On the Degrees-of-Freedom of the Large-Scale Interfering Two-Way Relay Network

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Abstract

Achievable degrees-of-freedom (DoF) of the *large-scale* interfering two-way relay network is investigated. The network consists of K pairs of communication nodes (CNs) and N relay nodes (RNs). It is assumed that $K \ll N$ and each pair of CNs communicates with each other through one of the N relay nodes without a direct link between them. Interference among RNs is also considered. Assuming local channel state information (CSI) at each RN, a distributed and opportunistic RN selection technique is proposed for the following three promising relaying protocols: amplify–forward, decode–forward, and compute–forward. As a main result, the asymptotically achievable DoF is characterized as N increases for the three relaying protocols. In particular, a sufficient condition on N required to achieve the certain DoF of the network is analyzed. Through extensive simulations, it is shown that the proposed RN selection techniques outperform conventional schemes in terms of achievable rate even in practical communication scenarios. Note that the proposed technique operates with a distributed manner and requires only local CSI, leading to easy implementation for practical wireless systems.

Index Terms

Degrees-of-freedom (DoF), interfering two-way relay channel, two-way $K \times N \times K$ channel, local channel state information, relay selection.

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I. Introduction

For a three-node relay network with a single pair of communication nodes (CNs) and a single relay node (RN), two-way relay (TWR) communication, where relays receive signals from two transmitters simultaneoulsy and then send signals to the two receivers, doubles the spectral efficiency of one-way relay (OWR) communications [1], [2]. The concept of the TWR communication has been extended to multi-node interference-limited relaying networks [3]. Recently, a combined technique of network coding and interference alignment (IA) was adopted to *interfering* TWR networks in order to reduce the effect of interference [4]–[7]. On the other hand, there have been few schemes that consider a general interfering TWR network which consists of K pairs of CNs and N RNs, also known as $K \times N \times K$ interfering TWR networks. In [1], Rankov and Wittneben showed that the amplify-and-forward (AF) relaying protocol with interference-neutralizing beamforming can achieve the optimal[1] DoF of the half-duplex $K \times N \times K$ interfering TWR network if $N \geq K(K-1)+1$ for a given K. However, the scheme in [1] requires global CSI at all nodes and full collaboration amongst all RNs. The authors of [8], [9] considered the achievable degrees-of-freedom of $K \times K \times K$ interfering OWR networks, where the number of CNs and RNs are the same. In particular, the interference neutralization technique of [1] was combined with the interference alignment technique to achieve the optimal DoF of the $2 \times 2 \times 2$ interfering OWR network [8] . However, the scheme in [8] cannot be applied to the general $K \times N \times K$ interfering TWR network with arbitrary numbers of K and N. In addition, the scheme in [8] works only with global CSI assumption at each node.

The internet-of-things (IoT) concept has recently received much attection from wireless researchers, where an extremely large number of devices are expected to exist. In addition, the fifth generation (5G) cellular network is expected to support more than 10,000 devices, each of which can communicate directly with others or operate as a relay [10]. Among many devices, a small number of devices may transmit at a time due to sparse traffic pattern in the IoT scenario. Several studies have defined and studied the (N, K)-user interference channel ($N \gg K$), in which K user pairs are selected to communicate at a time [11], [12].

In this correspondence, we consider a TWR network where the number of simultaneously transmitting nodes is relatively smaller than the number of relaying nodes, which is referred to as the large-scale interfering TWR network. Specifically, we investigate the achievable DoF of the $K \times N \times K$ interfering TWR network with local CSI at each node[2] and without collaboration among nodes in the network. Three-types of relay protocols are considered: i) AF, ii) decode–forward (DF), and iii) compute–forward (CF) with lattice codes. For each source-destination pair, one of N RNs is selected to help them, and thus, an opportunistic RN selection (ORS) technique is proposed to mitigate interference. The proposed ORS technique minimizes the sum of received interference at all nodes, and thereby maximizes the achievable DoF of the network. We show that the proposed ORS technique with AF or CF relaying asymptotically achieves the optimal DoF as the number of RNs, N, increases by rendering the overall network interference-free. In particular, for given signal-to-noise ratio (SNR) and K, we derive a sufficient condition on N required to achieve the optimal DoF for AF and CF relaying, which turns out to be $N = \omega\left(\text{SNR}^{2(K-1)}\right)$[3]. On the other hand, it is shown that the DoF with DF relaying is bounded by half of the optimal DoF. Simulation results show

[1]'Optimal' DoF implies the upper-bound on the DoF of the channel, which is usually derived from simple mathematical theorems.

[2]Each node is assumed to acquire the CSI of its own incoming or outgoing channels [13].

[3]The function $f(x)$ defined by $f(x) = \omega(g(x))$ implies that $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$.

that the proposed ORS technique outperforms the conventional max-min-SNR RN selection technique even in practical communication environments.

II. System and Channel Models

Consider the time-division dupex (TDD) half-duplex $K \times N \times K$ interfering TWR network composed of K pairs of CNs and N RNs, as depicted in Fig. 1. Each pair of the CNs attempts to communicate with each other through a single selected RN, and no direct paths between the CNs are assumed, i.e., separated TWR network [2]. The two sets of CNs at one and the other sides are referred to as Group 1 and 2, respectively, as shown in Fig. 1.

The channel coefficient between the i-th CN in Group n, $n \in \{1, 2\}$, and RN j is denoted by $h_{n(i),\mathrm{R}(j)}$, $i \in \{1, \ldots, K\} \triangleq \mathcal{K}$, $j \in \{1, \ldots, N\} \triangleq \mathcal{N}$, assuming TDD channel reciprocity. It is assumed that each channel coefficient is an identically and independently distributed (i.i.d.) complex Gaussian random variable with zero mean and unit variance. In addition, channel coefficients are assumed to be invariant during the T time slots, i.e. block fading.

In the first time slot, denoted by Time 1, the CNs transmit their signals to the RNs simultaneously. In the second time, Time 2, the selected RNs broadcast their signals to all CNs. The transmit symbol at the i-th CN in Group n in Time 1 is denoted by $x_{n(i)}$. The maximum average transmit power at the CN is defined by P, and thus the power constraint is given by

$$E|x_{n(i)}|^2 \le P, \quad n = 1, 2. \tag{1}$$

Suppose that RN j is selected to serve the i-th pair of CNs. Then, the transmit symbol at RN i is denoted by $x_{\mathrm{R}(j)}$, which includes the information of both $x_{1(i)}$ and $x_{2(i)}$, and the power constraint is given by

$$E|x_{\mathrm{R}(j)}|^2 \le P. \tag{2}$$

That is, the symmetric SNRs are assumed [3].

If we denote the achievable rate for transmitting and receiving $x_{n(i)}$ by $R_{n(i)}$, the total DoF is defined by

$$\mathrm{DoF} = \lim_{\mathrm{SNR} \to \infty} \frac{\sum_{i=1}^{K} R_{1(i)} + R_{2(i)}}{\log(\mathrm{SNR})}, \tag{3}$$

where $\mathrm{SNR} = P/N_0$ and N_0 is the received noise variance.

III. Distributed & Opportunistic Relay Selection

A. Overall Procedure

1) Step 1 - Scheduling Metric Calculation: From the pilots from the $2K$ CNs in Group 1 and 2, RN j, $j \in \mathcal{N}$, estimates the channels $h_{1(i),\mathrm{R}(j)}$ and $h_{2(i),\mathrm{R}(j)}$, $i = 1, \ldots, K$. Subsequently, RN j calculates the total interference levels (TILs), which account for the sums of received interference in Time 1 at RN j and leakage of interference that it generates in Time 2. As seen from Fig. 1, the TIL at RN j for the case where it serves the i-th pair of CNs, $i \in \mathcal{K}$, is given by

$$\eta_{i,\mathrm{R}(j)} = 2 \sum_{m=1, m \ne i}^{K} \left| h_{1(m),\mathrm{R}(j),} \right|^2 + \left| h_{2(m),\mathrm{R}(j)} \right|^2 . \tag{4}$$

2) Step 2 - RN Selection: For the RN selection, we extend the distributed RN selection algorithm used in [14] for the OWR network with a single pair of source and destination.

Upon calculating $\eta_{i,\mathrm{R}(j)}$, $i = 1, \ldots, K$, RN j initiates up to K different back-off timers, which are respectively proportional to $\eta_{i,\mathrm{R}(j)}$, if $\eta_{i,\mathrm{R}(j)} < \epsilon$, where $\epsilon > 0$ is the maximum allowable interference. Specifically, RN j initiates the back-off timers $\lambda_{i,\mathrm{R}(j)}$ given by

$$\lambda_{i,\mathrm{R}(j)} = \frac{\eta_{i,\mathrm{R}(j)}}{\epsilon} T_{\max}, \tag{5}$$

where $T_{\max}$ is the maximum back-off time duration. After the back-off time $\lambda_{i,\mathrm{R}(j)}$, if no RNs have been assigned to the i-th pair of CNs, RN j announces to serve the i-th pair of CNs to all the CNs and RNs in the network and terminates the selection. Upon acknowledging this announcement, all other unselected RNs deactivate the timers corresponding to the i-th pair of CNs, i.e., $\lambda_{k,\mathrm{R}(m)}$, $k \neq j$, $m \in \{\text{unselected RNs}\}$, to exclude the consideration of the selected CNs. In this way, the RN with the smallest TIL value can be selected in a distributed fashion for each i. Through the proposed RN selection, we assume without loss of generality that RN i is selected to serve the i-th pair of CNs for notational simplicity.

Since the RN selection is done only if $\lambda_{i,\mathrm{R}(j)} < \epsilon$, the total time required to select RNs for all CNs is not greater than $T_{\max}$. Noting that $\eta_{i,\mathrm{R}(j)}$ is independent for different i or j and has a continuous distribution, the probability of a collision between $\lambda_{i,\mathrm{R}(j)}$, $i = 1, \ldots, K$'s, $j = 1, \ldots, N$, is arbitrarily small. Thus, $T_{\max}$ can be chosen arbitrarily small compared to the block length T. The efficiency for the achievable rate is lower-bounded by $\frac{T}{T+T_{\max}}$, which tends to 1 by choosing $T_{\max}$ to be arbitrarily small compared to T which is relatively large in general [9], [13].

Note that the outage takes place if any RN cannot be assigned for one or more pairs of CNs because there was no RN with TIL smaller than ϵ during the selection process. In the sequel, we derive a condition on N to make the RN selection always successful for any given ϵ. In addition, we shall find practical values of ϵ for given N through numerical simulations, which makes the outage probabilities be almost zero.

3) Step 3 - Communication: In Time 1, the CNs transmit their signals to the RNs, and the received signal at RN i is expressed as

$$y_{\mathrm{R}(i)} = \underbrace{h_{1(i),\mathrm{R}(i)} x_{1(i)} + h_{2(i),\mathrm{R}(i)} x_{2(i)}}_{\text{desired signal}} + \underbrace{\sum_{k \neq i, k=1}^{K} \left(h_{1(k),\mathrm{R}(i)} x_{1(k)} + h_{2(k),\mathrm{R}(i)} x_{2(k)} \right)}_{\triangleq I_{\mathrm{R}(i)}, \text{interference}} + z_{\mathrm{R}(i)}, \tag{6}$$

where $z_{\mathrm{R}(i)}$ accounts for the additive white Gaussian noise (AWGN) at RN i with zero mean and the variance N_0. Upon receiving $y_{\mathrm{R}(i)}$, RN i generates the transmit symbol $x_{\mathrm{R}(i)}$ from

$$x_{\mathrm{R}(i)} = f_e(y_{\mathrm{R}(i)}), \tag{7}$$

where f_e is a discrete memoryless encoding function.

In Time 2, RN i then broadcasts $x_{\mathrm{R}(i)}$, and the received signal at the i-th CN in Group n, $n \in \{1, 2\}$, is written by

$$y_{n(i)} = \underbrace{h_{n(i),\mathrm{R}(i)} x_{\mathrm{R}(i)}}_{\text{desired signal}} + \underbrace{\sum_{m \neq i, m=1}^{K} h_{n(i),\mathrm{R}(m)} x_{\mathrm{R}(m)}}_{\triangleq I_{n(i)}, \text{interference}} + z_{n(i)}, \tag{8}$$

where $z_{n(i)}$ is the AWGN with zero mean and the variance N_0. With the side information of

$x_{n(i)}$, the i-th CN in Group n retrieves the symbol transmitted from the other side from

$$x_{\tilde{n}(i)} = f_d(y_{n(i)}, x_{n(i)}), \tag{9}$$

where $\tilde{n} = 3 - n$ and f_d is a discrete memoryless decoding function.

The encoding and decoding functions, f_e and f_d, respectively, differ from relaying protocols, i.e., AF, DF, and CF. We shall specify them in the sequel in terms of DoF achievability results. The overall procedure of the proposed scheme is illustrated in Fig. 2 for the case of $K = 2$ and $N = 3$.

IV. DoF Achievability

From (6) and (8), the sum of received interference at RN i in Time 1 and at the i-th pair of CNs in Time 2, normalized by the noise variance N_0, is expressed as

$$\begin{aligned} \Delta_i &\triangleq \frac{E\left|I_{\mathrm{R}(i)}\right|^2 + E\left|I_{1(i)}\right|^2 + E\left|I_{2(i)}\right|^2}{N_0} \\ &= \left(\sum_{k\neq i, k=1}^{K} \left|h_{1(k),\mathrm{R}(i)}\right|^2 + \left|h_{2(k),\mathrm{R}(i)}\right|^2\right)\mathrm{SNR} + \left(\sum_{m\neq i, m=1}^{K} \left|h_{1(i),\mathrm{R}(m)}\right|^2 + \left|h_{2(i),\mathrm{R}(m)}\right|^2\right)\mathrm{SNR} \end{aligned} \tag{10}$$

The following lemma establishes the condition for N required to decouple the network with constant received interference even for increasing interference-to-noise-ratio (INR). In particular, even though there exist a mismatch between the TIL of (4) calculated at RN i with the local CSI and the sum of received interference in (10), we shall show in the proof of the following lemma that the proposed ORS based on the TIL of (4) can minimize the sum of received interference at all nodes, thereby maximizing the achievable DoF.

Lemma 1: [**Decoupling Principle**] For any $\epsilon > 0$, define $\mathcal{P}_C$ as

$$\mathcal{P}_C \triangleq \Pr\left\{\sum_{i=1}^{K} \Delta_i < \epsilon\right\} \tag{11}$$

$$= \Pr\left\{\sum_{i=1}^{K} \left(E\left|I_{\mathrm{R}(i)}\right|^2 + E\left|I_{1(i)}\right|^2 + E\left|I_{2(i)}\right|^2\right) < \epsilon N_0\right\}. \tag{12}$$

Using the proposed ORS, we have

$$\lim_{\mathrm{SNR}\to\infty} \mathcal{P}_C = 1, \tag{13}$$

if

$$N = \omega\left(\mathrm{SNR}^{2(K-1)}\right). \tag{14}$$

Proof: From the fact that $\sum_{i=1}^{K}\Delta_i=\sum_{i=1}^{K}\eta_{i,\mathrm{R}(i)}\mathrm{SNR}$, $\mathcal{P}_C$ in the high SNR regime can be rewritten by

$$\lim_{\mathrm{SNR}\to\infty}\mathcal{P}_C=\lim_{\mathrm{SNR}\to\infty}\Pr\left\{\sum_{i=1}^{K}\eta_{i,\mathrm{R}(i)}\mathrm{SNR}<\epsilon\right\} \tag{15}$$

$$\geq\lim_{\mathrm{SNR}\to\infty}\Pr\left\{\eta_{i,\mathrm{R}(i)}<\frac{\epsilon\mathrm{SNR}^{-1}}{K},\forall i\in\{1,\dots,K\}\right\} \tag{16}$$

$$\geq\lim_{\mathrm{SNR}\to\infty}\left(\Pr\left\{\eta_{i,\mathrm{R}(i)}<\frac{\epsilon\mathrm{SNR}^{-1}}{K}\right\}\right)^{K}, \tag{17}$$

where (17) follows from the fact that $\eta_{i,\mathrm{R}(i)}$'s are independent for different i. Since the channel coefficients are independent complex Gaussian random variables with zero mean and unit variance, $\frac{\eta_{i,\mathrm{R}(i)}}{2}$ is a central Chi-square random variable with degrees-of-freedom $4(K-1)$. Consequently, the cumulative density function of $\eta_{i,\mathrm{R}(i)}$ is given by [15]

$$F_\eta(x)=\frac{\gamma\left(2(K-1),x/4\right)}{\Gamma(2(K-1))}, \tag{18}$$

where $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt$ is the Gamma function and $\gamma(s,x)=\int_0^x t^{s-1}e^{-t}dt$ is the lower incomplete Gamma function. In addition, from [15, Lemma 1], upper and lower bounds on $F_\eta(x)$ for $0<x<2$ are given by

$$C_1\cdot x^{2(K-1)}\leq F_\eta(x)\leq C_2\cdot x^{2(K-1)}, \tag{19}$$

where

$$C_1\triangleq\frac{e^{-1}2^{-4K+3}}{(K-1)\Gamma(2(K-1))}\quad\text{and}\quad C_2\triangleq\frac{2^{-4(K-1)}}{(K-1)\Gamma(2(K-1))}. \tag{20}$$

Recall that for notational simplicity, we assume without loss of generality that RN i is selected to serve the i-th pair of CNs. In addition, let us denote that the i-th RN is selected for the i-th pair of CNs in the $\pi(i)$-th selection, where $\pi(i)\in\{1,2,\dots,K\}$. Then, the probability $\Pr\left\{\eta_{i,\mathrm{R}(i)}<\frac{\epsilon\mathrm{SNR}^{-1}}{K}\right\}$ in (17) represents the case where at the $\pi(i)$-th RN selection, a RN is assigned to the i-th pair of CNs if and only if there exists at least one RN with the TIL smaller than $\frac{\epsilon\mathrm{SNR}^{-1}}{K}$ amongst $(N-\pi(i)+1)$ unselected RNs. If we denote the set of indices of the $(N-\pi(i)+1)$ unselected RNs at the $\pi(i)$-th RN selection by $\mathcal{R}_i$, it follows that

$$\Pr\left\{\eta_{i,\mathrm{R}(i)}<\frac{\epsilon\mathrm{SNR}^{-1}}{K}\right\}=1-\Pr\left\{\eta_{i,\mathrm{R}(j)}>\frac{\epsilon\mathrm{SNR}^{-1}}{K},\forall j\in\mathcal{R}_i\right\} \tag{21}$$

$$=1-\left(1-F_\eta\left(\frac{\epsilon\mathrm{SNR}^{-1}}{K}\right)\right)^{N-\pi(i)+1} \tag{22}$$

$$\geq1-\left(1-F_\eta\left(\frac{\epsilon\mathrm{SNR}^{-1}}{K}\right)\right)^{N-K+1} \tag{23}$$

$$\geq1-\frac{\left(1-C_1\left(\epsilon/K\right)^{2(K-1)}\cdot\mathrm{SNR}^{-2(K-1)}\right)^{N}}{\left(1-C_2\left(\epsilon/K\right)^{2(K-1)}\cdot\mathrm{SNR}^{-2(K-1)}\right)^{(K-1)}} \tag{24}$$

where (24) follows from (19). From the following Bernoulli's inequality

$$(1-x)^n \le \frac{1}{1+nx},\ x\in[0,1],\ n\in\mathbb{N}, \tag{25}$$

for sufficiently large SNR to satisfy $C_1\left(\epsilon/K\right)^{2(K-1)}\text{SNR}^{-2(K-1)} \le 1$, the last term of (24) can be bounded by

$$\frac{\left(1-C_1\left(\epsilon/K\right)^{2(K-1)}\cdot\text{SNR}^{-2(K-1)}\right)^N}{\left(1-C_2\left(\epsilon/K\right)^{2(K-1)}\cdot\text{SNR}^{-2(K-1)}\right)^{(K-1)}} \le \frac{\left(1-C_2\left(\epsilon/K\right)^{2(K-1)}\cdot\text{SNR}^{-2(K-1)}\right)^{-(K-1)}}{1+N\cdot C_1\left(\epsilon/K\right)^{2(K-1)}\cdot\text{SNR}^{-2(K-1)}}. \tag{26}$$

Therefore, for increasing SNR, the term $\frac{\left(1-C_1(\epsilon/K)^{2(K-1)}\cdot\text{SNR}^{-2(K-1)}\right)^N}{\left(1-C_2(\epsilon/K)^{2(K-1)}\cdot\text{SNR}^{-2(K-1)}\right)^{(K-1)}}$ tends to 0 if and only if $N\cdot\text{SNR}^{-2(K-1)}$ in the numerator of the right-hand side of (26) tends to infinity, i.e., $N=\omega\left(\text{SNR}^{2(K-1)}\right)$. In such a case, from (24), we get

$$\lim_{\text{SNR}\to\infty}\Pr\left\{\eta_{i,\text{R}(i)} < \frac{\epsilon\text{SNR}^{-1}}{K}\right\} = 1. \tag{27}$$

Otherwise, the term $\frac{\left(1-C_1(\epsilon/K)^{2(K-1)}\cdot\text{SNR}^{-2(K-1)}\right)^N}{\left(1-C_2(\epsilon/K)^{2(K-1)}\cdot\text{SNR}^{-2(K-1)}\right)^{(K-1)}}$ in (24) tends to 1 so that $\Pr\left\{\eta_{i,\text{R}(i)} < \frac{\epsilon\text{SNR}^{-1}}{K}\right\}$ is unbounded.

From (17), (24), and (27), we have

$$\lim_{\text{SNR}\to\infty}\mathcal{P}_C \ge \lim_{\text{SNR}\to\infty}\left(\Pr\left\{\eta_{i,\text{R}(i)} < \frac{\epsilon\text{SNR}^{-1}}{K}\right\}\right)^K = 1, \tag{28}$$

if and only if $N=\omega\left(\text{SNR}^{2(K-1)}\right)$ for any $\epsilon>0$, which proves the lemma. ■

Remark 1: From Lemma 1, the $K\times N\times K$ interfering TWR network becomes K isolated TWR networks with limited interference level even for increasing INR, if $N=\omega\left(\text{SNR}^{2(K-1)}\right)$. In the proposed scheme, the dimension extension of the time/frequency domain in the conventional IA technique [13], [16] is replaced by the dimension extension in the number of users.

Now the following theorem is our main result on the DoF achievability.

Theorem 1: Using the proposed ORS scheme, the AF, LC-CF, and DF schemes achieve

$$\text{DoF}_{\text{AF}}=K,\quad \text{DoF}_{\text{LC-CF}}=K,\quad \text{DoF}_{\text{DF}}=\frac{K}{2}, \tag{29}$$

respectively, with high probability if

$$N=\omega\left(\text{SNR}^{2(K-1)}\right). \tag{30}$$

Sections IV-A, IV-B, and IV-C prove Theorem 1 providing detailed encoding and decoding functions for each scheme. In addition, Section IV-D provides comprehensive comparisons among the AF, LC-DF, and DF schemes in terms of the DoF achievability.

Note that the overall procedure of the scheduling metric calculation, RN selection, and communication protocol is analogous for all the three schemes, and the only difference appears in the encoding function f_e in (7) for constructing $x_{\text{R}(i)}$ at the RN and the decoding function f_d in (9) for retrieving $x_{1(i)}$ and $x_{2(i)}$ at the CNs.

A. Proof of Theorem 1 for AF

In the AF scheme, the relay retransmits the received signal with a proper amplification. Specifically, from the received signal $y_{\text{R}(i)}$ in (6), RN i generates the transmit signal x_{R} from

$$x_{\text{R}(i)} = \gamma_i \cdot y_{\text{R}(i)}, \tag{31}$$

where $\gamma_i > 0$ is the amplifying coefficient defined such that the power constraint (2) is met. Thus, γ_i can be obtained from

$$\gamma_i = \frac{\sqrt{P}}{\sqrt{\sum_{n=1}^{2} |h_{n(i),\text{R}(i)}|^2 P + |I_{\text{R}(i)}|^2 + N_0}}. \tag{32}$$

Inserting (31) into (8) yields the received signal at the i-th CN in Group $\tilde{n}$, $\tilde{n} \in \{0, 1\}$, given by

$$y_{\tilde{n}(i)} = \gamma_i h_{\tilde{n}(i),\text{R}(i)} \left(h_{1(i),\text{R}(i)} x_{1(i)} + h_{2(i),\text{R}(i)} x_{2(i)} + I_{\text{R}(i)} + z_{\text{R}(i)} \right) + I_{\tilde{n}(i)} + z_{\tilde{n}(i)}. \tag{33}$$

The CN then subtracts the known interference signal from $y_{\tilde{n}(i)}$ to get

$$y_{\tilde{n}(i)} - \underbrace{\gamma_i \cdot h_{\tilde{n}(i),\text{R}(i)} h_{\tilde{n}(i),\text{R}(i)} x_{\tilde{n}(i)}}_{\text{known interference}} \tag{34}$$

$$= \gamma_i h_{\tilde{n}(i),\text{R}(i)} h_{n(i),\text{R}(i)} x_{n(i)} + \gamma_i h_{\tilde{n}(i),\text{R}(i)} I_{\text{R}(i)} + \gamma_i h_{\tilde{n}(i),\text{R}(i)} z_{\text{R}(i)} + I_{\tilde{n}(i)} + z_{\tilde{n}(i)}, \tag{35}$$

where $\tilde{n} = 3 - n$. Note here that unlike the DF or LC-CF scheme, the i-th pair of CNs should have the knowledge of the effective channel $\gamma_i \cdot h_{n(i),\text{R}(i)} h_{\tilde{n}(i),\text{R}(i)}$.

From (35), the achievable rate for $x_{n(i)}$ is given by

$$R_{n(i)} = \frac{1}{2} \log \left(1 + \frac{\gamma_i^2 |h_{n(i),\text{R}(i)}|^2 |h_{\tilde{n}(i),\text{R}(i)}|^2 P}{\gamma_i^2 |h_{\tilde{n}(i),\text{R}(i)}|^2 |I_{\text{R}(i)}|^2 + |I_{\tilde{n}(i)}|^2 + \left(\gamma_i^2 |h_{\tilde{n}(i),\text{R}(i)}|^2 + 1 \right) N_0} \right). \tag{36}$$

With $N = \omega \left(\text{SNR}^{2(K-1)} \right)$, Lemma 1 gives us

$$|I_{\text{R}(i)}|^2, |I_{1(i)}|^2, |I_{2(i)}|^2 < \epsilon N_0 \tag{37}$$

for any $\epsilon > 0$ with probability $\mathcal{P}_C$. Thus, for any $\epsilon > 0$, the achievable rate is bounded by

$$R_{n(i)} \geq \mathcal{P}_C \cdot \frac{1}{2} \log \left(1 + \frac{\gamma_i^2 |h_{n(i),\text{R}(i)}|^2 |h_{\tilde{n}(i),\text{R}(i)}|^2 P}{\left(\gamma_i^2 |h_{\tilde{n}(i),\text{R}(i)}|^2 + 1 \right) \epsilon N_0 + \left(\gamma_i^2 |h_{\tilde{n}(i),\text{R}(i)}|^2 + 1 \right) N_0} \right) \tag{38}$$

$$= \mathcal{P}_C \cdot \frac{1}{2} \log \left(1 + \underbrace{\frac{\gamma_i^2 |h_{n(i),\text{R}(i)}|^2 |h_{\tilde{n}(i),\text{R}(i)}|^2}{(\epsilon + 1) \left(\gamma_i^2 |h_{\tilde{n}(i),\text{R}(i)}|^2 + 1 \right)}}_{\triangleq I'} \cdot \frac{P}{N_0} \right), \tag{39}$$

where in (38), it is assumed that zero rate is achieved unless the condition $\sum_{i=1}^{K} \Delta_i < \epsilon$ holds as in Lemma 1. Inserting (37) into (32) gives us

$$\lim_{\text{SNR} \to \infty} \gamma_i \geq \lim_{\text{SNR} \to \infty} \frac{1}{\sqrt{\sum_{n=1}^{2} |h_{n(i),\text{R}(i)}|^2 + (\epsilon + 1)\text{SNR}^{-1}}} = \frac{1}{\sqrt{\sum_{n=1}^{2} |h_{n(i),\text{R}(i)}|^2}}, \tag{40}$$

while inserting $\left|I_{\mathrm{R}(i)}\right|^2 = 0$ into (32) yields $\lim_{\mathrm{SNR}\to\infty} \gamma_i \leq \lim_{\mathrm{SNR}\to\infty} \frac{1}{\sqrt{\sum_{n=1}^{2} |h_{n(i),\mathrm{R}(i)}|^2 + \mathrm{SNR}^{-1}}}$. Thus, we have $\lim_{\mathrm{SNR}\to\infty} \gamma_i = \frac{1}{\sqrt{\sum_{n=1}^{2} |h_{n(i),\mathrm{R}(i)}|^2}}$ and hence

$$\lim_{\mathrm{SNR}\to\infty} I' = \lim_{\mathrm{SNR}\to\infty} \frac{|h_{n(i),\mathrm{R}(i)}|^2 |h_{\tilde{n}(i),\mathrm{R}(i)}|^2}{(\epsilon+1)\left(|h_{\tilde{n}(i),\mathrm{R}(i)}|^2 + 1/\gamma_i^2\right)} \tag{41}$$

$$= \frac{|h_{n(i),\mathrm{R}(i)}|^2 |h_{\tilde{n}(i),\mathrm{R}(i)}|^2}{(\epsilon+1)\left(|h_{\tilde{n}(i),\mathrm{R}(i)}|^2 + \sqrt{\sum_{n=1}^{2} |h_{n(i),\mathrm{R}(i)}|^2}\right)} \triangleq \hat{I}. \tag{42}$$

Therefore, the achievable DoF for the AF scheme is given by

$$\mathrm{DoF}_{\mathrm{AF}} = \lim_{\mathrm{SNR}\to\infty} \frac{\sum_{i=1}^{K}\sum_{n=1}^{2} R_{n(i)}}{\log(\mathrm{SNR})} \tag{43}$$

$$\geq \frac{\sum_{i=1}^{K}\sum_{n=1}^{2} \left[\lim_{\mathrm{SNR}\to\infty} \mathcal{P}_C \cdot \lim_{\mathrm{SNR}\to\infty} \frac{1}{2}\log\left(1 + I' \cdot \mathrm{SNR}\right)\right]}{\lim_{\mathrm{SNR}\to\infty} \log \mathrm{SNR}} \tag{44}$$

$$= \frac{\sum_{i=1}^{K}\sum_{n=1}^{2} 1 \cdot \lim_{\mathrm{SNR}\to\infty} \frac{1}{2}\log\left(1 + I' \cdot \mathrm{SNR}\right)}{\lim_{\mathrm{SNR}\to\infty} \log \mathrm{SNR}} \tag{45}$$

$$= \frac{\sum_{i=1}^{K}\sum_{n=1}^{2} \left[\lim_{\mathrm{SNR}\to\infty} \frac{1}{2}\log\left(\mathrm{SNR}\right) + \lim_{\mathrm{SNR}\to\infty} \frac{1}{2}\log\left(\frac{1}{\mathrm{SNR}} + I'\right)\right]}{\lim_{\mathrm{SNR}\to\infty} \log \mathrm{SNR}} \tag{46}$$

$$= \frac{\sum_{i=1}^{K}\sum_{n=1}^{2} \left[\lim_{\mathrm{SNR}\to\infty} \frac{1}{2}\log\left(\mathrm{SNR}\right) + \frac{1}{2}\log\left(0 + \hat{I}\right)\right]}{\lim_{\mathrm{SNR}\to\infty} \log \mathrm{SNR}} \tag{47}$$

$$= K, \tag{48}$$

where (45) and (47) follow from Lemma 1 and (42), respectively. On the other hand, the cut-set outer bound [2], for which no inter-node interference is assumed, yields the upper bound $\mathrm{DoF}_{\mathrm{AF}} \leq K$. Therefore, the achievable DoF with the AF scheme is $\mathrm{DoF}_{\mathrm{AF}} = K$, which proves the theorem for (29).

B. Proof of Theorem 1 for LC-CF

The LC-CF scheme is a generalized version of the modulo-2 network coding, in which $x_{1(i)}, x_{2(i)} \in \{0,1\}$ and where $x_{\mathrm{R}(i)} = \left[x_{1(i)} + x_{2(i)}\right]_2$ is retransmitted in Time 2. Specifically, in Time 1, $x_{1(i)}$ and $x_{2(i)}$ are encoded using lattice codes such that $\left[h_{1(i),\mathrm{R}(i)} x_{1(i)} + h_{2(i),\mathrm{R}(i)} x_{2(i)}\right]_\Lambda$ falls into one of the lattice points in some lattice Λ. The encoding functions that generate $x_{1(i)}$ and $x_{2(i)}$ are dependent on the channel coefficients $h_{1(i),\mathrm{R}(i)}$ and $h_{2(i),\mathrm{R}(i)}$. Thus, it is reasonable to assume that the relay designs the encoding functions and forwards the information on them to the communication nodes, since the relay can easily acquire $h_{1(i),\mathrm{R}(i)}$ and $h_{2(i),\mathrm{R}(i)}$ using the pilot signals transmitted by the CNs.

Taking the modulo-Λ to the received signal $y_{\mathrm{R}(i)}$ in (6), the RN obtains

$$\left[y_{\mathrm{R}(i)}\right]_\Lambda = \left[h_{1(i),\mathrm{R}(i)} x_{1(i)} + h_{2(i),\mathrm{R}(i)} x_{2(i)} + I_{\mathrm{R}(i)} + z_{\mathrm{R}(i)}\right]_\Lambda, \tag{49}$$

and retrieves the estimate of $\left[h_{1(i),\mathrm{R}(i)} x_{1(i)} + h_{2(i),\mathrm{R}(i)} x_{2(i)}\right]_\Lambda$ via lattice decoding [2], [3]. More detailed procedures for constructing $x_{1(i)}$, $x_{2(i)}$, and Λ are omitted, since they are analogous to those for the three-node TWR channel [2], [17], except that the considered channel includes inter-node interference terms such as $I_{\mathrm{R}(i)}$, $I_{1(i)}$, and $I_{2(i)}$. The RN then transmits the retrieved signal $x_{\mathrm{R}(i)} = \left[h_{1(i),\mathrm{R}(i)} x_{1(i)} + h_{2(i),\mathrm{R}(i)} x_{2(i)}\right]_\Lambda$, and then the i-th CN in Group n obtains $x_{\tilde{n}(i)}$

in Time 2 following the two procedures: i) estimating $x_{\mathrm{R}(i)}$ from (8) via lattice decoding, ii) obtaining $x_{\tilde{n}(i)}$ with known $x_{\mathrm{R}(i)}$ and $x_{n(i)}$ from $x_{\tilde{n}(i)} = \frac{1}{h_{\tilde{n}(i),\mathrm{R}(i)}}\left[x_{\mathrm{R}(i)} - h_{n(i),\mathrm{R}(i)}x_{n(i)}\right]_{\Lambda}$.

For this lattice encoding and decoding, it is known that the achievable rates for Time 1 are given by [2]

$$R_{n(i)} \leq \left[\frac{1}{2}\log\left(\tau_{n(i)} + \frac{|h_{n(i),\mathrm{R}(i)}|^2 P}{|I_{\mathrm{R}(i)}|^2 + N_0}\right)\right]^+, \ n = 1, 2, \tag{50}$$

where $[x]^+ = \max\{x, 0\}$ and $\tau_{n(i)} \triangleq |h_{n(i),\mathrm{R}(i)}|^2 / \left(|h_{1(i),\mathrm{R}(i)}|^2 + |h_{2(i),\mathrm{R}(i)}|^2\right)$. In Time 2, the achievable rate is determined when estimating $x_{\mathrm{R}(i)}$ from (8) [2] as

$$R_{n(i)} \leq \frac{1}{2}\log\left(1 + \frac{\left|h_{\tilde{n}(i),\mathrm{R}(i)}\right|^2 P}{\left|I_{\tilde{n}(i)}\right|^2 + N_0}\right). \tag{51}$$

With $N = \omega\left(\mathrm{SNR}^{2(K-1)}\right)$, Lemma 1 gives us $|I_{\mathrm{R}(i)}|^2, |I_{1(i)}|^2, |I_{2(i)}|^2 < \epsilon N_0$ with probability $\mathcal{P}_C$. In addition, the maximum rate of $R_{n(i)}$ is bounded by the minimum of the two bounds in (50) and (51). Thus, for $N = \omega\left(\mathrm{SNR}^{2(K-1)}\right)$, the maximum rate is given by

$$R_{n(i)} = \min\left\{\left[\frac{1}{2}\log\left(\tau_{n(i)} + \frac{|h_{n(i),\mathrm{R}(i)}|^2 P}{|I_{\mathrm{R}(i)}|^2 + N_0}\right)\right]^+, \frac{1}{2}\log\left(1 + \frac{\left|h_{\tilde{n}(i),\mathrm{R}(i)}\right|^2 P}{\left|I_{\tilde{n}(i)}\right|^2 + N_0}\right)\right\} \tag{52}$$

$$\geq \min\left\{\mathcal{P}_C \cdot \frac{1}{2}\log\left(\tau_{n(i)} + \frac{|h_{n(i),\mathrm{R}(i)}|^2 P}{(1+\epsilon)N_0}\right), \mathcal{P}_C \cdot \frac{1}{2}\log\left(1 + \frac{|h_{\tilde{n}(i),\mathrm{R}(i)}|^2}{1+\epsilon}\mathrm{SNR}\right)\right\} \tag{53}$$

$$= \min\left\{\mathcal{P}_C \cdot \left(\frac{1}{2}\log(\mathrm{SNR}) + o_1(\mathrm{SNR})\right), \mathcal{P}_C \cdot \left(\frac{1}{2}\log(\mathrm{SNR}) + o_2(\mathrm{SNR})\right)\right\}, \tag{54}$$

where $o_1(\mathrm{SNR}) = \frac{1}{2}\log\left(\tau_{n(i)}\mathrm{SNR}^{-1} + \frac{|h_{n(i),\mathrm{R}(i)}|^2}{(1+\epsilon)}\right)$ and $o_2(\mathrm{SNR}) = \frac{1}{2}\log\left(\mathrm{SNR}^{-1} + \frac{|h_{\tilde{n}(i),\mathrm{R}(i)}|^2}{1+\epsilon}\right)$.

Therefore, with $N = \omega\left(\mathrm{SNR}^{2(K-1)}\right)$, inserting (54) to (3) and following the analogous derivation from (43) to (48) give us $\mathrm{DoF}_{\text{LC-CF}} = K$, which proves Theorem 1.

Remark 2: Optimal lattice coding that achieves Shannon's capacity bound of $\log(1+SNR)$ may require excessive computational complexity in the code construction [18]. Particularly, analytical methods for shaping the Voronoi region of each lattice point to be a hyper-sphere is unknown. However, sacrificing this shaping gain by 1.53 dB in SNR, one can easily design lattice codes with practical non-binary codes such as low-density parity check codes [19], or binary multilevel turbo codes [20]. For more detailed discussion on the implementation of lattice codes, the readers are referred to [21] and references therein, or to [22] and references therein for the effort to implement practically-tailored lattice codes in two-way relay channels.

C. *Proof of Theorem 1 for DF*

In the DF scheme, each of $x_{1(i)}$ and $x_{2(i)}$ is successively decoded at RN i in Time 1 from (6). That is, $x_{1(i)}$ is decoded first regarding the rest of the terms in (6), $h_{2(i),\mathrm{R}(i)}x_{2(i)} + I_{\mathrm{R}(i)} + z_{\mathrm{R}(i)}$, as a noise term, and then is subtracted from $y_{\mathrm{R}(i)}$ to decode $x_{2(i)}$. On the other hand, $x_{2(i)}$ can be decoded first regarding $h_{1(i),\mathrm{R}(i)}x_{1(i)} + I_{\mathrm{R}(i)} + z_{\mathrm{R}(i)}$ as a noise term, and then subtracted. For this successive decoding, the rates $R_{1(i)}$ and $R_{2(i)}$ are given by the multiple-access channel rate bound [1] as follows:

$$R_{n(i)} \le \frac{1}{2}\log\left(1+\frac{\left|h_{n(i),\mathrm{R}(i)}\right|^2 P}{\left|I_{n(i)}\right|^2+N_0}\right),\ n=1,2 \tag{55}$$

$$R_{1(i)}+R_{2(i)} \le \frac{1}{2}\log\left(1+\frac{\left(|h_{1(i),\mathrm{R}(i)}|^2+|h_{2(i),\mathrm{R}(i)}|^2\right)P}{\left|I_{\mathrm{R}(i)}\right|^2+N_0}\right). \tag{56}$$

In Time 2, from individually decoded $x_{1(i)}$ and $x_{2(i)}$, the network coding is used to construct $x_{\mathrm{R}(i)}$ at the RN as in the LC-CF scheme. Thus, the achievable rates for Time 2 are given again by (51). Combining (55), (56), and (51) together, we obtain the maximum sum-rate as

$$R_{1(i)}+R_{2(i)} = \min\left\{\sum_{n=1}^{2}\min\left\{\frac{1}{2}\log\left(1+\frac{\left|h_{n(i),\mathrm{R}(i)}\right|^2 P}{\left|I_{n(i)}\right|^2+N_0}\right),\frac{1}{2}\log\left(1+\frac{\left|h_{\tilde{n}(i),\mathrm{R}(i)}\right|^2 P}{\left|I_{\tilde{n}(i)}\right|^2+N_0}\right)\right\},\right.$$
$$\left.\frac{1}{2}\log\left(1+\frac{\left(|h_{1(i),\mathrm{R}(i)}|^2+|h_{2(i),\mathrm{R}(i)}|^2\right)P}{\left|I_{\mathrm{R}(i)}\right|^2+N_0}\right)\right\}. \tag{57}$$

From Lemma 1, with $N=\omega\left(\mathrm{SNR}^{2(K-1)}\right)$, we have $|I_{\mathrm{R}(i)}|^2, |I_{1(i)}|^2, |I_{2(i)}|^2 < \epsilon N_0$ with probability $\mathcal{P}_C$. In such a case, the maximum sum-rate is bounded by

$$R_{1(i)}+R_{2(i)} \ge \mathcal{P}_C\cdot\min\left\{\sum_{n=1}^{2}\min\left\{\frac{1}{2}\log\left(1+\frac{\left|h_{n(i),\mathrm{R}(i)}\right|^2}{\epsilon+1}\mathrm{SNR}\right),\frac{1}{2}\log\left(1+\frac{\left|h_{\tilde{n}(i),\mathrm{R}(i)}\right|^2}{\epsilon+1}\mathrm{SNR}\right)\right\},\right.$$
$$\left.\underbrace{\frac{1}{2}\log\left(1+\frac{\left(|h_{1(i),\mathrm{R}(i)}|^2+|h_{2(i),\mathrm{R}(i)}|^2\right)}{\epsilon+1}\mathrm{SNR}\right)}_{\triangleq\Delta_2}\right\} \tag{58}$$

$$=\mathcal{P}_C\cdot\min\left\{\underbrace{\log\left(1+\frac{\min\left\{\left|h_{1(i),\mathrm{R}(i)}\right|^2,\left|h_{2(i),\mathrm{R}(i)}\right|^2\right\}}{\epsilon+1}\mathrm{SNR}\right)}_{\triangleq\Delta_1},\Delta_2\right\} \tag{59}$$

For arbitrarily large SNR and with $h_{1(i),\mathrm{R}(i)}, h_{2(i),\mathrm{R}(i)} \neq 0$, we have $\Delta_1 > \Delta_2$ since

$$\left(1+\frac{\min\left\{\left|h_{1(i),\mathrm{R}(i)}\right|^2,\left|h_{2(i),\mathrm{R}(i)}\right|^2\right\}}{\epsilon+1}\mathrm{SNR}\right) > \left(1+\frac{|h_{1(i),\mathrm{R}(i)}|^2+|h_{2(i),\mathrm{R}(i)}|^2}{\epsilon+1}\mathrm{SNR}\right)^{1/2}. \tag{60}$$

Therefore, for large SNR, the sum-rate can be further expressed by

$$R_{1(i)}+R_{2(i)} \ge \mathcal{P}_C\cdot\frac{1}{2}\log\left(1+\frac{|h_{1(i),\mathrm{R}(i)}|^2+|h_{2(i),\mathrm{R}(i)}|^2}{\epsilon+1}\mathrm{SNR}\right). \tag{61}$$

Applying (61) to (3) and following the analogous derivation from (43) to (48), we can only achieve $\text{DoF}_{\text{DF}} = K/2$, even under the interference-limited condition, i.e., $N = \omega\left(\text{SNR}^{2(K-1)}\right)$.

D. Remark of Theorem 1: Comparison among the AF, DF, and LC-CF schemes

Since the AF scheme only performs power scaling at the RNs, it is the simplest for implementation but achieves the optimal DoF of the network. However, the CN-to-CN effective channel gain should be known by the CNs, and the scheme suffers from the noise propagation, particularly in the low SNR regime. The DF scheme requires the minimum of the CSI, and the conventional simple coding scheme can be used as in the AF scheme. Since the noise at the RNs is removed from the decoding at the RSs, it does not propagate the noise at the RSs. Nevertheless, the scheme only achieves the half of the optimal DoF. The LC-CF scheme attains benefits from both AF and DF schemes, i.e., the optimal DoF and removal of the noise at the RNs through decoding. On the other hand, the scheme requires lattice encoding and decoding, but the design of an optimal lattice code for given channel gains requires an excessive computational complexity [2]. The suboptimal design of lattice codes can be considered as discussed in Remark 2.

V. Numerical Examples

For comparison, two baseline schemes are considered: max-min-SNR and random selection schemes. In the max-min-SNR scheme, RN selection is done such that the minimum of the SNRs of the two channel links between the serving RN and two CNs is maximized at each selection.

Figure 3 shows the sum-rates versus SNR for $K = 2$, where N increases with respect to SNR according to Theorem 1, i.e., $N = \text{SNR}^{2(K-1)}$. As an upper-bound, the sum-rate of the proposed LC-CF ORS scheme but with no interference is also plotted, the DoF of which is K. It is seen that the proposed AF and LC-CF schemes achieve the DoF of K as derived in Theorem 1, whereas the max-min and random selection schemes achieve zero DoF due to non-vanishing interference. On the other hand, the DoF of the proposed DF scheme achieves only $K/2$, which also complies with Theorem 1. It is interesting to note that even the proposed LC-CF scheme cannot achieve K DoF if N scales slower than $\text{SNR}^{2(K-1)}$, as shown in the example of the $N = \text{SNR}^{(K-1)}$ case which is labeled as 'Prop. LC-CF ORS w/ $N = \text{SNR}^{(K-1)}$' in Fig. 3.

Figure 4 show the sum-rates versus SNR for $K = 2$ and (a) $N = 20$ or (b) $N = 50$. With fixed and small N, the max-min-SNR schemes outperform the proposed ORS schemes in the low SNR regime, where the noise is dominant compared to the interference. However, the sum-rates of the proposed schemes exceed those of the max-min schemes as the SNR increases, because the interference becomes dominant than the noise. As a consequence, there exist a crossover SNR point for each case. As seen from Fig. 4, these crossover points becomes low as N grows, since the proposed schemes exploit more benefit as N increases. The proposed schemes outperform the max-min-SNR schemes for the SNR greater than 7 dB with $N = 50$ as shown in Fig. 4(b).

Figure 5 shows the sum-rates versus N when $K = 2$ and SNR is 20 dB. It is seen that the proposed ORS scheme greatly enhances the sum-rate of the max-min-SNR scheme for all the cases. The LC-CF scheme exhibits the highest sum-rates amongst the three relay schemes for mid-to-large N regime, whereas it slightly suffers from the rate loss due to $\tau_{n(i)} \leq 1$ in (50) in the small N regime. The sum-rate of the proposed AF scheme becomes higher than that of the DF scheme as N increases, because the AF achieves higher DoF, as shown in Theorem 1.

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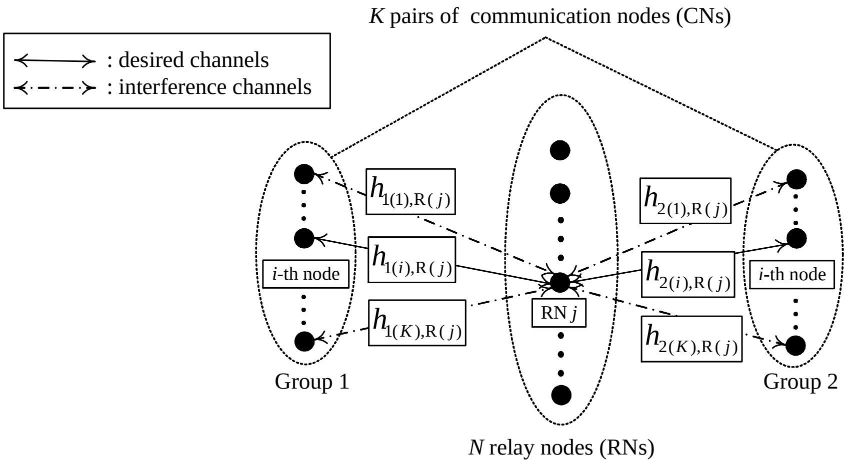

Fig. 1. The $K \times N \times K$ interfering two-way relay network.

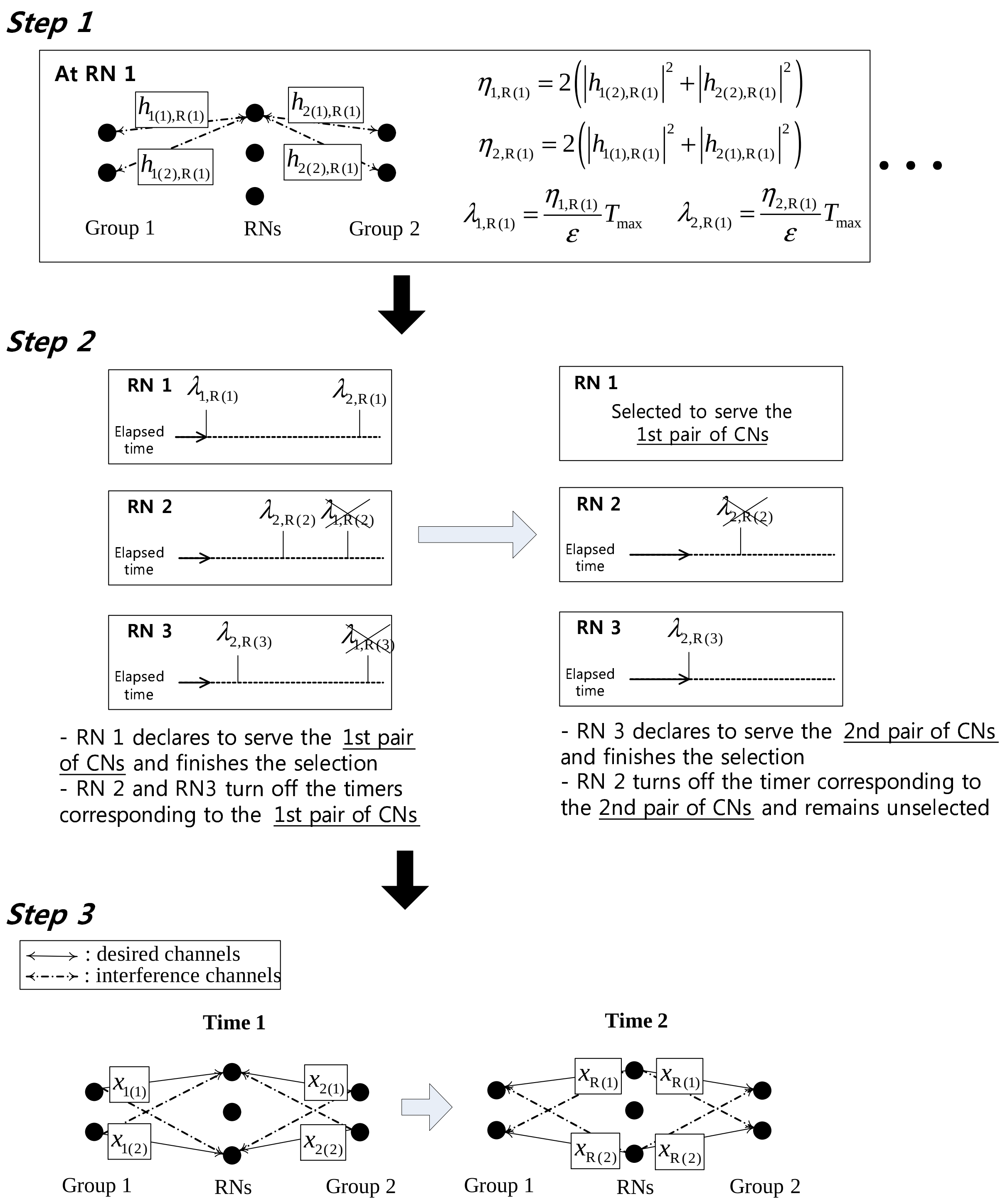

Fig. 2. Overall procedure of the proposed scheme for $2 \times 3 \times 2$ interfering two-way relay network.

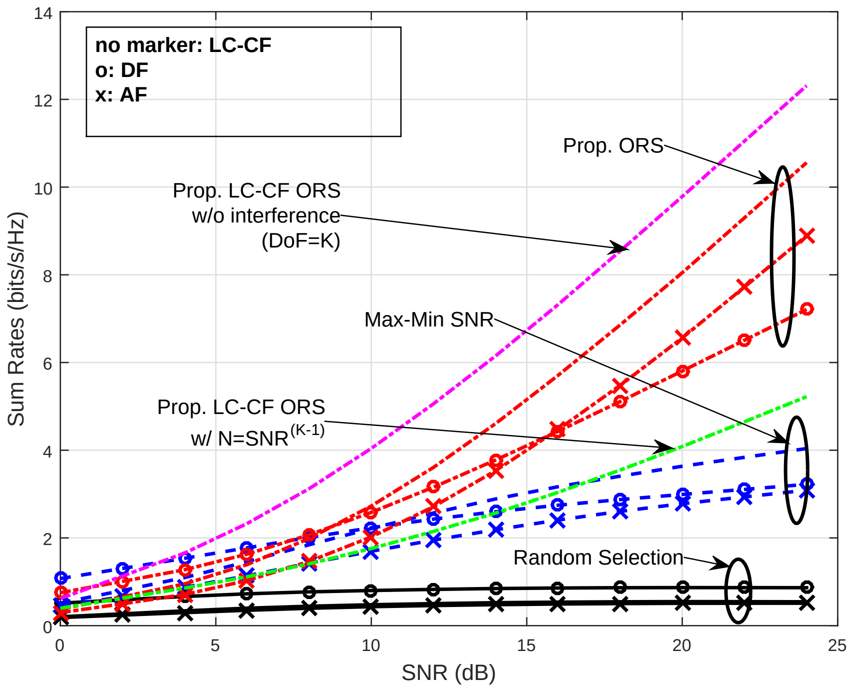

Fig. 3. Rates versus SNR with $K = 2$ and $N = \text{SNR}^{2(K-1)}$.

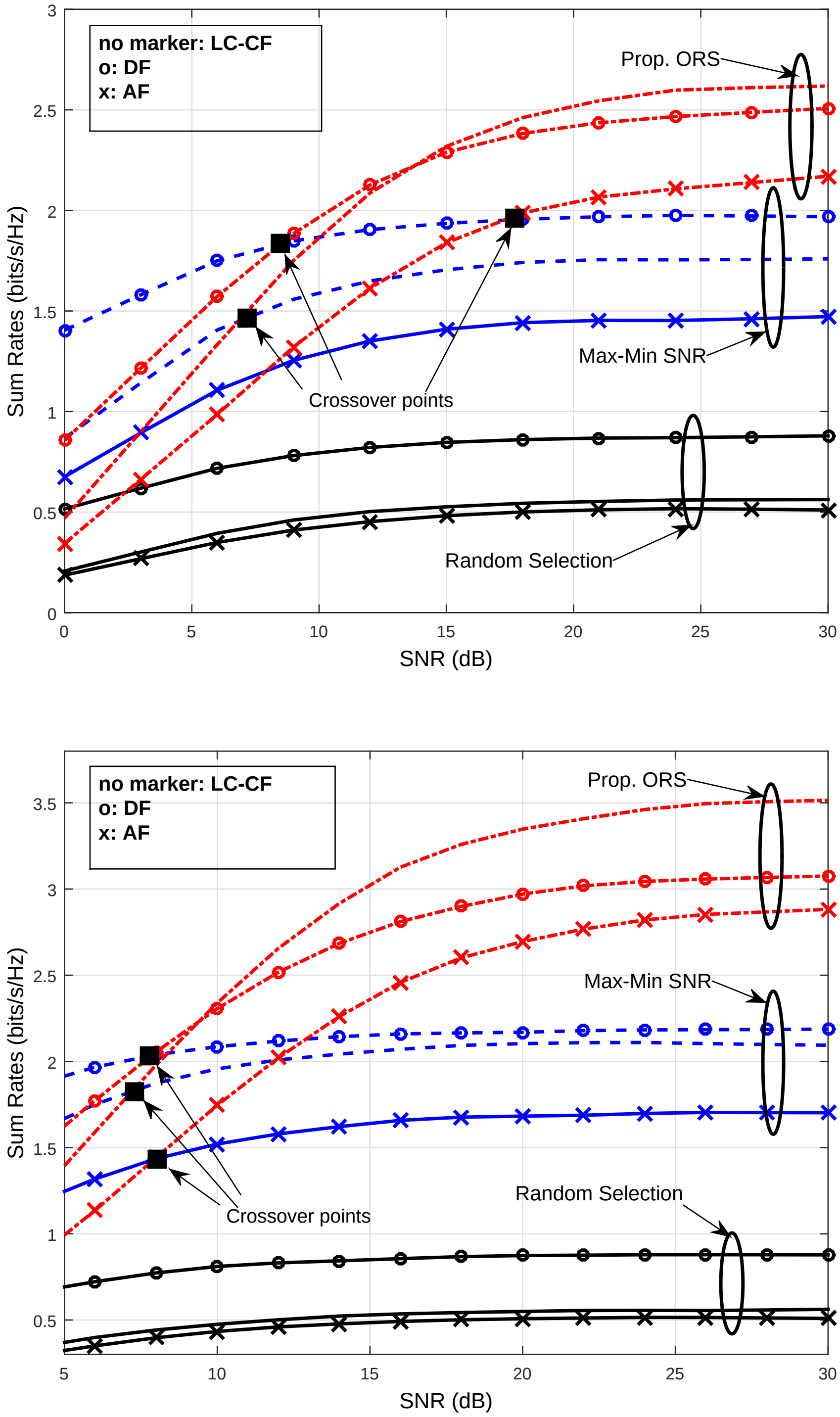

Fig. 4. Rates versus SNR when $K = 2$ and (a) $N = 20$ or (b) $N = 50$.

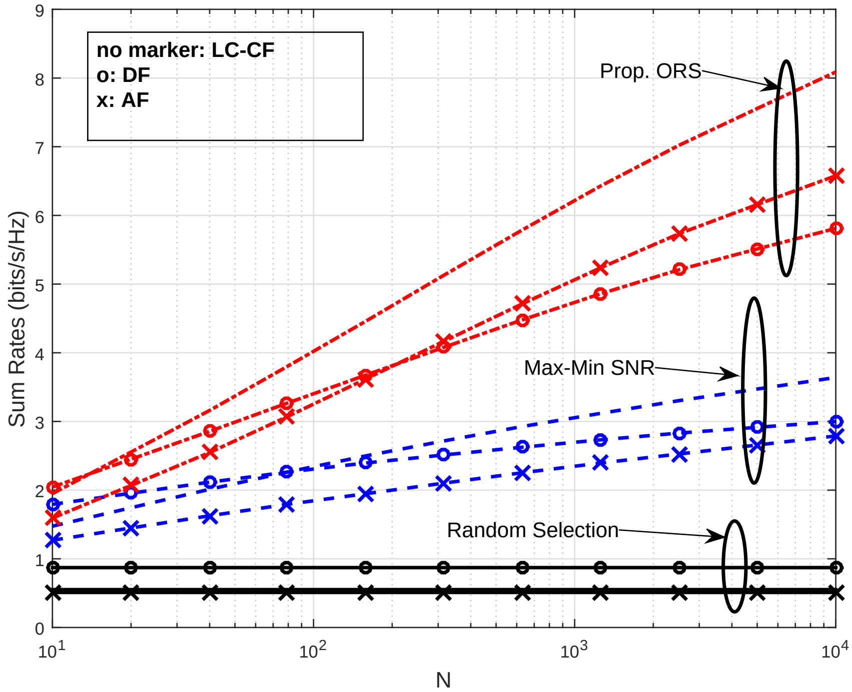

Fig. 5. Rates versus N when $K = 2$ and SNR=20dB.