


Parametric Timed Pattern Matching*

Masaki Waga 

Kyoto University, Japan

Étienne André 

Université de Lorraine, CNRS, Inria, LORIA, Nancy, France

Ichiro Hasuo 

National Institute of Informatics, Japan

The Graduate University for Advanced Studies, Japan

Abstract

Given a log and a specification, timed pattern matching aims at exhibiting for which start and end dates a specification holds on that log. For example, “a given action is always followed by another action before a given deadline”. This problem has strong connections with *monitoring* real-time systems. We address here timed pattern matching in the presence of an *uncertain* specification, i. e., that may contain timing parameters (e. g., the deadline can be uncertain or unknown). We want to know for which start and end dates, and for what values of the timing parameters, a property holds. For instance, we look for the minimum or maximum deadline (together with the corresponding start and end dates) for which the property holds. We propose two frameworks for *parametric* timed pattern matching. The first one is based on parametric timed model checking. In contrast to most parametric timed problems, the solution is effectively computable. The second one is a *dedicated* method; not only we largely improve the efficiency compared to the first method, but we further propose optimizations with skipping. Our experiment results suggest that our algorithms, especially the second one, are efficient and practically relevant.

1 Introduction

Monitoring real-time systems consists in deciding whether a log satisfies a specification. A problem of interest is to determine *for which segment* of the log the specification is satisfied or violated. This problem can be related to string

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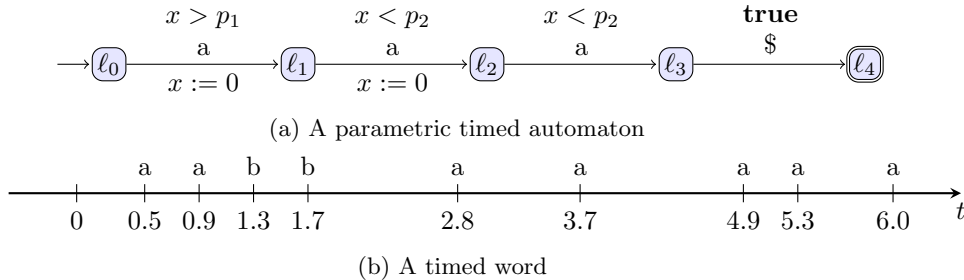


Figure 1: An example of parametric timed pattern matching [WHS17]

matching and pattern matching. The *timed pattern matching problem* was formulated in [Ulu+14], with subsequent works varying the setting and improving the technique (e. g., [Ulu+16; WAH16; Asa+17; WHS17]). The problem takes as input a log and a specification, and decides where in the log the specification is satisfied or violated. In [WAH16; WHS17], we introduced a solution to the timed pattern matching problem where the log is given in the form of a timed word (a sequence of events with their associated timestamps), and the specification in the form of a timed automaton (TA), an extension of finite-state automata with clocks [AD94].

Example 1. As a motivating example, consider the example in Fig. 1. Consider the automaton in Fig. 1a, and fix $p_1 = 1$ and $p_2 = 1$ —which gives a timed automaton [AD94]. Here $\$$ is a special terminal character. For this timed automaton (say \mathcal{A}) and the target timed word w in Fig. 1b, the output of the timed pattern matching problem is the set of matching intervals $\{(t, t') \mid w|_{(t, t')} \in \mathcal{L}(\mathcal{A})\} = \{(t, t') \mid t \in [3.7, 3.9), t' \in (6.0, \infty)\}$. We note that in our semantics, the automaton is forced to take a transition for each event, and thus, any intervals starting before 3.7 does not match.

While the log is by definition concrete, it may happen that the specification is subject to uncertainty. For example, we may want to detect *cyclic patterns* with a period d , without knowing the value of d with full certainty. Therefore, the more abstract problem of *parametric timed pattern matching* becomes of interest: **given a (concrete) timed log and an incomplete specification where some of the timing constants may be known with limited precision or completely unknown, what are the time intervals and the valuations of the parameters for which the specification holds?**

Coming back to Fig. 1, the question becomes to exhibit values for t, t', p_1, p_2 for which the specification holds on the log, i. e., $\{(t, t', v) \mid w|_{(t, t')} \in \mathcal{L}(v(\mathcal{A}))\}$, where v denotes a valuation of p_1, p_2 and $v(\mathcal{A})$ denotes the replacement of p_1, p_2 in \mathcal{A} with their respective valuation in v .

It is appreciated to conduct parametric timed pattern matching *online*, i. e., returning a partial result before obtaining the entire log. An online algorithm for parametric timed pattern matching can synthesize parameter valuations *in parallel* with the system execution under monitoring. This is useful, for example because the synthesized parameter valuation can tell how severely the monitored

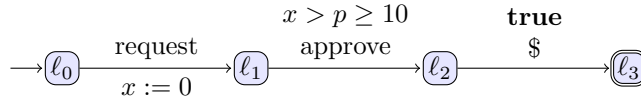


Figure 2: A parametric timed automaton to monitor a sequence of requests and approvals

system violates the timing constraints, and we can immediately react based on it.

Example 2. Consider monitoring a sequence of messages consisting of request and approve. We can detect too late approvals by the PTA in Fig. 2, which matches an approval at least 10 time units after the request. The severity of the violation, i. e., the delay between request and approve, is captured by the parameter p .

Contribution In this work, we introduce two approaches addressing the parametric timed pattern matching problem. Both use as underlying formalisms timed words and parametric timed automata (PTAs) [AHV93], two well-known formalisms in the real-time systems community. We show that the problem is decidable (which mainly comes from the fact that logs are finite).

Then, our algorithmic contribution is twofold. First, we propose a practical solution based on parametric timed model checking. We implement our method using IMITATOR [And21] and we perform a set of experiments on a set of automotive benchmarks.

Second, we propose a new *dedicated* technique for performing efficient parametric timed pattern matching. We propose optimizations based on *skipping*, in the line of [WHS17]. We implement our framework in a prototypical tool ParamMONAA, we perform a set of experiments on a set of automotive benchmarks, and show that we increase the efficiency compared to our first approach.

Both algorithms are suitable for online monitoring, as they do not need the whole run to be executed, and experiments show that they are fast enough to be applied at runtime.

About this manuscript This manuscript is an extension of [AHW18; WA19], with [AHW18] describing the first method of this manuscript (based on parametric timed model checking) while [WA19] describes the second dedicated method. We merged all concepts, and significantly improved the content, by notably modifying the problem formulation of [WA19] for the consistency, adding missing proofs, formalizing MakeSymbolic and TW2PTA (in Definitions 4 and 5 respectively), adding a new theoretical result on the correctness of our first approach (Section 3.4), almost rewriting the explanation of the skip values in Section 4.2 to make it more intuitive, adding new examples to illustrate the concept of the skip values in Section 4.2, and adding Section 5 to compare these two approaches.

Related work Several algorithms have been proposed for online monitoring of real-time temporal logic specifications. Online monitoring consists in mon-

itoring on-the-fly at runtime, while offline monitoring is performed after the execution is completed, with less hard constraints on the monitoring algorithm performance. An online monitoring algorithm for ptMTL (a past time fragment of MTL [Koy90]) was proposed in [RFB14] and an algorithm for MTL[U,S] (a variant of MTL with both forward and backward temporal modalities) was proposed in [HOW14]. In addition, a case study on an autonomous research vehicle monitoring [Kan+15] shows such procedures can be performed in an actual vehicle.

The approaches most related to ours are [Ulu+14; Ulu+16; Ulu17]. In that series of works, logs are encoded by *signals*, i.e., values that vary over time. This can be seen as a *state-based* view, while our timed words are *event-based*. The formalism used for specification in [Ulu+14; Ulu+16] is timed regular expressions (TREs). An offline monitoring algorithm is presented in [Ulu+14] and an online one is presented in [Ulu+16]. These algorithms are implemented in the tool *Montre* [Ulu17]. In [Bak+18], the setting is signals matched against a temporal pattern; the construction is automata-based as in [WAH16; WHS17].

Some algorithms have also been proposed for parameter identification of a temporal logic specification with uncertainty over a log. In the discrete time setting, an algorithm for an extension of LTL is proposed in [FR08]; and in the real-time setting, algorithms for parametric signal temporal logic (PSTL) are proposed in [Asa+11; Jha+17; BFM18; Bak+19]. Although these works are related to our approach, previous approaches do not focus on segments of a log but on a whole log. In contrast, we exhibit intervals together with their associated parameter valuations, in a fully symbolic fashion. We believe our matching-based setting is advantageous in many usage scenarios e.g., from hours of a log of a car, extracting timing constraints of a certain actions to cause slipping. Also, our setting allows the patterns with complex timing constraints (see the pattern in Fig. 6c for example).

Another related research direction is *specification mining*. In [Nar+18; SNF17; NF19], the authors propose algorithms to mine a TRE formula from a TRE *template* and a set of logs. Beyond the fact that we used a different formalism (an extension of timed automata vs. TRE), the main difference with ours is if the timing constraints are mined by a *statistical* approach or a *symbolic* approach: in [NF19], timing constraints are mined by clustering while we use symbolic analysis of polyhedra.

In [Bak+17], an offline algorithm for the robust pattern matching problem is considered over signal regular expressions, consisting in computing the *quantitative* (robust) semantics of a signal relative to an expression. For piecewise-constant and piecewise-linear signals, the problem can be effectively solved using a finite union of zones (linear constraints with a special form [BY03]). In [Wag19], an online algorithm for the robust pattern matching problem is presented over timed symbolic weighted automata, where the semantics is generalized with semiring. The algorithm in [Wag19] effectively solves the problem for piecewise-constant signals.

In [WAH19], we proposed a symbolic monitoring algorithm against specifications parametric in time and data, i.e., we considered logs extended with data

Table 1: Matching problems

	log, target	specification, pattern	output
string matching	a word $w \in \Sigma^*$	a word $pat \in \Sigma^*$	$\{(i, j) \in (\mathbb{N}_{>0})^2 \mid w(i, j) = pat\}$
pattern matching (PM)	a word $w \in \Sigma^*$	an NFA \mathcal{A}	$\{(i, j) \in (\mathbb{N}_{>0})^2 \mid w(i, j) \in \mathcal{L}(\mathcal{A})\}$
timed PM	a timed word $w \in (\Sigma \times \mathbb{R}_{>0})^*$	a TA \mathcal{A}	$\{(t, t') \in (\mathbb{R}_{>0})^2 \mid w _{(t, t')} \in \mathcal{L}(\mathcal{A})\}$
parametric timed PM	a timed word $w \in (\Sigma \times \mathbb{R}_{>0})^*$	a PTA \mathcal{A}	$\{(t, t', v) \mid w _{(t, t')} \in \mathcal{L}(v(\mathcal{A}))\}$

as the log formalism, and parametric timed data automata (an *ad-hoc* extension of timed automata) as the specification formalism.

In [WAH21], we proposed the *model-bounded monitoring* scheme to use prior knowledge about the monitored system in the interpolation of the observation obtained by discrete sampling. One of the procedures in [WAH21] is by reduction to the reachability analysis of a linear hybrid automaton [HPR94]. The idea of the reduction is essentially the same as ours in Section 3.

Further works attempted to quantify the distance between a specification and a signal temporal logic (STL) specification (e.g., [DFM13; DMP17; Jak+18]). The main difference with our work is that these works compute a distance w.r.t. to a whole log, while we aim at exhibiting where in the log is the property satisfied; our notion of parameters can also be seen as a relative time distance. However, our work is closer to the robust satisfaction of guards rather than signal values; in that sense, our contribution is more related to the time robustness in [DM10] or the distance in [ABD18].

Finally, while our work is related to parameter synthesis, in the sense that we identify parameter valuations in the property such that it holds (or not), the term “parameter synthesis” is also used in monitoring with a slightly different meaning: given a *model* with parameters, the goal is to find parameters that maximize the robustness of the specification, i.e., satisfying behaviors for a range of parameters for which the model robustly satisfies the property. A notable tool achieving this is BREACH [Don10].

A summary of various matching problems is recalled in Table 1.

Outline We introduce the necessary definitions and state our main objective in Section 2. We present and evaluate our method based on parametric timed model checking in Section 3. We then present and evaluate a *dedicated* method, enhanced with automata-based skipping, in Section 4. In Section 5, we discuss the comparison between the approaches in Sections 3 and 4. We conclude in Section 6.

2 Preliminaries and objective

Our target strings are *timed words* [AD94], that are time-stamped words over an alphabet Σ . Our patterns are given by parametric timed automata [AHV93].

2.1 Timed words and timed segments

For an alphabet Σ , a *timed word* is a sequence w of pairs $(a_i, \tau_i) \in (\Sigma \times \mathbb{R}_{>0})$ satisfying $\tau_i \leq \tau_{i+1}$ for any $i \in \{1, 2, \dots, |w| - 1\}$. We let $\tau_0 = 0$. For an alphabet Σ , we denote the set of the timed words on Σ by $\mathcal{T}(\Sigma)$. For an alphabet Σ and $n \in \mathbb{N}_{>0}$, we denote the set of the timed words of length n on Σ by $\mathcal{T}^n(\Sigma)$. Given a timed word w , we often denote it by $(\bar{a}, \bar{\tau})$, where \bar{a} is the sequence (a_1, a_2, \dots) and $\bar{\tau}$ is the sequence (τ_1, τ_2, \dots) . Let $w = (\bar{a}, \bar{\tau})$ be a timed word. We denote the subsequence $(a_i, \tau_i), (a_{i+1}, \tau_{i+1}), \dots, (a_j, \tau_j)$ by $w(i, j)$. For $t \in \mathbb{R}$ such that $-\tau_1 < t$, the t -*shift* of w is $(\bar{a}, \bar{\tau}) + t = (\bar{a}, \bar{\tau} + t)$ where $\bar{\tau} + t = \tau_1 + t, \tau_2 + t, \dots, \tau_{|\tau|} + t$. For timed words $w = (\bar{a}, \bar{\tau})$ and $w' = (\bar{a}', \bar{\tau}')$, their *absorbing concatenation* is $w \circ w' = (\bar{a} \circ \bar{a}', \bar{\tau} \circ \bar{\tau}')$ where $\bar{a} \circ \bar{a}'$ and $\bar{\tau} \circ \bar{\tau}'$ are usual concatenations, and their *non-absorbing concatenation* is $w \cdot w' = w \circ (w' + \tau_{|w|})$. The concatenations of subsets of $\mathcal{T}(\Sigma)$, i. e., sets of timed words, are also defined similarly. For a set $W \subseteq \mathcal{T}(\Sigma)$ of timed words, its untimed projection $\text{Untimed}(W) \in \Sigma^*$ is $\{\bar{a} \mid (\bar{a}, \bar{\tau}) \in W\}$.

For a timed word $w = (\bar{a}, \bar{\tau})$ on Σ and $t, t' \in \mathbb{R}_{\geq 0}$ satisfying $t < t'$, a *timed word segment* $w|_{(t, t')}$ is defined by the timed word $(w(i, j) - t) \circ (\$, t' - t)$ on the augmented alphabet $\Sigma \sqcup \{\$\}$, where i, j are chosen so that $\tau_{i-1} < t \leq \tau_i$ and $\tau_j \leq t' < \tau_{j+1}$. Here the fresh symbol $\$$ is called the *terminal character*.

2.2 Clocks, parameters and guards

We assume a set $\mathbb{X} = \{x_1, \dots, x_H\}$ of *clocks* (where $H \in \mathbb{N}$ denotes the clocks set cardinality), i. e., real-valued variables that evolve at the same rate. A clock valuation is a function $\nu : \mathbb{X} \rightarrow \mathbb{R}_{\geq 0}$. We write $\bar{0}$ for the clock valuation assigning 0 to all clocks. Given $d \in \mathbb{R}_{\geq 0}$, $\nu + d$ denotes the valuation s.t. $(\nu + d)(x) = \nu(x) + d$, for all $x \in \mathbb{X}$. Given $R \subseteq \mathbb{X}$, we define the *reset* of a valuation ν , denoted by $[\nu]_R$, as follows: $[\nu]_R(x) = 0$ if $x \in R$, and $[\nu]_R(x) = \nu(x)$ otherwise.

We assume a set $\mathbb{P} = \{p_1, \dots, p_M\}$ of *parameters*, i. e., unknown constants. A parameter *valuation* v is a function $v : \mathbb{P} \rightarrow \mathbb{Q}_+$. We assume $\bowtie \in \{<, \leq, =, \geq, >\}$. A guard g is a constraint over $\mathbb{X} \cup \mathbb{P}$ defined by a conjunction of inequalities of the form $x \bowtie d$, or $x \bowtie p$ with $d \in \mathbb{N}$ and $p \in \mathbb{P}$. Given g , we write $\nu \models v(g)$ if the expression obtained by replacing each x with $\nu(x)$ and each p with $v(p)$ in g evaluates to true.

A linear term over $\mathbb{X} \cup \mathbb{P}$ is of the form $\sum_{1 \leq i \leq H} \alpha_i x_i + \sum_{1 \leq j \leq M} \beta_j p_j + d$, with $x_i \in \mathbb{X}$, $p_j \in \mathbb{P}$, and $\alpha_i, \beta_j, d \in \mathbb{Z}$. A *constraint* C (i. e., a convex polyhedron) over $\mathbb{X} \cup \mathbb{P}$ is a conjunction of inequalities of the form $lt \bowtie 0$, where lt is a linear term. Given a set \mathbb{P} of parameters, we denote by $C \downarrow_{\mathbb{P}}$ the projection of C onto \mathbb{P} , i. e., obtained by eliminating the variables not in \mathbb{P} (e. g., using Fourier-Motzkin [Sch86]). \perp denotes the constraint over \mathbb{P} representing the empty set of parameter valuations.

2.3 Parametric timed automata

Parametric timed automata (PTAs) extend timed automata with parameters within guards in place of integer constants [AHV93].

2.3.1 Syntax

Definition 1 (PTA). A parametric timed automaton (PTA) \mathcal{A} is a tuple $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)^1$, where:

1. Σ is a finite set of actions,
2. L is a finite set of locations,
3. $\ell_0 \in L$ is the initial location,
4. $F \subseteq L$ is the set of final locations,
5. \mathbb{X} is a finite set of clocks,
6. \mathbb{P} is a finite set of parameters,
7. E is a finite set of edges $e = (\ell, g, a, R, \ell')$ where $\ell, \ell' \in L$ are the source and target locations, $a \in \Sigma$, $R \subseteq \mathbb{X}$ is a set of clocks to be reset, and g is a guard.

Given a parameter valuation v , we denote by $v(\mathcal{A})$ the non-parametric structure where all occurrences of a parameter p_i have been replaced by $v(p_i)$. In this work, we refer as a *timed automaton* to any structure $v(\mathcal{A})$.²

2.3.2 Synchronous product

The synchronous product (using strong broadcast, i. e., synchronization on shared actions) of several PTAs gives a PTA.

Definition 2. [synchronized product of PTAs] Let $N \in \mathbb{N}$. Given a set of PTAs $\mathcal{A}_i = (\Sigma_i, L_i, (\ell_0)_i, F_i, \mathbb{X}_i, \mathbb{P}_i, E_i)$, $1 \leq i \leq N$, the *synchronized product* of \mathcal{A}_i , $1 \leq i \leq N$, denoted by $\mathcal{A}_1 \parallel \mathcal{A}_2 \parallel \dots \parallel \mathcal{A}_N$, is the tuple $(\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$, where:

1. $\Sigma = \bigcup_{i=1}^N \Sigma_i$,
2. $L = \prod_{i=1}^N L_i$,
3. $\ell_0 = ((\ell_0)_1, \dots, (\ell_0)_N)$,

¹Following [WHS17], we do not allow invariants in the locations. Allowing invariants would be straightforward.

²Technically, a timed automaton requires non-negative integer constants, while we define non-negative *rational* valuations. So a TA can be obtained by assuming a rescaling of the constants: by multiplying all constants in $v(\mathcal{A})$ by the least common multiple of their denominators, we obtain an equivalent (integer-valued) TA, as defined in [AD94].

$$4. F = F_1 \times \dots \times F_N,$$

$$5. \mathbb{X} = \bigcup_{1 \leq i \leq N} \mathbb{X}_i,$$

$$6. \mathbb{P} = \bigcup_{1 \leq i \leq N} \mathbb{P}_i,$$

and E is defined as follows. For all $a \in \Sigma$, let ζ_a be the subset of indices $i \in 1, \dots, N$ such that $a \in \Sigma_i$. For all $a \in \Sigma$, for all $(\ell_1, \dots, \ell_N) \in L$, for all $(\ell'_1, \dots, \ell'_N) \in L$, $((\ell_1, \dots, \ell_N), g, a, R, (\ell'_1, \dots, \ell'_N)) \in E$ if:

- for all $i \in \zeta_a$, there exist g_i, R_i such that $(\ell_i, g_i, a, R_i, \ell'_i) \in E_i$, $g = \bigwedge_{i \in \zeta_a} g_i$, $R = \bigcup_{i \in \zeta_a} R_i$, and,
- for all $i \notin \zeta_a$, $\ell'_i = \ell_i$.

Note that we define the set of accepting locations to be the *product* of the individual accepting locations, i. e., a location of the product automaton is accepting if all its component locations are accepting.

2.3.3 Concrete semantics

Let us now recall the concrete semantics of TAs.

Definition 3 (Semantics of a TA). Given a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$ with clocks $\mathbb{X} = \{x_1, \dots, x_H\}$, and a parameter valuation v , the semantics of $v(\mathcal{A})$ is given by the timed transition system (TTS) (S, s_0, \rightarrow) , with

- $S = L \times \mathbb{R}_{\geq 0}^H$
- $s_0 = (\ell_0, \vec{0})$,
- \rightarrow consists of the discrete and (continuous) delay transition relations:
 1. discrete transitions: $(\ell, \nu) \xrightarrow{e} (\ell', \nu')$, if there exists $e = (\ell, g, a, R, \ell') \in E$, such that $\nu' = [\nu]_R$, and $\nu \models v(g)$.
 2. delay transitions: $(\ell, \nu) \xrightarrow{d} (\ell, \nu + d)$, with $d \in \mathbb{R}_{\geq 0}$.

Moreover we write $(\ell, \nu) \xrightarrow{(d, e)} (\ell', \nu')$ for a combination of a delay and discrete transition if $\exists \nu'' : (\ell, \nu) \xrightarrow{d} (\ell, \nu'') \xrightarrow{e} (\ell', \nu')$.

Given a TA $v(\mathcal{A})$ with concrete semantics (S, s_0, \rightarrow) , we refer to the states of S as the *concrete states* of $v(\mathcal{A})$. A *run* of $v(\mathcal{A})$ is an alternating sequence of concrete states of $v(\mathcal{A})$ and pairs of edges and delays starting from the initial state s_0 of the form $s_0, (d_0, e_0), s_1, (d_1, e_1), \dots$ with $i = 0, 1, \dots$, $e_i \in E$, $d_i \in \mathbb{R}_{\geq 0}$ and $s_i \xrightarrow{(d_i, e_i)} s_{i+1}$. Given such a run, the associated *timed word* is $(a_1, \tau_1), (a_2, \tau_2), \dots$, where a_i is the action of edge e_{i-1} , and $\tau_i = \sum_{0 \leq j \leq i-1} d_j$, for $i = 1, 2, \dots$.³ Given a state $s = (\ell, \nu)$, we say that s is reachable in $v(\mathcal{A})$ if

³The “-1” in indices comes from the fact that, following usual conventions in the literature, states are numbered starting from 0 while words are numbered from 1.

s appears in a run of $v(\mathcal{A})$. By extension, we say that ℓ is reachable; and by extension again, given a set T of locations, we say that T is reachable if there exists $\ell \in T$ such that ℓ is reachable in $v(\mathcal{A})$.

A finite run is *accepting* if its last state (ℓ, ν) is such that $\ell \in F$. The (timed) *language* $\mathcal{L}(v(\mathcal{A}))$ is defined to be the set of timed words associated with all accepting runs of $v(\mathcal{A})$.

2.4 Reachability synthesis

We use here reachability synthesis for the following two purposes.

- In Section 3: to solve parametric timed pattern matching
- In Section 4: to improve the dedicated parametric timed pattern matching algorithm (Algorithm 3) with a skipping optimization

This procedure, called *EFsynth*, takes as input a PTA \mathcal{A} and a set of target locations T , and attempts to synthesize all parameter valuations v for which T is reachable in $v(\mathcal{A})$. *EFsynth* was formalized in e. g., [JLR15] and is a procedure that may not terminate, but that computes an exact result (sound and complete) if it terminates. *EFsynth* traverses the *parametric zone graph* of \mathcal{A} , which is a potentially infinite extension of the well-known zone graph of TAs (see, e. g., [And+09; JLR15] for a formal definition).

Optimal parameter reachability synthesis In addition, for the specific case of pattern matching with optimization (Section 3.5), we will make use of *optimal parameter reachability synthesis* [And+19]. This procedure, called *OptParamSynth*, takes as input a PTA \mathcal{A} , a set of target locations T and a parameter p to minimize, and attempts to synthesize all parameter valuations v for which T is reachable in $v(\mathcal{A})$ and for which the value of p is optimal; *OptParamSynth* is a generic algorithm, that can be instantiated to *MinParamSynth* (where p should be minimized) or *MaxParamSynth* (where p should be maximized). *MinParamSynth* was studied in [And+19], and is essentially similar to *EFsynth*, with a condition to only keep the states leading to a minimal value of p .

2.5 Parametric timed pattern matching

Let us recall timed pattern matching [WAH16; WHS17; WHS18].

Timed pattern matching problem:

INPUT: a timed word w over an alphabet Σ and a TA \mathcal{A} over the augmented alphabet $\Sigma \sqcup \{\$\}$

PROBLEM: compute all the intervals (t, t') for which the segment $w|_{(t, t')}$ is accepted by \mathcal{A} . That is, it requires the *match set* $\mathcal{M}(w, \mathcal{A}) = \{(t, t') \mid w|_{(t, t')} \in \mathcal{L}(\mathcal{A})\}$.

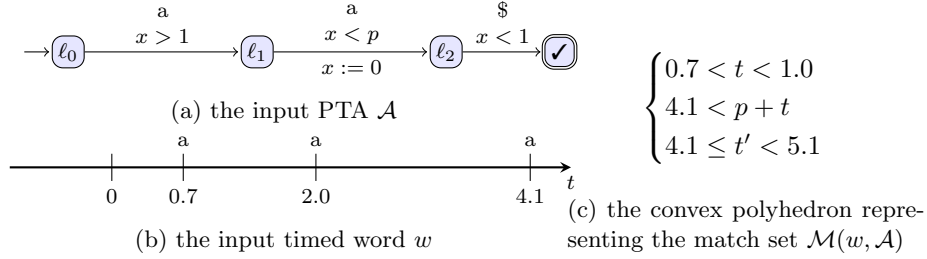


Figure 3: Example of parametric timed pattern matching

The match set $\mathcal{M}(w, \mathcal{A})$ is in general uncountable. For example, if \mathcal{A} accepts any timed word, the match set is $\{(t, t') \in \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \mid t < t'\}$, which is uncountable. However it allows finite representation, as a finite union of special polyhedra called *zones* (see [BY03; WAH16]). Roughly speaking, zones are made of constraints of the form $x \triangleleft c$ or $x - x' \triangleleft c$, with $x, x' \in \mathbb{X}$, $c \in \mathbb{Z}$ and $\triangleleft \in \{\leq, <\}$.

We now extend the timed pattern matching problem to parameters by allowing a specification expressed using PTAs. The problem now requires not only the start and end dates for which the property holds, but also the associated parameter valuations.

Parametric timed pattern matching problem:

INPUT: a timed word w over an alphabet Σ and a PTA \mathcal{A} over the augmented alphabet $\Sigma \sqcup \{\$\}$

PROBLEM: compute all the triples (t, t', v) for which the segment $w|_{(t, t')}$ is accepted by $v(\mathcal{A})$. That is, it requires the *match set* $\mathcal{M}(w, \mathcal{A}) = \{(t, t', v) \mid w|_{(t, t')} \in \mathcal{L}(v(\mathcal{A}))\}$.

Since parametric timed pattern matching is a generalization of (non-parametric) timed pattern matching, the match set $\mathcal{M}(w, \mathcal{A})$ is again in general uncountable; however, we will see that it can still be represented as a finite union of polyhedra, but in more dimensions, viz., $|\mathbb{P}| + 2$, i. e., the number of parameters + 2 further dimensions for t and t' . In addition, the form of the obtained polyhedra is more general than zones, as parameters may “accumulate” to produce sums of parameters with coefficients (e. g., $3 \times p_1 < p_2 + 2 \times p_3$).

Example 3. Fig. 3 shows an example of parametric timed pattern matching. Given the PTA \mathcal{A} and the timed word w , the parametric timed pattern matching problem asks for the match set $\mathcal{M}(w, \mathcal{A})$ in Fig. 3c.

The PTA \mathcal{A} shows that there must be at least 1 time unit between the beginning of the matching t and the first action a in the matching. Moreover, each matching consists of two actions a . Therefore, any matching must begin after 0.7 and at least 1 time unit before the action at time 2.0, and we have $0.7 < t < 2.0 - 1.0 (= 1.0)$. At location ℓ_1 , the value of the clock x is the elapsed time from the beginning of the matching. Therefore, at time T , the value of x is $T - t$. Since we observe an action a at 4.1 and we have $x < p$ at the edge from ℓ_1 to ℓ_2 , we have $4.1 - t < p$. Finally, \mathcal{A} shows that the matching must

end within 1 time unit after the second action a in the matching. Therefore, we have $4.1 \leq t' < 5.1$.

We observe that the convex polyhedron in the match set has three dimensions for the parameter p and for t and t' . We also observe that this convex polyhedron is not a zone because it contains the constraint $4.1 < p + t$.

In the rest of this paper, we make the following assumption on the PTA in parametric timed pattern matching to simplify the construction and the reasoning. This assumption is easy to remove in practice since we can simply remove the transitions that do not satisfy the second part of the assumption.

Assumption 1. *As in [WAH16; WHS17], we assume that all transitions to the accepting locations are labeled with $\$$, and actions that are not part of the timed word alphabet are removed, except for the special action $\$$.*

3 Parametric timed pattern matching using model checking

3.1 General approach

In addition to [Assumption 1](#), we make the following assumption, that does not impact the correctness of our method, but simplifies the subsequent reasoning.

Assumption 2. *We assume that the pattern automaton contains a single accepting location.*

[Assumption 2](#) is easy to remove in practice: if the pattern PTA contains more than one accepting location, they can be merged into a single accepting location.

We show using the following approach that parametric timed pattern matching can reduce to parametric reachability analysis.

1. We turn the pattern into a symbolic pattern, by allowing it to start anytime. In addition, we use two parameters to measure the (symbolic) starting time and the (symbolic) ending time of the pattern.
2. We turn the timed word into a (non-parametric) timed automaton that uses a single clock x_{abs} measuring the absolute time.
3. We consider the synchronized product of the symbolic pattern PTA and the timed word (P)TA.
4. We run the reachability synthesis algorithm `EFsynth` to derive all possible parameter valuations for which the accepting location of the pattern PTA and of the timed word TA is reachable.

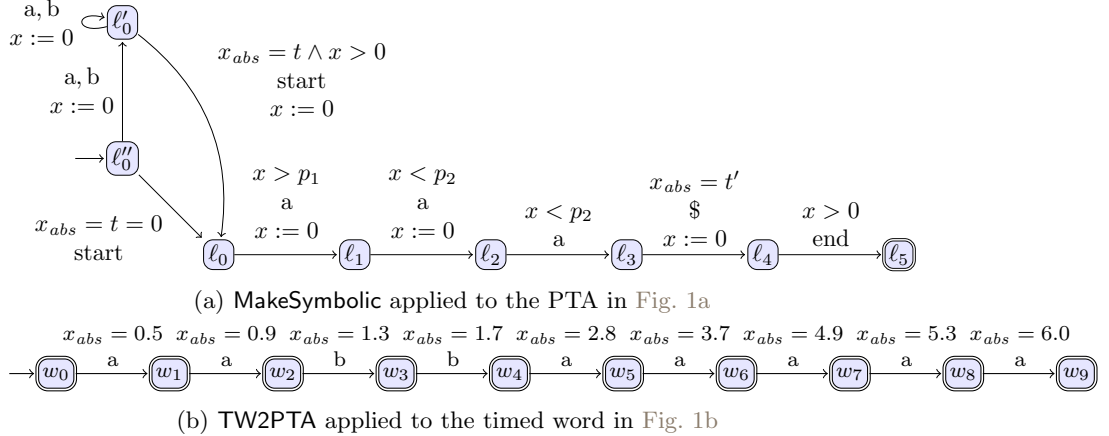


Figure 4: Our transformations exemplified on Fig. 1

3.2 Our approach step by step

3.2.1 Making the pattern symbolic

In this first step, we first add two parameters t and t' , which encode the (symbolic) start and end time where the pattern holds on the input timed word. This way, we will obtain a result in the form of a finite union of polyhedra in $|\mathbb{P}| + 2$ dimensions, where the 2 additional dimensions come from the addition of t and t' . We also add a clock x_{abs} measuring the absolute time, i. e., initially 0 and never reset (this clock is shared by the pattern PTA and the subsequent timed word TA). Then, we modify the pattern PTA as follows:

1. we add two fresh locations (say ℓ'_0 and ℓ''_0) prior to the initial location ℓ_0 ;
2. we add a fresh clock (say x); in practice, as this clock is used only in the initial location, an existing clock of the pattern may be reused;
3. we add an unguarded self-loop allowing any action of the timed word on ℓ'_0 , and resetting x ;
4. we add an unguarded transition from ℓ''_0 to ℓ'_0 allowing any action of the timed word on ℓ'_0 , and resetting x ;
5. we add a transition from ℓ'_0 to ℓ_0 guarded by $x_{abs} = t \wedge x > 0$, labeled with a fresh action start and resetting all clocks of the pattern (except x_{abs});
6. we add a transition from ℓ''_0 to ℓ_0 guarded by $x_{abs} = t \wedge x_{abs} = 0$, labeled with start;
7. we add a guard $x_{abs} = t'$ and reset x on the accepting transitions labeled with $\$$;
8. we add an extra location ℓ_F after the former (unique) accepting location, with a transition guarded by $x > 0$, labeled with end;

9. the initial location of the modified PTA becomes ℓ''_0 ;
10. the (only) final location of the modified PTA becomes ℓ_F .

Let us give the intuition behind our transformation. First, the two guards $x_{abs} = t$ and $x_{abs} = t'$ allow recording symbolically the value of the starting and ending dates. Second, the new locations ℓ''_0 and ℓ'_0 allow the pattern to “start anytime”; that is, it can synchronize with the timed word TA for an arbitrary long time while staying in the initial location ℓ''_0 (and therefore *not* matching the pattern), and start (using the transition from ℓ'_0 to ℓ_0) anytime. Third, due to the constraint $x > 0$, a non-zero time must elapse between the last action before the pattern start and the actual pattern start. The distinction between ℓ''_0 and ℓ'_0 is necessary to also allow starting the pattern at $x_{abs} = 0$ if no action occurred before. Finally, the guard $x > 0$ just before the accepting location ensures the next action of the system (if any) is taken after a non-zero delay, following our definitions of timed word and projection.

Let us formalize this procedure `MakeSymbolic` below.

Definition 4. Given a PTA \mathcal{A} over the augmented alphabet $\Sigma \sqcup \{\$, \text{start}, \text{end}\}$, $\text{MakeSymbolic}(\mathcal{A}) = (\Sigma \sqcup \{\$, \text{start}, \text{end}\}, L \sqcup \{\ell'_0, \ell''_0, \ell_F\}, \ell''_0, \{\ell_F\}, \mathbb{X} \sqcup \{x_{abs}, x\}, \mathbb{P} \sqcup \{t, t'\}, E')$, where E' is the following:

$$\begin{aligned}
E' = & \{(\ell, g, a, R, \ell') \mid (\ell, g, a, R, \ell') \in E, a \in \Sigma\} \\
& \sqcup \{(\ell, g \wedge x_{abs} = t', \$, R \sqcup \{x\}, \ell') \mid (\ell, g, \$, R, \ell') \in E\} \\
& \sqcup \{(\ell'_0, \top, a, \{x\}, \ell'_0) \mid a \in \Sigma\} \\
& \sqcup \{(\ell''_0, \top, a, \{x\}, \ell'_0) \mid a \in \Sigma\} \\
& \sqcup \{(\ell'_0, x_{abs} = t \wedge x > 0, \text{start}, \mathbb{X}, \ell_0), (\ell''_0, x_{abs} = 0 \wedge x_{abs} = 0, \text{start}, \mathbb{X}, \ell_0)\} \\
& \sqcup \{(\ell, x > 0, \text{end}, \emptyset, \ell_F) \mid \ell \in F\}.
\end{aligned}$$

Example 4. Consider the pattern PTA \mathcal{A} in Fig. 1a. The result of $\text{MakeSymbolic}(\mathcal{A})$ is given in Fig. 4a. Note that we use the same clock x for both the extra clock introduced by our construction and the original clock of the pattern automaton from Fig. 1a.

3.2.2 Converting the timed word into a (P)TA

In this second step, we convert the timed word into a (non-parametric) timed automaton. This is very straightforward, and simply consists in converting a timed word of the form $(a_1, \tau_1), \dots, (a_n, \tau_n)$ into a sequence of transitions labeled with a_i and guarded with $x_{abs} = \tau_i$ (recall that x_{abs} measures the absolute time and is shared by the timed word automaton and the pattern automaton). All locations are made accepting.⁴

Let us formalize this procedure `TW2PTA` below.

⁴This was not the case in [AHW18], but we added this requirement due to the modification of the definition of synchronized product (Definition 2), so as to have a unified framework in Sections 3 and 4.

Algorithm 1: PTPM(\mathcal{A}, w)

input : A pattern PTA \mathcal{A} with accepting location F , a timed word w
output: Constraint K over the parameters

- 1 $\mathcal{A}' \leftarrow \text{MakeSymbolic}(\mathcal{A})$
 - 2 $\mathcal{A}_w \leftarrow \text{TW2PTA}(w)$
 - 3 **return** $\text{EFsynth}(\mathcal{A}' \parallel \mathcal{A}_w, F)$
-

Definition 5. Given a timed word $w = (a_1, \tau_1), \dots, (a_n, \tau_n)$, the *transformation of this timed word into a PTA* is the PTA $\text{TW2PTA}(w) = (\Sigma, \{\ell_0, \ell_1, \dots, \ell_n\}, \ell_0, \{\ell_0, \ell_1, \dots, \ell_n\}, \{x_{abs}\}, \emptyset, E)$, where:

$$E = \bigcup_{i \in \{1, 2, \dots, n\}} (\ell_{i-1}, x_{abs} = \tau_i, a_i, \emptyset, \ell_i).$$

Example 5. Consider the timed word w in Fig. 1b. The result of $\text{TW2PTA}(w)$ is given in Fig. 4b.

3.2.3 Synchronized product

The last part of the method consists in performing the synchronized product of $\text{MakeSymbolic}(\mathcal{A})$ and $\text{TW2PTA}(w)$, and calling EFsynth on the resulting PTA.

We summarize our method $\text{PTPM}(\mathcal{A}, w)$ in Algorithm 1.

Example 6. Consider again the timed word w and the PTA pattern \mathcal{A} in Fig. 1. The result of $\text{PTPM}(\mathcal{A}, w)$ is as follows:

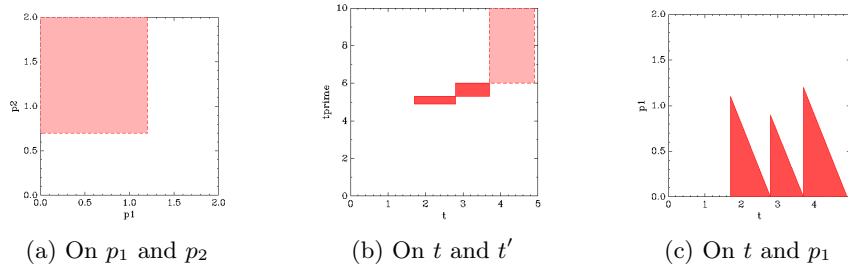
$$\begin{aligned} & 1.7 < t < 2.8 - p_1 \wedge 4.9 \leq t' < 5.3 \wedge p_2 > 1.2 \\ \vee & 2.8 < t < 3.7 - p_1 \wedge 5.3 \leq t' < 6 \wedge p_2 > 1.2 \\ \vee & 3.7 < t < 4.9 - p_1 \wedge t' \geq 6 \wedge p_2 > 0.7 \end{aligned}$$

Observe that, for the parameter valuation given in the introduction ($p_1 = p_2 = 1$), only the pattern corresponding to the last disjunct could be obtained, i.e., the pattern that matches the last three a of the timed word in Fig. 1b. In contrast, the first disjunct can match the first three a coming after the two bs, while the second disjunct allows to match the three as in the middle of the last five as in Fig. 1b.

We give various projections of this constraint onto two dimensions in Fig. 5 (the difference between plain red and light red is not significant—light red constraints denote unbounded constraints towards at least one dimension).

3.3 Termination

We state below the termination of our procedure.



(a) On p_1 and p_2 (b) On t and t' (c) On t and p_1
Figure 5: Projections of the result of parametric timed pattern matching on Fig. 1

Theorem 1 (termination). *Let \mathcal{A} be a PTA encoding a parametric pattern, and w be a timed word. Then $PTPM(\mathcal{A}, w)$ terminates.*

Proof. First, observe that there may be non-determinism in the pattern PTA, i. e., the timed word can potentially synchronize with two transitions labeled with the same action from a given location. Even if there is no syntactic non-determinism, nondeterminism can appear due to the interleaving of the initial start action: in Fig. 4, the first a of the timed word can either synchronize with the self-loop on ℓ'_0 , or the start action can first occur, and then the first a of the timed word synchronizes with the a labeling the transition from ℓ_0 to ℓ_1 of the pattern PTA. Second, the pattern PTA may well have loops (and this is the case in our experiments in Section 3.6.3), which yields an infinite parametric zone graph (for the pattern automaton not synchronized with the word automaton). However, let us show that only a finite part of the parametric zone graph is explored by EFsynth: indeed, since $TW2PTA(w)$ is only a finite sequence, and thanks to the strong synchronization between the pattern PTA and the timed word PTA and due to Assumption 1, only a finite number of finite discrete paths in the synchronized product will be explored. The only interleaving is due to the initial start action (which appears twice in the pattern PTA but can only be taken once at most due to the mutually exclusive guards $x = 0$ and $x > 0$), and due to the accepting $\$$ action, that only appears on the last transition to the last-but-one accepting location. As the pattern PTA is finitely branching, this gives a finite number of finite paths. The length of each path is clearly bounded by $|w| + 3$. Let us now consider the maximal number of such paths: given a location in $TW2PTA(w)$, the choice of the action (say a) is entirely deterministic. However, the pattern PTA may be non-deterministic, and can synchronize with B outgoing transitions labeled with a , which gives $B^{|w|}$ combinations. In addition, the start action can be inserted exactly once, at any position in the timed word (from before the first action to after the last action of the word—in the case of an empty pattern): this gives therefore $(|w| + 1) \times B^{|w|}$ different runs. (The $\$$ is necessarily the last-but-one action, and does not impact the number of runs, as the (potential) outgoing transitions from the accepting location are not explored.) Altogether, a total number of at most $(|w| + 3) \times (|w| + 1) \times B^{|w|}$ symbolic states is explored by EFsynth in the worst case. \square

Theorem 1 may not come as a surprise, as the input timed word is finite. But

it is worth noting that it comes in contrast with the fact that the wide majority of decision problems are undecidable for parametric timed automata, including the emptiness of the valuation set for which a given location is reachable both, for integer- and rational-valued parameters [AHV93; Mil00] (see [And19] for a survey).

3.4 Correctness

We show the correctness of our procedure using the following lemma on `MakeSymbolic` with the formal definition of `MakeSymbolic(\mathcal{A})`.

Lemma 1. *For any timed word w , $t < t' \in \mathbb{R}_{>0}$, PTA \mathcal{A} , and for any parameter valuation v over \mathbb{P} , we have $\mathcal{L}((v, t, t')(\text{MakeSymbolic}(\mathcal{A}))) = \mathcal{T}(\Sigma) \circ (\text{start}, t) \circ \{w + t \mid w \in \mathcal{L}(v(\mathcal{A}))\} \circ \{(\text{end}, t'') \mid t'' > t'\}$, where (v, t, t') is the parameter valuation such that $(v, t, t')(p) = v(p)$ for any $p \in \mathbb{P}$, $(v, t, t')(t) = t$, and $(v, t, t')(t') = t'$.*

Proof. Let $\mathcal{A} = (\Sigma \sqcup \{\$, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$ and $\text{MakeSymbolic}(\mathcal{A}) = (\Sigma \sqcup \{\$, \text{start}, \text{end}\}, L \sqcup \{\ell'_0, \ell''_0, \ell_F\}, \ell''_0, \{\ell_F\}, \mathbb{X} \sqcup \{x_{abs}, x\}, \mathbb{P} \sqcup \{t, t'\}, E')$, where E' is defined according to Definition 4.

Now, let \mathcal{A}' denote the PTA identical to $\text{MakeSymbolic}(\mathcal{A})$, except that the final location is ℓ_0 instead of ℓ_F . That is, $\mathcal{A}' = (\Sigma \sqcup \{\$, \text{start}, \text{end}\}, L \sqcup \{\ell'_0, \ell''_0, \ell_F\}, \ell''_0, \{\ell_0\}, \mathbb{X} \sqcup \{x_{abs}, x\}, \mathbb{P} \sqcup \{t, t'\}, E')$. For \mathcal{A}' , we have $\mathcal{L}((v, t, t')(\mathcal{A}')) = \mathcal{T}(\Sigma) \circ (\text{start}, t)$.

Now, let \mathcal{A}'' denote the PTA identical to $\text{MakeSymbolic}(\mathcal{A})$, except that the final locations are F (i. e., the final locations of the original automaton \mathcal{A}). That is, $\mathcal{A}'' = (\Sigma \sqcup \{\$, \text{start}, \text{end}\}, L \sqcup \{\ell'_0, \ell''_0, \ell_F\}, \ell''_0, F, \mathbb{X} \sqcup \{x_{abs}, x\}, \mathbb{P} \sqcup \{t, t'\}, E')$. Since all the clock variables \mathbb{X} are reset at all the transitions to ℓ_0 in E , for \mathcal{A}'' , we have the following.

$$\begin{aligned} \mathcal{L}((v, t, t')(\mathcal{A}'')) &= \mathcal{L}((v, t, t')(\mathcal{A}')) \cdot \mathcal{L}(v(\mathcal{A})) \\ &= (\mathcal{T}(\Sigma) \circ (\text{start}, t)) \cdot \mathcal{L}(v(\mathcal{A})) \\ &= \mathcal{T}(\Sigma) \circ (\text{start}, t) \circ \{w + t \mid w \in \mathcal{L}(v(\mathcal{A}))\} \end{aligned}$$

Moreover, since we have $x_{abs} = t'$ at all the transitions to F , we have

$$\begin{aligned} &\mathcal{L}((v, t, t')(\text{MakeSymbolic}(\mathcal{A}))) \\ &= \mathcal{L}((v, t, t')(\mathcal{A}'')) \cdot \{(\text{end}, t'') \mid t'' > 0\} \\ &= (\mathcal{T}(\Sigma) \circ (\text{start}, t) \circ \{w + t \mid w \in \mathcal{L}(v(\mathcal{A}))\}) \cdot \{(\text{end}, t'') \mid t'' > 0\} \\ &= \mathcal{T}(\Sigma) \circ (\text{start}, t) \circ \{w + t \mid w \in \mathcal{L}(v(\mathcal{A}))\} \circ \{(\text{end}, t'') \mid t'' > t'\} \end{aligned}$$

□

We state the correctness of the workflow in Section 3.2 as follows.

Theorem 2 (correctness). *For any PTA \mathcal{A} and timed word w , we have the following:*

$$\mathcal{M}(w, \mathcal{A}) = \{(t, t', v) \mid (v, t, t') \in \text{EFsynth}(\text{MakeSymbolic}(\mathcal{A}) \parallel \text{TW2PTA}(w), F)\}$$

where F denotes the set of final locations of the product automaton $\text{MakeSymbolic}(\mathcal{A}) \parallel \text{TW2PTA}(w)$.

Proof. Let \mathcal{A} be a PTA, and v be a parameter valuation. First note that, by definition of $\text{TW2PTA}(w)$, for any timed word w , $t < t' \in \mathbb{R}_{>0}$, we have $\mathcal{L}(\text{TW2PTA}(w)) = \{w(0, m) \mid m \in \{0, 1, \dots, |w|\}\}$.

Now, thanks to Lemma 1, we have the following, where $u \downarrow_{\Sigma}$ is the projection of the timed word u to an alphabet Σ , i.e., the timed word identical to u , except that all the actions not in Σ are removed. We also use the same notation for a set of timed words.

$$\begin{aligned}
& (v, t, t') \in \text{EFsynth}(\text{MakeSymbolic}(\mathcal{A}) \parallel \text{TW2PTA}(w), F) \\
\iff & \exists u \in \mathcal{T}(\Sigma \sqcup \{\text{start}, \text{end}, \$\}). u \in \mathcal{L}((v, t, t')(\text{MakeSymbolic}(\mathcal{A}) \parallel \text{TW2PTA}(w))) \\
\iff & \exists u \in \mathcal{T}(\Sigma \sqcup \{\text{start}, \text{end}, \$\}). u \in \mathcal{L}((v, t, t')(\text{MakeSymbolic}(\mathcal{A}))) \wedge u \downarrow_{\Sigma} \in \mathcal{L}(\text{TW2PTA}(w)) \\
\iff & \exists u \in \mathcal{L}((v, t, t')(\text{MakeSymbolic}(\mathcal{A}))). u \downarrow_{\Sigma} = w(0, n), \text{ where } \tau_n \leq t' < \tau_{n+1} \\
\iff & \exists u \in \mathcal{T}(\Sigma) \circ (\text{start}, t) \circ \{w' + t \mid w' \in \mathcal{L}(v(\mathcal{A}))\} \circ \{(\text{end}, t'') \mid t'' > t'\}. u \downarrow_{\Sigma} = w(0, n), \\
& \text{ where } \tau_n \leq t' < \tau_{n+1} \\
\iff & w(0, n) \in \left(\mathcal{T}(\Sigma) \circ (\text{start}, t) \circ \{w' + t \mid w' \in \mathcal{L}(v(\mathcal{A}))\} \circ \{(\text{end}, t'') \mid t'' > t'\} \right) \downarrow_{\Sigma}, \\
& \text{ where } \tau_n \leq t' < \tau_{n+1} \\
\iff & w|_{(t, t')} \in \mathcal{L}(v(\mathcal{A})) \\
\iff & (t, t', v) \in \mathcal{M}(w, \mathcal{A})
\end{aligned}$$

This concludes the proof. \square

3.5 Pattern matching with optimization

We also address the following optimization problem: given a timed word and a pattern containing parameters, what is the minimum or maximum value of a given parameter such that the pattern is matched by the timed word? Formally, this gives the following problem, where “optimal” denotes minimal or maximal.

Optimal parametric timed pattern matching problem:

INPUT: a timed word w over an alphabet Σ , a PTA \mathcal{A} over the augmented alphabet $\Sigma \sqcup \{\$\}$, a parameter p

PROBLEM: compute the optimal value o such that there exist v, t, t' such that $v(p) = o$ and the segment $w|_{(t, t')}$ is accepted by $v(\mathcal{A})$

That is, we are only interested in the *optimal value o of the given parameter p* , and not in the full list of matches as in PTPM.

While this problem can be solved using our solution from Section 3.1 (by computing the multidimensional constraint, and then eliminating all parameters but the target parameter, using variable elimination techniques), we use here a dedicated approach, with the hope it be more efficient. Instead of managing all symbolic matches (i.e., a finite union of polyhedra), we simply manage

Algorithm 2: $\text{PTPM}_{\text{opt}}(\mathcal{A}, w, p)$

input : A pattern PTA \mathcal{A} with accepting location F , a timed word w ,
a parameter p to be optimized

output: Constraint K over the parameters

- 1 $\mathcal{A}' \leftarrow \text{MakeSymbolic}(\mathcal{A})$
 - 2 $\mathcal{A}_w \leftarrow \text{TW2PTA}(w)$
 - 3 **return** $\text{OptParamSynth}(\mathcal{A}' \parallel \mathcal{A}_w, F, p)$
-

the current optimum; in addition, we cut branches that cannot improve the optimum, with the hope to reduce the number of states explored. For example, assume parameter p is to be minimized; if the current minimum is $p > 2$, and if a newly computed symbolic state is such that $p \geq 3$, then the branch starting from this new symbolic state will not improve the minimum, and can safely be discarded. This branch cutting is directly managed by the underlying algorithm OptParamSynth .

We give this procedure PTPM_{opt} in Algorithm 2. It is basically a refinement of Algorithm 1, which takes as additional argument the parameter p to be optimized, and where the call to EFSynth is replaced with a call to OptParamSynth .

Similarly to OptParamSynth recalled in Section 2.4, PTPM_{opt} is a “generic” algorithm, that can be “instantiated” to either

1. “pattern matching with minimization” (say PTPM_{min}), by replacing in Algorithm 2 OptParamSynth with MinParamSynth ; or
2. “pattern matching with maximization” (say PTPM_{max}), by replacing in Algorithm 2 OptParamSynth with MaxParamSynth .

Example 7. Consider again the timed word w and the PTA pattern \mathcal{A} in Fig. 1. Let us first minimize p_2 so that the pattern matches the timed word for at least one position: by calling $\text{PTPM}_{\text{min}}(\mathcal{A}, w, p_2)$, we obtain $p_2 > 0.7$. That is to say, the smallest valuation of p_2 allowing the pattern to match the timed word for at least one position is infinitesimally larger than 0.7, but 0.7 itself is not an admissible valuation. Let us then maximize p_1 so that the pattern matches the timed word for at least one position: by calling $\text{PTPM}_{\text{max}}(\mathcal{A}, w, p_1)$, we obtain $p_1 < 1.2$.

3.6 Experiments

We evaluated our approach against two standard benchmarks from [HAF14], already used in [WHS17], as well as a third *ad-hoc* benchmark specifically designed to test the limits of parametric timed pattern matching. We fixed no bounds for our parameters.

We used IMITATOR [And21] to perform the parameter synthesis (algorithm EFSynth). IMITATOR relies on the Parma Polyhedra Library (PPL) [BHZ08] to compute symbolic states. It was shown in [Bak+17] that polyhedra may be dozens of times slower than more efficient data structures such as DBMs

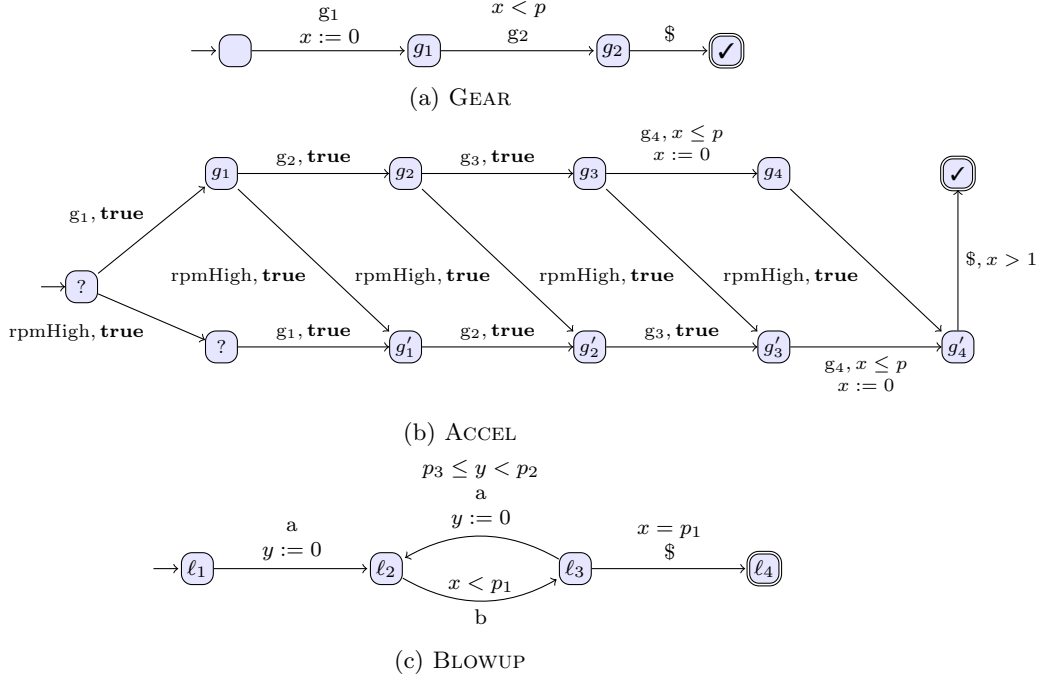


Figure 6: Experiments: patterns

(difference bound matrices); however, for parametric analyses, DBMs are not suitable, and parameterized extensions (e. g., in [Hun+02]) still need polyhedra in their representation.

We used a slightly modified version of IMITATOR for technical reasons: IMITATOR handles non-convex constraints (finite unions of polyhedra); while most case studies solved by IMITATOR in the past handle simple constraints (made of a few disjuncts), the experiments in this manuscript may handle up to *dozens of thousands* of such polyhedra. We therefore had to disable an inclusion test of a newly computed state into the already computed constraint: this test usually has a very interesting gain but, on our complex polyhedra, it had disastrous impact on the performance, due to the inclusion check of a (simple) new convex polyhedron into a disjunction of dozens of thousands of convex polyhedra. To disable this check, we added a new option⁵ (not set by default) to the master branch of IMITATOR, and used it in all our experiments.

We wrote a simple Python script to implement the TW2PTA procedure; the patterns (Fig. 6) were manually transformed following the MakeSymbolic procedure, and converted into the input language of IMITATOR.

We ran experiments using IMITATOR 2.10.4 “Butter Jellyfish” on a Dell Precision 3620 i7-7700 3.60 GHz with 64 GiB memory running Linux Mint 19 beta 64 bits.⁶

⁵Option “-no-inclusion-test-in-EF” in IMITATOR 2.10.4.

⁶Sources, binaries, models, logs can be found at imitator.fr/static/ICECCS18.

Model		PTPM				PTPM _{opt}	
Length	Time frame	States	Matches	Parsing (s)	Comp. (s)	States	Comp. (s)
1,467	1,000	4,453	379	0.02	1.60	3,322	0.94
2,837	2,000	8,633	739	0.33	2.14	6,422	1.70
4,595	3,000	14,181	1,247	0.77	3.63	10,448	2.85
5,839	4,000	17,865	1,546	1.23	4.68	13,233	3.74
7,301	5,000	22,501	1,974	1.94	5.88	16,585	4.79
8,995	6,000	27,609	2,404	2.96	7.28	20,413	5.76
10,316	7,000	31,753	2,780	4.00	8.38	23,419	6.86
11,831	8,000	36,301	3,159	5.39	9.75	26,832	7.87
13,183	9,000	40,025	3,414	6.86	10.89	29,791	8.61
14,657	10,000	44,581	3,816	8.70	12.15	33,141	9.89

Table 2: Experiments: GEAR

3.6.1 Gear

Benchmark GEAR is inspired by the scenario of monitoring the gear change of an automatic transmission system. We conducted simulation of the model of an automatic transmission system [HAF14]. We used BREACH [Don10] to generate an input sequence of gear change. A gear is chosen from $\{g_1, g_2, g_3, g_4\}$. The generated gear change is recorded in a timed word. The set W consists of 10 timed words; the length of each word is 1,467 to 14,657.

The pattern PTA \mathcal{A} , shown in Fig. 6a, detects the violation of the following condition: If the gear is changed to 1, it should not be changed to 2 within p seconds. This condition is related to the requirement ϕ_5^{AT} proposed in [HAF14] (the nominal value for p in [HAF14] is 2).

We tabulate our experiments in Table 2. We give from left to right the length of the timed word in terms of actions and time, then the data for PTPM (the number of symbolic states explored, the number of (symbolic) matches found, the parsing time and the computation time excluding parsing) and for PTPM_{opt} (number of symbolic states explored and computation time) using IMITATOR. The parsing time for PTPM_{opt} is almost identical to PTPM and is therefore omitted.

The corresponding chart is given in Fig. 7a (PTPM is given in plain black, and PTPM_{opt} in red dashed). PTPM_{opt} brings a gain in terms of memory (symbolic states) of about 25%, while the gain in time is about 20%.

3.6.2 Accel

The timed words W of benchmark ACCEL is also constructed from the Simulink model of the automatic transmission system [HAF14]. For this benchmark, the (discretized) value of three state variables are recorded in W : engine RPM (discretized to “high” and “low” with a certain threshold), velocity (discretized to “high” and “low” with a certain threshold), and 4 gear positions. We used BREACH [Don10] to generate an input sequence. Our set W consists of 10 timed words; the length of each word is 2,559 to 25,137.

The pattern PTA \mathcal{A} of this benchmark is shown in Fig. 6b. This pattern matches a part of a timed word that violates the following condition: If a gear changes from 1 to 2, 3, and 4 in this order in p seconds and engine RPM becomes large during this gear change, then the velocity of the car must be

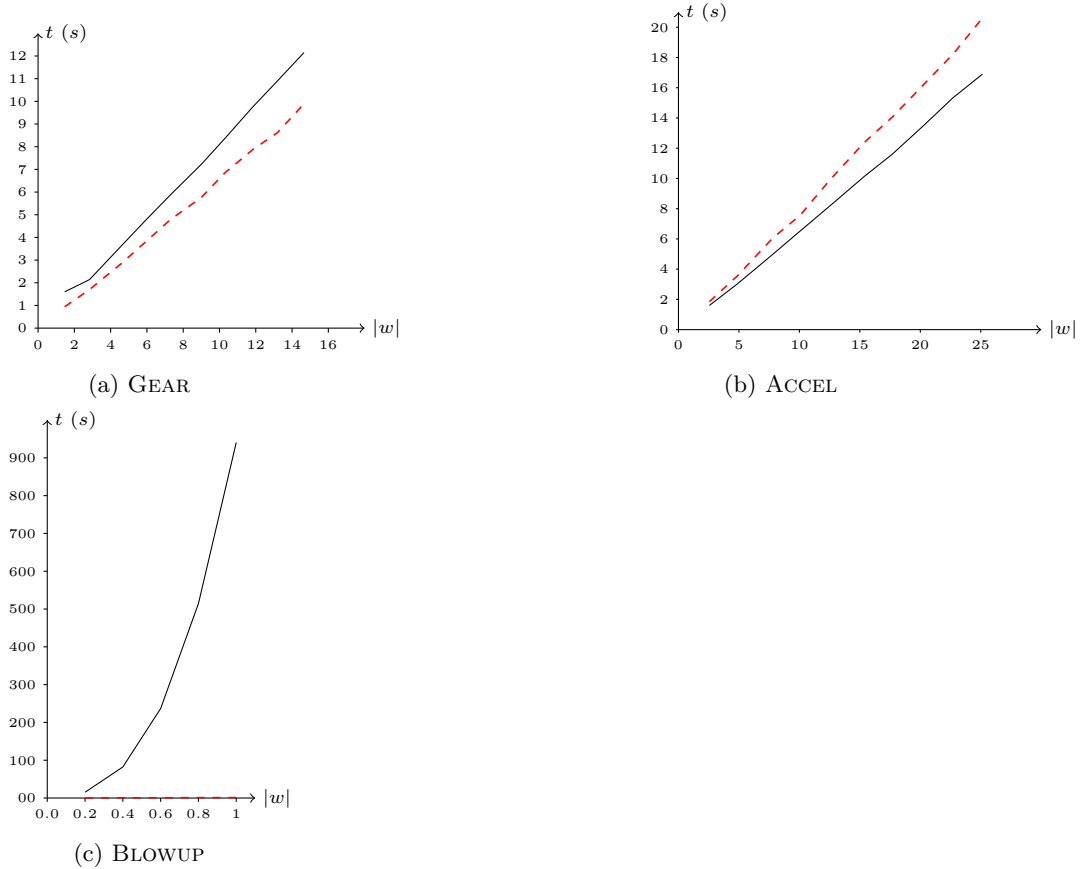


Figure 7: Experiments: charts (x -scale $\times 1,000$)

sufficiently large in one second. This condition models the requirement ϕ_8^{AT} proposed in [HAF14] (the nominal value for p in [HAF14] is 10).

Experiments are tabulated in Table 3. The corresponding chart is given in Fig. 7b. This time, PTPM_{opt} brings almost no gain in terms of states, and a loss of speed of about 15 to 20%, which may come from the additional polyhedra inclusion checks to test whether a branch is less good than the current optimum.

3.6.3 Blowup

As a third experiment, we considered an original (toy) benchmark that acts as a worst case situation for parametric timed pattern matching. Consider the PTA pattern in Fig. 6c, and assume a timed word consisting in an alternating sequence of “a” and “b”. Observe that the time from the pattern beginning (that resets x) to the end is exactly p_1 time units. Also observe that the duration of the loop through ℓ_2 and ℓ_3 has a duration in $[p_3, p_2]$; therefore, for values sufficiently small of p_2, p_3 , one can always match a larger number of loops. That is, for a timed word of length $2n$ alternating between “a” and “b”, there will

Model		PTPM				PTPM _{opt}	
Length	Time frame	States	Matches	Parsing (s)	Comp. (s)	States	Comp. (s)
2,559	1,000	6,504	2	0.27	1.60	6,502	1.85
4,894	2,000	12,429	2	0.86	3.04	12,426	3.57
7,799	3,000	19,922	7	2.21	4.98	19,908	6.06
10,045	4,000	25,520	3	3.74	6.51	25,514	7.55
12,531	5,000	31,951	9	6.01	8.19	31,926	9.91
15,375	6,000	39,152	7	9.68	10.14	39,129	12.39
17,688	7,000	45,065	9	13.40	11.61	45,039	14.06
20,299	8,000	51,660	10	18.45	13.52	51,629	16.23
22,691	9,000	57,534	11	24.33	15.33	57,506	18.21
25,137	10,000	63,773	13	31.35	16.90	63,739	20.61

Table 3: Experiments: ACCEL

Model		PTPM				PTPM _{opt}	
Length	Time frame	States	Matches	Parsing (s)	Comp. (s)	States	Comp. (s)
200	101	20,602	5,050	0.01	15.31	515	0.24
400	202	81,202	20,100	0.02	82.19	1,015	0.49
600	301	181,802	45,150	0.03	236.80	1,515	0.71
800	405	322,402	80,200	0.05	514.57	2,015	1.05
1,000	503	503,002	125,250	0.06	940.74	2,515	1.24

Table 4: Experiments: BLOWUP

be n possible matches from position 0 (with n different parameter constraints), $n - 1$ from position 1, and so on, giving a total number of $\frac{n(n+1)}{2}$ matches with different constraints in 5 dimensions.

Note that this worst case situation is not specific to our approach, but would appear independently of the approach chosen for parametric timed pattern matching.

We generated random timed words of various sizes, all alternating exactly between “a” and “b”. Our set W consists of 5 timed words of length from 200 to 1,000.

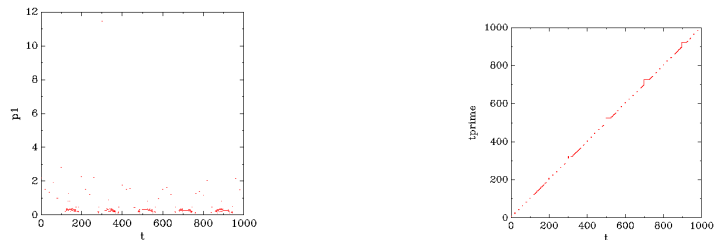
Experiments are tabulated in Table 4. The corresponding chart is given in Fig. 7c. PTPM becomes clearly non-linear as expected. This time, PTPM_{opt} brings a dramatic gain in both memory and time; even more interesting, PTPM_{opt} remains perfectly linear.

3.7 Discussion

A first positive outcome is that our method is effectively able to perform parametric pattern matching on words of length up to several dozens of thousands, and is able to output results in the form of several dozens of thousands of symbolic matches in several dimensions, in just a few seconds. Another positive outcome is that PTPM is perfectly linear in the size of the input word for GEAR and ACCEL: this was expected as these examples are linear, in the sense that the number of states explored by PTPM is linear as these patterns feature no loops.

Note that the parsing time is not linear, but it could be highly improved: due to the relatively small size of the models usually treated by IMITATOR, this part was never properly optimized, and it contains several quadratic syntax checking functions that could easily be avoided.

The performances do not completely allow yet for an *online* usage in the



(a) Projection onto t and p

(b) Projection onto t and t'

Figure 8: Visualizing a large number of matches for GEAR ($|w| = 1467$)

current version of our algorithm and implementation (in [WHS17], we pushed the ACCEL case study for timed words of length up to 17,280,002). A possible direction is to perform an on-the-fly computation of the parametric zone graph, more precisely to do an on-the-fly parsing of the timed word automaton; this will allow IMITATOR to keep in memory a single location at a time (instead of up to 25,137 in our experiments).

Finally, although this is not our original motivation, we believe that, if we are only interested in *robust* pattern matching, i. e., non-parametric pattern matching but with an allowed deviation (“guard enlargement”) of the pattern automaton, then using the efficient 1-dimensional parameterized DBMs of [San15] would probably be an interesting alternative: indeed, in contrast to classical parameterized DBMs [Hun+02] (that are made of a matrix and a parametric polyhedron), the structure of [San15] only needs an $H \times H$ matrix with a single parameter, and seems particularly efficient.

Remark 1. In the conference version of this work [AHW18], we describe this approach as an *offline* algorithm. In fact, it is essentially *online* in the sense that it can potentially run with only a portion of the log: it relies on parallel composition of a specification automaton and a log automaton, and this parallel composition can be achieved on-the-fly. However, as mentioned in [Bar+18], “a good online monitoring algorithm must:

1. be able to generate intermediate estimates of property satisfaction based on partial signals,
2. use minimal amount of data storage, and
3. be able to run fast enough in a real-time setting.”

So, at least for point 3, our algorithm may not really run in a real-time setting.

In contrast, we will present in the next section a contribution fast enough to run in a real-time setting, with runs of dozens of thousands of events being analyzable in less than a second.

4 Dedicated method

In this section, we present a *dedicated* online algorithm for parametric timed pattern matching (Section 4.1). We will then enhance it with skipping in Sec-

tion 4.2, and evaluate both versions with and without skipping in Section 4.3.

4.1 An online algorithm

Similarly to the online algorithm for timed pattern matching in [WAH16], our algorithm finds all the matching triples $(t, t', v) \in \mathcal{M}(w, \mathcal{A})$ by a breadth-first search. Our algorithm is online in the following sense: after reading the i -th element (a_i, τ_i) of the timed word $w = (\bar{a}, \bar{\tau})$, it immediately outputs all the matching triples (t, t', v) over the available prefix $(a_1, \tau_1), (a_2, \tau_2), \dots, (a_i, \tau_i)$ of w .

Firstly, we define the auxiliary functions for our online algorithm for parametric timed pattern matching. We introduce an additional variable t representing the absolute time of the beginning of the matching. We use a function $\rho: \mathbb{X} \rightarrow (\mathbb{R}_{>0} \sqcup \{t\})$ to represent the latest reset time of each clock variable $x \in \mathbb{X}$. Intuitively, $\rho(x) = \tau \in \mathbb{R}_{>0}$ means the latest reset of x is at τ , and $\rho(x) = t$ means x is not reset after the beginning of the matching.

Definition 6 ($\text{eval}(\rho, \tau)$). Let \mathbb{X} be the set of clock variables and t be the variable for the beginning of a matching. For a function $\rho: \mathbb{X} \rightarrow (\mathbb{R}_{>0} \sqcup \{t\})$ and the current time $\tau \in \mathbb{R}_{>0}$, $\text{eval}(\rho, \tau)$ is the following constraint on $\mathbb{X} \sqcup \{t\}$.

$$\text{eval}(\rho, \tau) = \bigwedge_{x \in \mathbb{X}} (x = \tau - \rho(x))$$

Definition 7 ($\text{reset}(\rho, R, \tau)$). For a function $\rho: \mathbb{X} \rightarrow (\mathbb{R}_{>0} \sqcup \{t\})$, the set $R \subseteq \mathbb{X}$ of clocks to be reset, and the current time $\tau \in \mathbb{R}_{>0}$, $\text{reset}(\rho, R, \tau): \mathbb{X} \rightarrow (\mathbb{R}_{>0} \sqcup \{t\})$ is the following function.

$$\text{reset}(\rho, R, \tau)(x) = \begin{cases} \tau & \text{if } x \in R \\ \rho(x) & \text{if } x \notin R \end{cases}$$

Definition 8 (ρ_\emptyset). By $\rho_\emptyset: \mathbb{X} \rightarrow (\mathbb{R}_{>0} \sqcup \{t\})$, we denote the function mapping each $x \in \mathbb{X}$ to t .

Intuitively, $\text{eval}(\rho, \tau)$ is the constraint corresponding to the clock valuation, $\text{reset}(\rho, R, \tau)$ is the operation to reset the clock variables $x \in R$ at τ , and ρ_\emptyset is the initial clock valuation.

Algorithm 3 shows our online algorithm for parametric timed pattern matching. In the pseudocode, we used *CurrConf*, *PrevConf*, and Z : *CurrConf* and *PrevConf* are finite sets of triples $(\ell, \rho, \mathcal{C})$ of a location $\ell \in L$, a mapping $\rho: \mathbb{X} \rightarrow (\mathbb{R}_{>0} \sqcup \{t\})$ denoting the latest reset of each clock, and a constraint \mathcal{C} over $\mathbb{P} \sqcup \{t\}$; and Z is a finite set of constraints over $\mathbb{P} \sqcup \{t, t'\}$. As a running example, we use the PTA and the timed word in Fig. 3 page 10.

At first, the counter i is 1 (line 2), and we start the matching trial from $t \in (\tau_0, \tau_1]$. At line 3, we add the new configuration $(\ell_0, \rho_\emptyset, (\tau_0 < t \leq \tau_1))$ to *CurrConf*, which means we are at the initial location ℓ_0 , we have no reset of the clock variables yet, and we can potentially start the matching from any

Algorithm 3: Online parametric timed pattern matching without skipping

Input: A timed word $w = (\bar{a}, \bar{\tau})$, and a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$.
Output: $\bigvee Z$ is the match set $\mathcal{M}(w, \mathcal{A})$

- 1 $CurrConf \leftarrow \emptyset$; $Z \leftarrow \emptyset$
- 2 **for** $i \leftarrow 1$ **to** $|w|$ **do**
- 3 **push** $(\ell_0, \rho_0, (\tau_{i-1} < t \leq \tau_i))$ **to** $CurrConf$ // start a matching trial
 from (τ_{i-1}, τ_i)
- 4 **for** $(\ell, \rho, \mathcal{C}) \in CurrConf$ **do** // lines 4 to 7 try to insert \$ in
 $[\tau_{i-1}, \tau_i)$
- 5 **for** $\ell_f \in F$ **do**
- 6 **for** $(\ell, g, \$, R, \ell_f) \in E$ **do**
- 7 **push** $(\mathcal{C} \wedge (\tau_{i-1} \leq t' < \tau_i) \wedge g \wedge \text{eval}(\rho, t')) \downarrow_{\mathbb{P} \sqcup \{t, t'\}}$ **to** Z
- 8 $(PrevConf, CurrConf) \leftarrow (CurrConf, \emptyset)$
- 9 **for** $(\ell, \rho, \mathcal{C}) \in PrevConf$ **do** // lines 9 to 13 try to go forward using
 (a_i, τ_i)
- 10 **for** $(\ell, g, a_i, R, \ell') \in E$ **do**
- 11 $\mathcal{C}' \leftarrow (\mathcal{C} \wedge g \wedge \text{eval}(\rho, \tau_i)) \downarrow_{\mathbb{P} \sqcup \{t\}}$
- 12 **if** $\mathcal{C}' \neq \perp$ **then**
- 13 **push** $(\ell', \text{reset}(\rho, R, \tau), \mathcal{C}')$ **to** $CurrConf$
- 14 **push** $(\ell_0, \rho_0, \{\tau_{|w|} < t < \infty\})$ **to** $CurrConf$ // for the trimming after the
 final event
- 15 **for** $(\ell, \rho, \mathcal{C}) \in CurrConf$ **do** // lines 15 to 18 try to insert \$ in $[\tau_{|w|}, \infty)$
- 16 **for** $\ell_f \in F$ **do**
- 17 **for** $(\ell, g, \$, R, \ell_f) \in E$ **do**
- 18 **push** $(\mathcal{C} \wedge (\tau_{|w|} \leq t' < \infty) \wedge g \wedge \text{eval}(\rho, t')) \downarrow_{\mathbb{P} \sqcup \{t, t'\}}$ **to** Z

$t \in (\tau_0, \tau_1]$. In lines 4 to 7, we try to insert $\$$ (i.e., the end of the matching) in $[\tau_0, \tau_1]$; in our running example in Fig. 3, since there is no edge from ℓ_0 to the final state, we immediately jump to line 8. Then, in lines 9 to 13, we consume $(a_1, \tau_1) = (a, 0.7)$ and try to transit from ℓ_0 to ℓ_1 . The guard $x > 1$ at the edge from ℓ_0 to ℓ_1 is examined at line 11. We take the conjunction of the current constraint \mathcal{C} , the guard g , and the constraints $\text{eval}(\rho, \tau_i)$ on the clock valuations. We take the projection to $\mathbb{P} \sqcup \{t\}$ because the constraint on the clock variables changes after time passing. Since no clock variable is reset so far, the constraint on the clock valuation is $x = \tau_1 - t$. The constraint $\mathcal{C} \wedge g \wedge \text{eval}(\rho, \tau_1) = (0 < t \leq 0.7) \wedge (x > 1) \wedge (x = 0.7 - t)$ is unsatisfiable, and we go back to line 3.

At line 3, we add the new configuration $(\ell_0, \rho_\emptyset, (\tau_1 < t \leq \tau_2))$ to *CurrConf*. Similarly, we immediately jump to line 8, and we try the edge from ℓ_0 to ℓ_1 in lines 9 to 13. This time, the constraint $\mathcal{C} \wedge g \wedge \text{eval}(\rho, \tau_2) = (0.7 < t \leq 2.0) \wedge (x > 1) \wedge (x = 2.0 - t)$ is satisfiable at line 12, and we push the next configuration $(\ell_1, \rho_\emptyset, \mathcal{C}')$ to *CurrConf* at line 13.

Similarly, we keep adding and updating configurations until the end of the input timed word w . Finally, in lines 15 to 18, we try to insert $\$$ in $[\tau_3, \infty) = [4.1, \infty)$. We can use the edge from ℓ_2 to the final state, and we add the constraint at the right of Fig. 3 to Z .

Algorithm 3 terminates because the size of *CurrConf* is always finite. Algorithm 3 is correct because it symbolically keeps track of all the runs of $v(\mathcal{A})$ over $w|_{(t, t')}$ for any $v \in (\mathbb{Q}_+)^{\mathbb{P}}$ and $(t, t') \subseteq \mathbb{R}_{\geq 0}$.

4.2 Parametric timed pattern matching enhanced with skipping

We now enhance Algorithm 3 with automata-based *skipping* that has been used for (non-parametric) timed pattern matching [WAH16; WHS17]. Skipping is an optimization for pattern matching by reducing the number of the matching trials. The high-level idea of skipping is to utilize the specification automaton and the information from the latest matching trial to decide if the current matching trial is necessary, and prevent the matching trial if it is unnecessary. More precisely, in the beginning of each matching trial (at line 2 of Algorithm 3), instead of increasing the counter i by one, we compute the smallest $\Delta \in \mathbb{N}_{>0}$ such that there may be a matching from $t \in (\tau_{i+\Delta-1}, \tau_{i+\Delta}]$, and update the counter i to $i + \Delta$. We call such Δ a *skip value*. We compute the skip value Δ using a function that we call a *skip value function*. When the skip value Δ is greater than one, some matching trials are prevented. Although the computation of such a skip value Δ requires some additional computational cost, a large part of the computation can be done beforehand thanks to the finiteness of the automata structure. Therefore, we can gain the runtime performance, which we confirm by our experiments in Section 4.3.

Following [WHS17], we employ *FJS-style*⁷ skipping [FJS07]. In FJS-style

⁷“FJS” is the acronym of the family names of the authors of [FJS07].

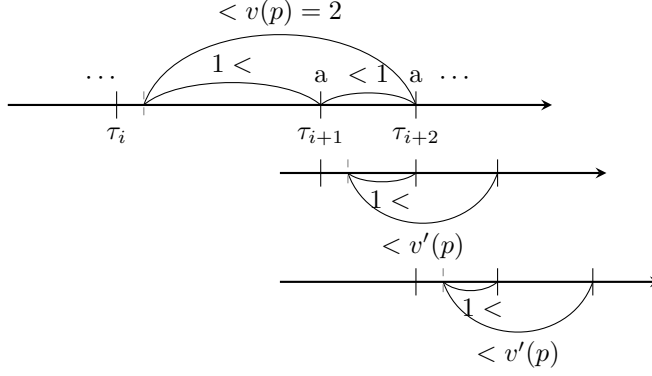


Figure 9: Illustration of the timing constraints in Example 8: constraints from the reached location ℓ_2 and the parameter valuation v (above), necessary timing constraints to have a matching after a shift by 1 (middle), and necessary timing constraints to have a matching after a shift by 2 (below)

skipping, the skip value Δ is computed by combining the result of two skip value functions: the KMP-style⁸ skip values function Δ_{KMP} [KJP77] and the Quick Search-style skip value function Δ_{QS} [Sun90]. Their combination in our FJS-style algorithm is presented later in Section 4.2.3. We remark that this optimization does not change the result thanks to the soundness of the skip value functions (Theorems 3 and 4).

The following are auxiliaries for the skip values. For a PTA \mathcal{A} and a parameter valuation v , the language without the last element is denoted by $\mathcal{L}_{-\$}(v(\mathcal{A})) = \{w(1, |w|-1) \mid w \in \mathcal{L}(v(\mathcal{A}))\}$. For a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$ and $\ell \in L$, \mathcal{A}_ℓ denotes the PTA $\mathcal{A}_\ell = (\Sigma, L, \ell_0, \{\ell\}, \mathbb{X}, \mathbb{P}, E)$.

4.2.1 KMP-style skip values

Given a location $\ell \in L$ and a set $V \subseteq (\mathbb{Q}_+)^{\mathbb{P}}$ of parameter valuations, the *KMP-style skip value function* Δ_{KMP} returns the skip value $\Delta_{\text{KMP}}(\ell, V) \in \mathbb{N}_{>0}$. The location ℓ and the parameter valuations V represent a set of the configurations in the latest matching trial. We utilize the pair (ℓ, V) to overapproximate the subsequence $w(i, j) - \tau_i$ of the timed word w examined in the latest matching trial. The following illustrates the idea of the KMP-style skipping.

Example 8. Let \mathcal{A} be the PTA in Fig. 3a. Suppose our latest matching trial from $t \in (\tau_i, \tau_{i+1}]$ finished at ℓ_2 with the parameter valuation v satisfying $v(p) = 2$. By the guards of \mathcal{A} , there is $t \in (\tau_i, \tau_{i+1}]$ satisfying $1 < \tau_{i+1} - t \wedge \tau_{i+2} - t < v(p) = 2 \wedge \tau_{i+2} - \tau_{i+1} < 1$. The above of Fig. 9 illustrates this timing constraint.

If we have a matching from $t \in (\tau_{i+1}, \tau_{i+2}]$, because of the guards in \mathcal{A} , there are $t \in (\tau_{i+1}, \tau_{i+2}]$ and a parameter valuation v' satisfying $1 < \tau_{i+2} - t \wedge \tau_{i+3} - t < v'(p)$. The middle of Fig. 9 illustrates this timing constraint. Since we have both $\tau_{i+2} - \tau_{i+1} < 1$ and $1 < \tau_{i+2} - t < \tau_{i+2} - \tau_{i+1}$, the timing constraints in the

⁸“KMP” is the acronym of the family names of the authors of [KJP77].

above and the middle of Fig. 9 does not hold at the same time. Therefore, we conclude that there is no matching from any point between τ_{i+1} and τ_{i+2} and we can skip this matching trial.

In contrast, if we have a matching from $t \in (\tau_{i+2}, \tau_{i+3}]$, there are $t \in (\tau_{i+2}, \tau_{i+3}]$ and a parameter valuation v' satisfying $1 < \tau_{i+3} - t \wedge \tau_{i+4} - t < v'(p)$. The below of Fig. 9 illustrates this timing constraint. Since the timing constraints in the above and the below of Fig. 9 are satisfiable at the same time, we may have a matching from $t \in (\tau_{i+2}, \tau_{i+3}]$, and we cannot skip this matching trial. Overall, we can increase the counter i in Algorithm 3 at most by 2 without changing the result.

We formalize this idea of the KMP-style skip value function with the languages of PTAs. Let $i \in \{1, 2, \dots, |w|\}$ and let ℓ be a location we reached in the end of the matching trial from i with the parameter valuation v . Since we reached ℓ with the parameter valuation v by reading a prefix of $w(i, |w|)$, there is an index $j \geq i$ satisfying $w(i, j) - \tau_i \in \mathcal{L}(v(\mathcal{A}_\ell))$.⁹ By appending an arbitrary timed word, we overapproximate the subsequence $w(i, |w|) - \tau_i$ by the language $\mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma)$.

If there is a matching from the i -th event (a_i, τ_i) of w with a parameter valuation v' , i. e., if there is $t \in (\tau_i, \tau_{i+1}]$ and $t' > t$ satisfying $w|_{(t, t')} \in \mathcal{L}(v(\mathcal{A}))$, there is an index $j \geq i$ and $t' \in [\tau_j, \tau_{j+1})$ satisfying $(w(i, j) \circ (\$, t')) - t \in \mathcal{L}(v(\mathcal{A}))$. By removing the terminal character $\$$, appending an arbitrary timed word, and shifting appropriately, we have $w(i, |w|) \in \{w'' + t \mid w'' \in \mathcal{L}_{-\$}(v'(\mathcal{A})), t \geq 0\} \cdot \mathcal{T}(\Sigma)$, and thus, for each $n \in \mathbb{N}_{>0}$, the matching after n -shift with a parameter valuation v' is overapproximated by $\mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\$}(v'(\mathcal{A})), t \geq 0\} \cdot \mathcal{T}(\Sigma)$. Therefore, we can skip the matching trial from $i + n$ if for any parameter valuation v' , and we have the following.

$$\mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma) \cap \mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\$}(v'(\mathcal{A})), t \geq 0\} \cdot \mathcal{T}(\Sigma) = \emptyset$$

Based on the above idea, the KMP-style skip value function Δ_{KMP} is defined as follows.

Definition 9 (Δ_{KMP}). Let \mathcal{A} be a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$. For a location $\ell \in L$ and $n \in \mathbb{N}_{>0}$, let $V_{\ell, n}$ be the set of parameter valuations v such that there is a parameter valuation $v' \in (\mathbb{Q}_+)^{\mathbb{P}}$ satisfying $\mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma) \cap \mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\$}(v'(\mathcal{A})), t \geq 0\} \cdot \mathcal{T}(\Sigma) \neq \emptyset$. The *KMP-style skip value function* $\Delta_{\text{KMP}}: L \times \mathcal{P}((\mathbb{Q}_+)^{\mathbb{P}}) \rightarrow \mathbb{N}_{>0}$ is $\Delta_{\text{KMP}}(\ell, V) = \min\{n \in \mathbb{N}_{>0} \mid V \subseteq V_{\ell, n}\}$.

Example 9. We continue Example 8. For a parameter valuation v , we have the following.

$$\mathcal{L}(v(\mathcal{A}_{\ell_2})) = \mathcal{L}_{-\$}(v(\mathcal{A})) = \{(a, \tau_1), (a, \tau_2) \mid 1 < \tau_1 < \tau_2 < v(p)\}$$

Therefore, for any parameter valuations v, v' , we have $\mathcal{L}(v(\mathcal{A}_{\ell_2})) \cdot \mathcal{T}(\Sigma) \cap \mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\$}(v'(\mathcal{A})), t \geq 0\} \cdot \mathcal{T}(\Sigma) \neq \emptyset$ if and only if the following is

⁹More precisely, we require $\ell \notin F$ because the transition to the final locations are labeled with the special terminal character $\$$, which is not in w .

satisfiable.

$$\exists t \in (\tau_n, \tau_{n+1}]. 1 < \tau_1 < \tau_2 < v(p) \wedge 1 < \tau_{n+1} - t < \tau_{n+2} - t < v'(p) \quad (1)$$

When $v(p) = 2$ and $n = 1$, we have $1 < \tau_2 - \tau_1 < 1$, and Eq. (1) is unsatisfiable. In contrast, when $v(p) = 2$ and $n = 2$, Eq. (1) is satisfiable, and we have $\Delta_{\text{KMP}}(\ell_2, \{v\}) = 2$, which coincides with the idea in Example 8.

We note that if we reached both ℓ and ℓ' , we have both $w(i, |w|) \in \mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma)$ and $w(i, |w|) \in \mathcal{L}(v(\mathcal{A}_{\ell'})) \cdot \mathcal{T}(\Sigma)$, and the intersection $(\mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma)) \cap (\mathcal{L}(v(\mathcal{A}_{\ell'})) \cdot \mathcal{T}(\Sigma))$ overapproximates $w(i, |w|)$. Therefore, we take the maximum of $\Delta_{\text{KMP}}(\ell, V)$ over the reached configurations.

Since $V_{\ell, n}$ is independent of the timed word w , we can compute it before the matching trials by reachability synthesis of PTAs. See Appendix A for the construction of the PTAs. During the matching trials, only the inclusion checking $V \subseteq V_{\ell, n}$ is necessary. This test can be achieved thanks to convex polyhedra inclusion.

Soundness For the KMP-style skip value function Δ_{KMP} , we have the following soundness result, and the use of Δ_{KMP} does not change the result of parametric timed pattern matching.

Theorem 3 (soundness of Δ_{KMP}). *Let $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$ be a PTA and let $w \in \mathcal{T}(\Sigma)$. For any subsequence $w(i, j)$ of w and for any $(\ell, v) \in L \times (\mathbb{Q}_+)^{\mathbb{P}}$, if there exists $t \in \mathbb{R}_{\geq 0}$ satisfying $w(i, j) - t \in \mathcal{L}(v(\mathcal{A}_\ell))$, for any $n \in \{1, 2, \dots, \Delta_{\text{KMP}}(\ell, \{v\}) - 1\}$, we have $([\tau_{i+n-1}, \tau_{i+n}] \times \mathbb{R}_{>0} \times (\mathbb{Q}_+)^{\mathbb{P}}) \cap \mathcal{M}(w, \mathcal{A}) = \emptyset$.*

First, we prove the following lemma.

Lemma 2. *Let \mathcal{A} be a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$ and let $w \in \mathcal{T}(\Sigma)$. For each subsequence $w(i, j)$ of w , let $C_{i, j} \subseteq L \times (\mathbb{Q}_+)^{\mathbb{P}}$ be $C_{i, j} = \{(\ell, v) \mid \exists t \in \mathbb{R}_{\geq 0}. w(i, j) - t \in \mathcal{L}(v(\mathcal{A}_\ell))\}$. For any subsequence $w(i, j)$ of w and $n \in \{1, 2, \dots, |w| - i\}$, if there exists $(\ell, v) \in C_{i, j}$ satisfying $v \notin V_{\ell, n}$, we have $([\tau_{i+n-1}, \tau_{i+n}] \times \mathbb{R}_{>0} \times (\mathbb{Q}_+)^{\mathbb{P}}) \cap \mathcal{M}(w, \mathcal{A}) = \emptyset$.*

Proof. For any $(\ell, v) \in C_{i, j}$ satisfying $v \notin V_{\ell, n}$, by definition of $C_{i, j}$ and $V_{\ell, n}$, we have the following.

- $\exists t \in \mathbb{R}_{\geq 0}. w(i, j) - t \in \mathcal{L}(v(\mathcal{A}_\ell))$
- $\forall v' \in (\mathbb{Q}_+)^{\mathbb{P}}. \mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma) \cap \mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\mathfrak{S}}(v'(\mathcal{A})), t > 0\} \cdot \mathcal{T}(\Sigma) = \emptyset$

Therefore, we have the following.

$$\begin{aligned}
& \exists(\ell, v) \in C_{i,j}. v \notin V_{\ell,n} \\
\Rightarrow & \exists t \in \mathbb{R}_{>0}. \forall v' \in (\mathbb{Q}_+)^{\mathbb{P}}. \\
& (w(i, j) - t) \cdot \mathcal{T}(\Sigma) \cap \mathcal{T}^n(\Sigma) \cdot \{w'' + t' \mid w'' \in \mathcal{L}_{-\mathfrak{s}}(v'(\mathcal{A})), t' > 0\} \cdot \mathcal{T}(\Sigma) = \emptyset \\
\Rightarrow & \forall v' \in (\mathbb{Q}_+)^{\mathbb{P}}. w(i, j) \cdot \mathcal{T}(\Sigma) \cap \mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\mathfrak{s}}(v'(\mathcal{A})), t > 0\} \cdot \mathcal{T}(\Sigma) = \emptyset \\
\Rightarrow & \forall v' \in (\mathbb{Q}_+)^{\mathbb{P}}. w(i, |w|) \cdot \mathcal{T}(\Sigma) \cap \mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\mathfrak{s}}(v'(\mathcal{A})), t > 0\} \cdot \mathcal{T}(\Sigma) = \emptyset \\
\Rightarrow & \forall v' \in (\mathbb{Q}_+)^{\mathbb{P}}. w(i, |w|) \notin \mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\mathfrak{s}}(v'(\mathcal{A})), t > 0\} \cdot \mathcal{T}(\Sigma) \\
\Rightarrow & \forall v' \in (\mathbb{Q}_+)^{\mathbb{P}}. w(i, |w|) \notin \mathcal{T}^n(\Sigma) \circ \{w'' + t \mid w'' \in \mathcal{L}_{-\mathfrak{s}}(v'(\mathcal{A})), t \in (\tau_{i+n-1}, \tau_{i+n}]\} \cdot \mathcal{T}(\Sigma) \\
\Rightarrow & \forall v' \in (\mathbb{Q}_+)^{\mathbb{P}}. w(i+n, |w|) \notin \{w'' + t \mid w'' \in \mathcal{L}_{-\mathfrak{s}}(v'(\mathcal{A})), t \in (\tau_{i+n-1}, \tau_{i+n}]\} \cdot \mathcal{T}(\Sigma) \\
\iff & \forall t \in (\tau_{i+n-1}, \tau_{i+n}], v' \in (\mathbb{Q}_+)^{\mathbb{P}}. w(i+n, |w|) - t \notin \{w'' \mid w'' \in \mathcal{L}_{-\mathfrak{s}}(v'(\mathcal{A}))\} \cdot \mathcal{T}(\Sigma) \\
\iff & \forall t \in (\tau_{i+n-1}, \tau_{i+n}], t' > t, v' \in (\mathbb{Q}_+)^{\mathbb{P}}. w|_{(t,t')} \notin \mathcal{L}(v'(\mathcal{A})) \\
\iff & ((\tau_{i+n-1}, \tau_{i+n}] \times \mathbb{R}_{>0} \times (\mathbb{Q}_+)^{\mathbb{P}}) \cap \mathcal{M}(w, \mathcal{A}) = \emptyset
\end{aligned}$$

□

Then, the proof of Theorem 3 is as follows.

Proof. By definition of $C_{i,j}$ in Lemma 2, for any subsequence $w(i, j)$ of w and $(\ell, v) \in L \times (\mathbb{Q}_+)^{\mathbb{P}}$, $\exists t \in \mathbb{R}_{\geq 0}. w(i, j) - t \in \mathcal{L}(v(\mathcal{A}_\ell))$ implies $(\ell, v) \in C_{i,j}$. By definition of Δ_{KMP} , for any $(\ell, v) \in C_{i,j}$ and $n \in \{1, 2, \dots, \Delta_{\text{KMP}}(\ell, \{v\}) - 1\}$, we have $v \notin V_{i,n}$. Therefore, Theorem 3 holds because of Lemma 2. □

Discussion Although $V_{\ell,n}$ can be computed before the matching trials, the KMP-style skip value function Δ_{KMP} requires checking $V \subseteq V_{\ell,n}$ after each matching trial, which means a polyhedral inclusion test in $|\mathbb{P}| + 2$ dimensions. To reduce this runtime overhead, we define the *non-parametric* KMP-style skip value function $\Delta'_{\text{KMP}}(\ell) = \min_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \Delta_{\text{KMP}}(\ell, \{v\})$. For comparison, we refer to Δ_{KMP} as the *parametric* KMP-style skip value function.

4.2.2 Quick Search-style skip values

In Quick Search-style skipping, we ignore the timing constraints and only utilize the actions. Given an action $a \in \Sigma$, the *Quick Search-style skip value function* Δ_{QS} returns the skip value $\Delta_{\text{QS}}(a) \in \mathbb{N}_{>0}$. Before the matching trial from the i -th element (a_i, τ_i) , we look ahead the action a_{i+N-1} , where N is the length of the shortest matching. If we observe that there is no potential matching, we also look ahead the action a_{i+N} and skip by $\Delta_{\text{QS}}(a_{i+N})$.

Example 10. Let \mathcal{A} be the PTA in Fig. 6b. The length of the shortest matching N is five. For any parameter valuation v and timed word $(a'_1, \tau_1), (a'_2, \tau'_2), \dots, (a'_n, \tau'_n) \in \mathcal{L}(v(\mathcal{A}))$, we have $a'_1 \in \{\mathfrak{g}_1, \text{rpmHigh}\}$, $a'_2 \in \{\mathfrak{g}_1, \mathfrak{g}_2, \text{rpmHigh}\}$, $a'_3 \in \{\mathfrak{g}_2, \mathfrak{g}_3, \text{rpmHigh}\}$, $a'_4 \in \{\mathfrak{g}_3, \mathfrak{g}_4, \text{rpmHigh}\}$, and $a'_5 \in \{\mathfrak{g}_4, \text{rpmHigh}\}$.

Suppose we are about to start the matching trial from a point between τ_0 and τ_1 (i. e., $i = 1$) and the actions in the timed word w against pattern matching are $a_1 = g_1$, $a_2 = g_2$, $a_3 = g_1$, $a_4 = g_2$, $a_5 = g_1$, and $a_6 = g_2$. First, we look ahead the action $a_{i+N-1} = a_5 = g_1$ and check if g_1 can be the 5th action in a matching, which is not the case because the 5th action in a matching must be in $\{g_4, \text{rpmHigh}\}$. Then, we move to the position where we can have a matching according to $a_{1+N} = a_6$. Since $a_6 = g_2$ cannot be the 4th nor 5th actions in a matching but can be the 3rd action in a matching, we move to the position where a_6 is the 3rd action in the matching trial, i. e., $i = 4$. Therefore, the Quick Search-style skip value is $\Delta_{\text{QS}}(g_2) = 4 - 1 = 3$.

We formalize this idea with the *untimed* language of PTAs. For $i \in \{1, 2, \dots, |w|\}$, the subsequence $w(i, |w|)$ is overapproximated by $\Sigma^N a_{i+N} \Sigma^*$. For $n \in \mathbb{N}_{>0}$, the matching from $i + n$ is overapproximated by $\bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \Sigma^n \text{Untimed}(\mathcal{L}_{-\$}(v(\mathcal{A})))$. Thus, we have no matching from $i + n$ if for any $v \in (\mathbb{Q}_+)^{\mathbb{P}}$, we have $\Sigma^N a_{i+N} \Sigma^* \cap \Sigma^n \text{Untimed}(\mathcal{L}_{-\$}(v(\mathcal{A}))) = \emptyset$. The definition of the Quick Search-style skip value function Δ_{QS} is as follows.

Definition 10 (Δ_{QS}). For a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$, the *Quick-Search-style skip value function* $\Delta_{\text{QS}}: \Sigma \rightarrow \mathbb{N}_{>0}$ is as follows, where $N \in \mathbb{N}_{>0}$ is $N = \min\{|w| \mid w \in \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \mathcal{L}_{-\$}(v(\mathcal{A}))\}$.

$$\Delta_{\text{QS}}(a) = \min\{n \in \mathbb{N}_{>0} \mid \exists v \in (\mathbb{Q}_+)^{\mathbb{P}}. \Sigma^N a \Sigma^* \cap \Sigma^n \text{Untimed}(\mathcal{L}_{-\$}(v(\mathcal{A}))) \neq \emptyset\}$$

The construction of the Quick Search-style skip value function Δ_{QS} is by reachability emptiness of PTAs, i. e., the emptiness of the valuation set reaching a given location.

Soundness For the Quick Search-style skip value function Δ_{QS} , we have the following soundness result, and the use of Δ_{QS} does not change the result of parametric timed pattern matching.

Theorem 4 (soundness of Δ_{QS}). *Let \mathcal{A} be a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$, let $w = (\bar{a}, \bar{\tau}) \in \mathcal{T}(\Sigma)$, and let $N = \min\{|w| \mid w \in \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \mathcal{L}_{-\$}(v(\mathcal{A}))\}$. For any index $i \in \{1, 2, \dots, |w|\}$ of w and for any $m \in \{1, 2, \dots, \Delta_{\text{QS}}(a_{i+N}) - 1\}$, we have $((\tau_{i+m-1}, \tau_{i+m}] \times \mathbb{R}_{>0} \times (\mathbb{Q}_+)^{\mathbb{P}}) \cap \mathcal{M}(w, \mathcal{A}) = \emptyset$.*

First, we prove the following lemma.

Lemma 3. *Let \mathcal{A} be a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$, let $w = (\bar{a}, \bar{\tau}) \in \mathcal{T}(\Sigma)$, and let $N = \min\{|w| \mid w \in \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \mathcal{L}_{-\$}(v(\mathcal{A}))\}$. For any index $i \in \{1, 2, \dots, |w|\}$ of w and for any $m \in \{1, 2, \dots, N\}$, if $a_{i+N} \neq a'_{N-m+1}$ holds for any $(\bar{a}', \bar{\tau}') \in \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \mathcal{L}(v(\mathcal{A}))$, we have $((\tau_{i+m-1}, \tau_{i+m}] \times \mathbb{R}_{>0} \times (\mathbb{Q}_+)^{\mathbb{P}}) \cap \mathcal{M}(w, \mathcal{A}) = \emptyset$.*

Proof. If $a_{i+N} \neq a'_{N-m+1}$ holds for any $(\bar{a}', \bar{\tau}') \in \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \mathcal{L}(v(\mathcal{A}))$, we have the following.

$$\text{Untimed}(\{w(i+m, |w|)\}) \not\subseteq \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \text{Untimed}(\mathcal{L}_{-\$}(v(\mathcal{A}))) \Sigma^*$$

Lemma 3 is proved by the following.

$$\begin{aligned}
& \text{Untimed}(\{w(i+m, |w|)\}) \not\subseteq \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \text{Untimed}(\mathcal{L}_{-\$}(v(\mathcal{A}))\Sigma^*) \\
& \Rightarrow \forall t \in (\tau_{i+m-1}, \tau_{i+m}], v \in (\mathbb{Q}_+)^{\mathbb{P}}. w(i+m, |w|) - t \notin \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \mathcal{L}_{-\$}(v(\mathcal{A})) \cdot \mathcal{T}(\Sigma) \\
& \Rightarrow \forall t \in (\tau_{i+m-1}, \tau_{i+m}], t' > t, v \in (\mathbb{Q}_+)^{\mathbb{P}}. w|_{(t,t')} \notin \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \mathcal{L}(v(\mathcal{A})) \\
& \iff ((\tau_{i+m-1}, \tau_{i+m}] \times \mathbb{R}_{>0} \times (\mathbb{Q}_+)^{\mathbb{P}}) \cap \mathcal{M}(w, \mathcal{A}) = \emptyset
\end{aligned}$$

□

Then, the proof of Theorem 4 is as follows.

Proof. Since $\Sigma^N a \Sigma^* \cap \Sigma^n \text{Untimed}(\mathcal{L}_{-\$}(v(\mathcal{A}))) \neq \emptyset$ holds for any $n \geq N+1$, we have $\Delta_{\text{QS}}(a_{i+n}) - 1 \leq N$. By definition of Δ_{QS} , $m < \Delta_{\text{QS}}(a_{i+N})$ implies the following.

$$\forall v \in (\mathbb{Q}_+)^{\mathbb{P}}. \Sigma^N a_{i+N} \Sigma^* \cap \Sigma^m \text{Untimed}(\mathcal{L}_{-\$}(v(\mathcal{A}))) = \emptyset$$

Therefore, Lemma 3 implies Theorem 4. □

4.2.3 Skipping enhanced algorithm

Algorithm 4 shows an improvement of Algorithm 3 enhanced by skipping. The loop in lines 2 to 13 of Algorithm 3 is used in the matching trial, i.e., lines 8 to 10 of Algorithm 4. Before each matching trial, we check if the $i+N$ -th action a_{i+N} of the timed word $w = (\bar{a}, \bar{\tau})$ can be the N -th action of a matching. In our implementation, for the efficiency, we compute the set of the actions that can be the N -th action of a matching beforehand. If there is no potential matching from the i -th element of w , we increase the counter i using the Quick Search-style skip value function Δ_{QS} . After reading the i -th element (a_i, τ_i) of w , Algorithm 4 does not immediately output the matching over the available prefix $(a_1, \tau_1), (a_2, \tau_2), \dots, (a_i, \tau_i)$ of w , but it still outputs the matching before obtaining the entire timed word with some delay. At line 11, we increase the counter i using the parametric KMP-style skip value $\Delta_{\text{KMP}}(\ell, V)$. We can employ the non-parametric KMP-style skip value by replacing $\Delta_{\text{KMP}}(\ell, V)$ with $\Delta'_{\text{KMP}}(\ell)$.

4.3 Experiments

We implemented our dedicated algorithms for parametric timed pattern matching in a tool `ParamMONAA`. We implemented the following three algorithms: the online algorithm without skipping (Algorithm 3, referred as “no skip”);

Algorithm 4: Parametric timed pattern matching with parametric skipping

Input: A timed word w and a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$
Output: Z is the match set $\mathcal{M}(w, \mathcal{A})$

```

1  $i \leftarrow 1$  //  $i$  is the position in  $w$  of the beginning of the current matching
   trial
2  $N = \min\{|w| \mid w \in \bigcup_{v \in (\mathbb{Q}_+)^{\mathbb{P}}} \mathcal{L}_{-\$}(v(\mathcal{A}))\}$ 
3 while  $i \leq |w| - N + 1$  do
4   while  $\forall v \in (\mathbb{Q}_+)^{\mathbb{P}}, (\bar{a}', \bar{\tau}') \in \mathcal{L}(v(\mathcal{A})). a_{i+N-1} \neq a'_N$  do
      // Try matching the  $N$ -th action of  $\mathcal{L}(v(\mathcal{A}))$ 
5      $i \leftarrow i + \Delta_{\text{QS}}(a_{i+N})$  // Quick Search-style skipping
6     if  $i > |w| - N + 1$  then
7       return
8    $Z \leftarrow Z \cup \{(t, t', v) \in (\tau_{i-1}, \tau_i] \times [\tau_{i-1}, \infty) \times (\mathbb{Q}_+)^{\mathbb{P}} \mid w|_{(t, t')} \in \mathcal{L}(v(\mathcal{A}))\}$ 
      // Try matching
9    $j \leftarrow \max\{j \in \{i, i+1, \dots, |w|\} \mid \exists \ell \in L, v \in (\mathbb{Q}_+)^{\mathbb{P}}, t \in \mathbb{R}_{>0}. w(i, j) - t \in \mathcal{L}(v(\mathcal{A}_\ell))\}$ 
10   $C \leftarrow \{(\ell, V) \in L \times \mathcal{P}((\mathbb{Q}_+)^{\mathbb{P}}) \mid \forall v \in V, \exists t \in \mathbb{R}_{>0}. w(i, j) - t \in \mathcal{L}(v(\mathcal{A}_\ell))\}$ 
11   $i \leftarrow i + \max_{(\ell, V) \in C} \Delta_{\text{KMP}}(\ell, V)$  // Parametric KMP-style skipping
12  $Z \leftarrow Z \cup \{(t, t', v) \in (\tau_{|w|}, \infty) \times [\tau_{|w|}, \infty) \times (\mathbb{Q}_+)^{\mathbb{P}} \mid w|_{(t, t')} \in \mathcal{L}(v(\mathcal{A}))\}$ 

```

the online algorithm with parametric skipping (Algorithm 4, referred as “parametric skip”); and the online algorithm with non-parametric skipping (Algorithm 4 where $\Delta_{\text{KMP}}(\ell, V)$ at line 11 is replaced with $\Delta'_{\text{KMP}}(\ell)$, referred as “non-parametric skip”). We note that in ParamMONAA, the definition of timed word segments is slightly different from Section 2.1. Namely, we use the timing constraints $\tau_{i-1} \leq t < \tau_i$ and $\tau_j < t \leq \tau_{j+1}$ instead of $\tau_{i-1} < t \leq \tau_i$ and $\tau_j \leq t < \tau_{j+1}$. This difference is minor and does not affect the performance evaluation.

In the skip value computation, we use reachability synthesis for PTAs. Since reachability synthesis is intractable in general (the emptiness problem, i. e., the (non-)existence of a valuation reaching a given location, is undecidable [AHV93]), we use the following overapproximation: after investigating 100 configurations, we speculate that all the inconclusive parameter valuations are reachable parameter valuations. We remark that this overapproximation does not affect the correctness of parametric timed pattern matching, as it potentially *decreases* the skip value. We conducted experiments to answer the following research questions.

RQ1 Which is the fastest algorithm for parametric timed pattern matching?

RQ2 Why is parametric timed pattern matching slower than non-parametric timed pattern matching? Namely, is it purely because of the difficulty of the problem itself or is it mainly because of the general implementation

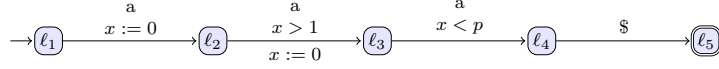


Figure 10: ONLYTIMING: the parameter p is substituted to 1 in ONLYTIMING-NP.

and data structure required by the general problem setting?

RQ3 How large is the overhead of the skip value computation? Namely, is it small and acceptable?

We implemented `ParamMONAA` in C++ and we compiled them using GCC 7.3.0. We conducted the experiments on an Amazon EC2 c4.large instance (2.9 GHz Intel Xeon E5-2666 v3, 2 vCPUs, and 3.75 GiB RAM) that runs Ubuntu 18.04 LTS (64 bit).¹⁰

Figure 6 shows the pattern PTAs we used in the experiments. We reuse the benchmarks `GEAR`, `ACCEL`, and `BLOWUP` from Section 3 as well as the new benchmark `ONLYTIMING`. The timed words for `GEAR` and `ACCEL` are generated by the automatic transmission system model in [HAF14]. `BLOWUP` and `ONLYTIMING` are toy examples. `BLOWUP` shows the worst case situation for parametric timed pattern matching (see Section 3.6.3). In `ONLYTIMING`, the parametric skip values are greater than the non-parametric skip values. In Sections 4.3.2 and 4.3.3, we also used the non-parametric variants `GEAR-NP`, `ACCEL-NP`, `BLOWUP-NP`, and `ONLYTIMING-NP` where the parameters are substituted to specific concrete values.

4.3.1 RQ1: Overall execution time

To answer RQ1, we compared the total execution time of `ParamMONAA` using `GEAR`, `ACCEL`, `BLOWUP`, and `ONLYTIMING`. As a baseline, we used our previous implementation of parametric timed pattern matching based on `IMITATOR` (“Butter Jellyfish”, version 2.10.4). Tables 5 to 8 and Fig. 11 show the execution time of our online algorithms compared with the `IMITATOR`-based implementation.

In Tables 5 to 8, we observe that our algorithms are faster than the `IMITATOR`-based implementation by orders of magnitude. Moreover, for `BLOWUP`, the `IMITATOR`-based implementation aborted due to out of memory. This is mainly because `ParamMONAA` is specific to parametric timed pattern matching while `IMITATOR` is a general tool for parametric verification. This shows the much better efficiency of our dedicated approach compared to Section 3.

In Fig. 11, we observe that the curve of “no skip” has the steepest slope and the curves of either “parametric skip” or “non-parametric skip” have the gentlest slope except for `BLOWUP`. `BLOWUP` is a benchmark designed on purpose to observe exponential blowup of the execution time, and it requires much time for all of the implementations.

¹⁰Experiment data can be found on github.com/MasWag/ParamMONAA/blob/master/doc/NFM2019.md.

Table 5: Execution time for GEAR [s]

$ w $	No Skip	Non-Param. Skip	Param. Skip	IMITATOR
1467	0.04	0.05	0.05	1.781
2837	0.0725	0.0805	0.09	3.319
4595	0.124	0.13	0.1405	5.512
5839	0.1585	0.156	0.17	7.132
7301	0.201	0.193	0.2115	8.909
8995	0.241	0.2315	0.2505	10.768
10315	0.2815	0.269	0.2875	12.778
11831	0.322	0.301	0.325	14.724
13183	0.3505	0.3245	0.353	16.453
14657	0.392	0.361	0.395	18.319

Table 6: Execution time for ACCEL [s]

$ w $	No Skip	Non-Param. Skip	Param. Skip	IMITATOR
2559	0.03	0.0515	0.06	2.332
4894	0.0605	0.0605	0.0705	4.663
7799	0.1005	0.071	0.08	7.532
10045	0.13	0.08	0.09	9.731
12531	0.161	0.09	0.1	12.503
15375	0.1985	0.1005	0.113	15.583
17688	0.2265	0.1095	0.1215	17.754
20299	0.261	0.115	0.1325	21.040
22691	0.288	0.121	0.143	23.044
25137	0.3205	0.1315	0.159	25.815

Table 7: Execution time for BLOWUP [s]

$ w $	No Skip	Non-Param. Skip	Param. Skip	IMITATOR
2000	66.75	68.0125	67.9735	OutOfMemory
4000	267.795	271.642	269.084	OutOfMemory
6000	601.335	611.782	607.58	OutOfMemory
8000	1081.42	1081.25	1079	OutOfMemory
10000	1678.15	1688.22	1694.53	OutOfMemory

Table 8: Execution time for ONLYTIMING [s]

$ w $	No Skip	Non-Param. Skip	Param. Skip	IMITATOR
1000	0.0995	0.1305	0.11	1.690
2000	0.191	0.23	0.191	3.518
3000	0.2905	0.3265	0.273	5.499
4000	0.3905	0.426	0.3525	7.396
5000	0.488	0.5225	0.4325	9.123
6000	0.588	0.6235	0.517	11.005

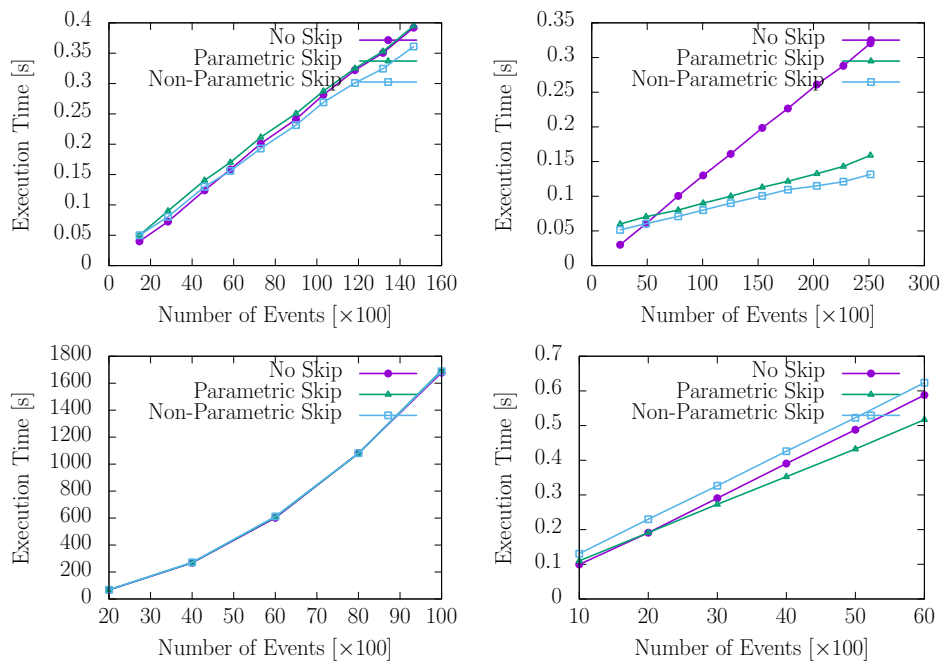


Figure 11: Execution time for the benchmarks with parameters which MONAA cannot handle: GEAR (above left), ACCEL (above right), BLOWUP (below left), and ONLYTIMING (below right)

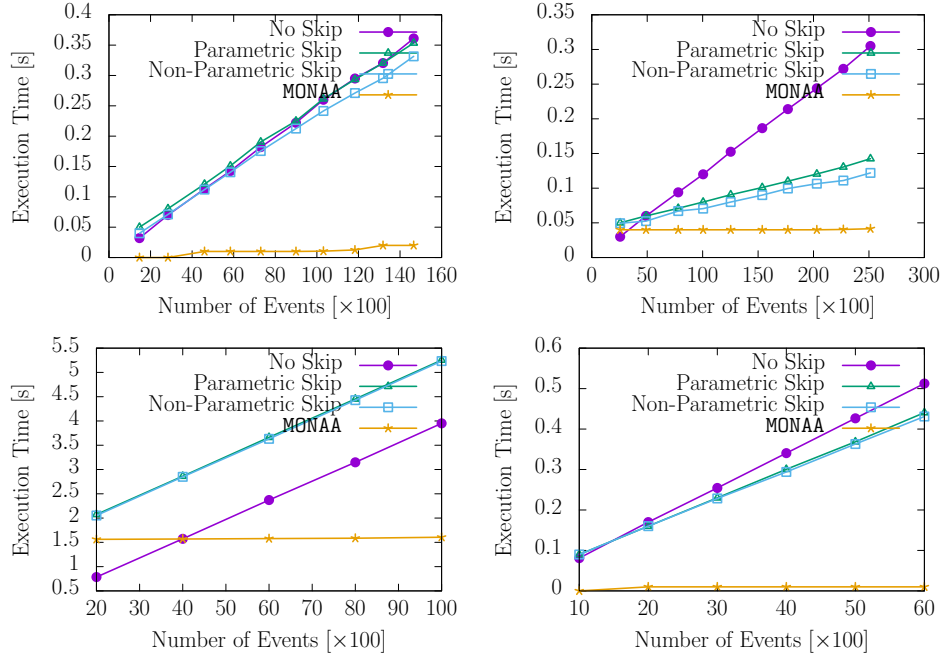


Figure 12: Execution time for the benchmarks without parameters: GEAR-NP (above left), ACCEL-NP (above right), BLOWUP-NP (below left), and ONLYTIMING-NP (below right)

For GEAR and ACCEL, the execution time of “non-parametric skip” increases the most gently. This is because the parametric KMP-style skip value $\Delta_{\text{KMP}}(\ell, V)$ and the non-parametric KMP-style skip value $\Delta'_{\text{KMP}}(\ell)$ are equal for these benchmarks, and “parametric skip” is slower due to the inclusion checking $V \subseteq V_{\ell, n}$.

For ONLYTIMING, we observe that the execution time of “parametric skip” increases the most gently because the parametric KMP-style skip value $\Delta_{\text{KMP}}(\ell, V)$ is larger than the non-parametric KMP-style skip value $\Delta'_{\text{KMP}}(\ell)$.

We conclude that skipping usually makes parametric timed pattern matching efficient. The preference between two skipping methods depends on the pattern PTA and it is a future work to investigate the tendency. Since the computation of the skip values does not take much time, the following work flow is reasonable:

1. compute the skip values for both of them; and
2. use “parametric skip” only if its skip values are strictly larger than that of “non-parametric skip”.

4.3.2 RQ2: Parametric vs. non-parametric timed pattern matching

To answer RQ2, we ran ParamMONAA using the non-parametric benchmarks (ACCEL-NP, GEAR-NP, BLOWUP-NP, and ONLYTIMING-NP) and compared the

Table 9: Execution time [s] for the skip value computation

	Non-Parametric Skip	Parametric Skip	MONAA
GEAR	0.0115	0.0175	n/a
GEAR-NP	0.01	0.01	< 0.01
ACCEL	0.042	0.0435	n/a
ACCEL-NP	0.04	0.04	0.0305
ONLYTIMING	0.03	0.03	n/a
ONLYTIMING-NP	0.02	0.02	< 0.01
BLOWUP	0.3665	0.381	n/a
BLOWUP-NP	1.268	1.2905	1.5455

execution time with a tool **MONAA** [WHS18] for non-parametric timed pattern matching.

In Fig. 12, we observe that our algorithms are slower than **MONAA** by orders of magnitude even though we solve the same problem (non-parametric timed pattern matching). This is presumably because our implementations rely on Parma Polyhedra Library (PPL) [BHZ08] to compute symbolic states, while **MONAA** utilizes DBMs (difference bound matrices) [Dil90]. It was shown in [Bak+17] that polyhedra may be dozens of times slower than DBMs; however, for parametric analyses, DBMs are not suitable, and parameterized extensions (e. g., in [Hun+02]) still need polyhedra in their representation.

Moreover, in Figs. 11 and 12, we observe that the execution time of our algorithms are not much different between each parametric benchmark and its non-parametric variant except **BLOWUP**. This observation shows that at least one additional parameter does not make the problem too difficult.

Therefore, we conclude that the lower efficiency of parametric timed pattern matching is mainly because of its general data structure required by the general problem setting.

4.3.3 RQ3: Overhead of skip value computation

To answer RQ3, we compared the execution time of our algorithms for an empty timed word using all the benchmarks. As a baseline, we also measured the execution time of **MONAA**.

In Table 9, we observe that the execution time for the skip values is less than 0.05 second except for **BLOWUP** and **BLOWUP-NP**. Even for the slowest pattern PTA **BLOWUP-NP**, the execution time for the skip values is less than 1.5 second and it is faster than that of **MONAA**. We conclude that the overhead of the skip value computation is small and acceptable in many usage scenarios.

5 Comparison Between Two Approaches

In Sections 3 and 4, we presented two approaches for parametric timed pattern matching. In Section 3, we reduced parametric timed pattern matching to parametric timed model checking of PTAs (Algorithm 1), using an existing

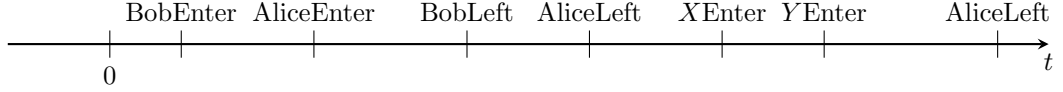


Figure 13: A log of entrance and leaving from a building. Timestamps are omitted for simplicity. We usually know who entered or left the building (e. g., BobEnter) but we sometimes do not know who (e. g., XEnter).

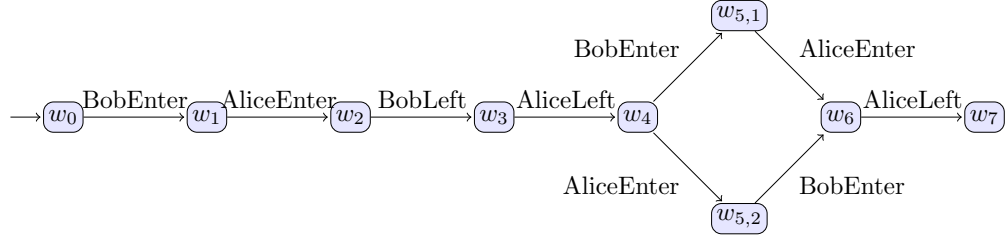


Figure 14: The TA constructed from the log in Fig. 13 using TW2PTA in Section 3.2.2. We assume $X, Y \in \{\text{Alice}, \text{Bob}\}$ and $X \neq Y$. The timing constraints are omitted for simplicity.

parametric timed model checker. Parametric timed model checking relies on the reachability analysis with symbolic abstraction by convex polyhedra. In Section 4, we solved parametric timed pattern matching by following the transitions of the PTA (Algorithm 3). Moreover, we optimized Algorithm 3 using *skipping* (Algorithm 4), which is a technique from string matching. Although the relationship between parametric timed pattern matching and parametric timed model checking is implicit in Section 4, Algorithms 3 and 4 utilize convex polyhedra to compute the *reachable* concrete states. Moreover, both IMITATOR and ParamMONAA rely on the Parma Polyhedra Library (PPL) [BHZ08] for convex polyhedra operations. In this sense, the underlying technique utilized in Section 3 is also used in Section 4.

For the performance, Section 4.3 shows that Algorithms 3 and 4 are much more efficient than Algorithm 1. This efficiency is thanks to the skipping and its direct implementation rather than an indirect approach via parametric timed model checking.

Therefore, someone might consider that the approach in Section 4 is strictly superior to the one in Section 3. However, we note that the model checking-based framework in Section 3 is generic, and it is robust to the modification of the problem definition. For example, consider the ill-shaped log shown in Fig. 13: we take a log of the entrance and leaving from a building with the name and the timestamp, but we sometimes fail to identify who passed the gate e. g., due to a sensor error. Since a timed word w is a sequence of pairs (a_i, τ_i) of an event a_i and a timestamp τ_i , it cannot directly represent the log in Fig. 13, and therefore, it is not straightforward to handle such a log using Algorithms 3 and 4. However, it is easy to generalize the conversion TW2PTA from a timed word to a (P)TA in Section 3.2.2 to a log with branching. An example of the result is shown in Fig. 14. Thus, it is rather straightforward to monitor such

an ill-shaped log using Algorithm 1. Moreover, the framework in Section 3 has been used for a different monitoring problem following the conference version of this work [AHW18], i. e., model-bounded monitoring in [WAH21].

In summary, Algorithms 3 and 4 are better at solving the parametric timed pattern matching problem, but the construction used in Algorithm 1 is general, and it is potentially applicable to extensions of the problem considered here.

6 Conclusion and perspectives

Conclusion We proposed two approaches to perform timed pattern matching in the presence of an uncertain specification. This allows us to synthesize parameter valuations and intervals for which the specification holds on an input timed word. Our implementation using IMITATOR may not completely allow for *online* timed pattern matching yet, but already gives an interesting feedback in terms of parametric monitoring. Our second algorithm aiming at finding minimal or maximal parameter valuations is less sensitive to state space explosion. While our algorithms should be further optimized, we believe they pave the way for a more precise monitoring of real-time systems with an output richer than just timed intervals.

In a second part, our dedicated method dramatically outperforms the previous approach using parametric timed model checking. In addition, we discussed an optimization using skipping, that brings an interesting speedup.

Perspectives Exploiting the polarity of parameters, as done in [Asa+11] or in lower-bound/upper-bound parametric timed automata [Hun+02; AL17], may help to improve the efficiency of PTPM_{opt} .

In addition, natural future works include more expressive specifications than (parametric) timed automata-based specifications, e. g., using more expressive logics such as [Bas+15].

Another challenge is the interpretation (and the visualization) of the results of parametric timed pattern matching. While the result of PTPM_{opt} is natural, the fully symbolic result of PTPM remains a challenge to be interpreted; for example, the 125,250 matches for BLOWUP means the union of 125,250 polyhedra in 5 dimensions. We give a possible way to visualize such results in Fig. 8 for GEAR ($|w| = 1,467$): in particular, observe in Fig. 8a that a single point exceeds 3, only a few exceed 2, while the wide majority remain in $[0, 1]$. This helps to visualize how fast the gear is changed from 1 to 2, and at what timestamps.

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A Construction of $V_{\ell,n}$

We present a construction of $V_{\ell,n}$ in Definition 9. We fix a PTA $\mathcal{A} = (\Sigma, L, \ell_0, F, \mathbb{X}, \mathbb{P}, E)$, a location $\ell \in L$, and $n \in \mathbb{N}_{>0}$. Since $V_{\ell,n}$ is the set of parameter valuations $v \in (\mathbb{Q}_+)^{\mathbb{P}}$ such that there is a $v' \in (\mathbb{Q}_+)^{\mathbb{P}}$ satisfying $\mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma) \cap \mathcal{T}^n(\Sigma) \cdot \{w'' + t \mid w'' \in \mathcal{L}_{-\$}(v'(\mathcal{A})), t > 0\} \cdot \mathcal{T}(\Sigma) \neq \emptyset$, we construct PTAs \mathcal{A}'_ℓ and \mathcal{A}'_{+n} satisfying $\mathcal{L}(v(\mathcal{A}'_\ell)) = \mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma)$ and $\mathcal{L}(v'(\mathcal{A}'_{+n})) = \mathcal{T}^n(\Sigma) \cdot \{w + t \mid w \in \mathcal{L}_{-\$}(v'(\mathcal{A})), t > 0\} \cdot \mathcal{T}(\Sigma)$.

We define \mathcal{A}'_ℓ as $\mathcal{A}_\ell = (\Sigma, L \sqcup \{\ell_{\text{fin}}\}, \ell_0, \{\ell, \ell_{\text{fin}}\}, \mathbb{X}, \mathbb{P}, E'_\ell)$, where $E'_\ell = E \cup \{(\ell, \top, a, \emptyset, \ell_{\text{fin}}) \mid a \in \Sigma\} \cup \{(\ell_{\text{fin}}, \top, a, \emptyset, \ell_{\text{fin}}) \mid a \in \Sigma\}$. For any $v \in (\mathbb{Q}_+)^{\mathbb{P}}$, $w' \in \mathcal{L}(v(\mathcal{A}_\ell))$, and $w'' \in \mathcal{T}(\Sigma)$, we have $\ell_0 \xrightarrow{w'} \ell \xrightarrow{w''} \ell_{\text{fin}}$ in $v(\mathcal{A}'_\ell)$ and $w' \cdot w'' \in \mathcal{L}(v(\mathcal{A}'_\ell))$. For any $v \in (\mathbb{Q}_+)^{\mathbb{P}}$ and $w \in \mathcal{L}(v(\mathcal{A}'_\ell))$, there exist timed words $w', w'' \in \mathcal{T}(\Sigma)$ satisfying $w = w' \cdot w''$ and $\ell_0 \xrightarrow{w'} \ell$ in $v(\mathcal{A}'_\ell)$, which implies $w' \in \mathcal{L}(v(\mathcal{A}_\ell))$. Therefore, we have $\mathcal{L}(v(\mathcal{A}'_\ell)) = \mathcal{L}(v(\mathcal{A}_\ell)) \cdot \mathcal{T}(\Sigma)$.

We define \mathcal{A}'_{+n} as $\mathcal{A}_{+n} = (\Sigma \sqcup \{\varepsilon\}, L', \ell_{n+1}, F', \mathbb{X}, \mathbb{P}, E')$, where

- ε is the unobservable character;
- $L' = L \sqcup \{\ell_i \mid i \in \{1, 2, \dots, n+1\}\} \sqcup \{\ell_{\text{fin}}\}$;
- $F' = \{\ell \mid \exists \ell' \in F. (\ell, g, a, R, \ell') \in E\} \sqcup \{\ell_{\text{fin}}\}$; and
- $E' = E \sqcup \{(\ell_{i+1}, \top, a, \emptyset, \ell_i) \mid a \in \Sigma, i \in \{1, 2, \dots, n\}\} \sqcup \{(\ell_1, \top, \varepsilon, \mathbb{X}, \ell_0)\} \sqcup \{(\ell, \top, a, \emptyset, \ell_{\text{fin}}) \mid a \in \Sigma, \ell \in F'\} \sqcup \{(\ell_{\text{fin}}, \top, a, \emptyset, \ell_{\text{fin}}) \mid a \in \Sigma\}$.

For any parameter valuation $v' \in (\mathbb{Q}_+)^{\mathbb{P}}$, timed words $w' \in \mathcal{T}^n(\Sigma)$, $w'' \in \mathcal{L}(v'(\mathcal{A}))$, $w''' \in \mathcal{T}(\Sigma)$, and $t \in \mathbb{R}_{>0}$, we have $\ell_{n+1} \xrightarrow{w'} \ell_1 \xrightarrow{(\varepsilon, t)} \ell_0 \xrightarrow{w''} \ell_f \xrightarrow{w'''} \ell_{\text{fin}}$ in $v'(\mathcal{A})$, where $\ell_f \in F'$. For any parameter valuation $v' \in (\mathbb{Q}_+)^{\mathbb{P}}$ and for any timed word $w \in \mathcal{L}(v'(\mathcal{A}'_{+n}))$, there exist $w' \in \mathcal{T}(\Sigma)$, $w'' \in \mathcal{T}(\Sigma)$, $w''' \in \mathcal{T}(\Sigma)$, and $t \in \mathbb{R}_{>0}$ satisfying $w = w' \cdot (\varepsilon, t) \cdot w'' \cdot w'''$ and $\ell_0 \xrightarrow{w'} \ell_f$ in $v(\mathcal{A}'_{+n})$, where $\ell_f \in F'$, and therefore, we have $w'' \in \mathcal{L}_{-\$}(v'(\mathcal{A}))$. Overall, for any $v \in (\mathbb{Q}_+)^{\mathbb{P}}$, we have $\mathcal{L}(v(\mathcal{A}'_{+n})) = \mathcal{T}^n(\Sigma) \cdot \{w + t \mid w \in \mathcal{L}_{-\$}(v'(\mathcal{A})), t > 0\} \cdot \mathcal{T}(\Sigma)$.

To take the intersection of $\mathcal{L}(v(\mathcal{A}'_\ell))$ and $\mathcal{L}(v(\mathcal{A}'_{+n}))$, we use the synchronous product.

For any $v, v' \in (\mathbb{Q}_+)^{\mathbb{P}}$, we have $\mathcal{L}((v \sqcup v')(\mathcal{A}'_\ell \parallel \mathcal{A}'_{+n})) = \mathcal{L}(v(\mathcal{A}'_\ell)) \cap \mathcal{L}(v'(\mathcal{A}'_{+n}))$. Therefore, we have $V_{\ell,n} = \{v \mid \exists v' \in (\mathbb{Q}_+)^{\mathbb{P}}. \mathcal{L}((v \sqcup v')(\mathcal{A}'_\ell \parallel \mathcal{A}'_{+n})) \neq \emptyset\}$, which can be computed by reachability synthesis of PTAs.