Reputation Cascade Model Over Social Connections in Online Social Networks

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Abstract

Reputation is turning to be one of the most important concerns in online social networks. Users in such environments extend their activity area with respect to the possible updates on their reputation level. In this paper, we propose a probabilistic model of reputation cascade through the network. This model helps users to select the best possible choice of interaction. Furthermore, we also provide a game theoretic approach to analyze the model.

1. Introduction

Understanding how users' reputation changes (increases or decreases) inside online communities is a fundamental question [1]. Agents' social relations inside Online Social Networks (OSNs) have crucial effect on their current and future level of reputation. Agents pass their reputation or their friends' reputation to other agents for many purposes (e.g. information gathering, recommendation, trust evaluation, etc.). In the last few years, some researchers proposed reputation mechanisms within the platform of OSNs [6], [4], [5].

In this paper, we introduce two main factors: 1) *local reputation*, which reflects current reputation status of the agent in his local network; and 2) *probability of reputation cascade*, which is the probability that the evaluating agent's reputation will cascade in other networks upon joining other agents' networks. This work presents a game theoretic formulation of agent interaction in the platform of OSNs with respect to the reputation of agents among their friends and the possibility of cascading this reputation. Beside having large friends community, agents have to consider the effectiveness of their new connections in terms of reputation propagation and maximization.

The rest of this paper is organized as follows. In Section 2, we develop a new reputation cascade model in OSN context. In Section 3, we define a reputation cascade game with incomplete information. In Section 4, we experimentally show the results of our prediction method on $epinions$ dataset.

2. Reputation Cascade Model

Reputation cascade through ONSs is known as a phenomenon rather than just an artifact. Such a natural phenomenon in large-scale social networks seems to be epidemic [7]. Therefore we propose a model of reputation propagation to analyze this natural phenomenon.

2.1. Model Background

In this section, we model the cascade of reputation in OSNs and emphasize the reputation effects, which are generated by word-of-mouth phenomenon. Here social ties between agents reflect agents' friendship relation and the weight of those social ties (rate that each agent associate to his friends) represents the level of mutual trust between them. In this model, friendship between agents is a bidirectional relation. This feature comes from the fact that friendship relation is always in the favor of two pairs and it is not considered independently for pairs. Depending on the structure of the network in which an agent is located, reputation level of his friends and agent's level of reputation among his friends this agent's reputation can cascade virally or fall into a network hole (can be considered as an isolated and local network cluster) and propagation process terminates. Thus, reputation is an important factor on the formation of social ties between agents. To this end, two main features are considered to measure the worth of new friendship establishment: First, probability of reputation cascade and second, current level of reputation.

2.2. Model Setting

In this section, we present the formal definitions of the reputation cascade model in a *Online Social Network*(OSN).

***Definition** 1: (OSN)* An OSN is a tuple $\Gamma = \langle \Omega, E, \tau \rangle$ which Ω is the set of agent profiles in the network, E is the set of agents' connections, and τ is the current living time of the social network.

***Definition** 2: (Agent Profile δ_a)* The Profile of an agent a is a tuple $\delta_a = \langle Rep, \phi, F \rangle$, where Rep is the reputation rank that each agent assigns to himself, ϕ is the set of agent's friends, and F is the reputation ranking function, $F : \phi \rightarrow [0, 1]$, that agent a uses to assign ranks to his friends .

***Definition** 3: (Profile Access Operator $' \rightarrow'$)* The profile access operator is a binary operator used to access an agent's profile elements. Let δ_i be the agent i's profile, $\delta_i \rightarrow \phi$ reflects i's set of friends, $\delta_i \rightarrow F(a)$ represents i's associated reputation rank to his friend a, and $\delta_i \rightarrow Rep$ represents reputation rank that i assignes to himself.

***Definition** 4:* (*Join Operator* $'\triangleright'$) Let a and b are agents in OSNs, $a \triangleright b$ represents the new friendship relation, means a joins b's friend list (reflexively b joins agent a's friends list $a \triangleright b \Longleftrightarrow b \triangleright a$). Relatively $a \not\triangleright b$ means that agent a and b could not establish mutual trust between each other and therefore agent a does not join b's friends list.

2.3. Friendship Formation Analysis

Friendship relation between pairs in OSNs is formed in a continuous way. Forming a new friendship relation generally has two main effects on (1) agents' local reputation among his friends; and (2) probability of cascade of agents' reputation in the network. We compute local reputation $L\text{Rep}(\delta_a \rightarrow \phi)$ of agent a (with agent profile δ_a) among his friends using equation 1.

$$L\text{Rep}(\delta_a \rightarrow \phi) = \frac{\sum\limits_{f \in \delta_a \rightarrow \phi} \delta_f \rightarrow F(a)}{|\delta_a \rightarrow \phi|} \tag{1}$$

In addition to the local reputation increase tendency, agents would take the probability of reputation cascade (PoC in equation 2) into account while they are making decisions for their further actions.

$$PoC(\delta_a | P(a \triangleright b)) = \mu + \nu \tag{2}$$

where

$$\mu = \beta \prod_{j \in \zeta} \delta_j \rightarrow F(b)$$

$$\nu = (1 - \beta) \prod_{j \in \zeta'} \delta_j \rightarrow F(b) P(a \triangleright b)$$

$$PoC(\delta_a | P(a \not\triangleright b)) = \prod_{j \in \delta_a \rightarrow \phi} \delta_j \rightarrow F(a).P(a \not\triangleright b) \tag{3}$$

where $\zeta = \{\delta_a \rightarrow \phi\} \Delta \{\delta_b \rightarrow \phi\} - \{j | \delta_j \rightarrow F(b) = 0\}$ and $\zeta' = \{\delta_a \rightarrow \phi\} \cap \{\delta_b \rightarrow \phi\}$.

Here we calculate the probability of joining $P(a \triangleright b)$ (similarly for $P(b \triangleright a)$), represented in equation 4.

$$\begin{aligned} \omega &= \{x | x \in \delta_a \rightarrow \phi - [\delta_b \rightarrow \phi \cap \delta_a \rightarrow \phi]\} \\ \Upsilon &= \{y | y \in \delta_b \rightarrow \phi - [\delta_b \rightarrow \phi \cap \delta_a \rightarrow \phi]\} \\ \Psi_a &= \{z \in \omega | \forall j \in \Upsilon, z \in \delta_j \rightarrow \phi\} \\ \Psi_b &= \{v \in \Upsilon | \forall k \in \omega, z \in \delta_k \rightarrow \phi\} \end{aligned}$$

$$P(a \triangleright b) = \begin{cases} \varepsilon & \text{if } \delta_b \rightarrow \phi = \emptyset; \\ \sigma.\theta & \quad otherwise. \end{cases} \tag{4}$$

where

$$\sigma = \alpha \frac{|\delta_a \rightarrow \phi \cap \delta_b \rightarrow \phi|}{|\delta_a \rightarrow \phi \cup \delta_b \rightarrow \phi|} + (1 - \alpha) \frac{|\Psi_a \cup \Psi_b|}{|\delta_b \rightarrow \phi \Delta \delta_a \rightarrow \phi|}$$

$$\theta = \frac{\sum\limits_{j \in \Upsilon \cup \{b\}} \delta_j \rightarrow F(a)}{|\Upsilon \cup \{b\}|}$$

We also calculate the probability that agent a does not join agent b's friends list by the following formula,

$$P(a \not\triangleright b) = 1 - P(a \triangleright b)$$

Without loss of generality, we assume that agents are self-interested, which makes them acting strategically in order to obtain high reputation level among their friends and thus, the network. Doing so, a particular agent a naturally selects a potential set of friends l_a (Tr is a proper threshold) as a choice of further expansion of friendship. This set is derived from equation 5.

$$l_a = \{x \in \delta_a \rightarrow \phi | \quad |\delta_x \rightarrow F(a) - \delta_a \rightarrow \text{Rep}| < Tr\} \tag{5}$$

As a selfish agent, agent a in order to maximize his $LRep$ and PoC searches for the best possible friends. Finding the best possible agent to be a friend, who would maximize agent a's $LRep$, is basically an optimization problem. We define this problem in equation 6 in order to find Lk^*. We also find Ck^* as an agent in the network of a friend of agent a that upon joining, would maximize a's PoC in equation 7 and similarly CLk^* as an agent that maximizes the combination of agent a's $LRep$ and PoC values in equation 8.

$$Lk^* = \underset{j \in l_a, k \in \delta_j \rightarrow \phi}{\arg\max} \quad \delta_k \rightarrow F(a) \tag{6}$$

$$Ck^* = \underset{j \in l_a, k \in \delta_j \rightarrow \phi}{\arg\max} \quad PoC(\delta_a | P(a \triangleright k)) \tag{7}$$

$$CLk^* = \underset{j \in l_i, k \in \delta_j \rightarrow \phi, 0 \leq \lambda \leq 1}{\arg\max} \quad G + G' \tag{8}$$

where

$$G = \lambda L\text{Rep}(\delta_i \rightarrow \phi \cup \{k\})$$

$$G' = (1 - \lambda) PoC(\delta_i | P(i \triangleright k))$$

After finding these maximizing agents (for the case of multiple solution in each maximization we randomly select one of them in order to have a unique answer for each one), the arising question is: which agent should be joined in order to obtain the most benefit (in terms of reputation formation) for agent a? We address this issue by providing a cascade game that is introduced in the following section.

3. Reputation Cascade Game with Imperfect Information

Properties such as scale of network, simultaneous actions of agents, privacy options and etc. increase the unobservability of agents behavior in the network. Let us assume that agents are rational entities who adopt actions in order to maximize their reputation (local reputation) and propagates (cascades) this reputation among network users. This leads us to the point of defining a game theoretic model of reputation formation and propagation inside OSNs. We

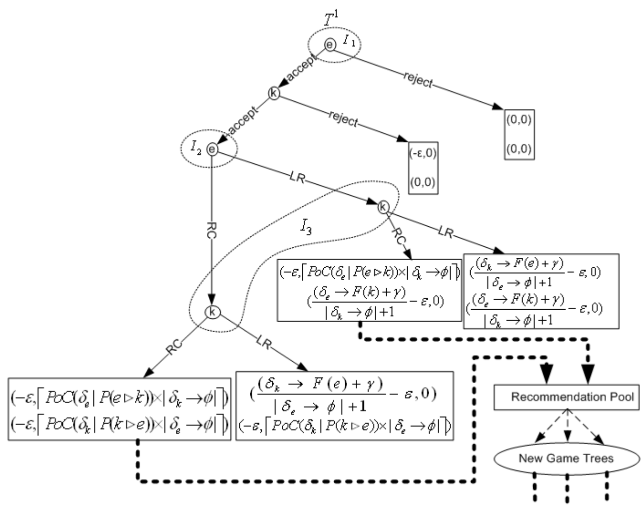

Figure 1. One shot interaction. Circles represent decision nodes (non-terminals), edge labels represent actions, dotted ovate represents an information set. Payoffs are represented in rectangles (terminals), the top row describes the payoff of the evaluator agent, the second row describes the target agent.

Consider an OSN Γ with the set of active, self-interested and rational agents. For modeling agents' interactions, we introduce the *Cascade Game* as follows:

***Definition** 5: (Cascade Game) An extensive form game with imperfect information for agent interaction is a tuple $\langle N, A, T, \pi, I \rangle$ where:*

- $N = \{1, 2, ..., n\}$ *is a finite set of n players;*
- $A = \{LR, RC, accept, reject\}$ *represents action profile of all agents in the game. These actions are: Local-Reputation (LR), Reputation-Cascade (RC), reject and accept friends. LR means increasing opponent's level of reputation with the portion of γ and RC means cascading the opponent player's reputation among the opponent's friends list (here we assume cascading an agent's reputation as recommending him to the proponent's friends).*
- $T = \{\langle T^1, T^2, T^3, ..., T^{n-1} \rangle\}$, *$T^i$=($V^i$, E^i) is an ordered set of directed trees, where nodes V^i and edges E^i are elements of directed tree T^i. We split V^i in two parts: TR^i and NTR^i, where TR^i represents set of terminal nodes and NTR^i represents set of non-terminal nodes in the tree.*
- $\pi = \{\pi_1, \pi_2, ..., \pi_n\}$, *where* $\pi_l : A \times A \rightarrow \mathbf{R} \times \mathbf{R}$ *represents payoff function of players in the game.*
- $I = \{I_1, I_2, ..., I_j\}$ *is the set of of information sets, available for agents in different steps of the game.*

In this game, we assume that player 1 plays the role of $evaluator$ agent e (agent who wants to find the proper friend), player 2 plays the role of *target agent* k (agent who is being evaluated). Agent k is the aggregation set of all Lk^*s, Ck^*s and CLk^*s (formation of this set will be discussed later in this section). We assume that agents only play pure strategies, which means that if an agent plays LR that means this agent does not want to cascade the $evaluator$ agent's reputation among his friends (reputation cascade can be interpreted as propagation of recommendation to join among his friends to motivate them in order to add $evaluator$ agent to their friends list) and therefore reputation propagation of the $evaluator$ agent will terminate. If agent plays RC that means he will propagate $evaluator$ agent's reputation to at least one of his friends, which we assume as a recommendation to add $evaluator$ agent to his friends list. We calculate the number of agent k's recommendations to his friends to join agent e after playing RC in the following formula:

$$\lceil PoC(\delta_e | P(e \triangleright k)) \times |\delta_k \rightarrow \varphi| \rceil$$

In general the game for $evaluator$ agent runs until the time that he considers the $reject$ action as a dominant strategy for the rest of his life time in the network or there is no agent in the network to accept his request. Upon adopting any action, players would obtain the corresponding payoffs represented in Figure 1. $Evaluator$ agent plays the game with the first 3 agents, LK^*, CK^*, and CLK^* in order to predict the best possible agent to join as well as best possible strategy to play. Therefore, in the beginning we have 3 different game trees. Figure 1 represents general game tree template in the sense that k could be LK^*, CK^*, and CLK^*, representing one-shot interaction, including agent payoffs.

Payoff for each player i (π_i) consists of a pair of two elements, the first element is the amount of local reputation increase and the second element represents the number of agents who adopt agents' reputation (clearly shown in figure 1). For agent e we have three information sets in T^1. The first information set is in the beginning of the game, once agent e decides to accept or reject to join. The second information set (I_2) is when the agent e wants to decide whether to play LR or CR given he observes action $accept$ from k. Third information set, I_3, is formed once agent e is in the situation that has to play RC or LR given that he observed RC or LR action from the other agent k.

4. Experimental Result

We employed $epinions.com$ dataset, gathered in the period of three weeks crawling [2]. We choose $epinions$ because it satisfies four properties of web based social networks discussed in Golbeck [3].

Since our evaluation environment is large scale, we have to randomly choose a set of agents from the network to test our model. Therefore we choose 100 agents for our evaluation. We apply our reputation cascade model on $epinions$ to

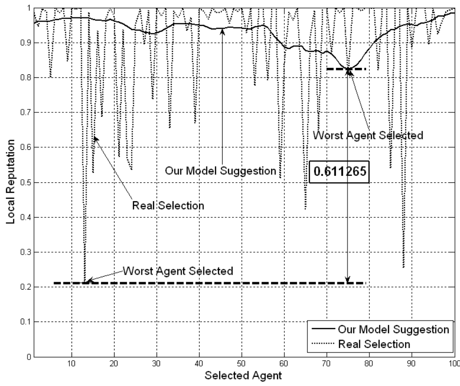

Figure 2. Local reputation of selected agents in Real-World data in comparison to the suggested agent.

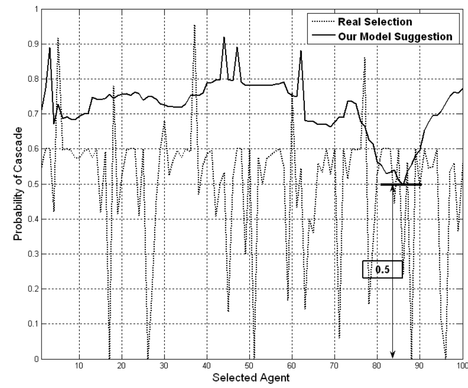

Figure 3. Probability of Reputation Cascade of selected agent in Real-World data in comparison to our suggested agent.

find the best possible agent to join in the next activity period of time. After that, we compare average of reputation and probability of reputation cascade of our suggested agent with the selected agent in the next activity period. We assume that $\varepsilon = 0.05$ (ε is the probability that an agent joins the other agent who does not have any friends). Figure 2represents the trend of average reputation change of our selections in three time steps in comparison with the real selection. As it is shown in figure 2, the difference between our selection out of 100 selection in the worst performance and the worst selection in reality out of 100 selections is 0.611255 which is a noticeable difference.

In figure 3, we show the trend of probability of cascade for 100 selection of our proposed model (following aforementioned criteria of selection) in comparison with the selection in the reality. As it was predictable according to the previous sections, our model is more efficient in terms of PoC because agents naturally do not consider this probability in the time of selection. This result is considerably important in the context of viral marketing since service providers in general want to increase the popularity of their services in the network. Therefore, in terms of advertisement and investment for obtaining more consumers in the future, suppliers normally look for the best possible agents who maximize distribution and reflection of their providing service among the other agents in the network.

5. Conclusion

We have formulated reputation formation and propagation in the context of OSNs using two concepts of current and future level of reputation. We proposed two factors: local reputation and probability of cascade of reputation, and analyzed the users tendency to increase each of these two factors in both experimental and theoretical settings. We employed a game theoretic approach to analyze rational agents strategic behaviors in selecting actions given the current reputation level of agents. In the experimental evaluation of our model, we analyzed $epinions$ network users' natural tendency on propagation and attraction of reputation. We tested our model on $epinions$ and we observed high performance of our prediction in comparison with the real selection that user made. This approach is general enough to be applied to a wide variety of online communities, such as online social network advertisement.

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