry out the same calculations as in section III with some modifications. Define $\boldsymbol{\alpha} \triangleq [\alpha_{g_0} \ldots \alpha_{g_N}\ \ \alpha_{h_1} \ldots \alpha_{h_N}]$. By applying Lemma 2 on (23), we have

$$\underline{d}_{\boldsymbol{H}}(r) = \inf_{\mathcal{O}(\boldsymbol{\alpha},r)} \left(\alpha_{g_0} + \sum_{i=1}^{N} (\alpha_{g_i} + \alpha_{h_i}) \right)$$

with

$$\mathcal{O}(\boldsymbol{\alpha}, r) = \left\{ \begin{array}{c} M(1-\alpha_{g_0})^+ < r; \\ k\sum_{i=1}^{N} (1-\alpha_{g_i}-\alpha_{h_i})^+ < r \end{array} \right\}.$$

Note that by using the variable changes $\alpha'_{g_i} \triangleq \alpha_{g_i} + \alpha_{h_i}$ for $i = 1, \ldots, N$, we get a linear programming problem with symmetry of $\alpha'_{g_1}, \ldots, \alpha'_{g_N}$. The optimum must satisfy $\alpha'_{g_1} = \ldots = \alpha'_{g_N} = \beta$, and the optimization problem reduces to

$$\underline{d}_{\boldsymbol{H}}(r) = \inf_{\mathcal{O}(\alpha_{g_0},\beta,r)} (\alpha_{g_0} + N\beta) \tag{24}$$

with

$$\mathcal{O}(\alpha_{g_0}, \beta, r) = \left\{ \begin{array}{c} M(1-\alpha_{g_0})^+ < r; \\ (M-1)\beta < r \end{array} \right\}.$$

Solving this problem, we get exactly (10).

In the $\tilde{N} = kN + m$ case, the RHS of (15) is directly revised as

$$\left(1 + \mathsf{SNR}\,|g_0|^2\right)^M + \left(\prod_{n=1}^{N} \left(1 + \mathsf{SNR}\,|g_n\,h_n|^2\right)^k\right) \prod_{i=1}^{m} \left(1 + \mathsf{SNR}\,|g_i\,h_i|^2\right). \tag{25}$$

Then, we have the same optimization problem (24) with different constraints, due to the relay ordering. Using the same variable changes, we have

$$\mathcal{O}(\boldsymbol{\alpha}, r) = \left\{ \begin{array}{c} M(1-\alpha_{g_0})^+ < r; \\ k\displaystyle\sum_{i=1}^{N} (1-\alpha'_{g_i})^+ + \sum_{i=1}^{m} (1-\alpha'_{g_i})^+ < r; \\ \max\{\alpha'_{g_1}, \ldots, \alpha'_{g_m}\} \leq \min\{\alpha'_{g_{m+1}}, \ldots, \alpha'_{g_N}\} \end{array} \right\},$$

where the third constraint comes from the fact that $C_i \doteq \mathsf{SNR}^{1-\alpha'_{g_i}}$ ($\mathsf{SNR}\,|b_i|^2 \doteq \mathsf{SNR}^0$). The second and the third constraints together are equivalent to

$$\left\{ k\sum_{i=1}^{N} (1-\alpha'_{g_i})^+ + \sum_{i=1}^{m} (1-\alpha'_{g_{\mathcal{S}(i)}})^+ < r, \forall\ \mathcal{S} \subseteq \{1, \ldots, N\} \text{ and } |\mathcal{S}| = m \right\}, \tag{26}$$

from which we get a symmetric problem for α'_{g_i}, $i = 1, \ldots, N$. We can then prove the same result as the previous case.

2) Smart scheduling: Using the two "best" relays, we can arrive at (26) with $N = 2$. Since our definition of "best" also corresponds to minimum value of α'_{g_i}, it is not difficult to verify that the outage region in this case is included in the region (26). Thus, the DMT is lower-bounded by that of the dumb scheduling and the achievability is proved. ■

D. Proof of Proposition 2

Fact 1: Let $\boldsymbol{f} \triangleq \begin{bmatrix} f_1 & f_2 \end{bmatrix}^\mathsf{T}$, $\boldsymbol{U} \triangleq \begin{bmatrix} u_{11} & 0 \\ u_{21} & u_{22} \end{bmatrix}$ and $\widetilde{\boldsymbol{H}}$ be a 3×3 upper-triangular matrix defined by

$$\widetilde{\boldsymbol{H}} \triangleq \begin{bmatrix} \boldsymbol{U} & \mathbf{0} \\ \boldsymbol{f}^\mathsf{T} & g \end{bmatrix}$$

with g being a scalar. Then, we have

$$\det(\mathbf{I} + \mathsf{SNR}\, \widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^{\dagger}) = (1 + \mathsf{SNR}\,|g|^2) \det\big(\mathbf{I} + \mathsf{SNR}\,\boldsymbol{U}\boldsymbol{U}^{\dagger}\big) + \mathsf{SNR}\,\|\boldsymbol{f}\|^2 + \mathsf{SNR}^2\,|f_2 u_{11}|^2 + \mathsf{SNR}^2\,|u_{22} f_1 - u_{21} f_2|^2 . \tag{27}$$

Since non-zero multiplicative constants independent of SNR do not appear in the high SNR regime analysis, from (13), we consider the following matrix

$$\boldsymbol{H} = \begin{bmatrix} g_0 & 0 & 0 \\ g_1\, h_1 & g_0 & 0 \\ g_2\, \gamma_{12}\, h_1 & g_2\, h_2 & g_0 \end{bmatrix}, \tag{28}$$

where the coefficients $\sqrt{\mathsf{SNR}}\, b_1$ and $\sqrt{\mathsf{SNR}}\, b_2$ are neglected ($\mathsf{SNR}\,|b_i|^2 \doteq \mathsf{SNR}^0$). With (27), we can now obtain the outage event set, in terms of the entries of $\boldsymbol{H}$.

In order to apply lemma 2, however, we must get the outage event set in the high SNR regime, in terms of α. To this end, we must rewrite $|u_{22} f_1 - u_{21} f_2|^2$ in (27) in a more convenient form of positive variables. Let us use the notation $V = |v|^2$ for v being any variable. Then, from (27) and (28), we have

$$F_1 \doteq G_2 H_1 \Gamma_{12}; \qquad F_2 \doteq G_2 H_2;$$
$$U_{11} \doteq U_{22} \doteq G_0; \qquad U_{21} \doteq G_1 H_1.$$

Let us rewrite

$$\begin{aligned} |u_{22} f_1 - u_{21} f_2|^2 &= U_{22} F_1 + U_{21} F_2 - 2\sqrt{U_{21} U_{22} F_1 F_2} \cos\theta \\ &= (1 - \cos\theta)(U_{22} F_1 + U_{21} F_2) \\ &\quad + \cos\theta \left| \sqrt{U_{22} F_1} - \sqrt{U_{21} F_2} \right|^2 \end{aligned}$$

with θ uniformly distributed in $[0, \pi]$ and is independent of the other random variables. The outage probability conditioned on θ is maximized when θ is close to 0^+, where $1 - \cos\theta \approx \frac{\theta^2}{2}$. In this region, we have

$$\begin{aligned} |u_{22} f_1 - u_{21} f_2|^2 \doteq{} & \frac{\theta^2}{2}(U_{22} F_1 + U_{21} F_2) \\ & + \left| \sqrt{U_{22} F_1} - \sqrt{U_{21} F_2} \right|^2 \end{aligned} \tag{29}$$

Then, from (27) and (29), we have the outage region $\mathcal{O}(\widetilde{\boldsymbol{H}}, r)$

$$\left\{ \begin{array}{rcl} (1+\mathsf{SNR}G_0)\det(\mathbf{I}+\mathsf{SNR}\,\boldsymbol{U}\boldsymbol{U}^\dagger) & \dot{\leq} & \mathsf{SNR}^r \\ 1+\mathsf{SNR}(F_1+F_2) & \dot{\leq} & \mathsf{SNR}^r \\ 1+\mathsf{SNR}^2 F_2 U_{11} & \dot{\leq} & \mathsf{SNR}^r \\ 1+\mathsf{SNR}^2\theta^2(U_{22}F_1+U_{21}F_2) & \dot{\leq} & \mathsf{SNR}^r \\ 1+\mathsf{SNR}^2\left|\sqrt{U_{22}F_1}-\sqrt{U_{21}F_2}\right|^2 & \dot{\leq} & \mathsf{SNR}^r \end{array} \right\} \tag{30}$$

The last inequality in (30) implies

$$1+\mathsf{SNR}^2(U_{22}F_1+U_{21}F_2)\ \dot{\leq}\, \mathsf{SNR}^r+2\mathsf{SNR}^2\sqrt{U_{21}U_{22}F_1F_2},$$

which means that, in the high SNR regime, the outage region $\mathcal{O}(\widetilde{\boldsymbol{H}}, r)$ is included[5] in the region $\mathcal{O}(\boldsymbol{\alpha}, r)$ defined by

$$\left\{ \begin{array}{rcl} 3(1-\alpha_{g_0}) & \leq & r \\ (1-\alpha_{g_0})+(1-\alpha_{g_1}-\alpha_{h_1}) & \leq & r \\ 2-\alpha_{g_0}-\alpha_{g_2}-\alpha_{h_2} & \leq & r \\ 1-\alpha_{g_2}-\alpha_{\gamma_{12}}-\alpha_{h_1} & \leq & r \\ 2-\alpha_{g_0}-\alpha_{g_2}-\alpha_{\gamma_{12}}-\alpha_{h_1}-\alpha_\theta & \leq & r \\ 2-\alpha_{g_1}-\alpha_{g_2}-\alpha_{h_1}-\alpha_{h_2}-\alpha_\theta & \leq & r \\ 2-\alpha_{g_0}-\alpha_{g_2}-\alpha_{\gamma_{12}}-\alpha_{h_1} & \leq & \max\{r,\phi(\boldsymbol{\alpha})\} \\ 2-\alpha_{g_1}-\alpha_{g_2}-\alpha_{h_1}-\alpha_{h_2} & \leq & \max\{r,\phi(\boldsymbol{\alpha})\} \end{array} \right\}$$

with $\phi(\boldsymbol{\alpha}) \triangleq 2-\frac{1}{2}(\alpha_{g_0}+\alpha_{g_1}+\alpha_{\gamma_{12}}+\alpha_{h_2})-\alpha_{h_1}-\alpha_{g_2}$. Let us define

$$\begin{aligned} \mathcal{O}_{\mathcal{T}}(\boldsymbol{\alpha}, r) &\triangleq \mathcal{O}(\boldsymbol{\alpha}, r)\cap\mathcal{T}(\boldsymbol{\alpha}, r) \\ \mathcal{O}_{\overline{\mathcal{T}}}(\boldsymbol{\alpha}, r) &\triangleq \mathcal{O}(\boldsymbol{\alpha}, r)\cap\overline{\mathcal{T}}(\boldsymbol{\alpha}, r) \end{aligned}$$

with

$$\mathcal{T}(\boldsymbol{\alpha}, r) \triangleq \{\boldsymbol{\alpha}\ :\quad r\leq\phi(\boldsymbol{\alpha})\}.$$

[5]In this case, we have $\mathcal{O}(\widetilde{\boldsymbol{H}}, r)\subseteq\mathcal{O}(\boldsymbol{\alpha}, r)$ but $\mathcal{O}(\boldsymbol{\alpha}, r)\nsubseteq\mathcal{O}(\widetilde{\boldsymbol{H}}, r)$

Since $\mathcal{O}(\boldsymbol{\alpha}, r) = \mathcal{O}_{\mathcal{T}}(\boldsymbol{\alpha}, r) \cup \mathcal{O}_{\overline{\mathcal{T}}}(\boldsymbol{\alpha}, r)$, we have

$$\inf_{\mathcal{O}(\boldsymbol{\alpha},r)} \varepsilon(\boldsymbol{\alpha}) = \min\left\{ \inf_{\mathcal{O}_{\mathcal{T}}(\boldsymbol{\alpha},r)} \varepsilon(\boldsymbol{\alpha}), \inf_{\mathcal{O}_{\overline{\mathcal{T}}}(\boldsymbol{\alpha},r)} \varepsilon(\boldsymbol{\alpha}) \right\},$$

with $\varepsilon(\boldsymbol{\alpha}) = \alpha_{g_0} + \alpha_{g_1} + \alpha_{g_2} + \alpha_{h_1} + \alpha_{h_2} + \alpha_{\gamma_{12}} + \frac{1}{2}\alpha_\theta$ by lemma 3 and the independence between the random variables. Thus, the DMT can be obtained with two linear optimizations. This problem can be solved numerically using sophisticated linear programming algorithms or softwares. If the relay ordering is such that $|h_2| > |h_1|$, we add $\alpha_{h_1} > \alpha_{h_2}$ to the constraints and carry out the same optimization problem. We can finally get the DMTs of Fig. 4.

$\blacksquare$

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