Novel Amplify-and-Forward Strategy Using MRC Reception over FSO Channels with Pointing Errors

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***Abstract*—Free-space optical (FSO) systems offer ultra-high data rates for a number of terrestrial applications. This paper presents the first characterization of average bit-error rate (BER) for a cooperative FSO system that uses channel side information (CSI)-assisted amplify-and-forward (AF) relaying when line-of-sight (LOS) is available. A novel feature of this paper is that maximum ratio combining (MRC) reception is used at destination node to maximize the output signal-to-noise ratio (SNR) under pointing error effects. An asymptotic closed-form expression for the BER is obtained over gamma-gamma (GG) fading channels with pointing errors. The developed asymptotic expression is used to analyze the diversity order gain in relation the direct path link. The maximum diversity order is determined to be $\beta_{SD}+\min(\beta_{SR},\beta_{RD})$, where β_{SD}, β_{SR} and β_{RD} are atmospheric turbulence parameters corresponding to the source-destination (S-D), source-relay (S-R) and relay-destination (R-D) links, respectively. Moreover, optimal relay placement, which plays an important roll in performance, is derived by taking full advantage of asymptotic expressions without the need for using optimization methods. Obtained results corroborate that a higher diversity order gain and, hence, a better BER performance is achieved in comparison with the direct path link and equivalent AF-dual-hop (DH) transmissions.**

***Keywords*—Free-space optical (FSO), cooperative diversity, bit-error rate (BER), amplify-and-forward (AF), line-of-sight (LOS), maximum ratio combining (MRC).**

I. Introduction

Free-space optical (FSO) communication systems have been thoroughly investigated for decades, which have traditionally been presented as an alternative solution to radio-frequency (RF) links. Nowadays, FSO technology is further considered as an emerging broadband wireless access solution. A significant number of applications can be provided by FSO technology such as last mile access networks, high data-rate links between buildings, next generation wireless broadband networks and back-haul for wireless cellular networks, among others [1]. Despite the advantages of FSO links, atmospheric turbulence, pointing errors and atmospheric effects such as fog greatly deteriorate systems performance. Different techniques have widely been proposed in the literature such as multiple-input/multiple-output (MIMO) systems and channel coding to mitigate the impact of atmospheric turbulence on terrestrial FSO links under the presence of pointing errors. Although MIMO techniques have been considered in FSO systems, these techniques may not be practical due to limited by hardware, size, or even cost. An alternative approach is the deployment of cooperative communications in order to extend the coverage area and achieve spatial diversity avoiding some issues related to MIMO FSO communication systems.

Cooperative strategies such as amplify-and-forward (AF) and detect-and-forward (DF) have been considered in-depth by different authors in both serial and parallel relaying [2]–[14]. Even though AF relaying has been a highlighted research topic in the last years, most contributions have been focused on multi-hop transmissions, i.e., without line-of-sight (LOS). As demonstrated in [3], [6], [11], [14] using DF relaying, the use of LOS can notably improve the performance in cooperative FSO systems even when this FSO link is strongly affected by atmospheric turbulence and pointing errors. In [4], a relay-assisted system that uses optical AF relaying and LOS is proposed to evaluate the error-rate performance over log-normal (LN) fading channels and equal gain combining (EGC) reception by using the photon-counting method, without considering pointing errors. Unlike [4], this paper provides a more realistic performance analysis in terms of error probability for practical cooperative FSO networks based on AF relaying with LOS and maximum ratio combining (MRC) reception under the presence of pointing errors. An MRC combiner has already been adopted in [15] to analyze the performance of RF systems over Nakagami-m fading channels using AF relaying.

In this paper, for the first time, we characterize the average bit-error rate (BER) for a cooperative FSO system that uses channel side information (CSI)-assisted AF relaying under pointing error effects. To the best of our knowledge, there is no other work regarding AF relaying scenarios which use LOS to improve the BER performance affected by pointing errors. In this way, a novel asymptotic closed-form expression for the BER is obtained by using MRC reception at destination, and electrical amplification at relay terminal. The MRC combiner is considered since this combining technique maximizes the output signal-to-noise ratio (SNR), which is specially important in cooperative FSO systems. The reason behind this is that cooperative communication systems need to maximize the SNR due to the fact that the SNR at the destination is significantly deteriorated as a consequence of the source-relay-destination path. In the light of obtained results,

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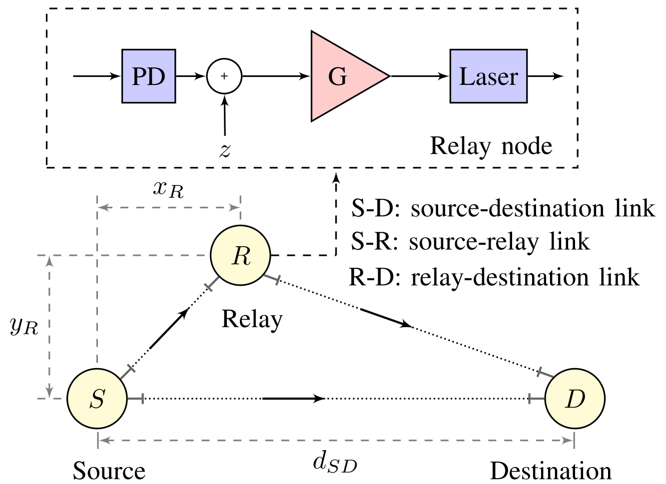


Fig. 1: Block diagram of a 3-nodes FSO system, where d_{SD} is S-D link distance, (x_R, y_R) is the relay location, G amplifies the received signal from S, and PD stands for photo-detector.

the relay location plays an essential roll in diversity order. On other hand, a greater robustness against larger amounts of misalignment is achieved when LOS is deployed. Monte Carlo simulations are further included in order to confirm the analytical results.

The remainder of the paper is organized as follows. System and channel models are described in Section II. In Section III, we obtain the asymptotic closed-form expression for the average BER corresponding to the AF cooperative protocol by using MRC reception over GG atmospheric turbulence channels with pointing errors. This asymptotic closed-form expression is totally valid for practical cooperative FSO networks and FSO scenarios with larger amounts of misalignment. We present some numerical results and discussions in Section IV, and conclude the paper in Section V.

II. System and Channel Models

A. System Model

Following Fig. 1, a 3-way FSO communications setup, a novel CSI-assisted AF cooperative protocol using MRC reception at destination (D) node and LOS is proposed here, hereinafter called AF-LOS cooperative protocol. Each of the FSO links are based on intensity modulation and direct detection (IM/DD), and on-off keying (OOK) modulation, as shown in Fig. 1. The AF-LOS cooperative protocol works in two stages. In the first stage, the source (S) node sends its own data to the destination node and the relay (R) node, i.e., the source node transmits the same information to the relay node and destination node. In the second stage, the relay node sends the received data from source node in the first stage to the destination node. The relay node uses an electrical amplifier with gain G to amplify the received signal from source node and then forwards it to destination node. It must be mentioned that in full-CSI relaying case, instantaneous CSI of the S-R link is assumed available at relay node in order to calculate the corresponding gain before forwarding the received signal. Also note that the CSI is only known at the receiver side of the relay and destination nodes. Finally, the received signals from source node and relay node are combined at destination node. Then, the combined output is detected. Note that the same information rate is obtained at destination node in comparison with the direct transmission (DT) or the direct path link without using a cooperative strategy.

As can be observed in Fig. 1, synchronization mechanisms are required at destination in order to detect data received from source and relay. In this way, data received from source can be stored in a buffer at destination for further detection. Without loss of generality, possible synchronization errors on BER performance are beyond the main objective of this study.

B. Channel Model

The received electrical signal for each FSO link of this cooperative system is given by

$$y_m = h_m R x + z_m, \quad (1)$$

where R is the detector responsivity, assumed hereinafter set to be the unity, x is the transmitted optical signal, h_m is the gain of the channel between the source and the receiver, and z_m is additive white Gaussian noise (AWGN) with zero mean and variance $\sigma_n^2 = N_0/2$ (A^2/Hz). The transmitted optical signal is either 0 or $2P_t$ where P_t is the average optical power. In the following, the subscript m is used to represent the different FSO links considered here, i.e., S-D, S-R and R-D. The channel gain is a product of three factors, i.e. $h_m = L_m \cdot h_a \cdot h_p$, atmospheric path loss L_m, atmospheric turbulence h_a, and geometric spread and pointing errors h_p. Note that both atmospheric turbulence and pointing errors can be assumed as statistically independent random variables. The path loss, L_m, is determined by the exponential Beers-Lambert law as $L_m = e^{-\Phi d_m}$, where d_m is the link distance and Φ is the atmospheric attenuation coefficient [16].

With regard to the statistical channel model, atmospheric turbulence is modeled according to the GG distribution to consider a wide range of turbulence conditions [17]. Pointing errors at the receiver are modeled assuming a model of misalignment where the effect of beam width, detector size and jitter variance are considered [18]. The attenuation due to geometric spread and pointing errors is approximated assuming a Gaussian spatial intensity profile of beam waist radius [18, eqn. (9)] as $h_p(r; z) \approx A_0 \exp\left(-2r^2/\omega_{z_{eq}^2}\right)$, where $v = \sqrt{\pi}a/\sqrt{2}\omega_z$, $A_0 = [\mathrm{erf}(v)]^2$ is the fraction of the collected power at $r = 0$, $\omega_{z_{eq}}^2 = \omega_z^2 \sqrt{\pi}\mathrm{erf}(v)/2v\exp(-v^2)$ is the equivalent beam width, and a is the radius of a circular detection aperture. The beam width ω_z can be approximated by $\omega_z = \theta_z \cdot d_m$, where θ_z is the divergence angle defining the increase in beam radius with link distance.

An exact closed-form expression for the composite fading channel was obtained in [19, eqn. (12)] in terms of the Meijer's G-function [20, eqn. (9.301)]. Here, an asymptotic closed-form expression for this probability density function (PDF) is adopted. Noting that the first term of the Mclaurin series expansion of $f_{h_m}(h)$ is $f_{h_m}(h) = a_m h^{b_m-1} + O(h^{b_m})$. This approximation is required to give a deeper understanding of the influence of atmospheric turbulence and pointing errors on BER performance corresponding to AF relay-assisted FSO systems. As presented in [21], [22], the PDF is approximated by a single polynomial term as

$$f_{h_m}(h) \doteq a_m h^{b_m-1} = \begin{cases} \frac{\varphi^2(\alpha\beta)^\beta\Gamma(\alpha-\beta)}{(A_0L_m)^\beta\Gamma(\alpha)\Gamma(\beta)(\varphi^2-\beta)} h^{\beta-1}, & \varphi^2 > \beta \\ \frac{\varphi^2(\alpha\beta)^{\varphi^2}\Gamma(\alpha-\varphi^2)\Gamma(\beta-\varphi^2)}{(A_0L_m)^{\varphi^2}\Gamma(\alpha)\Gamma(\beta)} h^{\varphi^2-1}, & \varphi^2 < \beta \end{cases} \tag{2}$$

where $b_m = \min(\beta_m, \varphi_m^2)$ and $\varphi^2 = \omega_{z_{eq}}^2/4\sigma_s^2$ is the ratio between the equivalent beam radius at the receiver and the corresponding pointing error displacement standard deviation (jitter) at the receiver. It is noteworthy to mention that the asymptotic expression given in Eq. (2) is dominated by $b_m - 1$. Moreover, $\Gamma(\cdot)$ is the Gamma function, and (α, β) can be directly linked to physical parameters through the following expressions [17]:

$$\alpha = \left[\exp\left(0.49\sigma_R^2/(1+1.11\sigma_R^{12/5})^{7/6}\right) - 1\right]^{-1}, \tag{3a}$$

$$\beta = \left[\exp\left(0.51\sigma_R^2/(1+0.69\sigma_R^{12/5})^{5/6}\right) - 1\right]^{-1}, \tag{3b}$$

where $\sigma_R^2 = 1.23C_n^2\kappa^{7/6}d_m^{11/6}$ is the Rytov variance assuming plane wave propagation, which is a measure of optical turbulence strength. Here, $\kappa = 2\pi/\lambda$ is the optical wave number and C_n^2 stands for the altitude-dependent index of the refractive structure parameter. It is demonstrated that the relationship $\alpha > \beta$ is always satisfied, and β is lower bounded above 1 as turbulence strength increases [23]. Here, we assume that h_{SD}, h_{SR} and h_{RD} are statistically independent.

III. Asymptotic Bit-Error Rate Analysis

Asymptotic closed-form expressions are obtained in order to quantify the average BER at high SNR for this cooperative FSO system by taking full advantage of the asymptotic expression given in Eq. (2). According to MRC reception, the combined output can easily be expressed assuming CSI at the receiver as follows

$$\begin{aligned} y_T^{AF} &= w_1 y_{SD} + w_2 y_{SRD} \\ &= \frac{x}{2}\left(w_1 h_{SD} + w_2 h_{SR} h_{RD} G\right) \\ &\quad + w_1 z_{SD} + w_2\left(z_{SR} h_{RD} G + z_{RD}\right), \end{aligned} \tag{4}$$

where w_1 and w_2 are the optimum combining weights corresponding to the received SNR at destination node, and they can readily be derived as $w_1 = h_{SD}$ and $w_2 = h_{SR}h_{RD}G/(1+h_{RD}^2G^2)$. Due to the fact that the source node transmits the same information to the relay node and destination node during the first stage, the division by 2 is considered in Eq. (4) in order to hold the average optical power at a constant level of P_t. In this sense, each of lasers corresponding to the source node transmits an average optical power of $P_t/2$. The variable gain, G, is selected at relay node to satisfy its power constraint, and is given by

$$G = 2\sqrt{\frac{P_t^2}{P_t^2 h_{SR}^2 + N_0}} \approx \frac{2}{h_{SR}}. \tag{5}$$

The multiplication by 2 is also considered in Eq. (5) to ensure an average optical power of P_t since only one laser is used at relay node. By using the optimum combining weights, the instantaneous received SNR at destination node is given by

$$\gamma_T^{\text{AF-LOS}} = \gamma\left(h_{SD}^2 + \frac{4h_{SR}^2 h_{RD}^2}{h_{SR}^2 + 4h_{RD}^2}\right) = \gamma h_T^2, \tag{6}$$

where $\gamma = P_t^2 T_b/N_0$ is the received electrical SNR in absence of turbulence and pointing error,s where T_b is the bit period. In this way, the asymptotic BER performance corresponding to the AF-LOS cooperative protocol is obtained as follows

$$P_b^{\text{AF-LOS}} \doteq \int_0^\infty Q\left(\frac{\gamma_{\text{opt}}}{\sqrt{2}} h_T\right) f_{h_T}(h_T) dh_T, \tag{7}$$

where $Q(\cdot)$ is the Gaussian Q-function, and γ_{opt} is the optical SNR in absence of turbulence given by $\gamma_{\text{opt}} = \sqrt{\gamma} = P_t\sqrt{T_b}/\sqrt{N_0}$. Next, the asymptotic closed-form expression for the PDF of h_T^2 is derived. Notice that h_T^2 is expressed as in Eq. (6) as

$$h_T^2 = h_{SD}^2 + \frac{4h_{SR}^2 h_{RD}^2}{h_{SR}^2 + 4h_{RD}^2} = h_{SD}^2 + h_{SRD}^2. \tag{8}$$

According to Eq. (2), we firstly derive the asymptotic PDF of h_{SD}^2 as follows

$$f_{h_{SD}^2}(h) \doteq \frac{a_{SD}}{2} h^{\frac{b_{SD}}{2}-1}. \tag{9}$$

Now, we derive the asymptotic PDF of h_{SRD}^2. To the best of the authors' knowledge, a closed-form analytical derivation of the h_{SRD}^2 statistics given in Eq. (8) is mathematically intractable. Therefore, we approximate h_{SRD}^2 as in [24] as

$$h_{SRD}^2 = \frac{4h_{SR}^2 h_{RD}^2}{h_{SR}^2 + 4h_{RD}^2} \approx \min\left(h_{SR}^2, 4h_{RD}^2\right). \tag{10}$$

The asymptotic PDF of h_{SRD}^2 is derived from its cumulative density function (CDF) as $f_{h_{SRD}^2}(h) = \frac{d}{dh}\left[F_{h_{SRD}^2}(h)\right]$. Note that an asymptotic expression for the CDF can easily be derived from Eq. (2) as $F_{h_m}(h) \doteq (a_m/b_m)h^{b_m}$. Then, the CDF of $\min\left(h_{SR}^2, 4h_{RD}^2\right)$ can be written as in [24, eqn. (4)] as follows

$$\begin{aligned} &F_{h_{SRD}^2}(h) \\ &\simeq F_{h_{SR}^2}(h) + F_{h_{RD}^2}(h/4) - F_{h_{SR}^2}(h)F_{h_{RD}^2}(h/4), \end{aligned} \tag{11}$$

where $F_{h_{SR}^2}(h)$ and $F_{h_{RD}^2}(h)$ are the CDFs of h_{SR}^2 and h_{RD}^2, respectively. Similar to Eq. (9), $f_{h_{SR}^2}(h)$ and $f_{h_{RD}^2}(h)$ are derived and, then, $F_{h_{SR}^2}(h)$ and $F_{h_{RD}^2}(h)$ are also derived. After some algebraic manipulations, we can obtain the asymptotic expression for the CDF of h_{SRD}^2. Finally, the corresponding PDF is obtained via $f_{h_{SRD}^2}(h) = \frac{d}{dh}\left[F_{h_{SRD}^2}(h)\right]$ as follows

$$f_{h_{SRD}^2}(h) \doteq \frac{a_{SR}}{2} h^{\frac{b_{SR}}{2}-1} + \frac{a_{RD}}{2^{b_{RD}+1}} h^{\frac{b_{RD}}{2}-1} + \frac{a_{SR}a_{RD}\left(b_{SR}+b_{RD}\right) h^{\frac{b_{SR}}{2}+\frac{b_{RD}}{2}-1}}{2^{b_{RD}+1} b_{SR} b_{RD}}. \quad (12)$$

Since h_{SD}^2 and h_{SRD}^2 are statistically independent, the resulting PDF of $h_T^2 = h_{SD}^2 + h_{SRD}^2$ is derived as follows

$$f_{h_T^2}(h) = \mathcal{TL}^{-1}\left\{\mathcal{M}_{h_{SD}^2}(-t) \cdot \mathcal{M}_{h_{SRD}^2}(-t)\right\}, \quad (13)$$

where the $\mathcal{M}_{h_{SD}^2}(t)$ and $\mathcal{M}_{h_{SRD}^2}(t)$ are the moment-generating functions (MGF) of h_{SD}^2 and h_{SRD}^2, respectively. Note that $\mathcal{M}_{h_{SD}^2}(-t)$ and $\mathcal{M}_{h_{SRD}^2}(-t)$ are the two-sided Laplace transforms of $f_{h_{SD}^2}(h)$ and $f_{h_{SRD}^2}(h)$, respectively. Hence, the PDF of h_T^2 is obtained via inverse Laplace transform. Finally, taking into account that the PDF of h_T is derived as $f_{h_T}(h) \doteq \frac{d}{dh}\left[F_{h_T^2}(h^2)\right]$, the corresponding asymptotic closed-form expression for the PDF of h_T, $f_{h_T}(h)$, is given by

$$f_{h_T}(h) \doteq \frac{\Gamma\left(b_{SD}/2\right)\Gamma\left(b_{SR}/2\right) h^{b_{SD}+b_{SR}-1}}{2(a_{SD}a_{SR})^{-1}\Gamma\left(\left(b_{SD}+b_{SR}\right)/2\right)} + \frac{\Gamma\left(b_{SD}/2\right)\Gamma\left(b_{RD}/2\right) h^{b_{SD}+b_{RD}-1}}{2^{b_{RD}+1}(a_{SD}a_{RD})^{-1}\Gamma\left(\left(b_{SD}+b_{RD}\right)/2\right)}. \quad (14)$$

Clearly, the above asymptotic PDF is dominated by $b_{SD} + \min(b_{SR}, b_{RD}) - 1$. It is noteworthy to mention that a_m and b_m depend on the relationship between φ^2 and β for any FSO link as can be observed in Eq. (2), corroborating that the diversity order does not depend on pointing errors when the condition $\varphi^2 > \beta$ holds. Now, substituting Eq. (14) into Eq. (7), we can evaluate the integral in Eq. (7) making use of the relationship $\mathrm{erfc}(x) = 2Q(\sqrt{2}x)$ [20, eqn. (6.287)]. Then, we compute the integral with the help of [20, eqn. (6.281)], obtaining an asymptotic closed-form solution for the BER corresponding to the proposed AF-LOS cooperative protocol as can be seen in Eq. (15) at the top of the next page. Interestingly, it is simple to prove that the average BER behaves asymptotically as $(G_c\gamma)^{-G_d}$, where G_c and G_d denote coding gain and diversity order, respectively [25]. At high SNR, the diversity order represents the slope of the BER versus SNR curve and the coding gain the shift of the curve in SNR. According to Eq. (15), the diversity order gain, $G_d^{\text{AF-LOS}}$, corresponding to the AF-LOS cooperative protocol in relation to the direct transmission is given by

$$G_d^{\text{AF-LOS}} = \frac{b_{SD} + \min(b_{SR}, b_{RD})}{b_{SD}} = 1 + \frac{\min(b_{SR}, b_{RD})}{b_{SD}}. \quad (16)$$

Note that the above diversity gain is always greater than 1 regardless of pointing error effects and atmospheric turbulence considered in this study.

At this point, it can be interesting to compare and contrast the BER performance derived here for cooperative strategies based on AF-LOS using MRC reception with a DH transmission based on AF relaying (hereinafter called AF-DH transmission), where LOS is not available. It should be noted that the asymptotic closed-form expression obtained for AF-DH transmission is not the contribution of this paper and is reproduced here for convenience in order to make a fair comparison between both cooperative strategies. A multi-hop transmission employing CSI-assisted AF relaying over GG fading channels with pointing errors was analyzed in [9], [10], where closed-form asymptotic expressions were not obtained to study the impact of this 3-way communications setup on the diversity order gain. Similar to the proposed AF-LOS cooperative protocol, the asymptotic closed-form expression for the PDF of h_T corresponding to the AF-DH transmission is directly related to h_{SRD} but taking into account that both S-R and R-D links transmit the same average optical power, i.e., P_t. Hence, the variable gain at relay node is redefined as $G = 1/h_{SR}$ for a AF-DH transmission. Therefore, we can readily obtain the asymptotic closed-form solution for the BER corresponding to the AF-DH transmission as follows

$$P_b^{\text{AF-DH}} \doteq \frac{a_{\text{min}}\Gamma\left(\left(1+b_{\text{min}}\right)/2\right)}{2b_{\text{min}}\sqrt{\pi}} \gamma_{\text{opt}}^{-b_{\text{min}}}, \quad (17)$$

where $b_{\text{min}} = \min(b_{SR}, b_{RD})$. As in Eq. (15), it can be deduced from Eq. (17) that the diversity order gain corresponding to the AF-DH transmission is given by

$$G_d^{\text{AF-DH}} = \min(b_{SR}, b_{RD})/b_{SD}. \quad (18)$$

Unlike Eq. (16), the above diversity order gain is not always greater than 1. It is noteworthy to mention that both the asymptotic closed-form expression in Eq. (17) and diversity order gain in Eq. (18) had not been derived in any earlier work [9], [10].

Finally, we can conclude that the asymptotic closed-form expressions obtained in this work allow us to carefully study the BER performance of practical FSO networks based on AF relaying and affected by atmospheric turbulence and pointing errors. These expressions are very useful and accurate to compute the BER performance due to the fact that a typical BER performance target is set to 10^{-6} for most practical FSO links as we will see in the section.

IV. Numerical Results

In this Section, we show some numerical results for the asymptotic BER performance corresponding to the proposed cooperative strategy under clear visibility conditions of 16 km with $C_n^2 = 1.7 \times 10^{-14}\ m^{-2/3}$. The parameters α and β are calculated from Eq. (3). Pointing errors are present here assuming a transmit divergence (θ_z) at $1/e^2$ of 1 mrad [26], and different jitter angles (θ_s), which can take values up to 0.4 mrad [27]. Note that the system configuration adopted in Table I is used in most practical terrestrial FSO links [26].

With the goal of illustrating the importance of the AF-LOS cooperative strategy on BER performance, the diversity order gain in Eq. (16) is plotted in Fig. 2(a) as a function

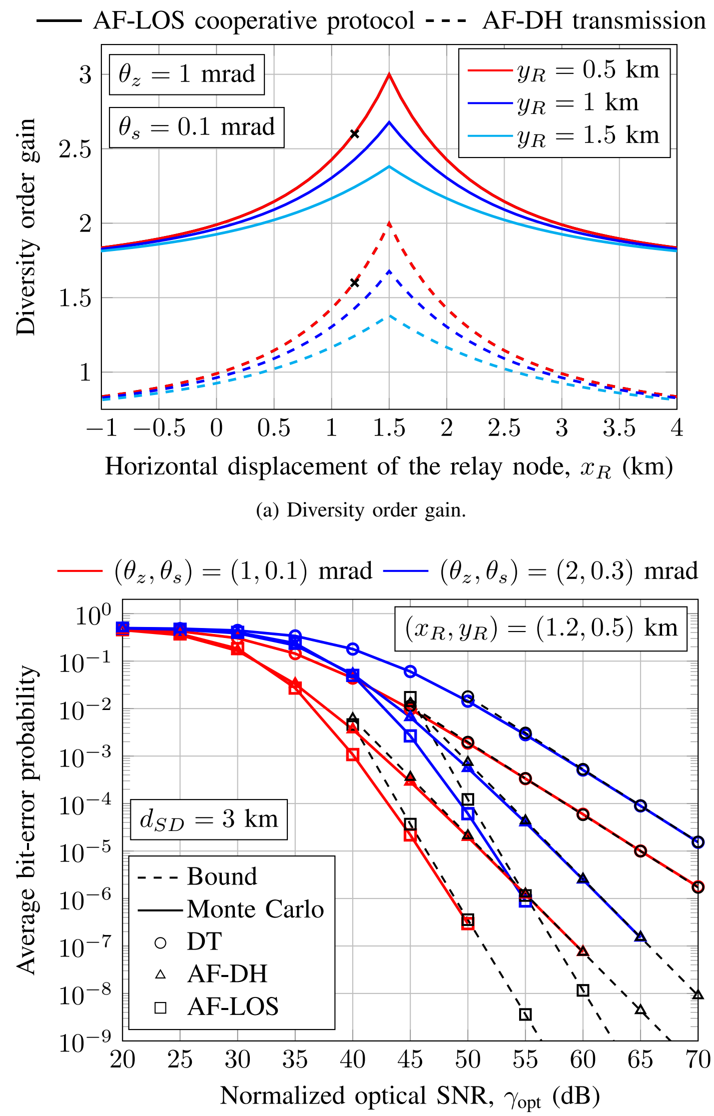
$$P_b^{\text{AF-LOS}} \doteq \frac{a_{SR}a_{SD}\Gamma\left(b_{SR}/2\right)\Gamma\left(b_{SD}/2\right)\Gamma\left((b_{SD}+b_{SR}+1)/2\right)}{2^{3-b_{SD}-b_{SR}}\Gamma\left((b_{SD}+b_{SR}+2)/2\right)\sqrt{\pi}}\gamma_{\text{opt}}^{-(b_{SD}+b_{SR})}, \qquad b_{SR}<b_{RD} \tag{15a}$$

$$P_b^{\text{AF-LOS}} \doteq \frac{a_{RD}a_{SD}\Gamma\left(b_{RD}2\right)\Gamma\left(b_{SD}/2\right)\Gamma\left((b_{SD}+b_{RD}+1)/2\right)}{2^{3-b_{SD}}\Gamma\left((b_{SD}+b_{RD}+2)/2\right)\sqrt{\pi}}\gamma_{\text{opt}}^{-(b_{SD}+b_{RD})}, \qquad b_{SR}>b_{RD} \tag{15b}$$

TABLE I: FSO System Settings

Parameter	Symbol	Value
S-D link distance	d_{SD}	3 km
Index of the refractive structure parameter	C_n^2	$1.7\times10^{-14}\ m^{-2/3}$
Visibility (clear sky)	V	16 km
Wavelength	λ	1550 nm
Responsivity	R	1 A/W
Receiver aperture diameter	$D=2a$	10 cm
Transmit divergence	θ_z	1 mrad
Normalized beam width	ω_z/a	$\simeq 20\cdot d_m$(km)
Jitter angle	θ_s	0.1-0.4 mrad
Normalized jitter	σ_s/a	$\simeq$ {2-8} $\cdot\, d_m$(km)

of the relay location (x_R, y_R) for a source-destination link distance of $d_{SD}=3$ km. The source node is located at origin of a Cartesian system. Note that the condition $\varphi_m^2>\beta_m$ is satisfied for each FSO link in Fig. 2(a) and, hence, these results are independent of pointing errors, i.e., atmospheric turbulence is the dominant effect when pointing error values of $(\theta_z,\theta_s)=(1,0.1)$ mrad are considered. It must be mentioned that most practical FSO systems operate under this desirable condition [26]. These curves are obtained from the intersection of two expressions: $(\beta_{SD}+\beta_{RD})/\beta_{SD}$ and $(\beta_{SD}+\beta_{SR})/\beta_{SD}$, as deduced from Eq. (16). At the same time, the diversity order gain corresponding to the AF-DH transmission obtained in Eq. (18) is also included in Fig. 2(a) to make a fair comparison and highlight the advantages of using LOS in cooperative FSO systems, as demonstrated in [6], [11], [14] using DF relaying. It can be observed that the maximum diversity order is obtained when $d_{SR}=d_{RD}=d_{SD}/2$. As expected, the available diversity order strongly depends on the relay placement, achieving a much higher diversity order gain than the AF-DH transmission as a consequence of using LOS. Additionally, these results are totally valid for any pair of (θ_z,θ_s) as long as the condition $\varphi_m^2>\beta_m$ holds for each link. The latter will be checked in Fig. 2(b). A notable improvement in BER performance is observed when comparing to the two-transmitter case (diversity order gain is always two) as well as an alternative to other cooperative protocols based on AF relaying [9], [10], presenting a diversity order gain quite superior to 2 or even 3 for some relay locations. It must also be mentioned that the diversity order gain corresponding to the AF-LOS cooperative protocol tends to a constant level equals $1+\beta_{SR}/\beta_{SD}$ on the right side, i.e. when $x_R>d_{SD}/2$, and $1+\beta_{RD}/\beta_{SD}$ on the left side, i.e. when $x_R<d_{SD}/2$. This constant level is greater in strong turbulence than moderate turbulence due to the fact that β tends to 1 faster as the strength of the atmospheric turbulence increases and, hence, the relations β_{SR}/β_{SD} and β_{RD}/β_{SD} are approximately equal to 1 in strong turbulence.



(a) Diversity order gain.

(b) BER performance.

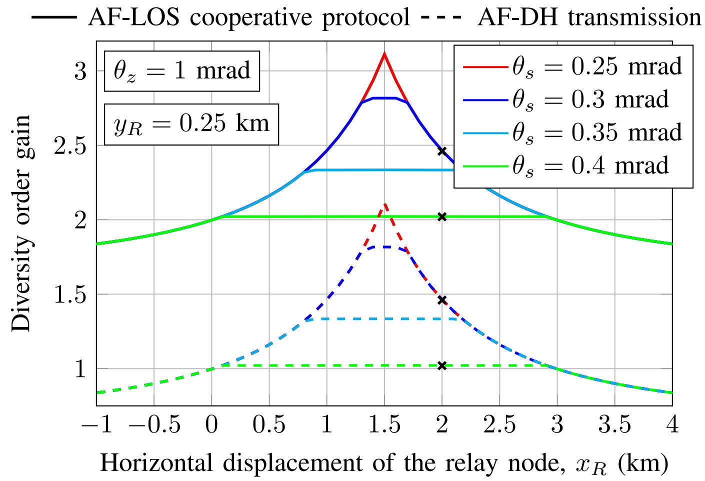
Fig. 2: (a) Diversity order gain, and (b) BER performance for a S-D link distance of $d_{SD}=3$ km and a relay location of $(x_R,y_R)=(1.2,0.5)$ km under different pointing error values.

The results corresponding to this asymptotic BER performance analysis are depicted in Fig. 2(b) as a function of the normalized optical SNR $\gamma_{\text{opt}}[dB]$. A source-destination link distance of $d_{SD} = 3$ km is considered for a relay location of $(x_R, y_R) = (1.2, 0.5)$ km together with pointing error values of $(\theta_z, \theta_s) = \{(1, 0.1), (2, 0.3)\}$ mrad. In order to confirm the accuracy and usefulness of the derived expression, Monte Carlo simulation results, where the FSO link is modeled using Eq. (1), are included by generating the corresponding random variables from the exact combined PDF and no approximations including turbulence conditions and pointing errors. Due to long time involved, Monte Carlo simulation results only up to 10^{-7} are considered in this analysis. As can be seen in Fig. 2(b), the asymptotic expression derived in Eq. (15) for the average BER corresponding to the AF-LOS cooperative protocol is in good agreement with these simulation results as well as the asymptotic expression derived in Eq. (17) for the AF-DH transmission. At the same time, we also consider the BER performance for DT in order to establish a benchmark, whose asymptotic BER performance was derived in [12, eqn. (8)] as

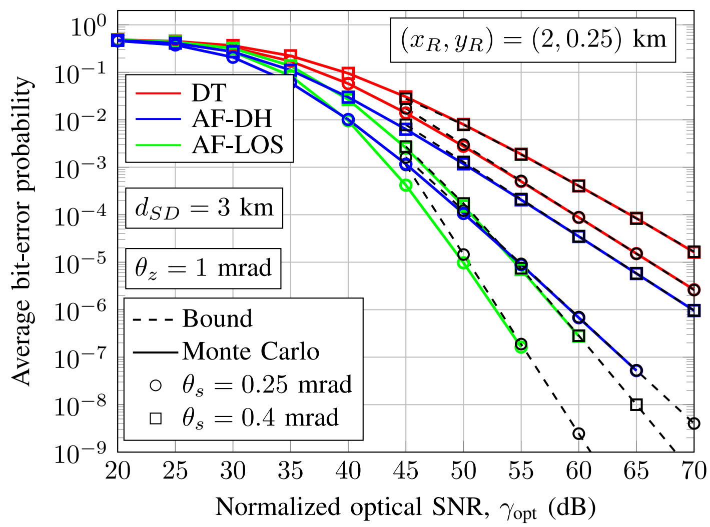
$$P_b^{\text{DT}} \doteq \frac{a_{SD}\Gamma\left((b_{SD}+1)/2\right)}{2b_{SD}\sqrt{\pi}}\gamma_{\text{opt}}^{-b_{SD}}. \tag{19}$$

Importantly, it can be proved that these BER results are also in good agreement with previous results shown in Fig. 2(a) in relation to the diversity order gain. It can be seen diversity gains of 2.6 and 1.6 (indicated by black '×') when the relay node is located at $(x_R, y_R) = (1.2, 0.5)$ km for AF-LOS cooperative protocol and AF-DH transmission, respectively. As commented before, these diversity gain values can be obtained for any pair of (θ_z, θ_s) as long as the condition $\varphi_m^2 > \beta_m$ holds for each FSO link, i.e., for $(\theta_z, \theta_s) = (1, 0.1)$ mrad and $(\theta_z, \theta_s) = (2, 0.3)$ mrad, where the only difference is that pointing error values of $(\theta_z, \theta_s) = (1, 0.1)$ mrad offer a coding gain of ≈12 dB in relation to $(\theta_z, \theta_s) = (2, 0.3)$ mrad in this case.

In order to observe how the diversity order gain derived in Eq. (16) corresponding to the AF-LOS cooperative protocol is affected by larger amounts of misalignment, the results are repeated in Fig. 3(a) over a variety of θ_s values for a source-destination link distance of $d_{SD} = 3$ km and a vertical displacement of $y_R = 0.25$ km. A transmit divergence value of $\theta_z = 1$ mrad together with jitter angle values of $\theta_s = \{0.25, 0.3, 0.35, 0.4\}$ mrad are considered. Unlike Fig. 2(a), the condition $\varphi_m^2 > \beta_m$ might not be always satisfied for each FSO link as jitter values increase. Actually, this condition is not satisfied when the relay node is located at a nearby placement between source node and destination node for jitter angle values of $\theta_s \geq 0.3$ (blue, cyan and green colors) in S-R and R-D links. The diversity order gain depends on pointing errors, i.e. $1 + \varphi_{SR}^2/\beta_{SD}$ or $1 + \varphi_{RD}^2/\beta_{SD}$, and remains constant when the condition $\varphi_m^2 > \beta_m$ does not hold. Notice that the range of x_R values for which the condition $\varphi_m^2 > \beta_m$ is not satisfied, increases as θ_s. Moreover, $b_{SD} = \beta_{SD}$ for all jitter values considered in Fig. 3(a) since a greater normalized beam width is obtained in S-D link and, hence, the relationship $\varphi_{SD}^2 > \beta_{SD}$ always holds. As expected, the BER performance is considerably



(a) Diversity order gain.



(b) BER performance.

Fig. 3: (a) Diversity order gain and (b) BER performance for a S-D link distance of $d_{SD} = 3$ km and a relay location of $(x_R, y_R) = (2, 0.25)$ km under larger amounts of misalignment.

decreased when pointing errors become dominant in relation to atmospheric turbulence. These comments are contrasted in Fig. 3(b) for BER performance when pointing error values of $(\theta_z, \theta_s) = \{(1, 0.25), (1, 0.4)\}$ mrad are assumed for a relay location of $(x_R, y_R) = (2, 0.25)$ km. It must be highlighted that the AF-LOS cooperative protocol is able to keep a much greater robustness than the AF-DH transmission for larger amounts of misalignment such as $(\theta_z, \theta_s) = (1, 0.4)$ mrad. As in previous figures (using black marks), it can be seen diversity gains of 2.46 and 2.02 for AF-LOS cooperative protocol as well as 1.46 and 1.02 for AF-DH transmission when pointing errors values of $(\theta_z, \theta_s) = (1, 0.25)$ mrad and

$(\theta_z, \theta_s) = (1, 0.4)$ mrad are assumed, respectively.

V. Conclusions

A research of a 3-way FSO communications setup with AF relaying using MRC reception is carried out over GG atmospheric turbulence channels with zero boresight pointing errors. A novel asymptotic closed-form solution for the BER is derived, which has been validated through Monte Carlo simulations with a very high precision.

We can conclude that a cooperative strategy based on AF relaying and making use of LOS is an interesting approach to mitigate the combined effect of atmospheric turbulence and pointing errors. This kind of relaying technique can be applied to extend the coverage area and achieve spatial diversity without investing in extra hardware, i.e., to achieve a much higher diversity order as well as a greater robustness against larger amounts of misalignment. Hence, LOS always helps to achieve a better BER performance regardless of atmospheric turbulence and pointing errors. These gains in BER performance occur despite the fact that the atmospheric turbulence and pointing errors are more severe due to the increased distances traversed compared to S-R and R-D links. A remarkable improvement is obtained not only compared to the equivalent AF-DH transmission but also to the non-cooperative case with two transmitters.

In light of the asymptotic closed-form expression obtained in this analysis, it is also concluded that the diversity order is strongly dependent on the relay placement. As expected and proved in [8] by using optimization methods, the optimal relay location is obtained when the relation $d_{SR} = d_{RD} = d_{SD}/2$ holds. This latter is drawn from a diversity order gain point of view, which was derived by taking full advantage of the asymptotic behavior of FSO systems without the need for using optimization methods. In this case, the diversity order is maximum for the AF-LOS cooperative protocol and the equivalent AF-DH transmission, being further atmospheric turbulence the dominant effect in relation to pointing errors. Once again, it is corroborated that practical FSO networks operating under this desirable condition provide not only a higher diversity order but also a higher coding gain.

From relevant results obtained in this paper, we believe that investigating the impact of adding more relays on the diversity order of AF relaying combined with techniques based on optical path selection using LOS can be interesting topics for future research.

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