

Project Management Contracts with Delayed Payments

H. Dharma Kwon*, Steven A. Lippman, Kevin F. McCardle, and Christopher S. Tang[†]

August 5, 2009

Abstract

In project management, most manufacturers (project managers) offer no delayed payment contracts under which each supplier (contractor) will receive a pre-specified payment when she completes her task. However, some manufacturers impose delayed payment contracts under which each supplier is paid only after all suppliers have completed their tasks. In this paper, we investigate whether or not the manufacturer ought to demand such a delayed payment contract. In our model with one manufacturer and $n \geq 2$ suppliers, we compare the impact of both a delayed payment regime and a no delayed payment regime on each supplier's effort level and on the manufacturer's net profit in equilibrium. When the suppliers' completion times are exponentially distributed, we show that the delayed payment regime is more (less) profitable than the no delayed payment regime if the manufacturer's revenue is below (above) a certain threshold. Also, we show the delayed payment regime is dominated by the no delayed payment regime when the number of suppliers exceeds a certain threshold. By considering a different setting in which each supplier has information about the progress of all other suppliers' tasks, we obtain similar structural results except that the delayed payment regime is more profitable than the no delayed payment regime when the number of suppliers exceeds a certain threshold.

Keywords: Project Management, Time-Based Contracts, Delayed Payments, Stackelberg game, Nash equilibrium.

*Kwon was partially supported by the UCLA Dissertation Year Fellowship and by the Harold and Pauline Price Center for Entrepreneurial Studies.

[†]Direct correspondence to any of the authors. Kwon, Department of Business Administration, University of Illinois at Urbana-Champaign, 1206 South Sixth St., Champaign, IL 61820; Email: dhkwon@illinois.edu. Lippman, McCardle and Tang: UCLA Anderson School, UCLA, 110 Westwood Plaza, Los Angeles, CA 90095; Email: slippman@anderson.ucla.edu, kmccardl@anderson.ucla.edu, ctang@anderson.ucla.edu. The authors are grateful to the editor, the associate editor, three anonymous reviewers, as well as seminar participants at the INFORMS meeting in Washington DC, Duke University, Hong Kong University of Science and Technology, London Business School, Purdue University, University of California at San Diego, University of Chicago, Singapore Management University, and Washington University in St. Louis and for their constructive comments.

1 Introduction

The growing importance of efficient and effective project management has led to the development and introduction of many project management tools since the 1950s such as Critical Path Method (CPM), Project Evaluation and Review Techniques (PERT), and cost-time tradeoff analysis (Klasterin (2004)). These tools are effective when there is little uncertainty in project completion times and/or operating costs. However, relatively little is known about ways to manage projects with considerable uncertainty such as those arising in construction, defense, management consultancy, and new product development. Although we have witnessed an increased research interest in examining supply contracts under uncertainty (Cachon (2003)), little research has been done in the area of project management contracts under uncertainty. Survey studies conducted by Simister (1994) and Akintoye and MacLeod (1997) indicated that insurance and project contract design are two most common mechanisms for mitigating project risks. This paper focuses on project contract design.

We consider a manufacturer manages a project consisting of $n \geq 2$ separate and independent tasks that can be performed in parallel. Due to different requisite skills associated with different tasks, each task is performed by a different supplier. The manufacturer's contract with each supplier specifies both the payment to the supplier and the payment terms. We consider two different payment regimes: no delayed payment and delayed payment. Under the conventional or no delayed payment regime, a supplier receives her payment immediately after she has completed her task. Under the delayed payment regime, however, each supplier receives her payment only after *all* suppliers have completed their tasks.

We offer three examples to illustrate the existence of both payment regimes in practice. First, consider a translation agency that offers one-stop written translation services to customers who need to translate customer-specific materials such as employee handbooks, safety manuals, and web site content from a source language (e.g., English) to multiple target languages (e.g., Spanish and Italian). Typically, the agency receives full payment from the customer upon the completion of the entire translation project. Most agencies outsource the translation work associated with each target language to an external translator. According to our discussion with the managing director of Inline Translations Services (www.inlinela.com) based in Los Angeles, both payment regimes are common in practice. Second, consider a home warranty company that offers comprehensive home repair services to home owners. Upon receiving a repair service request from a customer, the company outsources the actual repair tasks to different independent contractors who specialize in different types of repair services (e.g., electrical, plumbing, flooring). For example, when one of the authors requested a home warranty company to repair his kitchen after an accidental flood,

the home repair company managed his request by coordinating different repair tasks performed by a plumber, an electrician, a carpenter, and a carpet installer. According to a manager of First American Home Buyers Protection Corporation (www.homewarranty.firstam.com) based in Los Angeles, both payment regimes are common in practice. Third, when Boeing developed its 737 and 747 aircrafts, Boeing offered the no delayed payment regime to its suppliers. When developing the 787 aircraft, however, Boeing imposed the delayed payment regime (also known as the “risk-sharing” contract) upon its strategic suppliers. As reported in Greising and Johnsson (2007), the risk-sharing contracts stipulate that these strategic suppliers will not receive payments from Boeing to recoup their development costs until the *first* 787 plane is developed, certified, and delivered to Boeing’s first customer (Japan’s All Nippon Airways). Boeing’s risk-sharing contract captures a key element of the delayed payment regime: a supplier receives her payment for her development task only after all suppliers have completed their development tasks.¹

Even though both payment regimes exist in practice, we are not aware of any formal study regarding the rationale behind each payment regime. Based on our discussion with two translation agencies, various translators, and two major Boeing suppliers who request anonymity, we learned of the following issues. First, all suppliers believe that the no delayed payment regime is fair because each supplier gets paid immediately after completing her task. Because the timing of the payment to each supplier depends on the completion times of all suppliers under the delayed payment regime, there is a consistent perception among suppliers that the delayed payment regime penalizes those suppliers who finish early. Consequently, some suppliers may work slower under the delayed payment regime. Second, because each supplier is paid when she completes her own task, the no delayed payment regime can create potential cash flow problems for the manufacturer, especially when the last supplier completes her task very late. As a way to reduce the manufacturer’s financial risks, some manufacturers believe that the delayed payment scheme may provide an incentive for the suppliers to coordinate their tasks better so as to complete the entire project earlier. (Perhaps this is one of the reasons why Boeing called its delayed payment regime the “risk sharing” contract.) Third, all suppliers prefer to receive their payments earlier, while all manufacturers prefer to issue their payments later. This sentiment suggests that both suppliers and manufacturers discount the value of future payments, either through mental calculations or actual financial discounting. Accordingly, we shall assume that there exists an “imputed” continuous time discount rate in our model. Also, we shall consider the case when the manufacturer and the suppliers are interested in maximizing their own expected discounted profit. (When the completion time of each task is deterministic, various researchers have used expected profits as objective functions to examine

¹The *quid pro quo* for accepting the delayed payment contract appears to be vesting intellectual property rights associated with the systems developed by the suppliers with the suppliers rather than with Boeing (Horng and Bozdogan (2007)).

different issues arising from project management (e.g., Smith-Daniels and Aquilano (1987), Smith-Daniels and Smith-Daniels (1987), and Vanhoucke et al. (2001).)

As an initial attempt to analyze these two payment regimes in the context of project contracts with uncertain completion times, we consider the case in which one manufacturer engages $n \geq 2$ suppliers in the project. By considering an abstraction of the aforementioned industry examples, we propose a stylized model to capture the salient features of the two regimes in order to gain intuition as to which regime yields shorter project completion time and which regime imparts the larger manufacturer's profit. Although we compare the manufacturer's profits associated with two payment regimes that are simple and common in practice, there may be other payment regimes that dominate these two regimes. As such, our intent is to develop a basic model which can be used as a building block to examine more general settings. We sketch an analysis of two slightly more general payment regimes in Section 5.1.

Our model consists of one risk-neutral manufacturer and $n \geq 2$ risk-neutral external suppliers. The manufacturer will receive a revenue of nq from his customer upon delivering the product or service which comes into being when all suppliers complete their tasks. The manufacturer acts as the leader in a Stackelberg game. (Similar to the spirit of most supply contract models in the operations management and economics literature (Cachon (2003)), ours is a single-period model, and we do not consider adaptive learning in a multi-period game setting.) The game starts when the manufacturer specifies the time-based contract: he selects not only the payment p to be paid to each of the suppliers but also the payment regime, either the no delayed payment regime N or the delayed payment regime D . Each supplier is a follower in this Stackelberg game. Given the payment p and the regime, each supplier selects her optimal work rate. The completion time of each task is uncertain. Because each supplier receives her payment only after all suppliers have completed their tasks under the delayed payment regime D , each supplier needs to take the other suppliers' work rates into consideration when selecting her own work rate. In the base model, we assume that each supplier has no information regarding other suppliers' progress and that each supplier works at her selected rate until she completes her task. In a later section, we relax this assumption so that each supplier has information about other suppliers' progress and each supplier can adjust her work rate over time. Our analysis answers the following questions:

1. Given the payment p , what is the supplier's optimal work rate under regime N and regime D ?
2. Given the payment p , which regime yields a shorter expected project completion time?
3. Given the manufacturer's revenue q , what contract (*i.e.*, payment p and regime N or D) will the manufacturer select in order to maximize his profit?

4. What are the conditions that render the no delayed payment regime more profitable for the manufacturer?
5. How would information regarding other suppliers' progress affect a supplier's optimal work rate? the manufacturer's optimal profit? the selection of the manufacturer's optimal payment regime?

The primary contributions of this paper are four-fold. First, our paper is the first to construct a model of a project management contract with and without delayed payments with uncertain completion times. Second, by exploring the underlying mathematical structure, we obtain insights regarding the optimal payment p under each regime. Third, we derive conditions under which one payment regime dominates the other. Specifically, we show that the delayed payment regime is more profitable (less profitable) than the no delayed payment regime when the manufacturer's revenue is below (above) a certain threshold. Fourth, by considering a different setting in which each supplier has information about the other suppliers' progress and each supplier can adjust her work rate over time, we obtain two additional interesting structural results under the delayed payment regime: (1) it is optimal for each supplier to begin with a slower work rate and then switch to a faster rate when another supplier completes her task; and (2) it is optimal for the manufacturer to offer the delayed payment regime when the number of suppliers exceeds a certain threshold.

This paper is organized as follows. Section 2 provides a brief review of related literature. Section 3 presents the base model: for each regime, we determine the supplier's optimal work rate, the expected project completion time, the optimal payment offered by the manufacturer, and the corresponding profit. In Section 4, we consider a different setting in which each supplier has information about the other suppliers' progress. The analysis is more complex because it involves the analysis of an n -stage non-cooperative game. Despite certain technical challenges, we establish analytical conditions under which one payment regime dominates the other in equilibrium. We conclude in Section 5 with a brief summary of our results, a sketch of the analysis associated with two payment regimes that are slightly more general than regimes N and D , and a discussion of the limitations of our model and potential future research topics. In order to streamline the presentation, all proofs are given in the Online Appendix.

2 Literature Review

To our knowledge, a time-based project contract with delayed payment has not been examined previously in the project management literature. In particular, there are three features of the time-based contract analyzed in this article which differ markedly from the existing supply contract

literature (Cachon (2003) and Tang (2006)). First, under the delayed payment regime, each supplier receives payment at the time when all suppliers have completed their tasks. Consequently, each supplier needs to take into account the other supplier's behavior when selecting her own work rate. It is through this interaction among suppliers that the several underlying supply contracts are, in effect, transformed into a single joint supply contract between the manufacturer and his multiple suppliers. This linking of the several suppliers is a fundamental and crucial departure from the traditional supply contract. A related interaction among suppliers has been examined by Cachon and Zhang (2007). For an exogenously given price p , they consider the case when the manufacturer allocates randomly arriving jobs to different suppliers, and they develop a queueing game to evaluate the expected lead time for different allocation policies. In their model, each supplier selects her work rate so as to optimize her expected profit by taking other supplier's behavior into consideration. Their model differs from ours in that they focus on different allocation policies while we concentrate on pricing policies under different payment regimes. In addition, their model is based on substitutable tasks while ours focuses on complementary tasks.

The notion of substitutable tasks (or technologies) has been examined in the economics literature. For example, Reinganum (1982) analyzes a search game among competing firms who conduct new product R&D. The underlying technology of the new product is substitutable in that the profit of a given firm decreases as the costs of the other firms decrease. She establishes the existence of a Nash equilibrium in which each firm searches until it finds a cost below its reservation threshold. Naturally the R&D efforts of a given firm decreases as the other firms increase their efforts. In the same vein, the R&D model in Lippman and Mamer (1993) represents the extreme in substitutability. The firms engage in R&D, and the first firm to make the decision to bring its product to market wins the entire market. Bringing a low quality product to market results in a low firm profit, which spoils the market for the other firms. These R&D models are based on substitutable tasks (or technologies) while ours focuses on complementary tasks.

Wang and Gerchak (2003) present a model that deals with complementary tasks in the context of assembly operations: a manufacturer sells a product that requires different assembly components produced by different suppliers. To produce the components, the suppliers need to construct their individual component production capacities before observing the actual order quantities to be placed by the manufacturer. In this case, the effective production capacity of the product is dictated by the minimum of the component production capacities. As a way to induce proper component capacity installation, the manufacturer offers a unit price to each supplier for its component; however, the manufacturer delays its order-quantity until demand uncertainty is resolved. By solving a Stackelberg game in which the manufacturer acts as the leader who specifies the unit price of each component and the suppliers act as followers who install the component production capacities, Wang and Gerchak (2003) first determine each supplier's best response; i.e., the optimal

production capacity in equilibrium for any given unit price. By anticipating the supplier’s best response, they determine the manufacturer’s optimal unit price. Their model differs from ours in that they focus on the suppliers’ production capacities while we concentrate on the suppliers’ work rates under time-based contracts with different payment regimes.

The economics literature on multi-agent incentive contract theory is vast: some seminal papers include Holmstrom (1982), Demski and Sappington (1984), Mookherjee (1984), McAfee and McMillan (1991), and Itoh (1991). While our model deals with multiple suppliers (agents), our setting and our focus are different from multi-agent incentive contract theory in the following sense. First, our model is intended to compare two common payment regimes in the context of project management contracts with uncertain completion times, while the multi-agent models focus on examining the existence of Nash equilibrium and general characteristics of optimal incentive contracts (e.g., Holmstrom (1982), Mookherjee (1984), and McAfee and McMillan (1991)). Second, in our model, the manufacturer receives his revenue at the instant when all suppliers have completed their tasks; i.e., the “maximum” of the completion times of all tasks performed by different suppliers. Hence, in our model, the manufacturer’s expected profit is a non-separable function of the suppliers’ outputs (i.e., the completion times of different tasks). In most multi-agent models, the manufacturer’s (principal’s) expected profit function is a separable function of the suppliers’ outputs (e.g., Itoh (1991)). Third, in our model, the completion time of each task is a continuous random variable, while in most economic models, the outcome of each task takes on discrete values (e.g., Demski and Sappington (1984) and Itoh (1991)).

3 Base Model

The manufacturer will receive a revenue nq from a customer when the project is complete. (To focus our analysis on the interaction between the manufacturer and n suppliers and to obtain tractable results, we assume that the revenue nq is given exogenously. In essence, we do not model the contract design between the customer and the manufacturer; i.e., we implicitly assume that the revenue nq is agreeable to the customer and the manufacturer a priori. Without this simplifying assumption, one needs to analyze a 3-level Stackelberg game with $n+2$ players, which is beyond the scope of this paper.) The project consists of $n \geq 2$ parallel tasks, each of which is to be performed by a distinct external supplier. Throughout this paper, we assume the tasks are of equal difficulty and the suppliers have equal capability so that the manufacturer can offer an identical payment p to all suppliers.² In addition to the payment p , the manufacturer specifies the payment regime

²In many instances, the assumption of identical suppliers is reasonable and innocuous. For example, in translation services, the price for translating a document into Spanish or Italian is usually the same because the difficulty of

N or D . Under the no delayed payment regime N , each supplier is paid immediately after she completes her own task. Under D , each supplier is paid when all n suppliers have completed their tasks. We assume that the completion time of development task X_i is exponentially distributed with parameter r_i , where the work rate $r_i > 0$ is selected by supplier i at time 0, $i = 1, \dots, n$.³

In the base model, we assume that the suppliers do not have information regarding the progress of other suppliers. Due to the memoryless property of the exponential distribution, this assumption implies that each supplier has no updated information as time progresses. Hence, it is optimal for each supplier to continue to work at her initial rate r_i selected at time 0 until she completes her task. Therefore, the project completion time T satisfies: $T = \max\{X_i : i = 1, \dots, n\}$. (In Section 4, we relax this assumption so that each supplier has information about the progress of other suppliers. Due to the memoryless property of the exponential distribution, the only time that a supplier should change her work rate is when another supplier completes her task. This observation leads us to analyze an n -stage game in which each continuing supplier adjusts her work rate at the beginning of each stage that occurs at the instant when another supplier completes her task.)

To capture the sentiment that all suppliers prefer to receive their payments earlier and the manufacturer prefers to issue his payments later, let $\alpha > 0$ be the “imputed” continuous time discount rate. The expected discount factor associated with the project completion time $T = \max\{X_i : i = 1, \dots, n\}$ (or the time for the suppliers to receive their payments under regime D) is denoted by $\beta_n(r_1, \dots, r_n) = E(e^{-\alpha T})$. Because the distribution of X_i is $F_i(t) = 1 - e^{-r_i t}$, the distribution of T is $F(t) \equiv \prod_{i=1}^n F_i(t)$. Hence, the discount factor $\beta_n(r_1, \dots, r_n) = E[e^{-\alpha T}] = \int_0^\infty e^{-\alpha t} d(F(t)) = \alpha \cdot \int_0^\infty e^{-\alpha t} F(t) dt$, where the last equality is obtained via integration by parts. Similarly, the expected discount factor associated with the completion time of task i (or the time for supplier i to receive her payment under regime N) is denoted by $\beta(r_i)$: $\beta(r_i) = E(e^{-\alpha X_i}) = \int_0^\infty r_i e^{-(r_i + \alpha)t} dt = \frac{r_i}{r_i + \alpha}$. Our analysis utilizes the following properties of $\beta_n(r_1, \dots, r_n)$.

Lemma 1 *For any positive integer n , the expected discount factor $\beta_n(r_1, \dots, r_n)$ satisfies:*

translation is quite similar. While an approach similar to ours can be used to analyze the case of non-identical suppliers, the analysis is highly complex due to asymmetric equilibria and is beyond the scope of this paper.

³In the project management literature, it is commonly assumed that the completion time of a development task is exponentially distributed (e.g., Adler et al. (1995), Maggott and Skudlarski (1993), Pennings and Lint (1997), and Cohen et al. (2004)). Besides the fact that exponential completion times enable us to obtain analytical results and insights, Dean et al. (1969) argue that an exponential completion time is more realistic in the context of project management than the Normally distributed completion times that are commonly assumed (e.g., Bayiz and Corbett (2005)). In project management, it was observed that the uncertain completion time is usually caused by the occurrence of an unforeseen situation. Hence, the distribution of the completion time should be positively skewed, which is a property of the exponential distribution. Cohen et al. (2004) cite empirical evidence for exponential completion times in project management.

1. $\beta_n(r_1, \dots, r_n) = E(e^{-\alpha T}) \leq E(e^{-\alpha X_i}) = \beta(r_i)$ for $i = 1, \dots, n$.
2. $\beta_n(r_1, \dots, r_n)$ is increasing and concave in r_i for $i = 1, \dots, n$.
3. $\beta_n(r_1, \dots, r_n)$ is a submodular function of (r_1, \dots, r_n) : $\frac{\partial^2 \beta_n(r_1, \dots, r_n)}{\partial r_j \partial r_i} > 0$ for $i \neq j$.
4. When $r_i = r \forall i$, $\beta_n(r_1, \dots, r_n) = \sum_{j=0}^n \binom{n}{j} (-1)^j \frac{\alpha}{\alpha + jr}$. By letting $e^{-rt} = x$, we can express $\beta_n(r_1, \dots, r_n) = \frac{\alpha}{r} \cdot \int_0^1 x^{(\alpha-r)/r} (1-x)^n dx = \frac{\alpha}{r} \cdot B(\frac{\alpha}{r}, n+1) = \prod_{j=1}^n \frac{jr}{jr+\alpha}$, where $B(., .)$ is the Beta function (Chap. 6 of Abramowitz and Stegun, 1965).
5. When $r_i = r \forall i$, $\beta_n(r_1, \dots, r_n)$ is decreasing in n and increasing in r .

Because each supplier gets paid only after all suppliers have completed their tasks under regime D , statement 1 asserts that each supplier's payment is discounted more heavily under regime D . Statement 2 asserts that each supplier can reduce this "discounting penalty" under regime D by working faster, and statement 5 asserts that each supplier's payment is discounted more heavily under regime D as the number of suppliers n increases.

Throughout this paper, we assume that each supplier i will participate in the development project when the payment $p > \theta \geq 0$. The threshold θ is an exogenously specified parameter based on different factors including the minimum payment established by the market, the supplier's outside opportunity, and the supplier's internally established *hurdle rate*.⁴ Knowing that each supplier will set her work rate $r_i = 0$ when $p \leq \theta$, we assume without loss of generality that the manufacturer will always set $p > \theta$.

The supplier's operating cost $\kappa(r)$ per unit time associated with work rate r is a convex increasing function. To simplify our analysis, we assume that $\kappa(r) = kr^2$ with $k > 0$. Hence, supplier i 's expected discounted total operating cost equals $E[\int_0^{X_i} \kappa(r_i) \cdot e^{-\alpha t} dt] = \int_0^\infty [\int_0^x \kappa(r) \cdot e^{-\alpha t} dt] r_i e^{-r_i x} dx = kr_i^2 / (r_i + \alpha)$.

3.1 Profit functions

We now determine the supplier's and the manufacturer's expected discounted profit functions. Under regime N , supplier i gets paid immediately when she completes her task. Given the manufacturer's payment p , supplier i 's expected discounted profit $\Pi_i^N(p; r_i)$ under regime N satisfies:

$$\Pi_i^N(p; r_i) = p \cdot \beta(r_i) - \frac{kr_i^2}{r_i + \alpha}, \quad \text{for } i = 1, \dots, n. \quad (3.1)$$

⁴We include this exogenously given participation constraint $p > \theta$ to ensure tractability. The analysis will be extremely complex if one imposes an endogenous participation constraint, say, the supplier's expected profit in equilibrium exceeds a certain threshold. The analysis associated with an endogenous participation constraint is intractable and is beyond the scope of this paper.

Under regime D , supplier i gets paid when all suppliers have completed their tasks. Given p , the supplier's expected discounted profit $\Pi_i^D(p; r_1, \dots, r_n)$ under regime D satisfies:

$$\Pi_i^D(p; r_1, \dots, r_n) = p \cdot \beta_n(r_1, \dots, r_n) - \frac{kr_i^2}{r_i + \alpha}, \quad \text{for } i = 1, \dots, n. \quad (3.2)$$

Given payment $p > \theta$, the manufacturer's expected discounted profit under regimes N and D satisfy:

$$\Pi_m^N(p; q) = nq \cdot \beta_n(r_1, \dots, r_n) - p \sum_{i=1}^n \beta(r_i), \quad \text{and} \quad (3.3)$$

$$\Pi_m^D(p; q) = n(q - p) \cdot \beta_n(r_1, \dots, r_n). \quad (3.4)$$

Suppose the imputed discount rate α were 0 so that $\beta(r_i) = \beta_n(r_1, \dots, r_n) = 1$. Then $\Pi_i^N(p; r_i) = \Pi_i^D(p; r_i)$, and $\Pi_m^N(p; q) = \Pi_m^D(p; q)$ so the suppliers would not mind receiving their payments later, and the manufacturer would not mind paying the suppliers earlier. This is the complete opposite of what we learned from our discussion with various manufacturers and suppliers: they care a lot about the timing of the payments. Hence, we assume that $\alpha > 0$.

Let us now compare the supplier's and the manufacturer's profit functions when $p > \theta$ and the work rates r_i are given exogenously. Using $\beta_n(r_1, \dots, r_n) \leq \beta(r_i)$ given in Lemma 1, it is easy to check that $\Pi_i^N(r_i) > \Pi_i^D(r_i)$ for all i , and that $\Pi_m^N(p; q) < \Pi_m^D(p; q)$. These observations confirm a basic intuition: when the price p and the work rates r_i are the same under both regimes, supplier i prefers regime N (i.e., receives her payment earlier), while the manufacturer prefers regime D (i.e., issues his payments later).

Suppose the manufacturer changes the payment regime from N to D and keeps the payment p unchanged. Then each supplier can adjust her work rate. Specifically, if she increases her work rate under regime D , her expected total operating cost will increase while her expected discounted payment will increase only if other suppliers also increase their work rates. On the other hand, if she reduces her work rate under regime D , her expected operating cost and her expected discounted payment will both decrease. Hence, it is unclear whether it is optimal for the supplier to reduce her work rate when the manufacturer changes the payment regime from N to D . Moreover, by anticipating the supplier's response to the payment regime as well as the payment p , it is unclear whether the manufacturer should change his payment p when he changes the payment regime from N to D . These questions motivate us to analyze a Stackelberg game in which the manufacturer has the first move and the n suppliers simultaneously move second. The manufacturer starts by selecting the regime (either N or D) and the payment p . As in a backward recursion, each supplier determines her work rate r_i given the regime and p . Anticipating each supplier i 's work rate r_i , the

manufacturer selects the payment p^* that maximizes his profit. By selecting the regime that yields the higher expected profit, the manufacturer informs the suppliers of the regime and the price p^* ; in response, the suppliers select their optimal work rates.

3.2 N : The No Delayed Payment Regime

Utilizing supplier i 's expected discounted profit $\Pi_i^N(p; r_i)$ given in (3.1), we determine the supplier i 's optimal work rate and her optimal expected profit under regime N .

Proposition 1 *Given $p > \theta$, supplier i 's profit function $\Pi_i^N(p; r)$ is concave in r , and $r_i^N(p)$, supplier i 's optimal rate, is given by*

$$r_i^N(p) = r^N(p) = \alpha \left(\sqrt{1 + \frac{p}{\alpha k}} - 1 \right). \quad (3.5)$$

Supplier i 's optimal profit $\Pi_i^N(p) \equiv \Pi_i^N(p; r^N(p))$ is given by

$$\Pi_i^N(p) = \frac{k}{\alpha} \cdot [r^N(p)]^2 = k\alpha \left(\sqrt{1 + \frac{p}{\alpha k}} - 1 \right)^2. \quad (3.6)$$

Observe from (3.5) that supplier i 's optimal work rate $r_i^N(p) > 0$ when $p > \theta$. Also, we can use the optimal work rate $r_i^N(p)$ to determine the expected project completion time $E(T^N(p)) = E(\max\{X_i : i = 1, \dots, n\})$, where X_i is exponentially distributed with parameter $r_i^N(p)$.

Corollary 1 *Given payment $p > \theta$, the expected project completion time $E(T^N(p))$ under regime N satisfies:*

$$E(T^N(p)) = \frac{1}{r^N(p)} [\psi(n+1) - \psi(1)], \quad (3.7)$$

where $\psi(x)$ is the Digamma function.⁵ Also, $E(T^N(p))$ is increasing in n and decreasing in p .

Corollary 1 confirms two basic intuitions. As the number of suppliers n increases, the expected completion time increases. Also, the manufacturer can shorten the expected completion time if he offers a larger payment p to induce the suppliers to work faster.

By anticipating the supplier's optimal work rate $r^N(p)$ given in (3.5), one can determine the optimal payment p^N that maximizes the manufacturer's profit function $\Pi_m^N(p; q)$ given in (3.3).

⁵The Digamma function $\psi(x)$ is the derivative of the logarithm of the Gamma function: $\psi(x) \equiv \frac{d}{dx} \ln(\Gamma(x)) = \int_0^\infty \left(\frac{e^{-t}}{t} - \frac{e^{-xt}}{(1-e^{-t})} \right) dt$. When n is a positive integer, $\psi(n+1) - \psi(1) = \sum_{k=1}^n k^{-1}$ (see Chap. 6 of Abramowitz and Stegun (1965)).

Due to the complexity of $\beta_n(r_1, \dots, r_n)$, there is no explicit analytical expression for the optimal payment p^N or the manufacturer's optimal expected profit $\Pi_m^N(q) \equiv \Pi_m^N(q, p^N)$. Nevertheless, it is important to note that the manufacturer's optimal profit $\Pi_m^N(q)$ can be negative under regime N . This is because, under regime N , the manufacturer has to pay each supplier when she completes her own task, but he receives his revenue after all suppliers have completed their tasks. This series of cash flows can cause the manufacturer to suffer a loss unless he proactively takes this "time delay" into consideration when negotiating the project revenue nq with his customer. This observation motivates us to determine a lower bound for the revenue q to ensure that the manufacturer's optimal expected profit $\Pi_m^N(q) \equiv \Pi_m^N(q, p^N)$ is strictly positive. Using the optimal rate $r_i^N(p)$ given in (3.5) along with the implicit expression for p^N , we establish the following result:

Lemma 2 *For any $n \geq 2$, there exists a threshold q_n such that $\Pi_m^N(q) > 0$ if and only if $q > \max\{q_n, \theta\}$, where q_n is increasing in n . Moreover, when n is sufficiently large, $q_n = \frac{k\alpha}{4}(\ln n)^2 + O(\ln n)$.*

Because the time delay becomes more severe as the number of suppliers n increase, it is intuitive that the threshold q_n is increasing in n . Also, the above lemma implies that, for any fixed revenue q , the manufacturer will suffer from a loss (i.e., $\Pi_m^N(q) < 0$) if the number of suppliers n exceeds a threshold τ_n^N , where $\tau_n^N \equiv \operatorname{argmin}_{n>0} \{q_n > q\}$. Therefore, when the number of suppliers n increase, the manufacturer should negotiate a higher revenue q , where $q > \max\{q_n, \theta\}$, so that his optimal profit $\Pi_m^N(q) > 0$. In other words, to avoid making a loss, the manufacturer should not accept the project and earn nothing when the revenue $q \leq \max\{q_n, \theta\}$.

3.3 D : The Delayed Payment Regime

We now examine regime D under which each supplier receives her payment when all suppliers have completed their tasks: each supplier's expected discounted profit depends on all suppliers' work rates. We now show that there exists a symmetric Nash equilibrium in which all suppliers work at the same rate. By examining the supplier's profit function $\Pi_i^D(p; r_1, \dots, r_n)$ given in (3.2) along with Lemma 1, we have:

Lemma 3 *Given $(r_1, \dots, r_{i-1}, r_{i+1}, \dots, r_n)$, supplier i 's expected discounted profit $\Pi_i^D(p; r_1, \dots, r_n)$ given in (3.2) is concave in r_i . Also, supplier i 's best response r_i^* (i.e., the value of r_i^* that maximizes $\Pi_i^D(p; r_1, \dots, r_n)$) is increasing in r_j for $j \neq i$.*

Proposition 2 *There are no asymmetric Nash equilibria. There is a threshold p_n such that: if $p > \max\{\theta, p_n\}$, then there are multiple symmetric Nash equilibria in which all suppliers work at the same rate r , where r satisfies:*

$$\frac{p\alpha}{r^2}B\left(\frac{\alpha+r}{r}, n\right)\left[\psi\left(\frac{\alpha+r}{r} + n\right) - \psi\left(\frac{\alpha+r}{r}\right)\right] = \frac{(2k\alpha r + kr^2)}{(\alpha+r)^2}. \quad (3.8)$$

Among all possible equilibria, the Nash equilibrium with the largest work rate $r^D(n; p)$ has the following properties: both $r^D(n; p)$ and its corresponding expected discounted profit for the supplier $\Pi_i^D(n; p)$ are decreasing in n .

To ease our exposition, we defer our discussion of the threshold p_n till Lemma 4 below. Proposition 2 has three implications. First, observe from the last statement that the largest work rate in equilibrium $r^D(n; p)$ satisfies: $r^D(n; p) < r^D(n-1; p) < \dots < r^D(2; p) < r^D(1; p) = r^N(p)$. This implies that, due to the “gaming effect” associated with regime D , the supplier’s optimal work rate under regime D is lower than the optimal work rate under regime N ; i.e., $r^D(n; p) < r^N(p)$. This result is intuitive because, under regime D , each supplier is penalized for completing her task before other suppliers. Second, using the same proof of Corollary 1, it is easy to show that the expected completion time $E(T^D(p))$ can be expressed as:

$$E(T^D(p)) = \frac{1}{r^D(n; p)}[\psi(n+1) - \psi(1)]. \quad (3.9)$$

Because $r^D(n; p) < r^N(p)$, it is easy to check from (3.7) and (3.9) that $E(T^D(p)) > E(T^N(p))$: the expected project completion time is longer under regime D . This result is expected because the supplier’s optimal work rate under regime D is lower than the optimal work rate under regime N . Third, because $r^D(n; p) < r^N(p)$, the lower work rate $r^D(n; p)$ reduces supplier i ’s discounted operating cost $\frac{kr_i^2}{r_i + \alpha}$ as well as her discounted payment $p \cdot \beta_n(r_1, \dots, r_n)$. Hence, it is not clear if supplier i ’s expected profit $\Pi_i^D(n, p) \equiv \Pi_i^D(p; r^D(n, p), \dots, r^D(n, p))$ is lower under regime D . However, by combining the fact that $\Pi_i^D(n, p) = \Pi_i^N(p)$ when $n = 1$ (because $r^D(1; p) = r^N(p)$) with $\Pi_i^D(n; p)$ decreasing in n , we can conclude that $\Pi_i^D(n; p) < \dots < \Pi_i^D(1; p) = \Pi_i^N(p)$. Therefore, given p , the supplier’s profit under regime D is indeed lower than under regime N .

While there is no simple closed form expression for the work rate in equilibrium $r^D(n; p)$ that solves (3.8) for any $n \geq 2$, we obtain a closed form expression when $n = 2$. When $n = 2$, (3.8) reduces to:

$$\left[\frac{p\alpha}{(\alpha+r)^2} - \frac{p\alpha}{(\alpha+2r)^2}\right] - \frac{(2k\alpha r + kr^2)}{(\alpha+r)^2} = -\frac{r \cdot k \cdot h(r)}{(r+\alpha)^2(2r+\alpha)^2} = 0,$$

where $h(r) = 4r^3 + 12\alpha r^2 + 9\alpha^2 r - 3\frac{p}{k}\alpha r + 2\alpha^3 - 2\frac{p}{k}\alpha^2$. By close examination of the cubic equation $h(r) = 0$, we determine the work rate $r^D(2; p)$ in Corollary 2.

Corollary 2 *Suppose $n = 2$. Then $r^D(2; p) = 0$ if $p \leq \max\{\theta, p_2\}$, where $p_2 = k\alpha$. If $p > \max\{\theta, p_2\}$, then $r^D(2; p) = 0$ is an equilibrium, and the only Nash equilibrium with $r^D(2; p) > 0$ satisfies:*

$$r^D(2; p) = \alpha \left[\sqrt{1 + \frac{p}{k\alpha}} \cos(\phi/3) - 1 \right], \text{ where} \quad (3.10)$$

$$\phi \equiv \pi - \arctan \sqrt{\frac{p}{k\alpha}}. \quad (3.11)$$

Corollary 2 informs us that there is a unique Nash equilibrium with positive work rate so that the suppliers earn positive profits (the supplier's profit is 0 when her work rate is 0). Consequently, it is Pareto optimal for the suppliers to select the equilibrium $r^D(2; p) > 0$ when $p > \max\{\theta, p_2\}$.

By anticipating the supplier's best response (i.e., the optimal work rate $r^D(n; p)$ that satisfies (3.8)), one can determine the optimal payment p^D that maximizes the manufacturer's profit function $\Pi_m^D(p; q)$ given in (3.4). However, because there is no explicit expression for the supplier's equilibrium work rate $r^D(n; p)$, there is no explicit expression for the optimal payment p^D or for the manufacturer's optimal expected profit $\Pi_m^D(q) \equiv \Pi_m^D(q, p^D)$. However, observe from (3.4) that the manufacturer's optimal profit $\Pi_m^D(q) = n(q - p) \cdot \beta_n(r_1, \dots, r_n) = 0$ when the supplier's work rate in equilibrium $r^D(n; p)$ drops to zero. In particular, Corollary 2 reveals that, when $n = 2$, the supplier's equilibrium work rate $r^D(2; p)$ will drop to zero when the payment $p \leq \max\{\theta, p_2\}$, where $p_2 = k\alpha$. Hence, in order for the manufacturer to obtain a positive profit, he has to make sure that his revenue q and his payment p satisfy $q > p > \max\{\theta, p_2\}$. This observation motivates us to establish a lower bound for the payment p to ensure that the supplier's equilibrium work rate $r^D(n; p) > 0$ and a lower bound for the revenue q so as to ensure that the manufacturer's optimal expected profit $\Pi_m^D(q) \equiv \Pi_m^D(q, p^D) > 0$ for each $n \geq 2$.

Lemma 4 *For $n \geq 2$, there exists a threshold p_n such that $r^D(n; p) > 0$ if and only if $p > \max\{p_n, \theta\}$, and $\Pi_m^D(q) > 0$ if and only if $q > \max\{p_n, \theta\}$. Also, p_n is increasing in n . Moreover, when n is sufficiently large, $p_n = k\alpha n(\ln n + O(1))$.*

By noting from Proposition 2 that the supplier's work rate in equilibrium $r^D(n; p)$ is decreasing in n , it is intuitive that the threshold p_n is increasing in n so as to ensure $r^D(n; p) > 0$ and $\Pi_m^D(q) > 0$. Also, Lemma 4 implies that for a fixed revenue q , the manufacturer will earn zero (i.e., $\Pi_m^D(q) = 0$) if the number of suppliers n exceeds τ_n^D , where $\tau_n^D \equiv \operatorname{argmin}_{n>0} \{p_n > q\}$. Therefore, when the number of suppliers n increases, the manufacturer should negotiate a higher revenue q to ensure $\Pi_m^D(q) > 0$.

3.4 Choosing the Payment Regime

Proposition 2 asserts that for any given p , each supplier works slower in equilibrium (i.e., $r^D(n; p) < r^N(p)$) and earns a lower profit (i.e., $\Pi_i^D(n; p) < \Pi_i^N(p)$) under regime D . Hence, it would be natural to conjecture that the manufacturer earns a higher profit under regime D for each given p ; i.e., $\Pi_m^D(p; q) > \Pi_m^N(p; q)$. However, it is not clear if this speculation is correct. To elaborate, let us first combine statements 5 and 1 of Lemma 1 and the fact $r^D(n; p) < r^N(p)$ to show that

$$\beta_n(r^D(n; p), \dots, r^D(n; p)) < \beta_n(r^N(p), \dots, r^N(p)) \leq \beta(r^N(p)).$$

Now we can compare the manufacturer's profit functions given in (3.4) and (3.3) to show that the profit comparison depends on two countervailing forces. The first force is based on the fact $nq \cdot \beta_n(r^D(n; p), \dots, r^D(n; p)) < nq \cdot \beta_n(r^N(p), \dots, r^N(p))$. Hence, the manufacturer's discounted revenue is lower under regime D due to a lower expected discount factor, which in turn is caused by the fact that $r^D(n; p) < r^N(p)$. The countering force stems from the fact that $np \cdot \beta_n(r^D(n; p), \dots, r^D(n; p)) < np \cdot \beta(r^N(p))$: the manufacturer's discounted cost is lower under regime D because the manufacturer benefits from not having to pay any of the suppliers until he receives his own revenue. Due to these two countervailing forces, it is inconclusive whether $\Pi_m^D(q; p) > \Pi_m^N(q; p)$ when the manufacturer offers the same payment p under both regimes. This observation poses another challenge: can one make any conclusive statement about the comparison between the manufacturer's optimal expected discounted profit under regimes N and D ?

Because there is no implicit expression for the optimal payment p^N and p^D that maximize the manufacturer's profit functions $\Pi_m^N(q; p)$ and $\Pi_m^D(q; p)$ given in (3.3) and (3.4); respectively, it is technically challenging to compare $\Pi_m^N(q; p^N)$ and $\Pi_m^D(q; p^D)$. Despite this challenge, we did manage to establish two characteristics of the manufacturer's optimal profits $\Pi_m^N(q)$ and $\Pi_m^D(q)$. First, rather trivially, when the revenue q is sufficiently small, say, $q < \max\{\theta, q_n\}$ and $q < \max\{\theta, p_n\}$, Lemmas 2 and 4 suggest that $\Pi_m^N(q) = \Pi_m^D(q) = 0$. The second characteristic is based on the following Lemma.

Lemma 5 *For any $n \geq 2$, the manufacturer's optimal profits $\Pi_m^N(q)$ and $\Pi_m^D(q)$ are convex and non-decreasing in q .*

We now compare the manufacturer's optimal profit functions $\Pi_m^N(q)$ and $\Pi_m^D(q)$ analytically for the case when q is sufficiently large and for the case when q is sufficiently small in Propositions 3 and 4, respectively. (For the case when the revenue q is in the intermediate range, we can only compare the manufacturer's profits numerically. Due to space limitations, the reader is referred to Kwon et al. (2008) for details.)

Proposition 3 *When the revenue q exceeds a certain threshold, say, $q > \tau_l$, (1) the manufacturer's optimal price is smaller under regime N : $p^N(q) < p^D(q)$; (2) the supplier's optimal work rate is larger under regime N : $r^N(p^N(q)) > r^D(p^D(q))$; (3) the expected completion time of the project is shorter under regime N : $E(T^N(p^N(q))) < E(T^D(p^D(q)))$; and (4) the manufacturer obtains a larger profit under regime N : $\Pi_m^N(q) > \Pi_m^D(q)$.*

Even though the manufacturer offers a higher optimal payment under regime D (i.e., $p^D > p^N$) when the revenue q is sufficiently large, Proposition 3 reveals that the manufacturer can afford to offer a sufficiently high payment under regime N so that the resulting work rate in equilibrium is higher under regime N (i.e., $r^D(n; p^D) < r^N(p^N)$). Applying Lemma 1, we have

$$\beta_n(r^D(n; p^D), \dots, r^D(n; p^D)) < \beta_n(r^N(p^N), \dots, r^N(p^N)) \leq \beta(r^N(p^N)).$$

Combining this fact with the manufacturer's profit functions given in (3.4) and (3.3), we can make the following observations. First, the manufacturer's discounted revenue is lower under regime D when the manufacturer offers the optimal payments; i.e., $nq \cdot \beta_n(r^D(n; p^D), \dots, r^D(n; p^D)) < nq \cdot \beta_n(r^N(p^N), \dots, r^N(p^N))$. Second, it is unclear if the manufacturer's discounted cost is higher or lower under regime D because of two opposing forces: $p^D > p^N$ and $\beta_n(r^D(n; p^D), \dots, r^D(n; p^D)) < \beta(r^N(p^N))$. On balance, statement (4) of Proposition 3 establishes that the manufacturer earns a larger profit under regime N when the revenue q is sufficiently large.

Next, we investigate the case when the revenue q is sufficiently small. In this case, the manufacturer can only afford to offer small payments to the suppliers under either regime. Consequently, the supplier's equilibrium work rates must be small and the project completion times are long under either regime. However, under regime D , the manufacturer's optimal profit is strictly positive (albeit small) because the manufacturer issues payments to the suppliers and receives his revenue after all suppliers have completed their tasks. However, under regime N , the manufacturer has to pay each supplier when she completes her own task, but he will receive his revenue after all suppliers have completed their tasks. Essentially, these "early" payments to the suppliers severely impair the manufacturer's discounted profit under regime N . Consequently, it is natural to speculate that the manufacturer would earn a higher profit under regime D when q is sufficiently small. The following Proposition asserts that this speculation is indeed correct.

Proposition 4 *Suppose $\theta > \max\{q_n, p_n\}$. Then, when the revenue q is below a certain threshold, say, $q < \tau_s$, (1) the manufacturer's optimal price is larger under regime D : $p^D(q) > p^N(q)$; (2) the supplier's optimal work rate is larger under regime D : $r^D(p^D(q)) > r^N(p^N(q))$; (3) the expected completion time of the project is shorter under regime D : $E(T^D(p^D(q))) < E(T^N(p^N(q)))$; and (4) the manufacturer obtains a higher optimal profit under regime D : $\Pi_m^D(q) > \Pi_m^N(q)$.*

When the revenue q is sufficiently small, Proposition 4 reveals that the manufacturer will offer a higher optimal payment under regime D (i.e., $p^D > p^N$) to ensure that the supplier's work rate in equilibrium is higher under regime D (i.e., $r^D(n; p^D) > r^N(p^N)$). Also, statement (4) of Proposition 4 establishes that the manufacturer earns a larger profit under regime D when the revenue q is sufficiently small.

We now compare the manufacturer's optimal profits $\Pi_m^N(q)$ and $\Pi_m^D(q)$ as the number of suppliers n increases. First, under regime N , there is a time delay between the manufacturer's accounts payable and account receivable, and this time delay becomes more severe as the number of suppliers n increases. Hence, as implied by Lemma 2, the manufacturer's optimal profit under regime N (i.e., $\Pi_m^N(q)$) will drop to zero when the number of suppliers n is sufficiently large, say, when $n > \tau_n^N$, where $\tau_n^N \equiv \operatorname{argmin}_{n>0} \{q_n > q\}$. Next, under regime D , due to her concern about further delay in payment when the number of suppliers increase, Proposition 2 and Lemma 4 assert that each supplier's optimal work rate $r^D(n; p)$ given in (3.8) and the manufacturer's optimal profit $\Pi_m^D(q)$ will drop to 0 when n is sufficiently large. Specifically, $\Pi_m^D(q)$ will drop to 0 when $n > \tau_n^D$, where $\tau_n^D \equiv \operatorname{argmin}_{n>0} \{p_n > q\}$. In summary, under both regimes N and D , the manufacturer's optimal profits will drop to 0 when n is sufficiently large. Therefore, the comparison of the manufacturer's optimal profits $\Pi_m^N(q)$ and $\Pi_m^D(q)$ hinges upon how fast these profits drop to 0 as n increases. By observing from Lemmas 2 and 4 that the threshold p_n grows faster than q_n in terms of n , we can conclude that $\tau_n^D < \tau_n^N$. Hence, the manufacturer's optimal profit $\Pi_m^D(q)$ drops to 0 before the manufacturer's profit $\Pi_m^N(q)$ drops to 0 as n increases. This proves the following result:

Proposition 5 *Given any fixed value of θ and q , the manufacturer obtains a higher optimal profit under regime N when the number of suppliers n exceeds a certain threshold, say, when $n > \tau_n$.*

Based on the results presented in Propositions 3, 4 and 5, we can conclude that as the revenue q or the number of suppliers n increases, the manufacturer's optimal profit functions associated with regimes N and D (i.e., $(\Pi_m^N(q)$ and $\Pi_m^D(q))$) will cross at least once. We are, however, unable to prove that there is exactly one crossing mainly because the analytical comparison between $\Pi_m^N(q)$ and $\Pi_m^D(q)$ is intractable. However, as reported in Kwon et al. (2008), numerical examples suggest that there is exactly one crossing between these two profit functions when we increase q or n . Combining our analytical results stated in Propositions 3, 4 and 5 with our numerical results, we make the following conjectures regarding the existence of two thresholds τ_q and τ_n as follows:

1. For any given n , there exists a threshold τ_q so that the manufacturer earns a higher profit under regime D if and only if the revenue $q \leq \tau_q$.

2. For any given q , there exists a threshold τ_n so that the manufacturer earns a higher profit under regime N if and only if the number of suppliers $n \geq \tau_n$.

Overall, the results presented in this section will be useful to the manufacturer when designing a project contract that involves a payment p and the choice of a payment regime.

4 Adjustable work rates under complete information

In the base model, we assume that the suppliers do not have information regarding the progress of other suppliers whence it is optimal for each supplier to continue to work at her initial rate until she completes her task. We now relax this assumption by considering a situation when each supplier is informed of the process of the other suppliers. (For example, under the Boeing's 787 development program, the completion time as well as the progress of each task are commonly observed by all suppliers (Nolan and Kotha (2005)).) Our goal is to examine the supplier's behavior under complete information and to examine whether the results established in Section 3 continue to hold. Ultimately, we are interested to investigate whether it behooves the manufacturer to provide this additional information to the suppliers. While it is clear that neither the suppliers nor the manufacturer can benefit from this additional information under the no delayed payment regime, the impact of this additional information is unclear under the delayed payment regime. Based on our analysis of the delayed payment regime with this additional information, we obtain the following results: (1) it is optimal for each supplier to work at a slower rate initially and then increase her rate when another supplier completes her task; and (2) the manufacturer prefers the delayed payment regime when the number of suppliers is sufficiently large.

4.1 *NI*: The No Delayed Payment Regime with Complete Information

Under regime *NI*, each supplier i receives p when her task is completed so that her expected discounted profit is independent of the other suppliers' completion times. Obviously, complete information has no impact on supplier behavior in the no delayed payment regime: it is optimal for each supplier to behave exactly the same way as in the case with no information under regime N . Consequently, all results reported in Section 3.2 continue to hold when the manufacturer provides complete information. Complete information provides no benefit to the suppliers or to the manufacturer.

4.2 *DI*: The Delayed Payment Regime with Complete Information

The information about other suppliers' progress can affect the behavior of each supplier under regime *DI* because each supplier i 's expected discounted profit depends on the completion times of other suppliers. To elaborate, observe the following situation. At time 0, each of the n suppliers selects her work rate and begins working on her own task. Due to the memoryless property of the exponential distribution, there is no valuable information to update until one of the n suppliers completes her task so there is no incentive for any supplier to change her work rate until one of the n suppliers completes her task. This observation suggests that each continuing supplier will alter her work rate only at the beginning of stage j , $j = n, (n-1), \dots, 1, 0$, where stage n begins at time 0 with n continuing suppliers, stage $(n-1)$ begins (and stage n ends) at the instant when one of the n suppliers completes her task so that there are $(n-1)$ continuing suppliers, and so forth.

Because the work rate decision is made only at the beginning of each of the n stages, we can formulate the supplier's problem as an n -stage game. Specifically, at the beginning of stage j ($j = n, (n-1), \dots, 1$), we analyze a non-cooperative game among j continuing suppliers. Due to the dynamic nature of the n -stage game and the delayed payment regime, each of the j continuing suppliers needs to take the other continuing suppliers' work rates at stage j and future stages (i.e., stages $(j-1), \dots, 1$) into consideration when determining her work rate at stage j . Akin to the backward induction approach for solving a dynamic programming problem, we now solve this n -stage game backward in time: solve stage 1 first, solve stage 2 second, and finally solve stage n . Then we determine the supplier's and the manufacturer's optimal profit functions in equilibrium. Finally, we compare the manufacturer's profits under regimes *DI* and *N*.

4.2.1 Analysis of the supplier's problem at stage 1 and stage 2

Let us consider the game at stage 1. At the beginning of stage 1, there is only 1 continuing supplier, say, supplier i , who needs to determine her work rate λ (a decision variable) that maximizes her expected profit discounted back to the beginning of stage 1. Because the beginning of stage 1 marks the end of stage 2 that occurs at the instant when the "second to last" supplier completed her task, there are $(n-1)$ "idle" suppliers who have completed their tasks earlier and are waiting for supplier i to complete her task so they can receive their payments. The only continuing supplier i at stage 1 will receive her payment p when she completes her task. Hence, for any work rate λ , supplier i 's optimal expected profit discounted back to the beginning of stage 1 can be expressed as $R_i^{(1)}$, where $R_i^{(1)} \equiv \max_{\lambda} R_i^{(1)}(\lambda) = \max_{\lambda} [-\frac{k\lambda^2}{\lambda+\alpha} + p \cdot \frac{\lambda}{\lambda+\alpha}]$. (We use the superscript (j) to denote stage j , where $j = n, (n-1), \dots, 1$.) It follows from the fact that the objective function is identical to

(3.1), the optimal work rate for supplier i at stage 1 is $\lambda^{(1)} = \alpha(\sqrt{1 + \frac{p}{\alpha k}} - 1)$, which equals $r^N(p)$ given in (3.5). By substituting $\lambda^{(1)}$ into $R_i^{(1)}(\lambda)$, it is easy to check that $R_i^{(1)} = p \cdot \frac{\lambda^{(1)}}{\lambda^{(1)} + \alpha} - \frac{k[\lambda^{(1)}]^2}{\lambda^{(1)} + \alpha}$.

Given the fact that the only continuing supplier i will work at rate $\lambda^{(1)}$ throughout stage 1 and that the operating costs of those $(n - 1)$ idle suppliers have already been incurred prior to the beginning of stage 1, the expected payment discounted back to the beginning of stage 1 for each of the $(n - 1)$ idle suppliers, say, supplier i' , can be expressed as $S_{i'}^{(1)}$, where $S_{i'}^{(1)} = \frac{\lambda^{(1)}}{\lambda^{(1)} + \alpha} \cdot p$. This completes our analysis of stage 1.

We now analyze the game at stage 2. At the beginning of stage 2, there are 2 continuing suppliers i and i' who need to decide on their work rates for stage 2. The remaining $(n - 2)$ suppliers are idle. Suppose supplier i works at rate λ (a decision variable) and supplier i' works at rate μ (a decision variable) throughout stage 2. Then the duration of stage 2, denoted by $\tau^{(2)}$, can be expressed as $\tau^{(2)} = \min\{X_i, X_{i'}\}$, where X_i and $X_{i'}$ are exponentially distributed with parameters λ and μ , respectively.

Using the property of the exponential distribution, the probability that supplier i finishes before supplier i' satisfies: $Prob\{X_i < X_{i'}\} = \frac{\lambda}{\lambda + \mu}$. Also, $Prob\{X_{i'} < X_i\} = \frac{\mu}{\lambda + \mu}$. First, consider the case when supplier i is the first to finish. In this case, stage 2 ends and stage 1 begins at the instant when supplier i finishes, supplier i will become an idle supplier at stage 1, and supplier i will earn $S_i^{(1)}$ during the game at stage 1. Second, suppose supplier i' is the first to finish. Then stage 2 ends and stage 1 begins at the instant when supplier i' finishes. In this case, supplier i will become the only continuing supplier at stage 1 and will earn $R_i^{(1)}$ during the game at stage 1. Combining these observations along with the fact that the expected discount factor associated with stage 2 is given by $E(e^{-\alpha \cdot \tau^{(2)}}) = \frac{\lambda + \mu}{\lambda + \mu + \alpha}$, we can express the expected profit of supplier i discounted back to the beginning of stage 2 (for any given work rate μ of the other supplier i') as:

$$\begin{aligned} R_i^{(2)}(\lambda, \mu) &= \max_{\lambda} [-E[\int_0^{\tau^{(2)}} k\lambda^2 \cdot e^{-\alpha t} dt] + E(e^{-\alpha \cdot \tau^{(2)}}) \cdot \frac{\lambda}{\lambda + \mu} \cdot S_i^{(1)} + E(e^{-\alpha \cdot \tau^{(2)}}) \cdot \frac{\mu}{\lambda + \mu} \cdot R_i^{(1)}] \\ &= \max_{\lambda} [-\frac{k\lambda^2}{\lambda + \mu + \alpha} + \frac{\lambda}{\lambda + \mu + \alpha} S_i^{(1)} + \frac{\mu}{\lambda + \mu + \alpha} R_i^{(1)}]. \end{aligned} \quad (4.1)$$

Similarly, we can express the expected profit of the other supplier i' discounted back to the beginning of stage 2 (for any given supplier i 's work rate λ) as:

$$R_{i'}^{(2)}(\lambda, \mu) = \max_{\mu} [-\frac{k\mu^2}{\lambda + \mu + \alpha} + \frac{\mu}{\lambda + \mu + \alpha} S_{i'}^{(1)} + \frac{\lambda}{\lambda + \mu + \alpha} R_{i'}^{(1)}].$$

By considering the first-order conditions associated with both continuing suppliers, we have:

Proposition 6 *At stage 2, there exists a symmetric unique equilibrium in which both continuing suppliers will work at rate $\lambda^{(2)}$, where*

$$\lambda^{(2)} = \frac{[(S_i^{(1)} - R_i^{(1)}) - 2k\alpha] + \sqrt{[(S_i^{(1)} - R_i^{(1)}) - 2k\alpha]^2 + 12k\alpha S_i^{(1)}}}{6k}. \quad (4.2)$$

Also, $0 < \lambda^{(2)} < \lambda^{(1)} = r^N(p)$ and $0 < R_i^{(2)} < R_i^{(1)}$.

By substituting $\lambda = \mu = \lambda^{(2)}$ into (4.1), we obtain $R_i^{(2)} \equiv R_i^{(2)}(\lambda^{(2)}, \lambda^{(2)})$. Also, for each of those $(n - 2)$ idle suppliers, say, supplier i , who is waiting to receive her payment, her equilibrium expected payment discounted back to the beginning of stage 2 can be expressed as $S_i^{(2)}$, where $S_i^{(2)} = E(e^{-\alpha\tau^{(2)}}) \cdot S_i^{(1)} = \frac{2\lambda^{(2)}}{2\lambda^{(2)} + \alpha} S_i^{(1)}$. This completes the analysis of the game associated with stage 2.

When $n = 2$. Proposition 6 states that both suppliers will work at rate $\lambda^{(2)}$ in equilibrium. Then, as soon as one of the suppliers completes her task, it is optimal for the remaining supplier to expedite her task by increasing her work rate from $\lambda^{(2)}$ to $\lambda^{(1)} = r^N(p)$. This result reveals that under the delayed payment regime *DI*, the information regarding the progress of other suppliers provides an incentive for each supplier to begin with a slower work rate and then switch to a faster work rate when another supplier completes her task. Eventually, the only remaining supplier will work at the fastest work rate that is equal to the optimal work rate under the no-delayed payment regime.

4.2.2 Analysis of the supplier's problem at stage j

Using the same approach as described above, we can solve the games associated with stages 1 through $(j - 1)$ by determining the equilibrium work rate $\lambda^{(m)}$ and the expected profits discounted back to the beginning of stage m (i.e., $R_i^{(m)}$ and $S_i^{(m)}$), where $m = 1, \dots, (j - 1)$. We now solve the game at stage j , where $j = n, (n - 1), \dots, 2$. At the beginning stage j , there are j continuing suppliers and $(n - j)$ idle suppliers. Without loss of generality, we index those j continuing suppliers $1, 2, \dots, j$ so that we can simplify our exposition.

Let λ_i (a decision variable) be each continuing supplier i 's work rate throughout stage j , where $i = 1, \dots, j$. In this case, the duration of stage j , denoted by $\tau^{(j)}$, satisfies: $\tau^{(j)} = \min\{X_1, X_2, \dots, X_j, \}$, where X_i is exponentially distributed with parameter λ_i . Using the property of the exponential distribution, the probability that supplier i is the first to finish is $\frac{\lambda_i}{\sum_{i'=1}^j \lambda_{i'}}$. Suppose supplier i is the first to finish. Then stage j ends and stage $(j - 1)$ begins the instant when supplier i finishes. At this moment, supplier i will become an idle supplier at stage $(j - 1)$, and

supplier i will earn $S_i^{(j-1)}$ during the game at stage $(j-1)$. If supplier i is not the first to finish, then stage j ends and stage $(j-1)$ begins at the instant when another continuing supplier finishes. In this case, supplier i will become one of the continuing suppliers at stage $(j-1)$, and supplier i will earn $R_i^{(j-1)}$ during the game at stage $(j-1)$. Combining these observations with the fact that the expected discount factor associated with stage j is given by $E(e^{-\alpha\tau^{(j)}}) = \frac{\sum_{i'=1}^j \lambda_{i'}}{\sum_{i'=1}^j \lambda_{i'} + \alpha}$, we can express the expected profit of supplier i discounted back to the beginning of stage j (for any given work rates $(\lambda_1, \dots, \lambda_{i-1}, \lambda_{i+1}, \dots, \lambda_j)$) as:

$$R_i^{(j)}(\lambda_1, \dots, \lambda_j) = \max_{\lambda_i} \left[-\frac{k\lambda_i^2}{\sum_{i'=1}^j \lambda_{i'} + \alpha} + \frac{\lambda_i}{\sum_{i'=1}^j \lambda_{i'} + \alpha} S_i^{(j-1)} + \frac{\sum_{i' \neq i}^j \lambda_{i'}}{\sum_{i'=1}^j \lambda_{i'} + \alpha} R_i^{(j-1)} \right], \quad (4.3)$$

where $i = 1, 2, \dots, j$. By considering the first-order conditions associated with the work rate for each of the j continuing suppliers at stage j , we have:

Proposition 7 *Under regime DI, there exists a symmetric and unique Nash equilibrium at stage j in which each continuing supplier will work at rate $\lambda^{(j)}$, where*

$$\lambda^{(j)} = \frac{[(j-1)(S_i^{(j-1)} - R_i^{(j-1)}) - 2k\alpha] + \sqrt{[(j-1)(S_i^{(j-1)} - R_i^{(j-1)}) - 2k\alpha]^2 + 4(2j-1)k\alpha S_i^{(j-1)}}}{2(2j-1)k} \quad (4.4)$$

To verify Proposition 7, observe that (4.4) yields $\lambda^{(1)} = \alpha(\sqrt{1 + \frac{p}{\alpha k}} - 1)$ when $j = 1$, and that (4.4) reduces to (4.2) when $j = 2$. By observing from (4.4) and (4.2) that $S_i^{(j-1)}$ and $R_i^{(j-1)}$ are functions of $\lambda^{(j-1)}$, Proposition 7 exhibits that we can compute $\lambda^{(j)}$ in a recursive manner. Once $\lambda^{(j)}$ is determined, we can compute the equilibrium expected discounted profits $R_i^{(j)}$ and $S_i^{(j)}$ for each stage j accordingly, where $j = n, (n-1), \dots, 1$. This completes our analysis of the n -stage game.

4.2.3 Profit functions under regime DI

Once we solve the n -stage game, we obtain the supplier's equilibrium work rate $\lambda^{(j)}$ at stage j and the equilibrium expected discounted profits back at the beginning of stage j (i.e., $R_i^{(j)}$ and $S_i^{(j)}$), where $j = n, (n-1), \dots, 1$. We now use these quantities to determine the supplier's equilibrium and the manufacturer's expected discounted profits under regime DI.

First, at the beginning of stage n (i.e., time 0), $R_i^{(n)}$, the expected profit discounted back to the beginning of stage n , is the equilibrium expected discounted profit for each supplier i under regime DI. Therefore, for any given p , each supplier i 's expected discounted profit can be expressed as:

$$\Pi_i^{DI}(n; p) = R_i^{(n)}, \text{ for } i = 1, \dots, n. \quad (4.5)$$

Next, before we determine the expected project completion time, let us examine $\tau^{(j)}$, the duration of stage j . By observing that all j continuing suppliers work at rate $\lambda^{(j)}$ at stage j , $\tau^{(j)} = \min\{X_1, X_2, \dots, X_j\}$, where X_i , $i = 1, \dots, j$, are independent and exponentially distributed with parameter $\lambda^{(j)}$ so $E(\tau^{(j)}) = \frac{1}{j\lambda^{(j)}}$. Combining this observation with the fact that the project completion time is equal to the sum of the duration of all n stages, the project completion time under regime DI satisfies:

$$E(T^{DI}(p)) = \sum_{j=1}^n E(\tau^{(j)}) = \sum_{j=1}^n \frac{1}{j\lambda^{(j)}}. \quad (4.6)$$

Also, it is easy to check that

$$E(e^{-\alpha \cdot T^{DI}(p)}) = \prod_{j=1}^n \frac{j \cdot \lambda^{(j)}}{j \cdot \lambda^{(j)} + \alpha}. \quad (4.7)$$

Finally, the manufacturer's expected discounted profit in equilibrium under regime DI satisfies:

$$\Pi_m^{DI}(q, p) = n(q - p) \cdot E(e^{-\alpha \cdot T^{DI}(p)}) = n(q - p) \cdot \prod_{j=1}^n \frac{j \cdot \lambda^{(j)}}{j \cdot \lambda^{(j)} + \alpha} \quad (4.8)$$

Hence, the manufacturer's optimal expected discounted profit under regime DI is equal to $\Pi_m^{DI}(q) \equiv \max_{p > \theta} \Pi_m^{DI}(q, p)$, where the optimal price is denoted by p^{DI} .

4.3 Choosing the Payment Regime

Due to the recursive formula (4.4), there is no closed-form expression for the supplier's optimal work rate $\lambda^{(j)}$, the supplier's optimal expected profit function $\Pi_i^{DI}(n, p)$ given in (4.5), or the manufacturer's optimal profit function $\Pi_m^{DI}(q, p)$ given in (4.8). Consequently, there is no explicit expression for the optimal payment p^{DI} or the manufacturer's optimal discounted profit $\Pi_m^{DI}(q)$ under regime DI . Even so, we manage to use the same approach as presented in Section 3.4 to compare $\Pi_m^{DI}(q)$ and $\Pi_m^N(q)$ analytically for the case when q is sufficiently large (small).

Proposition 8 *For $n \geq 2$ and for sufficiently large q , the manufacturer obtains a higher profit under regime N : $\Pi_m^N(q) > \Pi_m^{DI}(q)$. Also, for sufficiently small q , the manufacturer obtains a higher profit under regime DI : $\Pi_m^{DI}(q) > \Pi_m^N(q)$.*

The above proposition establishes the same characteristics as reported in Propositions 3 and 4 when we compare regime D and N : it is beneficial for the manufacturer to choose regime DI when q is sufficiently small and to choose regime N when q is sufficiently large.

We now compare the manufacturer's profit under regimes DI , D and N when the number of suppliers n is large. In preparation, observe from (4.4) that the supplier's work rate $\lambda^{(j)} > 0$ for stage $j = 1, \dots, n$ if $p > \theta$. This is because under regime DI with complete information, the suppliers can adjust their rates so they can start slow and work faster later so as to lower the discounted total cost and to generate a positive profit as long as payment $p > \theta$. Because each supplier works at a strictly positive rate $\lambda^{(j)}$ at each stage j under regime DI , we can make two conclusions: (1) each supplier i will obtain a positive expected profit $\Pi_i^{DI}(n; p) > 0$ if $p > \theta$ under regime DI ; and (2) the manufacturer will obtain a positive expected profit $\Pi_m^{DI}(q) > 0$ if $q > \theta$. The latter conclusion follows from (4.8) and the fact that $\lambda^{(j)} > 0$ for stage $j = 1, \dots, n$ when $q > p > \theta$.

We now use these two conclusions to compare the manufacturer's optimal profits under regimes DI , D , and N . First, under regime D , each supplier will continue to work at her optimal work rate $r^D(n; p)$ given in (3.8) until she completes her task. When there is no information, each supplier is concerned about further delay in payment when the number of suppliers n increases. As such, each supplier will work slower as n increase. This intuition is captured in Proposition 2: the suppliers optimal work rate $r^D(n; p)$ is decreasing in the number of suppliers n . By using this logic, we can apply Lemma 4 to show that the supplier's optimal work rate $r^D(n; p)$ and the manufacturer's optimal profit $\Pi_m^D(q)$ will drop to 0 under regime D when the number of suppliers $n > \tau_n^D$, where $\tau_n^D \equiv \operatorname{argmin}_{n>0} \{p_n > q\}$. In contrast, the supplier's optimal work rate and the manufacturer's optimal profit are positive for any n under regime DI . Hence, we can conclude that regime DI dominates D when the number of suppliers n is sufficiently large; i.e., $\Pi_m^{DI}(q) > \Pi_m^D(q)$ when $n > \tau_n^D$.

Under regime N , each supplier will continue to work at her optimal work rate $r^N(p)$ given in (3.5) until she completes her task. Even though each supplier will work at a positive rate when $p > \theta$ under regime N , the manufacturer must deal with a potential cash flow problem because he pays each supplier when she completes her task, but he receives his own revenue only when all suppliers have completed their tasks. Because this 'time delay' between the manufacturer's stream of payments and the time he gets paid increases as the number of suppliers n increases, we can apply Lemma 2 to show that the manufacturer's optimal profit $\Pi_m^N(q)$ will drop to 0 under regime N when the number of suppliers $n > \tau_n^N$, where $\tau_n^N \equiv \operatorname{argmin}_{n>0} \{q_n > q\}$. In contrast, the supplier's optimal work rate and the manufacturer's optimal profit are positive for any n under regime DI : regime DI dominates N when the number of suppliers n is sufficiently large; i.e., $\Pi_m^{DI}(q) > \Pi_m^N(q)$ when $n > \tau_n^N$. In summary, we have established the following result:

Proposition 9 *Given $q > \theta$, the manufacturer obtains a higher expected profit under regime DI when the number of suppliers $n > \max\{\tau_n^N, \tau_n^D\}$; i.e., regime DI dominates both regimes N and D*

when n is sufficiently large.

Coupling Proposition 9 and Proposition 5 reveals that for any fixed value of $q > \theta$, $\Pi_m^{DI}(q) > \Pi_m^N(q) > \Pi_m^D(q)$ when n is sufficiently large. This result suggests that, when the number of suppliers is large, the manufacturer can obtain a higher profit under the delayed payment regime by providing suppliers information regarding the progress of the project so as to induce the suppliers to increase their work rates as time progresses.

Combining our analytical results stated in Propositions 8 and 9 with our numerical results presented in Kwon et al. (2008), we conjecture the existence of two thresholds τ_q and τ_n as follows:

1. For any given n , there exists a threshold τ_q so that the manufacturer earns a higher profit under regime DI if and only if the revenue $q \leq \tau_q$.
2. For any given $q > \theta$, there exists a threshold τ_n so that the manufacturer earns a higher profit under regime DI if and only if the number of suppliers $n \geq \tau_n$.

Overall, the results presented in this section can be useful to the manufacturer when designing a project contract that involves a payment p , a choice of a the payment regime, and the information to be provided to the suppliers.

5 Discussion and Concluding Remarks

Our model enabled us to examine how a delayed payment affects the supplier's optimal work rate, the manufacturer's optimal payment, the supplier's and the manufacturer's expected discounted profits, and the expected project completion time. Relative to the no delayed payment regime N , we have shown that each supplier would operate at a slower rate and obtain a lower expected profit under regime D for any given p . Consequently, for any given p , use of regime D lengthens the project completion time. To induce suppliers to increase their work rates under regime D , we have shown that the manufacturer will offer a higher price. In addition, we have established conditions under which one regime dominates the other.

We have investigated how information regarding the suppliers' progress would affect supplier behavior and the profits of the suppliers and the manufacturer. We showed that information has no value to the suppliers under regime N ; however, information is definitely beneficial to the suppliers under regime DI . This is mainly because, with complete information, each supplier can utilize this information and adjust her work rate accordingly. By modeling this scenario as an n -stage game,

we have shown that under the delayed payment regime, there exists a symmetric equilibrium at each stage so that all continuing suppliers work at the same rate. Also, we have developed recursive formulae for computing the supplier's and the manufacturer's profits as well as the expected project completion time in equilibrium. We have shown that when the number of suppliers is sufficiently large, the manufacturer obtains a higher profit under the delayed payment regime if he provides the suppliers with information: regime DI dominates both regimes N and D .

5.1 Other Payment Schemes

The model presented in this paper is motivated by two simple payment regimes commonly observed in practice. Essentially, both regimes N and D are based on a single decision variable p and the timing of the payment. However, if the manufacturer (and the suppliers) are willing to entertain other time-based contracts with more decision variables, then many other forms of contracts deserve attention.

5.1.1 A Combined Payment Regime $N + D$

Let us consider a regime $N + D$ that combines regimes N and D . Under regime $N + D$, each supplier receives a portion of her payment δp when she completes her own task and then receives the remaining portion of her payment $(1 - \delta)p$ after all suppliers have completed their task. In this case, the manufacturer has to make two decisions: δ and p , where $\delta \in [0, 1]$.⁶

To begin, consider the base model in which suppliers have no information about other suppliers' progress. For any given δ and p , we can utilize (3.1) and (3.2) to determine supplier i 's expected discounted profit, getting:

$$\Pi_i^{N+D}(n; \delta; p; r_1, \dots, r_n) = \delta \cdot \Pi_i^N(p; r_i) + (1 - \delta) \cdot \Pi_i^D(p; r_1, \dots, r_n)$$

By using the same approach as presented in Section 3.3, one can show that there are multiple symmetric Nash equilibria in which all suppliers work at the same rate $r^{N+D}(\delta; p)$ in equilibrium. In this case, for any given payment $p > \theta$, we can utilize (3.3) and (3.4) to show that the manufacturer's expected discounted profit under regime $N + D$ satisfies:

$$\begin{aligned} \Pi_m^{N+D}(\delta; p; q) &= \delta \cdot \Pi_m^N(p; q; r^{N+D}(\delta; p)) + (1 - \delta) \cdot \Pi_m^D(p; q; r^{N+D}(\delta; p)) \\ &= nq \cdot \beta_n(r^{N+D}, \dots, r^{N+D}) - \delta np \cdot \beta(r^{N+D}) - (1 - \delta)np \cdot \beta_n(r^{N+D}, \dots, r^{N+D}). \end{aligned} \tag{5.1}$$

⁶We would like to thank the Editor for suggesting this combined regime.

By anticipating the supplier's optimal work rate $r^{N+D}(n; \delta; p)$, the manufacturer can determine his optimal δ^* and p^* that maximizes $\Pi_m^{N+D}(\delta; p; q)$. While the analysis is complex, it is clear that regime $N + D$ dominates both regimes N and D because regimes N and D are special cases of regime $N + D$ when $\delta = 1$ and $\delta = 0$, respectively. As such, the manufacturer's optimal profit is higher under regime $N + D$.

Next, consider the case when suppliers have information about the other suppliers' progress, and refer to this combined regime with information as regime $N + D + I$. The analysis of regime $N + D + I$ is complex because it involves the analysis of an n -stage game that is similar to the one presented in Section 4.2. By considering the fact that each supplier will first receive a portion of her payment δp when she completes her own task and then receive the remaining portion of her payment $(1 - \delta)p$ after all suppliers have completed their task, we can utilize the n -stage game to model the supplier's problem under regime $N + D + I$ as follows. First, the supplier's problem at stage 1 is exactly the same as the one presented in Section 4.2.1 because there is only one continuing supplier at the beginning of stage 1. For any intermediate stage j , $j = n, (n - 1), \dots, 2$, the supplier's problem at stage j is the same as the one presented in Section 4.2.2 except for the following: to capture the fact that each supplier i will receive δp when she completes her task, we need to replace the discounted payment $S_i^{(j-1)}$ by $\tilde{S}_i^{(j-1)}$, where $\tilde{S}_i^{(j-1)} = \delta \cdot p + (1 - \delta) \cdot S_i^{(j-1)}$. Then we can apply Proposition 7 and (4.4) to show the following: at stage j , all continuing suppliers will work at the same rate $\tilde{\lambda}^{(j)}$ under regime $N + D + I$, where:

$$\tilde{\lambda}^{(j)} = \frac{[(j-1)(\tilde{S}_i^{(j-1)} - R_i^{(j-1)}) - 2k\alpha] + \sqrt{[(j-1)(\tilde{S}_i^{(j-1)} - R_i^{(j-1)}) - 2k\alpha]^2 + 4(2j-1)k\alpha\tilde{S}_i^{(j-1)}}}{2(2j-1)k} \quad (5.2)$$

Clearly, (5.2) reduces to (4.4) when $\delta = 0$. As discussed in Section 4.2.2, the terms $\tilde{\lambda}^{(j)}$, $\tilde{S}_i^{(j)}$, and $R_i^{(j)}$ can be determined in a recursive manner. We omit the details. Also, by using the same approach as presented in Section 4.2.3 and the fact that the manufacturer has to pay δp to the supplier who completes her task at the end of each stage j , $j = n, (n - 1), \dots, 1$, and $(1 - \delta)p$ to each of the n suppliers at the end of stage 1, the manufacturer's expected discounted profit in equilibrium under regime $N + D + I$ is:

$$\Pi_m^{N+D+I}(q, \delta, p) = n(q - (1 - \delta) \cdot p) \cdot \prod_{j=1}^n \frac{j \cdot \tilde{\lambda}^{(j)}}{j \cdot \tilde{\lambda}^{(j)} + \alpha} - \sum_{m=1}^n (\delta \cdot p) \cdot \prod_{j=1}^m \frac{j \cdot \tilde{\lambda}^{(j)}}{j \cdot \tilde{\lambda}^{(j)} + \alpha}. \quad (5.3)$$

Notice that (5.3) reduces to (4.8) when $\delta = 0$. In this case, the manufacturer can determine his optimal $\tilde{\delta}$ and \tilde{p} that maximize $\Pi_m^{N+D+I}(q, \delta, p)$. It is clear that regime $N + D + I$ dominates both regimes NI and DI because regimes NI and DI are special cases of regime $N + D$ when $\delta = 1$ and $\delta = 0$, respectively.

Although there is no closed form expressions for the manufacturer's optimal profits under regimes $N + D$ and $N + D + I$, we can utilize the following observations to compare the man-

manufacturer's optimal profits under regime $N + D$ and $N + D + I$ as the number of suppliers n increases for the case when $\delta \in (0, 1)$. First, when n becomes very large, the time it takes for each supplier to receive her remaining portion of the payment $(1 - \delta)p$ becomes very long. As such, each supplier will behave as if she will only receive δp when she completes her own task and will simply neglect the remaining portion $(1 - \delta)p$. Formally, one can show that, as n becomes very large, the supplier's optimal work rate $r^{N+D}(\delta; p) \approx r^N(\delta p)$. Second, by using the fact that $r^{N+D}(\delta; p) \approx r^N(\delta p)$ as n becomes very large, we can apply Statement 5 of Lemma 1 to show that $\beta_n(r^{N+D}, \dots, r^{N+D}) \approx 0$ and $\beta(r^{N+D}) \approx \frac{r^N(\delta p)}{r^N(\delta p) + \alpha} > 0$ when n is very large. In this case, one can observe from (5.1) that $\Pi_m^{N+D}(\delta; p; q)$, the manufacturer's optimal profit under regime $N + D$, will drop below 0 as the number of suppliers n is sufficiently large. On the contrary, observe from (5.2) that the supplier's work rate $\tilde{\lambda}^{(j)} > 0$ as n increases. Consequently, the manufacturer's optimal profit given in (5.3) is always positive. Hence, we can conclude that regime $N + D + I$ dominates regime $N + D$ when the number of suppliers n is sufficiently large. This result is consistent with Proposition 9.

5.1.2 Performance Based Payment Regimes

In addition to regimes $N + D$ and $N + D + I$, there are other payment regimes that can be of interest. For instance, recall from Section 4 that, under regime DI , each continuing supplier will work at a faster rate when another supplier completes her task. However, if the manufacturer wants to incentivize the continuing suppliers to work even faster, then he needs to provide additional incentives for the continuing suppliers to expedite their tasks. For example, the manufacturer can pay the suppliers according to the order of their completion times: pay $p_{(1)}$ to the supplier who finishes first, pay $p_{(2)}$ to the supplier who finishes second, and pay $p_{(n)}$ to the supplier who finishes last.⁷ Clearly, to implement such payment scheme, each supplier needs to know the completion times of the other suppliers. We shall refer this incentive payment with information as regime II .

Given the payments $p_{(1)}, p_{(2)}, \dots, p_{(n)}$, the manufacturer can implement regime II by postponing his payments until all suppliers have completed their tasks or by issuing his payments without delay; i.e., each supplier will receive her payment when she completes her task. By considering the optimal payments $p_{(1)}^*, p_{(2)}^*, \dots, p_{(n)}^*$ that maximize the manufacturer's expected discounted profit, we are able to establish the following results for the 2-supplier case. (The exact analysis for the general n case is intractable, but the analysis for the 2-supplier case is available upon request.) First, when the manufacturer postpones his payments $p_{(1)}$ and $p_{(2)}$ until both suppliers have completed their tasks, we can show that the manufacturer's optimal profit under regime II is strictly larger than

⁷We would like to thank one of the reviewers for suggesting this payment regime.

under regime *DI*. Second, when the payments $p_{(1)}$ and $p_{(2)}$ are issued without delay, we show that the manufacturer's optimal profit under regime *II* is strictly larger than under regime *N*.

The results associated with regimes *N+D* and *II* reveals that the manufacturer can benefit from payment regimes that involve more decision variables. Therefore, it will be of interest to explore the general form of performance based payment regimes in the future. For instance, consider a situation when the manufacturer offers supplier i a payment that depends on the completion times of all other suppliers; i.e., $p_i(X_1, X_2, \dots, X_n)$. It is easy to check that all aforementioned regimes are special cases of this general form. For example, this general payment structure reduces to regime *II* when $p_i(X_1, X_2, \dots, X_n) = p_{(1)}$ if X_i is the smallest among (X_1, X_2, \dots, X_n) , $p_i(X_1, X_2, \dots, X_n) = p_{(2)}$ if X_i is the second smallest among (X_1, X_2, \dots, X_n) , and so forth. Clearly, the analysis of this general payment regime will be highly complex. We leave this for future research.

5.2 Future Research Topics

In addition to various payment schemes that deserve attention in the future, there are many research opportunities for addressing the limitations of the model presented in this paper. First, our model is based on the assumption that the completion time of each task is exponentially distributed. This assumption ensures that a 'static' policy is optimal in the following sense: (1) under regimes *N* and *D*, it is optimal for each supplier to continue to work at her initial rate selected at time 0 until she completes her task; and (2) under regime *DI*, it is optimal for each supplier to continue to work at her rate selected at the beginning of stage j , $j = n, (n - 1), \dots, 1$, until the end of stage j . These static policies enabled us to obtain tractable results and closed form expressions for the suppliers' optimal work rates as well as other performance metrics. As a research direction, it would be of interest to examine other probability distributions, develop near-optimal heuristics for the suppliers' time-varying work rates, and conduct simulation experiments to examine the robustness of the results presented in this paper. Second, we have assumed that the operating costs of all n suppliers are identical. This assumption is critical to establish the existence of symmetric equilibria and to establish the analytical results presented in this paper. One potential future research direction is to examine the case of non-identical suppliers and to investigate the robustness of the results presented in this paper numerically. Third, our model assumes that all parties are risk-neutral. It would be of interest to examine the behavior and the performance metrics when the suppliers are risk-averse. Fourth, our model is based on the assumption that the manufacturer has perfect information about the supplier's cost structure, say, the value of k . In reality, the manufacturer will not possess perfect information. Because imperfect information can create another technical challenge for the manufacturer to design an effective project contract, it would be of interest to explore the use of mechanism design theory to develop effective project contracts. Fifth, when

the information regarding each supplier's cost structure is private, it would be of interest for the manufacturer to consider using auction mechanisms instead of incentive contracts. Sixth, even though supply contracts have been well studied, the issue of channel coordination in the context of project management contracts is not well-understood. This is another potential future research topic.

References

- [1] Abramowitz, M., and Stegun, I.A., *Handbook of Mathematical Functions*, Dover Publications, 1965.
- [2] Adler, P., Mandelbaum, A., Nguyen, V., and Schwerer, E., "From Project to Process Management: An Empirical-based Framework for Analyzing Product Development Time," *Management Science*, Vol. 41, 3, pp. 458-482, 1995.
- [3] Akintoye, A.S., and MacLeod, M.J., "Risk Analysis and Management in Construction," *International Journal of Project Management*, Vol. 15, 1, pp. 31-38, 1997.
- [4] Bayiz, M., and Corbett, C.J., "Coordination and Incentive Contracts in Project Management under Asymmetric Information," working paper, UCLA Anderson School, 2005.
- [5] Cachon, G., "Supply Chain Coordination with Contracts," in *Handbooks in Operations Research and Management Science*, (Eds. De Kok, A.G., and Graves, S.), Elsevier Publisher, 2003.
- [6] Cachon, G., and Zhang, F., "Obtaining Fast Service in a Queueing System via Performance-Based Allocation of Demand," *Management Science*, Vol. 53, pp. 408-420, 2007.
- [7] Cohen, I., Mandelbaum, A., and Shtub, A., "Multi-Project Scheduling and Control: A Project-Based Comparative Study of the Critical Chain Methodology and Some Alternatives," *Project Management Journal*, Vol. 35, 2, pp. 39-50, 2004.
- [8] Dean, B., Merterl Jr. S., and Roepke, L., "Research Project Cost Distribution and Budget Forecasting," *IEEE Transactions on Engineering Management*, Vol. 16, 4, pp. 176-191, 1969.
- [9] Dempski, J., and Sappington, D., "Optimal Incentive Contracts with Multiple Agents," *Journal of Economic Theory*, Vol. 17, pp. 152-171, 1984.
- [10] Gradshteyn, I.S., and Ryzhik, I.M., *Table of Integrals, Series, and Products*, 7th ed., Eds Alan Jeffrey and Daniel Zwillinger, Academic Press, 2007.

- [11] Greising, D., and Johnsson, J., "Behind Boeing's 787 Delays," *Chicago Tribune*, December 8, 2007.
- [12] Holmstrom, B., "Moral Hazard in Teams," *The Bell Journal of Economics*, Vol. 13, 2, pp. 324-340, 1982.
- [13] Horng, T.C., and Bozdogan, K., "Comparative Analysis of Supply Chain Management Practices by Boeing and Airbus: Long-Term Strategic Implications," MIT Lean Institute presentation, 2007.
- [14] Itoh, H., "Incentives to Help in Multi-agent Situations," *Econometrica*, Vol. 59, 3, pp. 611-636, 1991.
- [15] Klastorin, T., *Project Management: Tools and Trade-offs*, John Wiley and Sons, Inc., Somerset, New Jersey, 2004.
- [16] Kwon, D., Lippman, S.A., McCardle, K.F., and Tang, C.S., "Managing Time-Based New Product Development Contracts with Delayed Payments," unpublished manuscript, 2008.
- [17] Lippman, S.A., and Mamer, J.W., "Pre-emptive Innovation," *Journal of Economic Theory*, Vol. 61, pp. 104-119, 1993.
- [18] Magott, J., and Skudlarski, K., "Estimating the mean completion time of PERT networks with exponentially distributed durations of activities," *European Journal of Operational Research*, Vol. 71, 8, pp. 7079, 1993.
- [19] McAfee, R.P., and McMillan, J., "Optimal Contracts for Teams," *International Economic Review*, Vol. 32, 3, pp. 561-577, 1991.
- [20] Mookherjee, D., "Optimal Incentive Schemes with Many Agents," *Review of Economic Studies*, Vol. 51, pp. 433-446, 1984.
- [21] Nolan, R., and Kotha, S., "Boeing 787: The Dreamliner," Harvard Business School case 9-305-101, 2005.
- [22] Pennings, E., and Lint, O., "The Option Value of advance R & D," *European Journal of Operational Research*, Vol. 103, 16, pp. 83-94, 1997.
- [23] Reinganum, J.F., "Strategic Search Theory," *International Economic Review*, Vol. 23, 1, pp. 1 - 17, 1982.
- [24] Ross, S., *Introduction to Stochastic Dynamic Programming*, Academic Press, 1983.

- [25] Simister, S.J., "Usage and benefits of project risk analysis and management," *International Journal of Project Management*, Vol. 12, pp. 5-8, 1994.
- [26] Smith-Daniels, D.E., and Smith-Daniels, V.L., "Maximizing the Net Present Value of a Project subject to Materials and Capital Constraints," *Journal of Operations Management*, vol. 7, 1, pp. 33-45, 1987.
- [27] Smith-Daniels, D.E., and Aquilano, N.J., " Using a Late-Start Resource-Constrained Project Schedule to Improve Project Net Present Value," *Decision Sciences*, vol. 18, 4, pp. 617-630, 1987.
- [28] Tang, C.S., "Perspectives in Supply Chain Risk Management," *International Journal of Production Economics*, Vol. 103, pp. 451-488, 2006.
- [29] Tavares, L.V., "A Review of the Contribution of Operations Research to Project Management," *European Journal of Operational Research*, Vol. 136, pp. 1-18, 2002.
- [30] Vanhoucke, M., Demeulemeester, E., and Herroelen, W., "On Maximizing the Net Present Value of a Project Under Renewable Resource Constraints," *Management Science*, vol. 47, 8, pp. 1113-1121, 2001.
- [31] Wang, Y., and Gerchak, Y., "Capacity Games in Assembly Systems with Uncertain Demand," *Manufacturing and Service Operations Management*, Vol. 5, 3, pp. 252-267, 2003.