Comments and Corrections

Comments on "New Inner and Outer Bounds for the Memoryless Cognitive Interference Channel and Some New Capacity Results"

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***Abstract*—In a recent paper [1], Rini *et al.* proved a capacity result for the discrete memoryless cognitive interference channel, under the condition named "better cognitive decoding." We show that this capacity region is the same as the capacity region characterized by Wu *et al.* in [2].**

***Index Terms*—Better cognitive decoding, capacity region, cognitive interference channel, strong interference, weak interference.**

I. Introduction

The capacity region of the discrete memoryless *cognitive interference channel* (DM-CIFC) is known in the "weak interference" [2] and "strong interference" [3] regimes. In a recent paper, titled "New Inner and Outer Bounds for the Memoryless Cognitive Interference Channel and Some New Capacity Results," Rini *et al.* derived a new capacity result for this channel in the "better cognitive decoding" regime, which includes the previously known capacity results [1, Th. 10].

We prove that the capacity result presented in [1, Th. 10] is the same as the capacity region established in [2, Th. 3.4] for the so-called "weak interference" regime. This is proved by showing that the "weak interference" and "better cognitive decoding" conditions are equivalent. We also determine the relationship between the "weak interference" and "strong interference" conditions in this paper.

Throughout this paper, we use the same setting and notation as in [1]. Particularly, in a two-user DM-CIFC, transmitter/receiver 1 is referred to as the *cognitive* pair and transmitter/receiver 2 is referred to as the *primary* pair.

II. Preliminaries

In what follows, we present the definitions of the "better cognitive decoding," "weak interference," and "strong interference" regimes, and their corresponding capacity results that are established by Rini–Tuninetti–Devroye (RTD) [1, Th. 10], Wu–Vishwanath–Arapostathis (WVA) [2, Th. 3.4], and Maric–Yates–Kramer (MYK) [3, Th. 5], respectively.

Definition 1. [1, eq. (15)]: The DM-CIFC is said to be in the "better cognitive decoding" regime if

$$I(U, X_2; Y_2) \leq I(U, X_2; Y_1) \tag{1}$$

for all $p(u, x_1, x_2)$.

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Definition 2. [1, eq. (6)]: The DM-CIFC is said to be in the "weak interference"[1] regime if

$$I(U; Y_2|X_2) \leq I(U; Y_1|X_2) \tag{2a}$$

$$I(X_2; Y_2) \leq I(X_2; Y_1) \tag{2b}$$

for all $p(u, x_1, x_2)$.

Definition 3. [3, eqs. (87) and (88)]: The DM-CIFC is said to be in the "strong interference" regime if

$$I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2) \tag{3a}$$

$$I(X_1, X_2; Y_2) \leq I(X_1, X_2; Y_1) \tag{3b}$$

for all $p(x_1, x_2)$.

Throughout this paper, the conditions defined in Definitions 1, 2, and 3, respectively, will be referred to as RTD, WVA, and MYK conditions.

We next present the capacity results of the DM-CIFC corresponding to the RTD, WVA, and MYK conditions.

Proposition 1 [1, Th. 10]: The capacity region of the DM-CIFC satisfying the condition (1) is the set of rate pairs (R_1, R_2) such that

$$R_1 \leq I(X_1; Y_1|X_2) \tag{4a}$$

$$R_2 \leq I(U, X_2; Y_2) \tag{4b}$$

$$R_1 + R_2 \leq I(U, X_2; Y_2) + I(X_1; Y_1|U, X_2) \tag{4c}$$

for some joint distribution $p(u, x_1, x_2)$.

Proposition 2 [2, Th. 3.4]: The capacity region of the DM-CIFC satisfying the conditions in (2) is the set of rate pairs (R_1, R_2) such that

$$R_1 \leq I(X_1; Y_1|U, X_2) \tag{5a}$$

$$R_2 \leq I(U, X_2; Y_2) \tag{5b}$$

for some $p(u, x_1, x_2)$.

Proposition 3 [3, Th. 5]: The capacity region of the DM-CIFC satisfying the conditions in (3) is the set of rate pairs (R_1, R_2) such that

$$R_1 \leq I(X_1; Y_1|X_2) \tag{6a}$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_2) \tag{6b}$$

for some $p(x_1, x_2)$.

In the following, we prove that what Proposition 1 [1, Th. 10] characterizes is merely a different representation of Proposition 2 [2, Th. 3.4]. We show this by proving that the conditions required for these two capacity regions are equivalent.

III. Main Results

In this section, we show that the "better cognitive decoding" and "weak interference" conditions are equivalent. In addition, we determine the relationship between the "weak interference" and "strong interference" conditions.

[1] In [1], this condition is referred to as "very weak interference" whereas in the original paper [2], it is called "weak interference." We use the latter in this paper.

To this end, it is useful to define two different sets of conditions, namely MYK' and MYK'', which will be shown to be equivalent to the conditions in MYK. Let MYK' be the set of constraints defined by

$$I(X_1;Y_2|X_2) = I(X_1;Y_1|X_2) \tag{7a}$$
$$I(X_2;Y_2) \le I(X_2;Y_1) \tag{7b}$$

for all $p(x_1, x_2)$. Also, let MYK'' denote the set of constraints characterized by

$$I(U;Y_2|X_2) = I(U;Y_1|X_2) \tag{8a}$$
$$I(X_2;Y_2) \le I(X_2;Y_1) \tag{8b}$$

for all $p(u, x_1, x_2)$.

Now, we are ready to point out the main contribution of this paper.

Claim 1: $\mathrm{MYK}'' \iff \mathrm{MYK}' \iff \mathrm{MYK} \Rightarrow \mathrm{WVA} \iff \mathrm{RTD}$.

Proof: We first prove that $\mathrm{RTD} \iff \mathrm{WVA}$, i.e., the "better cognitive decoding" condition implies the "weak interference" conditions and vice versa. To prove the direct implication, we show that (1) implies both inequalities (2a) and (2b). The latter inequality is achieved by setting $U = \emptyset$ in (1). To prove the former, similar to [3, Lemma 5], we can write

$$\begin{aligned} I(U;Y_2|X_2) &= I(U,X_2;Y_2|X_2) \\ &= \sum_{x_2} p(x_2) I(U,X_2;Y_2|X_2 = x_2) \\ &\le \sum_{x_2} p(x_2) I(U,X_2;Y_1|X_2 = x_2) \\ &= I(U,X_2;Y_1|X_2) \\ &= I(U;Y_1|X_2) \end{aligned} \tag{9}$$

in which the inequality follows because $I(U,X_2;Y_2) \le I(U,X_2;Y_1)$ holds for all joint distributions $p(u,x_1,x_2)$ and in particular when X_2 is a constant x_2. This completes the proof of the direct part. That is, if (1) holds for all $p(u,x_1,x_2)$, then (2a) and (2b) hold as well. The proof of the converse part is rather simple; it suffices to add up the inequalities (2a) and (2b) to get (1). Therefore, both directions are established; i.e., $\mathrm{RTD} \iff \mathrm{WVA}$.

We next show that

$$\mathrm{MYK}'' \iff \mathrm{MYK}' \iff \mathrm{MYK} \Rightarrow \mathrm{WVA}. \tag{10}$$

In consideration of the previous arguments, the only nontrivial part of (10) is to show that $\mathrm{MYK}' \Rightarrow \mathrm{MYK}''$. That is, we need to prove (7a) $\Rightarrow$ (8a). To this end, it is enough to prove that (7a) implies both

$$I(X_1;Y_2|X_2,U) = I(X_1;Y_1|X_2,U) \tag{11}$$

and

$$I(X_1,U;Y_2|X_2) = I(X_1,U;Y_1|X_2) \tag{12}$$

for all $p(u,x_1,x_2)$ because (8a) is obtained taking the differences on both sides of (12) and (11).

Suppose that $I(X_1;Y_2|X_2) = I(X_1;Y_1|X_2)$ holds for all $p(x_1,x_2)$; then, $I(X_1;Y_2|X_2,U=u) = I(X_1;Y_1|X_2,U=u)$ holds for all $p(x_1,x_2|u)$, and we can average both sides of this identity with respect to $p(u)$ in order to obtain (11). On the other hand, applying the chain rule to (12) and taking into account the Markov relation $U \to (X_1,X_2) \to (Y_1,Y_2)$, it is obvious that (12) and (7a) are equivalent. That is, (7a) implies both (11) and (12), and hence (8a). This completes the proof that $\mathrm{MYK}' \Rightarrow \mathrm{MYK}''$. ∎

From Claim 1, one can see that the "better cognitive decoding" and "weak interference" conditions are equivalent. Further, Claim 1 proves that if a DM-CIFC is in the "strong interference" regime, it will be in the "better cognitive decoding" regime, as well. But is the converse true? The answer is no, as stated in the following.

Claim 2: $\mathrm{WVA} \nRightarrow \mathrm{MYK}$.

Proof: Consider a physically degraded DM-CIFC $p(y_1|x_1,x_2)p(y_2|y_1)$, where $p(y_2|y_1)$ is nontrivial. In such a channel, (7a) cannot hold, but (2a) can. ∎

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