Journal of Intelligent & Fuzzy Systems 35 (2018) 3213–3231
DOI:10.3233/JIFS-17166
IOS Press

Interval neutrosophic hesitant fuzzy choquet integral in multicriteria decision making

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Abstract. We define the *Interval Neutrosophic Hesitant Fuzzy Choquet Integral* (INHFCI) operator as a useful tool for multicriteria decision making (MCDM). The INHFCI operator generalizes both the interval neutrosophic hesitant fuzzy ordered weighted averaging operator and the interval neutrosophic hesitant fuzzy weighted averaging operator. A modified version of the score function to make comparison among Interval Neutrosophic Hesitant Fuzzy elements is proposed. We develop an approach for multicriteria decision making based on the interval neutrosophic hesitant fuzzy choquet integral operator that applies to our proposed score function. Finally the model is illustrated with the help of an example.

Keywords: Interval neutrosophic hesitant fuzzy set (INHFS), Interval neutrosophic hesitant fuzzy choquet integral (INHFCI), Multicriteria decision making (MCDM)

1. Introduction

In decision making process under multiple criteria, governing information are often incomplete, indeterminate and inconsistent. To deal with such imprecise information, fuzzy set [1] was introduced by Zadeh in 1965. A fuzzy set is characterized by a membership function which represents the degree of acceptance in a decision making problem. A fuzzy set, thus, converts the impreciseness or vagueness by attributing a degree to which a certain object belongs to a set. In real situation, however, there may be a hesitation or uncertainty about the membership degree of the object in that set. So, as its consequence, Atanassov [2, 3] introduced the intuitionistic fuzzy sets (IFSs) in 1983 that is characterized by the degrees of membership and non-membership with the condition that sum of these two degrees should not exceed 1. In the case of IFS, the non-membership grade expresses the degree of rejection in a decision making problem. Later, Atanassov and Gargov [4] introduced interval valued intuitionistic fuzzy set (IVIFS) as a further generalization of IFS in which intervals in [0, 1] are used for membership and non-membership values rather than exact numerical values. Although, IFSs and IVIFSs have the ability to handle incomplete information like acceptance and non-acceptance, the issue of indeterminate and inconsistent information remains in paucity. To overcome this, Smarandache [5] introduced neutrosophic sets (NSs). A neutrosophic set generalizes the concept of a fuzzy set [1], intuitionistic fuzzy set [2], interval valued intuitionistic fuzzy set [4], paraconsistent set [5], dialetheist set [5], paradoxist set [5], tautological set [5] to name a few. In the neutrosophic set, indeterminacy is quantified explicitly, and truth, indeterminacy, and falsity memberships are expressed independently. Wang et al. [11, 12] proposed the concepts of a single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS), which are the

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subclasses of a neutrosophic set. Ye [13] proposed a single-valued neutrosophic cross-entropy measure and used it to multicriteria decision making(MCDM) problems. It is observed that, in fuzzy multicriteria decision making (MCDM) problems, representation of membership degrees of objects to a certain set is not unique. To deal with such type of difficulty, Torra and Narukawa [14] and Torra [15] defined a hesitant fuzzy set (HFS) as an extension of the fuzzy set (FS). Chen et al. [16] extended this to include interval valued HFS (IVHFS) in which the membership degrees of an element to a given set are not exactly defined but denoted by several possible interval values. Ye [17] defined the single valued neutrosophic hesitant fuzzy sets (SVNHFS) by combining the single valued neutrosophic set with the hesitant fuzzy set and developed some weighted averaging and weighted geometric operators for SVNHFS. Further, Liu and Shi [22] proposed the concept of interval neutrosophic hesitant fuzzy sets (INHFSs) by combining the IVHFS and INS, and then developed interval neutrosophic hesitant fuzzy generalized weighted average (INHFGWA) operator, interval neutrosophic hesitant fuzzy generalized ordered weighted average (INHFGOWA) operator and an interval neutrosophic hesitant fuzzy generalized hybrid weighted average (INHFGHWA) operator. Most of these aggregation operators however specify to situations, where criteria and preferences of decision makers are independent of one another, therefore their combined effect is additive in nature. However, in real life decision-making problems, the criteria of the problems are often interdependent or interactive. Choquet integrals [23] have been used as an aggregation mechanism in various MCDM problems involving ordinary fuzzy sets in order to describe the relative importance of decision criteria and their interactions.

In the present study, we introduce the interval neutrosophic hesitant fuzzy choquet integral (INHFCI) operator as a tool for multicriteria decision making (MCDM), and discuss the relevant properties. It is shown that the interval neutrosophic hesitant fuzzy choquet integral (INHFCI) operator generalizes the interval neutrosophic hesitant fuzzy OWA operator, and the interval neutrosophic hesitant fuzzy weighted averaging operator. An approach for multicriteria decision making is also developed based on the interval neutrosophic hesitant fuzzy choquet integral operator.

The rest of the paper proceeds as follows. Section 2 deals with the preliminarily ideas of the model formulation, Section 3 describes an ordering approach for the INHFEs. Section 4 discusses the notion of interval neutrosophic hesitant fuzzy choquet operator. Section 5 discusses some of the properties of the interval neutrosophic hesitant fuzzy choquet operator. Section 6 proposes a multicriteria decision making approach based on the interval neutrosophic hesitant fuzzy choquet operator. Section 7 illustrates the model with an example followed by the concluding remarks in Section 8.

2. Preliminaries

We compile in this section the relevant notion required for the development of the present paper.

Definition 1. [8, 10] Let $\tilde{a} = [a^L, a^U] = \{x \mid 0 \leq a^L \leq x \leq a^U\}$, then $\tilde{a}$ is called a non-negative interval number. Especially, if $a^L = a^U$ then $\tilde{a}$ is a nonnegative real number. Consider two nonnegative interval numbers $\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$, then some of their basic operations are as follows:

(i) $\tilde{a} = \tilde{b} \Leftrightarrow a^L = b^L, a^U = b^U$
(ii) $\tilde{a} + \tilde{b} = [a^L + b^L, a^U + b^U]$
(iii) $k\tilde{a} = [ka^L, ka^U], k > 0$
(iv) $\tilde{a}^k = [(a^L)^k, (a^U)^k], k > 0$.

Definition 2. [10] Let $\tilde{a} = [a^L, a^U]$ and $\tilde{b} = [b^L, b^U]$, $l_{\tilde{a}} = \tilde{a}^U - \tilde{a}^L$ and $l_{\tilde{b}} = \tilde{b}^U - \tilde{b}^L$. Then the degree of possibility of $\tilde{a} \geqslant \tilde{b}$ is denoted by $p(\tilde{a} \geqslant \tilde{b})$ and is defined by

$$p(\tilde{a} \geqslant \tilde{b}) = \max\left\{1 - \max\left(\frac{\tilde{b}^U - \tilde{a}^L}{l_{\tilde{a}} + l_{\tilde{b}}}, 0\right), 0\right\}. \tag{2.1}$$

If $0 \leqslant p(\tilde{a} \geqslant \tilde{b}) < 0.5$, then $\tilde{a} < \tilde{b}$; if $p(\tilde{a} \geqslant \tilde{b}) = 0.5$, then $\tilde{a} = \tilde{b}$; if $0.5 < p(\tilde{a} \geqslant \tilde{b}) \leqslant 1$, then $\tilde{a} > \tilde{b}$.

Suppose that there are n interval numbers $\tilde{a}_i = [a_i^L, a_i^U]\,(i = 1, 2, \ldots, n)$, then each interval number $\tilde{a}_i$ is compared to all interval numbers $\tilde{a}_j\,(i = 1, 2, \ldots, n)$ by using Equation (2.1), as

$$\begin{aligned} p_{ij} &= p(\tilde{a}_i \geqslant \tilde{a}_j) \\ &= \max\left\{1 - \max\left(\frac{\tilde{a}_j^U - \tilde{a}_i^L}{l_{\tilde{a}_i} + l_{\tilde{a}_j}}, 0\right), 0\right\}. \end{aligned} \tag{2.2}$$

Then a complementary matrix can be constructed as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ & & \vdots & \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}, \tag{2.3}$$

where $p_{ij} \geqslant 0$, $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$.

A neutrosophic set [5] characterizes each logical statement in a three dimensional space, where each dimension represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration with not necessarily any connection between T, F and I. Suppose x is a generic element of a set X, then x belongs to the set in the following way: it is t true in the set, i indeterminate in the set, and f false, where t, i, and f are real numbers taken from the subsets T, I, and F of $\mathbb{R}$ with no restriction on T, I, F, nor on their sum $n = t + i + f$. But it is difficult to apply neutrosophic set in practical problems without specifying T, I and F. In the following we give a formal definition of a neutrosophic set taking the subsets T, I and F as the unit intervals $[0, 1]$.

Definition 3. A neutrosophic set A in X can be characterized by a truth-membership function $T_A : X \to [0, 1]$, an indeterminacy-membership function $I_A : X \to [0, 1]$, and a falsity-membership function $F_A : X \to [0, 1]$. Note that $0 \leq T_A(x), I_A(x), F_A(x) \leq 1$, and so, in this case, we have $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. This is known as a single valued neutrosophic set (SVNS) [11].

The complement of a neutrosophic set A, denoted by $c(A)$, is defined by $T_{c(A)}(x) = 1 - T_A(x)$, $I_{c(A)}(x) = 1 - I_A(x)$, and $F_{c(A)}(x) = 1 - F_A(x)$ for all x in X.

An interval neutrosophic set (INS) [12] gives value that ranges for the truth, indeterminacy and the falsity rather than single values for each of these quantities. Formally we have,

Definition 4. [12] Given a set X, an object of the form $A = \{\langle x, \tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)\rangle | x \in X\}$, where $\tilde{T}_A(x) \subset [0, 1]$, $\tilde{I}_A(x) \subset [0, 1]$ and $\tilde{F}_A(x) \subset [0, 1]$ are intervals is called an INS over X.

Definition 5. [12] An interval neutrosophic number (INN) $\tilde{a}$ is an INS given by the expression $\tilde{a} = \langle [T^L_{\tilde{a}}, T^U_{\tilde{a}}], [I^L_{\tilde{a}}, I^U_{\tilde{a}}], [F^L_{\tilde{a}}, F^U_{\tilde{a}}]\rangle$, where each component of $\tilde{a}$ is an interval number.

Following operations on INNs due to [12] are important for the development of our model. Let $\tilde{a}$ and $\tilde{b}$ be two INNs, and let $\lambda > 0$ be a real number, then

(i) $\tilde{a} \oplus \tilde{b} = \langle [T^L_{\tilde{a}} + T^L_{\tilde{b}} - T^L_{\tilde{a}} \cdot T^L_{\tilde{b}},$
$T^U_{\tilde{a}} + T^U_{\tilde{b}} - T^U_{\tilde{a}} \cdot T^U_{\tilde{b}}], [I^L_{\tilde{a}} \cdot I^L_{\tilde{b}}, I^U_{\tilde{a}} \cdot I^U_{\tilde{b}}],$
$[F^L_{\tilde{a}} \cdot F^L_{\tilde{b}}, F^U_{\tilde{a}} \cdot F^U_{\tilde{b}}]\rangle$

(ii) $\tilde{a} \otimes \tilde{b} = \langle [T^L_{\tilde{a}} \cdot T^L_{\tilde{b}}, T^U_{\tilde{a}} \cdot T^U_{\tilde{b}}],$
$[I^L_{\tilde{a}} + I^L_{\tilde{b}} - I^L_{\tilde{a}} \cdot I^L_{\tilde{b}}, I^U_{\tilde{a}} + I^U_{\tilde{b}} - I^U_{\tilde{a}} \cdot I^U_{\tilde{b}}],$
$[F^L_{\tilde{a}} + F^L_{\tilde{b}} - F^L_{\tilde{a}} \cdot F^L_{\tilde{b}}, F^U_{\tilde{a}} + F^U_{\tilde{b}} - F^U_{\tilde{a}} \cdot F^U_{\tilde{b}}]\rangle$

(iii) $\lambda\tilde{a} = \langle [1 - (1 - T^L_{\tilde{a}})^\lambda, 1 - (1 - T^U_{\tilde{a}})^\lambda],$
$[(I^L_{\tilde{a}})^\lambda, (I^U_{\tilde{a}})^\lambda], [(F^L_{\tilde{a}})^\lambda, (F^U_{\tilde{a}})^\lambda]\rangle$

(iv) $\tilde{a}^\lambda = \langle [(T^L_{\tilde{a}})^\lambda, (T^U_{\tilde{a}})^\lambda],$
$[1 - (1 - I^L_{\tilde{a}})^\lambda, 1 - (1 - I^U_{\tilde{a}})^\lambda],$
$[1 - (1 - F^L_{\tilde{a}})^\lambda, 1 - (1 - F^U_{\tilde{a}})^\lambda]\rangle.$

Definition 6. [14, 15] A hesitant fuzzy set (HFS) which is defined in terms of a function that returns a set of membership values for each element in the domain is defined as an object of the form $A = \{\langle x, h_A(x)\rangle | x \in X\}$, where $h_A(x)$ is a set of some distinct values in $[0, 1]$ representing the possible membership degrees of the element $x \in X$ to A. We call $h_A(x)$ a hesitant fuzzy element (HFE), denoted simply by h if no ambiguity arises on A.

For three hesitant fuzzy elements h, h_1 and h_2, Torra [15] defined three basic operations as follows.

(i) $h^c = \bigcup_{\gamma \in h} \{1 - \gamma\}$

(ii) $h_1 \bigcup h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\max\{\gamma_1, \gamma_2\}\}$

(iii) $h_1 \bigcap h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\min\{\gamma_1, \gamma_2\}\}.$

Also, Xia and Xu [28] defined four operations on the HFEs h, h_1, h_2 with a positive scale n as follows.

(i) $h^n = \bigcup_{\gamma \in h} \{\gamma^n\}$

(ii) $nh = \bigcup_{\gamma \in h} \{1 - (1 - \gamma)^n\}$

(iii) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$

(iv) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}.$

An interval-valued hesitant fuzzy set (IVHFS) [16, 29] on X is defined as: $E = \{\langle x, \tilde{h}_E(x)\rangle | x \in E\}$, where $\tilde{h}_E(x)$ is a set of some distinct interval values in [0, 1], that denote the possible membership degrees of the element $x \in X$ to the set E. We call $\tilde{h}_E(x)$ an interval valued hesitant fuzzy element (IVHFE). Let us simplify our notation by taking $\tilde{h}$ instead of $\tilde{h}_E$ so that $\tilde{h} = \{\tilde{\gamma} | \tilde{\gamma} \in \tilde{h}\}$, where $\tilde{\gamma} = [\gamma^L, \gamma^U]$ is an interval number.

For three IVHFEs $\tilde{h}, \tilde{h}_1, \tilde{h}_2$ and a positive scale n, Chen et al. [16] introduced the following operations.

(i) $\tilde{h}^n = \bigcup_{\tilde{\gamma}\in\tilde{h}} \{[(\gamma^L)^n, (\gamma^U)^n]\}$

(ii) $n\tilde{h} = \bigcup_{\tilde{\gamma}\in\tilde{h}} \{[1-(1-\gamma^L)^n, 1-(1-\gamma^U)^n]\}$

(iii) $\tilde{h}_1 \oplus \tilde{h}_2 = \bigcup_{\tilde{\gamma}_1\in\tilde{h}_1, \tilde{\gamma}_2\in\tilde{h}_2} \{[\gamma_1^L + \gamma_2^L - \gamma_1^L\gamma_2^L, \gamma_1^U + \gamma_2^U - \gamma_1^U\gamma_2^U]\}$

(iv) $\tilde{h}_1 \otimes \tilde{h}_2 = \bigcup_{\tilde{\gamma}_1\in\tilde{h}_1, \tilde{\gamma}_2\in\tilde{h}_2} \{[\gamma_1^L\gamma_2^L, \gamma_1^U\gamma_2^U]\}$.

Definition 7. [22] An interval neutrosophic hesitant fuzzy set (INHFS) on X is given by the object $N = \{\langle x, \tilde{t}(x), \tilde{i}(x), \tilde{f}(x)\rangle | x \in X\}$, where $\tilde{t}(x) = \{\tilde{\gamma}|\tilde{\gamma} \in \tilde{t}(x)\}, \tilde{i}(x) = \{\tilde{\delta}|\tilde{\delta} \in \tilde{i}(x)\}$ and $\tilde{f}(x) = \{\tilde{\eta}|\tilde{\eta} \in \tilde{f}(x)\}$ are three sets of some interval values in the real unit interval [0, 1], that denote the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees and the falsity-membership hesitant degrees of the element $x \in X$ to the set N, respectively. The expression $\tilde{n} = \{\tilde{t}(x), \tilde{i}(x), \tilde{f}(x)\}$ is called an interval neutrosophic hesitant fuzzy element (INHFE).

Definition 8. [22] Let $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$ and $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ be two INHFEs in a non-empty finite set X and $k > 0$ is a positive scale; then we have the following operations:

(i) $\tilde{n}_1 \cup \tilde{n}_2 = \{\tilde{t}_1 \cup \tilde{t}_2, \tilde{i}_1 \cap \tilde{i}_2, \tilde{f}_1 \cap \tilde{f}_2\}$

(ii) $\tilde{n}_1 \cap \tilde{n}_2 = \{\tilde{t}_1 \cap \tilde{t}_2, \tilde{i}_1 \cup \tilde{i}_2, \tilde{f}_1 \cup \tilde{f}_2\}$

(iii) $\tilde{n}_1 \oplus \tilde{n}_2 = \{\tilde{t}_1 \oplus \tilde{t}_2, \tilde{i}_1 \otimes \tilde{i}_2, \tilde{f}_1 \otimes \tilde{f}_2\}$

$$= \bigcup_{\substack{\tilde{\gamma}_1\in\tilde{t}_1,\tilde{\gamma}_2\in\tilde{t}_2,\\ \tilde{\delta}_1\in\tilde{i}_1,\tilde{\delta}_2\in\tilde{i}_2,\\ \tilde{\eta}_1\in\tilde{f}_1,\tilde{\eta}_2\in\tilde{f}_2}} \{[\gamma_1^L + \gamma_2^L - \gamma_1^L\gamma_2^L, \gamma_1^U + \gamma_2^U - \gamma_1^U\gamma_2^U], [\delta_1^L\delta_2^L, \delta_1^U\delta_2^U], [\eta_1^L\eta_2^L, \eta_1^U\eta_2^U]\}$$

(iv) $\tilde{n}_1 \otimes \tilde{n}_2 = \{\tilde{t}_1 \otimes \tilde{t}_2, \tilde{i}_1 \oplus \tilde{i}_2, \tilde{f}_1 \oplus \tilde{f}_2\}$

$$= \bigcup_{\substack{\tilde{\gamma}_1\in\tilde{t}_1,\tilde{\gamma}_2\in\tilde{t}_2,\\ \tilde{\delta}_1\in\tilde{i}_1,\tilde{\delta}_2\in\tilde{i}_2,\\ \tilde{\eta}_1\in\tilde{f}_1,\tilde{\eta}_2\in\tilde{f}_2}} \{[\gamma_1^L\gamma_2^L, \gamma_1^U\gamma_2^U], [\delta_1^L + \delta_2^L - \delta_1^L\delta_2^L, \delta_1^U + \delta_2^U - \delta_1^U\delta_2^U], [\eta_1^L + \eta_2^L - \eta_1^L\eta_2^L, \eta_1^U + \eta_2^U - \eta_1^U\eta_2^U]\}$$

(v) $$k\tilde{n}_1 = \bigcup_{\substack{\tilde{\gamma}_1\in\tilde{t}_1,\\ \tilde{\delta}_1\in\tilde{i}_1,\\ \tilde{\eta}_1\in\tilde{f}_1}} \{[1-(1-\gamma_1^L)^k, 1-(1-\gamma_1^U)^k], [(\delta_1^L)^k, (\delta_1^U)^k], [(\eta_1^L)^k, (\eta_1^U)^k]\}$$

(vi) $$\tilde{n}_1^k = \bigcup_{\substack{\tilde{\gamma}_1\in\tilde{t}_1,\\ \tilde{\delta}_1\in\tilde{i}_1,\\ \tilde{\eta}_1\in\tilde{f}_1}} \{[(\gamma_1^L)^k, (\gamma_1^U)^k], [1-(1-\delta_1^L)^k, 1-(1-\delta_1^U)^k], [1-(1-\eta_1^L)^k, 1-(1-\eta_1^U)^k]\}.$$

In order to develop an MCDM using interval neutrosophic hesitant fuzzy elements, the score function for making their comparisons is defined in [22] as follows.

Definition 9. [22] The score function of an interval neutrosophic hesitant fuzzy element (INHFE) $\tilde{n}$ is given as:

$$\mathcal{S}(\tilde{n}) = \left[\frac{1}{l}\sum_{i=1}^{l}\tilde{\gamma}_i + \left(\frac{\sum_{i=1}^{p}(1-\tilde{\delta}_i)}{p}\right) + \left(\frac{\sum_{i=1}^{q}(1-\tilde{\eta}_i)}{q}\right)\right] \Big/ 3, \tag{2.4}$$

where l, p, q are the numbers of the interval values in $\tilde{\gamma}, \tilde{\delta}, \tilde{\eta}$, respectively. It has been observed that $\mathcal{S}(\tilde{n})$ is an interval value included in [0, 1], and so for two INHFEs $\tilde{n}_1, \tilde{n}_2$, the score functions $\mathcal{S}(\tilde{n}_1)$ and $\mathcal{S}(\tilde{n}_2)$ are comparable using the degrees of possibility. If $\mathcal{S}(\tilde{n}_1) \geq \mathcal{S}(\tilde{n}_2)$ then $\tilde{n}_1 \succcurlyeq \tilde{n}_2$, i.e. $\tilde{n}_1$ is superior than or equal to $\tilde{n}_2$.

Liu and Shi [22] further improved the score function using a simple average method as follows.

$$
\begin{aligned}
\mathcal{S}(\tilde{n}_j) \\
&= \frac{1}{3}\left[\frac{1}{l_j}\sum_{j=1}^{l_j}\left(\frac{\gamma_j^L+\gamma_j^U}{2}\right) + \frac{\sum_{j=1}^{p_j}\left(1-\frac{\delta_j^L+\delta_j^U}{2}\right)}{p_j} + \frac{\sum_{j=1}^{q_j}\left(1-\frac{\eta_j^L+\eta_j^U}{2}\right)}{q_j}\right] \\
&= \frac{1}{6}\left[\frac{1}{l_j}\sum_{j=1}^{l_j}(\gamma_j^L+\gamma_j^U) + \frac{1}{p_j}\sum_{j=1}^{p_j}\big((1-\delta_j^L)+(1-\delta_j^U)\big) + \frac{1}{q_j}\sum_{j=1}^{q_j}\big((1-\eta_j^L)+(1-\eta_j^U)\big)\right] \\
&= \frac{1}{6}\left[\frac{1}{l_j}\sum_{j=1}^{l_j}(\gamma_j^L+\gamma_j^U) + \frac{1}{p_j}\sum_{j=1}^{p_j}\big(2-(\delta_j^L+\delta_j^U)\big) + \frac{1}{q_j}\sum_{j=1}^{q_j}\big(2-(\eta_j^L+\eta_j^U)\big)\right], \qquad (2.5)
\end{aligned}
$$

where l_j, p_j, q_j are the numbers of the interval values in $\tilde{\gamma}_j, \tilde{\delta}_j, \tilde{\eta}_j$.

Remark 1. Note that there are instances where even the extended score function given by Equation (2.5) also fails to differentiate interval neutrosophic fuzzy elements (INHFEs). Take for example, $\tilde{n}_1 = \{[0.4, 0.5], [0.2, 0.3], [0.3, 0.4]\}$ and $\tilde{n}_2 = \{[0.4, 0.7], [0.1, 0.4], [0.4, 0.5]\}$.

Their scores using Equation (2.5) are found to be equal i.e., $\mathcal{S}(\tilde{n}_1) = \mathcal{S}(\tilde{n}_2) = 0.6167$. But clearly $\tilde{n}_1$ and $\tilde{n}_2$ are not equal. Therefore it is necessary to further improve the ranking procedure of INHFEs. In Section 3 an improved ranking method for INHFEs is proposed to address this issue. This is applied in the further development of our proposed model.

Following weighted aggregation operators defined on interval neutrosophic hesitant fuzzy elements are deemed important to draw comparison with our proposed model.

Definition 10. [22] Let $\lambda > 0$ and let $\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\}\,(j = 1, 2, \ldots, n)$ be a collection of interval neutrosophic hesitant fuzzy numbers. Let the weight vector $w = (w_1, w_2, \ldots, w_n)^T$ be such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$; then an interval neutrosophic hesitant fuzzy generalized weighted average (INHFGWA) operator of dimension n is a mapping INHFGWA : $\Omega^n \longrightarrow \Omega$ given by

$$
\mathrm{INHFGWA}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \left(\sum_{j=1}^{n} w_j \tilde{n}_j^{\lambda}\right)^{1/\lambda}, \qquad (2.6)
$$

where Ω is the set of all the interval neutrosophic hesitant fuzzy numbers.

Definition 11. [22] Let $\lambda > 0$ and let $\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\}\,(j = 1, 2, \ldots, n)$ be a collection of interval neutrosophic hesitant fuzzy elements (INHFEs). Let $w = (w_1, w_2, \ldots, w_n)^T$ be a weight vector such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$; then an interval neutrosophic hesitant fuzzy generalized order weighted average (INHFGOWA) operator of dimension n is a mapping INHFGOWA : $\Omega^n \to \Omega$ and

$$
\mathrm{INHFGOWA}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \left(\sum_{j=1}^{n} w_j \tilde{n}_{(j)}^{\lambda}\right)^{1/\lambda}, \qquad (2.7)
$$

where Ω is the set of all the INHFEs and $\{(1), (2), \ldots, (n)\}$ a permutation of $\{1, 2, \ldots, n\}$ such that $\tilde{n}_{(j)} \succeq \tilde{n}_{(j-1)}$ for all $j = 1, 2, \ldots, n$.

Definition 12. [23, 24, 30] Let $X = \{x_1, x_2, \ldots, x_n\}$ be a non-empty set, let $P(X)$ be the power set of X. A fuzzy measure on X is a set function $m : P(X) \longrightarrow [0, 1]$, satisfying the following conditions:

(i) $m(\emptyset) = 0, m(X) = 1$ (boundary conditions)
(ii) if $A \subseteq B \subseteq X$, then $m(A) \leq m(B)$ (monotonicity).

A fuzzy measure is additive if for any two disjoints subsets $A, B \subseteq X$, we have $m(A \cup B) = m(A) + m(B)$. Further studies on fuzzy measures and their properties can be found in [32]. A fuzzy measure is symmetric if for any subsets $A, B \subseteq X$ with $|A| = |B|$ implies $m(A) = m(B)$. If $E = \{x_i\}$, then $m(E) = w_i$ denotes the subjective weight of criterion x_i in the set of criteria X. Thus a fuzzy measure represents the weight of each criterion as well as combination of criteria in which all of the $w_i (i = 1, 2, \ldots, n)$'s are not necessarily equal to one. Therefore, in order to determine fuzzy measures on $X = \{x_1, x_2, \ldots, x_n\}$, we need to find $2^n - 2$ values for n criteria, except the values $m(\emptyset)$ and $m(X)$ which are always equal to 0 and 1, respectively. So the evaluation model obtained becomes quite difficult. To avoid this difficulty, Sugeno [24] proposed a special kind of fuzzy measure called λ-fuzzy measure which is defined as follows:

Definition 13. [24, 30] Let $X = \{x_1, x_2, \ldots, x_n\}$ be a non-empty set, let $P(X)$ be the power set of X. Given a real number $\lambda > -1$, a λ-fuzzy measure m on X is a function $m : P(X) \longrightarrow [0, 1]$, satisfying the followings

(i) $m(\emptyset) = 0, m(X) = 1$
(ii) if $A \subseteq B \subseteq X$, then $m(A) \leq m(B)$
(iii) $m(A \cup B) = m(A) + m(B) + \lambda m(A)m(B),\ \lambda \in [-1, +\infty)\, \forall A, B \in P(X)$, $A \cap B = \emptyset, \lambda > -1$.

Note that if X represents a set of criteria, the parameter λ determines an interaction level between the criteria. If $\lambda = 0$, then the fuzzy measure reduces to an additive measure, and A and B have no interaction between them. For negative and positive λ, a λ-fuzzy measure reduces to the sub-additive ($m(A \cup B) \leq m(A) + m(B)$) and the super-additive measure ($m(A \cup B) \geq m(A) + m(B)$), respectively.

Let $X = \bigcup_{i=1}^{n} \{x_i\}$ be a finite set. To determine a normalized measure on X, Sugeno [24] provides the following expression.

$$m(X) = \begin{cases} \frac{1}{\lambda}\left(\prod_{i=1}^{n}[1 + \lambda m(x_i)] - 1\right), & \text{if } \lambda \neq 0 \\ \sum_{i=1}^{n} m(x_i), & \text{if } \lambda = 0. \end{cases} \tag{2.8}$$

Also, for every subset $A \subseteq X$, we have

$$m(A) = \begin{cases} \frac{1}{\lambda}\left(\prod_{x_i \in A}[1 + \lambda m(x_i)] - 1\right), & \text{if } \lambda \neq 0 \\ \sum_{x_i \in A} m(x_i), & \text{if } \lambda = 0. \end{cases} \tag{2.9}$$

Using equation Equation (2.8) the parameter λ can be uniquely determined from the boundary condition $m(X) = 1$ which is equivalent to solving the following equation

$$\lambda + 1 = \prod_{i=1}^{n}(1 + \lambda m(x_i)). \tag{2.10}$$

If there is no ambiguity in the parameter λ, we call a λ-fuzzy measure simply a fuzzy measure.

Definition 14. [25] Let f be a positive real-valued function on $X = \{x_1, x_2, \ldots, x_n\}$, and let m be a fuzzy measure on X. The discrete choquet integral of f with respective to m, denoted by $C_m(f)$ is defined as follows.

$$C_m(f) = \sum_{i=1}^{n} f(x_{(i)})\Big(m(A_{(i)}) - m(A_{(i-1)})\Big), \tag{2.11}$$

where $(\cdot)$ indicates a permutation on X such that $f(x_{(1)}) \geq f(x_{(2)}) \geq \cdots \geq f(x_{(n)})$, and $A_{(i)} = \{x_{(1)}, x_{(2)}, \ldots, x_{(i)}\}$ for $i \geq 1$ and $A_{(0)} = \emptyset$.

In what follows next we define a modified score function to compare INHFEs that applies to our proposed MCDM procedure using INHFCI.

3. An ordering between INHFEs

Following the ambiguities of comparing two INHFEs by using the score functions given by Equations (2.4) and (2.5) we propose here an alternative ordering for INHFEs. This ordering approach is based on the possibility degree ranking (PDR) for the interval numbers [9, 10], the interval neutrosophic numbers (INNs) [18] and the interval valued hesitant fuzzy elements (IVHFEs) [19, 20].

Definition 15. Let

$$\tilde{n}_i = \bigcup_{\substack{[\gamma_i^L, \gamma_i^U] \in \tilde{t}_i, \\ [\delta_i^L, \delta_i^U] \in \tilde{i}_i, \\ [\eta_i^L, \eta_i^U] \in \tilde{f}_i}} \left\{[\gamma_i^L, \gamma_i^U], [\delta_i^L, \delta_i^U], [\eta_i^L, \eta_i^U]\right\}$$

and

$$\tilde{n}_j = \bigcup_{\substack{[\gamma_j^L, \gamma_j^U] \in \tilde{t}_j, \\ [\delta_j^L, \delta_j^U] \in \tilde{i}_j, \\ [\eta_j^L, \eta_j^U] \in \tilde{f}_j}} \left\{[\gamma_j^L, \gamma_j^U], [\delta_j^L, \delta_j^U], [\eta_j^L, \eta_j^U]\right\}$$

be two INHFEs. Then the possibility degree of $\tilde{n}_i \succcurlyeq \tilde{n}_j$ is defined as follows:

$$P(\tilde{n}_i \succcurlyeq \tilde{n}_j) = \frac{1}{3}\left[\begin{array}{l} \frac{1}{l_i l_j}\sum_{i=1}^{l_i}\sum_{j=1}^{l_j} P([\gamma_i^L, \gamma_i^U] \succcurlyeq [\gamma_j^L, \gamma_j^U]) \\ + \frac{1}{p_i p_j}\sum_{i=1}^{p_i}\sum_{j=1}^{p_j} P([\delta_j^L, \delta_j^U] \succcurlyeq [\delta_i^L, \delta_i^U]) \\ + \frac{1}{q_i q_j}\sum_{i=1}^{q_i}\sum_{j=1}^{q_j} P([\eta_j^L, \eta_j^U] \succcurlyeq [\eta_i^L, \eta_i^U]) \end{array}\right] \tag{3.1}$$

where l_i, p_i, q_i are the numbers of the interval values in $\tilde{\gamma}_i$, $\tilde{\delta}_i$, $\tilde{\eta}_i$, and l_j, p_j, q_j are the numbers of the interval values in $\tilde{\gamma}_j$, $\tilde{\delta}_j$, $\tilde{\eta}_j$. Now, using Equation (2.2), we can observe that $0 \leq P(\tilde{n}_i \succcurlyeq \tilde{n}_j) \leq 1$, $P(\tilde{n}_i = \tilde{n}_j) = 0.5$, and $P(\tilde{n}_i \succcurlyeq \tilde{n}_j) + P(\tilde{n}_j \succcurlyeq \tilde{n}_i) = 1$.

Also,

$$P([\gamma_i^L, \gamma_i^U] \succcurlyeq [\gamma_j^L, \gamma_j^U]) = \max\left\{1 - \max\left(\frac{\gamma_j^U - \gamma_i^L}{\gamma_i^U - \gamma_i^L + \gamma_j^U - \gamma_j^L}, 0\right), 0\right\} \tag{3.2}$$

$$P([\delta_j^L, \delta_j^U] \succcurlyeq [\delta_i^L, \delta_i^U]) = \max\left\{1 - \max\left(\frac{\delta_i^U - \delta_j^L}{\delta_i^U - \delta_i^L + \delta_j^U - \delta_j^L}, 0\right), 0\right\} \tag{3.3}$$

and,

$$P([\eta_j^L, \eta_j^U] \succcurlyeq [\eta_i^L, \eta_i^U]) = \max\left\{1 - \max\left(\frac{\eta_i^U - \eta_j^L}{\eta_i^U - \eta_i^L + \eta_j^U - \eta_j^L}, 0\right), 0\right\}. \tag{3.4}$$

Suppose there are n INHFEs

$$\tilde{n}_i = \bigcup_{\substack{[\gamma_i^L, \gamma_i^U] \in \tilde{\gamma}_i, \\ [\delta_i^L, \delta_i^U] \in \tilde{\delta}_i, \\ [\eta_i^L, \eta_i^U] \in \tilde{\eta}_i}} \left\{[\gamma_i^L, \gamma_i^U], [\delta_i^L, \delta_i^U], [\eta_i^L, \eta_i^U]\right\},$$

for $i = 1, 2, \ldots, n$. Then the possibility degree $P_{ij} = P(\tilde{n}_i \succcurlyeq \tilde{n}_j)$ of each pair of INHFE $\tilde{n}_i$, $(i = 1, 2, \ldots, n)$ is given by Equation (3.1); thus the matrix of possibility degrees $P = (P_{ij})_{n \times n}$ can be constructed, where $P_{ij} \geq 0$, $P_{ij} + P_{ji} = 1$, and $P_{ii} = 0.5$. Also, the ranks r_i of the $\tilde{n}_i\,(i = 1, 2, \ldots, n)$ is given in the line of [6, 7] as follows

$$r_i = \frac{\left(\sum_{j=1}^{n} P_{ij} + \frac{n}{2} - 1\right)}{n(n-1)}. \tag{3.5}$$

Using Equation (3.5), the INHFEs $\tilde{n}_i\,(i = 1, 2, \ldots, n)$ can be ranked in a descending order in accordance with the values of $r_i\,(i = 1, 2, \ldots, n)$. We call the ranking : a Possibility Degree Ranking (PDR). It is evident from the above formulations that the PDR generalizes the ordering given by the score function over the class of INNs and IVHFEs. However it is worth mentioning here that the ordering among INHFEs in terms of the r_i's is only a partial ordering and therefore this ranking is also not unique.

Example 1. As already mentioned in the previous section, for the INHFEs, $\tilde{n}_1 = \{[0.4, 0.5], [0.2, 0.3], [0.3, 0.4]\}$ and $\tilde{n}_2 = \{[0.4, 0.7], [0.1, 0.4], [0.4, 0.5]\}$, their scores are equal to $\mathcal{S}(n_1) = \mathcal{S}(n_2) = 0.6167$, although $\tilde{n}_1$ and $\tilde{n}_2$ are two distinct INHFEs. Now using Equation (3.1) the PDR of $\tilde{n}_1$ and $\tilde{n}_2$ is calculated as follows.

$$P_{12} = P(\tilde{n}_1 \succcurlyeq \tilde{n}_2) = 0.5833$$

Since, $P_{12} + P_{21} = 1$, therefore $P_{21} = 1 - 0.5833 = 0.4167$. Also, $P_{11} = P_{22} = 0.5000$. Then, the matrix of possibility degrees for $\tilde{n}_1$ and $\tilde{n}_2$ can be constructed as follows.

$$P = \begin{bmatrix} 0.5000 & 0.5833 \\ 0.4167 & 0.5000 \end{bmatrix}$$

Now, the PDR values are given by, $r_1 = 0.5417$, $r_2 = 0.4584$. Since $r_1 \geq r_2$, therefore, $\tilde{n}_1 \succcurlyeq \tilde{n}_2$. Thus it appears that the PDR is more efficient for ranking INHFEs.

Remark 2. The main advantage of adopting this possibility degree ranking (PDR) approach is that the possibility degrees between any two adjacent INHFEs can be also be obtained from the possibility degree matrix in Definition 15. Also as explained in Example 1, in some typical situations it seems that ranking orders ($\succcurlyeq$) of INHFEs with PDR approach is more effective than the other ranking approach like score function. Moreover the possibility degree $P(\tilde{n}_1 \succcurlyeq \tilde{n}_2)$ of any two INHFEs $\tilde{n}_1, \tilde{n}_2$ can be interpreted from the probability point of view for ranking INHFEs. As for instance if $P(\tilde{n}_1 \succcurlyeq \tilde{n}_2) \approx 1$ then there is the more possibility that $\tilde{n}_1 \succcurlyeq \tilde{n}_2$, whereas no such interpretation can be seen in the ranking approach like score function.

4. The interval neutrosophic hesitant fuzzy choquet integral (INHFCI) operator

In this section, we propose the interval neutrosophic hesitant fuzzy choquet integral (INHFCI) operator on the interval hesitant fuzzy elements (INHFEs) which are ranked by the PDR of INHFEs given by Definition 15. We also discuss some of the properties of INHFCI operator.

Definition 16. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of criteria and let m be a fuzzy measure on X. Let,

$$\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\}\ (j = 1, 2, \ldots, n)$$
$$= \bigcup_{\substack{[\gamma_j^L, \gamma_j^U] \in \tilde{t}_j,\\ [\delta_j^L, \delta_j^U] \in \tilde{i}_j,\\ [\eta_j^L, \eta_j^U] \in \tilde{f}_j}} \left\{[\gamma_j^L, \gamma_j^U], [\delta_j^L, \delta_j^U], [\eta_j^L, \eta_j^U]\right\}$$

be a collection of INHFEs, where $(\cdot)$ is a permutation such that $r_{(1)} \geq r_{(2)} \geq \cdots \geq r_{(n)}$, and

$$A_{(j)^+} = \left\{x_{(i)} \middle| r_{(i)} \geq r_{(j)}\right\},$$
$$A_{(j)^-} = \left\{x_{(i)} \middle| r_{(i)} > r_{(j)}\right\}.$$

Denote by $card(A)$ the cardinality of set A. Then the INHFCI operator is defined as:

$$\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \bigoplus_{j=1}^{n} \left(\frac{m(A_{(j)^+}) - m(A_{(j)^-})}{card(A_{(j)^+}) - card(A_{(j)^-})}\right) \tilde{n}_{(j)} \quad (4.1)$$

Note that $\oplus$ denotes here the sum operation on INHFEs as defined in the rule (iii) of Definition 8, and that the Equation (4.1) does not depend on the considered permutaion $(\cdot)$. Moreover, the ranking of $\tilde{n}_j\ (j = 1, 2, \ldots, n)$ is done with respect to the PDR of INHFEs. If there are no ties between values $r_{(1)}, .., r_{(n)}$, then Equation (4.1) can be rewritten as

$$\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \bigoplus_{j=1}^{n} \left(m(A_{(j)}) - m(A_{(j-1)})\right) \tilde{n}_{(j)}, \quad (4.2)$$

where $A_{(j)} = \{(1), .., (j)\}$, with the convention $A_{(0)} = \emptyset$. Following theorem is an immediate consequence.

Theorem 1. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of criteria and let m be a fuzzy measure on X. Let,*

$$\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\}\ (j = 1, 2, \ldots, n)$$
$$= \bigcup_{\substack{[\gamma_j^L, \gamma_j^U] \in \tilde{t}_j,\\ [\delta_j^L, \delta_j^U] \in \tilde{i}_j,\\ [\eta_j^L, \eta_j^U] \in \tilde{f}_j}} \left\{[\gamma_j^L, \gamma_j^U], [\delta_j^L, \delta_j^U], [\eta_j^L, \eta_j^U]\right\}$$

be a collection of INHFEs. *Then the value aggregated by the* INHFCI *is also an* INHFE, *moreover,*

$$\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \bigcup_{\substack{[\gamma_j^L, \gamma_j^U] \in \tilde{t}_j,\\ [\delta_j^L, \delta_j^U] \in \tilde{i}_j,\\ [\eta_j^L, \eta_j^U] \in \tilde{t}_j}} \left\{\left[1 - \prod_{j=1}^{n}(1 - \gamma_{(j)}^L)^{\lambda_j}, 1 - \prod_{j=1}^{n}(1 - \gamma_{(j)}^U)^{\lambda_j}\right], \left[\prod_{j=1}^{n}(\delta_{(j)}^L)^{\lambda_j}, \prod_{j=1}^{n}(\delta_{(j)}^U)^{\lambda_j}\right], \left[\prod_{j=1}^{n}(\eta_{(j)}^L)^{\lambda_j}, \prod_{j=1}^{n}(\eta_{(j)}^U)^{\lambda_j}\right]\right\} \quad (4.3)$$

where,

$$\lambda_j = \left(\frac{m(A_{(j)^+}) - m(A_{(j)^-})}{card(A_{(j)^+}) - card(A_{(j)^-})}\right).$$

Proof. We use mathematical induction to prove this theorem. For $n = 1$ from Equation (4.1),

$$\text{INHFCI}(\tilde{n}_1) = \lambda_1 \tilde{n}_{(1)} = \bigcup_{\substack{[\gamma_1^L, \gamma_1^U] \in \tilde{t}_1,\\ [\delta_1^L, \delta_1^U] \in \tilde{i}_1,\\ [\eta_1^L, \eta_1^U] \in \tilde{f}_1}} \left\{\left[1 - (1 - \gamma_{(1)}^L)^{\lambda_1}, 1 - (1 - \gamma_{(1)}^U)^{\lambda_1}\right], \left[(\delta_{(1)}^L)^{\lambda_1}, (\delta_{(1)}^U)^{\lambda_1}\right], \left[(\eta_{(1)}^L)^{\lambda_1}, (\eta_{(1)}^U)^{\lambda_1}\right]\right\}.$$

Therefore the result holds for $n = 1$. Next suppose that the result hold for $n = k$, i.e.,

$$\begin{aligned}
&\mathrm{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_k) \\
&= \bigcup_{\substack{[\gamma_j^L,\gamma_j^U]\in\tilde{t}_j,\\ [\delta_j^L,\delta_j^U]\in\tilde{i}_j,\\ [\eta_j^L,\eta_j^U]\in\tilde{t}_j}} \left\{ \left[1-\prod_{j=1}^{k}(1-\gamma_{(j)}^L)^{\lambda_j},\, 1-\prod_{j=1}^{k}(1-\gamma_{(j)}^U)^{\lambda_j} \right], \right. \\
&\quad \left. \left[\prod_{j=1}^{k}(\delta_{(j)}^L)^{\lambda_j},\, \prod_{j=1}^{k}(\delta_{(j)}^U)^{\lambda_j} \right], \left[\prod_{j=1}^{k}(\eta_{(j)}^L)^{\lambda_j},\, \prod_{j=1}^{k}(\eta_{(j)}^U)^{\lambda_j} \right] \right\}
\end{aligned}$$

Then for $n = k + 1$, we have

$$\begin{aligned}
&\mathrm{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_k, \tilde{n}_{k+1}) \\
&= \mathop{\oplus}_{j=1}^{k} \lambda_j \tilde{n}_{(j)} \oplus \lambda_k \tilde{n}_{(k)} \\
&= \bigcup_{\substack{[\gamma_j^L,\gamma_j^U]\in\tilde{t}_j,\\ [\delta_j^L,\delta_j^U]\in\tilde{i}_j,\\ [\eta_j^L,\eta_j^U]\in\tilde{t}_j}} \left\{ \left[1-\prod_{j=1}^{k}(1-\gamma_{(j)}^L)^{\lambda_j},\quad 1-\prod_{j=1}^{k}(1-\gamma_{(j)}^U)^{\lambda_j} \right], \right. \\
&\quad \left[\prod_{j=1}^{k}(\delta_{(j)}^L)^{\lambda_j},\, \prod_{j=1}^{k}(\delta_{(j)}^U)^{\lambda_j} \right], \left[\prod_{j=1}^{k}(\eta_{(j)}^L)^{\lambda_j},\, \prod_{j=1}^{k}(\eta_{(j)}^U)^{\lambda_j} \right] \\
&\quad \oplus \bigcup_{\substack{[\gamma_{k+1}^L,\gamma_{k+1}^U]\in\tilde{t}_{k+1},\\ [\delta_{k+1}^L,\delta_{k+1}^U]\in\tilde{i}_{k+1},\\ [\eta_{k+1}^L,\eta_{k+1}^U]\in\tilde{t}_{k+1}}} \left\{ \left[1-(1-\gamma_{(k+1)}^L)^{\lambda_{k+1}},\, 1-(1-\gamma_{(k+1)}^U)^{\lambda_{k+1}} \right], \right. \\
&\quad \left. \left[(\delta_{(k+1)}^L)^{\lambda_{k+1}},\, (\delta_{(k+1)}^U)^{\lambda_{k+1}} \right], \left[(\eta_{(k+1)}^L)^{\lambda_{k+1}},\, (\eta_{(k+1)}^U)^{\lambda_{k+1}} \right] \right\} \\
&= \bigcup_{\substack{[\gamma_j^L,\gamma_j^U]\in\tilde{t}_j,\\ [\delta_j^L,\delta_j^U]\in\tilde{i}_j,\\ [\eta_j^L,\eta_j^U]\in\tilde{t}_j}} \left\{ \left[1-\prod_{j=1}^{k}(1-\gamma_{(j)}^L)^{\lambda_j} \;+1-(1-\gamma_{(k+1)}^L)^{\lambda_{k+1}} \right. \right. \\
&\quad -\left(1-\prod_{j=1}^{k}(1-\gamma_{(j)}^L)^{\lambda_j} \right) \cdot \left(1-(1-\gamma_{(k+1)}^L)^{\lambda_{k+1}} \right), \\
&\quad 1-\prod_{j=1}^{k}(1-\gamma_{(j)}^U)^{\lambda_j} + 1-(1-\gamma_{(k+1)}^U)^{\lambda_{k+1}} - \left(1-\prod_{j=1}^{k}(1-\gamma_{(j)}^U)^{\lambda_j} \right) \\
&\quad \left. \cdot \left(1-(1-\gamma_{(k+1)}^U)^{\lambda_{k+1}} \right) \right], \\
&\quad \left[\prod_{j=1}^{k}(\delta_{(j)}^L)^{\lambda_j} \cdot (\delta_{(k+1)}^L)^{\lambda_{k+1}},\, \prod_{j=1}^{k}(\delta_{(j)}^U)^{\lambda_j)} \cdot (\delta_{(k+1)}^U)^{\lambda_{k+1}} \right],
\end{aligned}$$

$$\left[\prod_{j=1}^{k}(\eta_{(j)}^{L})^{\lambda_j}\cdot(\eta_{(k+1)}^{L})^{\lambda_{k+1}},\ \prod_{j=1}^{k}(\eta_{(j)}^{U})^{\lambda_j}\cdot(\eta_{(k+1)}^{U})^{\lambda_{k+1}}\right]\Bigg\}$$

$$= \bigcup_{\substack{[\gamma_j^L,\gamma_j^U]\in\tilde{t}_j,\\ [\delta_j^L,\delta_j^U]\in\tilde{i}_j,\\ [\eta_j^L,\eta_j^U]\in\tilde{f}_j}} \left\{\left[1-\prod_{j=1}^{k+1}(1-\gamma_{(j)}^{L})^{\lambda_j},\ 1-\prod_{j=1}^{k+1}(1-\gamma_{(j)}^{U})^{\lambda_{k+1}}\right],\right.$$

$$\left.\left[\prod_{j=1}^{k+1}(\delta_{(j)}^{L})^{\lambda_j},\ \prod_{j=1}^{k+1}(\delta_{(j)}^{U})^{\lambda_j}\right],\ \left[\prod_{j=1}^{k+1}(\eta_{(j)}^{L})^{\lambda_j},\ \prod_{j=1}^{k+1}(\eta_{(j)}^{U})^{\lambda_j}\right]\right\}$$

Hence the theorem holds for $n = k + 1$, which completes the proof. □

Note that in proving all the following results relating to the INHFCI operators, there is no loss of generality in considering the case when the ordering among INHFEs is unique i.e., using Equation (4.2) for the INHFCI as the related results using Equation (4.1) follows exactly in the same manner.

Proposition 1. *Let $X = \{x_1, x_2, \ldots, x_n\}$, m a fuzzy measure on X and $(\cdot)$ an ordering on the* INHFE*s $\tilde{n}_1, \tilde{n}_2, \cdots \tilde{n}_n$. If $m(A) = \sum_{x_{(j)}\in A} m(\{x_{(j)}\})$ for all $A \subseteq X$ with $\sum_{j=1}^{n} m(\{x_{(j)}\}) = 1$, then*

$$m(\{x_{(j)}\}) = m(A_{(j)}) - m(A_{(j-1)}). \tag{4.4}$$

In this case the INHFCI *operator reduces to an interval neutrosophic hesitant fuzzy weighted average* (INHFWA) *operator [22], i.e.,*

$$\mathrm{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \mathrm{INHFWA}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n). \tag{4.5}$$

Proof. Using $m(\{x_{(j)}\}) = m(A_{(j)}) - m(A_{(j-1)})$ and $\sum_{j=1}^{n} m(\{x_{(j)}\}) = 1$ in Equation (4.3), we have

$$\mathrm{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \bigcup_{\substack{[\gamma_j^L,\gamma_j^U]\in\tilde{t}_j,\\ [\delta_j^L,\delta_j^U]\in\tilde{i}_j,\\ [\eta_j^L,\eta_j^U]\in\tilde{f}_j}} \left\{\left[1-\prod_{j=1}^{n}(1-\gamma_{(j)}^{L})^{m(\{x_{(j)}\})},\ 1-\prod_{j=1}^{n}(1-\gamma_{(j)}^{U})^{m(\{x_{(j)}\})}\right],\right.$$

$$\left.\left[\prod_{j=1}^{n}(\delta_{(j)}^{L})^{m(\{x_{(j)}\})},\ \prod_{j=1}^{n}(\delta_{(j)}^{U})^{m(\{x_{(j)}\})}\right],\ \left[\prod_{j=1}^{n}(\eta_{(j)}^{L})^{m(\{x_{(j)}\})},\ \prod_{j=1}^{n}(\eta_{(j)}^{U})^{m(\{x_{(j)}\})}\right]\right\}$$

$$= \bigoplus_{j=1}^{n}\tilde{n}_j\cdot m(\{x_{(j)}\}) = \mathrm{INHFWA}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n). \tag{4.6}$$

It follows that,

$$\mathrm{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \mathrm{INHFWA}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) \qquad \square$$

Corollary 1. *Let $X = \{x_1, x_2, \ldots, x_n\}$ and let m be a fuzzy measure on X and (.) an ordering on the set of INHFEs $\{\tilde{n}_j\}_{j=1}^{n}$. If $m(\{x_{(j)}\}) = \frac{1}{n}$, $(j = 1, 2, \ldots, n)$, then the* INHFCI *operator reduces to an interval neutrosophic hesitant fuzzy averaging operator* (INHFA operator) *namely,*

$$\mathrm{INHFA}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \frac{1}{n}\bigoplus_{j=1}^{n}\tilde{n}_j. \tag{4.7}$$

Corollary 1 follows directly from Proposition 1.

Corollary 2. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set, m a fuzzy measure on X and (.) an ordering on the set of INHFEs $\{\tilde{n}_j\}_{j=1}^{n}$. If $m(\{x_{(j)}\}) = \sum_{j=1}^{Card(A)} w_j$ for all $A \subseteq X$, then $\sum_{j=1}^{n} w_j = 1$ with*

$$w_j = m(A_{(j)}) - m(A_{(j-1)}),\ (j = 1, 2, \ldots, n).$$

In this case the INHFCI *operator reduces to an interval neutrosophic hesitant fuzzy ordered weighted average* (IVIHFOWA) *operator [22], i.e.,*

$$\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \text{INHFOWA}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n). \quad (4.8)$$

Corollary 2 follows directly from Proposition 1.

Remark 3. In view of Corollaries 1 and 2, it is clear that the interval neutrosophic hesitant choquet integral operator generalizes both the interval neutrosophic hesitant fuzzy weighted averaging operator (INHFWA) operator and interval neutrosophic hesitant fuzzy ordered weighted average (INHFOWA) operator.

5. Properties of the INHFCI operator

Some of the properties of INHFCI operator are as follows.

Theorem 2. *(Idempotency). Let*

$$\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\} = \bigcup_{\substack{[\gamma_j^L,\gamma_j^U]\in\tilde{t}_j,\\ [\delta_j^L,\delta_j^U]\in\tilde{i}_j,\\ [\eta_j^L,\eta_j^U]\in\tilde{f}_j}} \left\{[\gamma_j^L, \gamma_j^U], [\delta_j^L, \delta_j^U], [\eta_j^L, \eta_j^U]\right\} \quad (5.1)$$

be a collections of INHFEs *on X, and let m a fuzzy measure on X. If all $\tilde{n}_j (j = 1, 2, \ldots, n)$ are equal, i.e., $\tilde{n}_j = \tilde{n}$ for all j, then* $\text{INHFCI}(\tilde{n}, \tilde{n}, \ldots, \tilde{n}) = \tilde{n}$.

Proof. From Theorem 1, if $\tilde{n}_j = \tilde{n}$ for all j, we have,

$$\begin{aligned}
&\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) \\
&= \text{INHFCI}(\tilde{n}, \tilde{n}, \ldots, \tilde{n}) \\
&= \bigcup_{\substack{[\gamma^L,\gamma^U]\in\tilde{t},\\ [\delta^L,\delta^U]\in\tilde{i},\\ [\eta^L,\eta^U]\in\tilde{f}}} \Bigg\{\Bigg[1 - \prod_{j=1}^{n}(1-\gamma^L)^{m(A_{(j)})-m(A_{(j-1)})}, \\
&\quad 1 - \prod_{j=1}^{n}(1-\gamma^U)^{m(A_{(j)})-m(A_{(j-1)})}\Bigg], \\
&\quad \Bigg[\prod_{j=1}^{n}(\delta^L)^{m(A_{(j)})-m(A_{(j-1)})}, \\
&\quad \prod_{j=1}^{n}(\delta^U)^{m(A_{(j)})-m(A_{(j-1)})}\Bigg], \\
&\quad \Bigg[\prod_{j=1}^{n}(\eta^L)^{m(A_{(j)})-m(A_{(j-1)})}, \\
&\quad \prod_{j=1}^{n}(\eta^U)^{m(A_{(j)})-m(A_{(j-1)})}\Bigg]\Bigg\} \\
&= \bigcup_{\substack{[\gamma^L,\gamma^U]\in\tilde{t},\\ [\delta^L,\delta^U]\in\tilde{i},\\ [\eta^L,\eta^U]\in\tilde{f}}} \Bigg\{\Bigg[1-(1-\gamma^L)^{\sum_{j=1}^{n} m(A_{(j)})-m(A_{(j-1)})}, \\
&\quad 1-(1-\gamma^U)^{\sum_{j=1}^{n} m(A_{(j)})-m(A_{(j-1)})}\Bigg], \\
&\quad \Bigg[(\delta^L)^{\sum_{j=1}^{n} m(A_{(j)})-m(A_{(j-1)})}, \\
&\quad (\delta^U)^{\sum_{j=1}^{n} m(A_{(j)})-m(A_{(j-1)})}\Bigg], \\
&\quad \Bigg[(\eta^L)^{\sum_{j=1}^{n} m(A_{(j)})-m(A_{(j-1)})}, \\
&\quad (\eta^U)^{\sum_{j=1}^{n} m(A_{(j)})-m(A_{(j-1)})}\Bigg]\Bigg\}.
\end{aligned}$$

It follows from

$$\sum_{j=1}^{n}(m(A_{(j)}) - m(A_{(j-1)})) = m(A_{(n)}) - m(A_{(0)}) = 1,$$

that

$$\text{INHFCI}(\tilde{n}, \tilde{n}, \ldots, \tilde{n}) = \bigcup_{\substack{[\gamma^L,\gamma^U]\in\tilde{t},\\ [\delta^L,\delta^U]\in\tilde{i},\\ [\eta^L,\eta^U]\in\tilde{f}}} \left\{[\gamma^L, \gamma^U], [\delta^L, \delta^U], [\eta^L, \eta^U]\right\} = \tilde{n}.$$

□

Theorem 3. *(Monotonicity). Let*

$$\tilde{n}_j = \{\tilde{t}, \tilde{i}, \tilde{f}\} = \bigcup_{\substack{[\gamma_j^L,\gamma_j^U]\in\tilde{t}_j,\\ [\delta_j^L,\delta_j^U]\in\tilde{i}_j,\\ [\eta_j^L,\eta_j^U]\in\tilde{f}_j}} \left\{[\gamma_j^L, \gamma_j^U], [\delta_j^L, \delta_j^U], [\eta_j^L, \eta_j^U]\right\}$$

and

$$\tilde{n}'_j = \{\tilde{t}', \tilde{i}', \tilde{f}'\} = \bigcup_{\substack{[\gamma'^L_j, \gamma'^U_j] \in \tilde{t}'_j,\\ [\delta'^L_j, \delta'^U_j] \in \tilde{i}'_j,\\ [\eta'^L_j, \eta'^U_j] \in \tilde{f}'_j}} \left\{ [\gamma'^L_j, \gamma'^U_j], [\delta'^L_j, \delta'^U_j], [\eta'^L_j, \eta'^U_j] \right\}$$

be two collections of INHFEs *on X, and let m be a fuzzy measure on X such that* $\gamma^L_j \le \gamma'^L_j$, $\gamma^U_j \le \gamma'^U_j$, $\delta^L_j \ge \delta'^L_j$, $\delta^L_j \ge \delta'^L_j$, $\eta^L_j \ge \eta'^L_j$, $\eta^U_j \ge \eta'^U_j$ *for all j; then*

$$\mathrm{INHFCI}(\tilde{n}'_1, \tilde{n}'_2 \ldots, \tilde{n}'_n) \succcurlyeq \mathrm{INHFCI}(\tilde{n}_1, \tilde{n}_2 \ldots, \tilde{n}_n). \tag{5.2}$$

Proof. Since $A_{(j-1)} \subseteq A_{(j)}$, therefore $m(A_{(j)}) - m(A_{(j-1)}) \ge 0$. Given that $\gamma^L_j \le \gamma'^L_j$, $\gamma^U_j \le \gamma'^U_j$, $\delta^L_j \ge \delta'^L_j$, $\delta^L_j \ge \delta'^L_j$, $\eta^L_j \ge \eta'^L_j$, $\eta^U_j \ge \eta'^U_j$ for all j. It follows that

$$1 - \prod_{j=1}^{n} (1 - \gamma^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \le 1 - \prod_{j=1}^{n} (1 - \gamma'^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}, \tag{5.3}$$

$$1 - \prod_{j=1}^{n} (1 - \gamma^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \le 1 - \prod_{j=1}^{n} (1 - \gamma'^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}, \tag{5.4}$$

$$\prod_{j=1}^{n} (\delta^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \ge \prod_{j=1}^{n} (\delta'^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}, \tag{5.5}$$

$$\prod_{j=1}^{n} (\delta^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \ge \prod_{j=1}^{n} (\delta'^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}, \tag{5.6}$$

$$\prod_{j=1}^{n} (\eta^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \ge \prod_{j=1}^{n} (\eta'^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}, \tag{5.7}$$

$$\prod_{j=1}^{n} (\eta^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \ge \prod_{j=1}^{n} (\eta'^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}. \tag{5.8}$$

Now,

$$\mathrm{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \bigcup_{\substack{[\gamma^L_j, \gamma^U_j] \in \tilde{t}_j,\\ [\delta^L_j, \delta^U_j] \in \tilde{i}_j,\\ [\eta^L_j, \eta^U_j] \in \tilde{f}_j}} \left\{ \left[1 - \prod_{j=1}^{n} (1 - \gamma^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}, 1 - \prod_{j=1}^{n} (1 - \gamma^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \right], \left[\prod_{j=1}^{n} (\delta^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}, \prod_{j=1}^{n} (\delta^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \right], \left[\prod_{j=1}^{n} (\eta^L_{(j)})^{m(A_{(j)})-m(A_{(j-1)})}, \prod_{j=1}^{n} (\eta^U_{(j)})^{m(A_{(j)})-m(A_{(j-1)})} \right] \right\}$$

Combining Inequalities (5.3) to (5.8) we have

$\mathrm{INHFCI}(\tilde{n}'_1, \tilde{n}'_2 \ldots, \tilde{n}'_n) \succcurlyeq \mathrm{INHFCI}(\tilde{n}_1, \tilde{n}_2 \ldots, \tilde{n}_n)$. □

Theorem 4. *(Boundedness). Let*

$$\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\}\ (j = 1, 2, \ldots, n) = \bigcup_{\substack{[\gamma^L_j, \gamma^U_j] \in \tilde{t}_j,\\ [\delta^L_j, \delta^U_j] \in \tilde{i}_j,\\ [\eta^L_j, \eta^U_j] \in \tilde{f}_j}} \left\{ [\gamma^L_j, \gamma^U_j], [\delta^L_j, \delta^U_j], [\eta^L_j, \eta^U_j] \right\}$$

be a collection of INHFEs *on X, let m a fuzzy measure on X, and let*

$$\tilde{n}^- = \left\{ \left[\gamma^L_{\min}, \gamma^U_{\min} \right], \left[\delta^L_{\max}, \delta^U_{\max} \right], \left[\eta^L_{\max}, \eta^U_{\max} \right] \right\}$$
$$\tilde{n}^+ = \left\{ \left[\gamma^L_{\max}, \gamma^U_{\max} \right], \left[\delta^L_{\min}, \delta^U_{\min} \right], \left[\eta^L_{\min}, \eta^U_{\min} \right] \right\},$$

where,

$$\gamma^L_{\min} = \min_j(\gamma^L_j),\ \delta^L_{\min} = \min_j(\delta^L_j),\ \eta^L_{\min} = \min_j(\eta^L_j),$$

$$\gamma^U_{\min} = \min_j(\gamma^U_j),\ \delta^U_{\min} = \min_j(\delta^U_j),\ \eta^U_{\min} = \min_j(\eta^U_j),$$

$$\gamma^L_{\max} = \max_j(\gamma^L_j),\ \delta^L_{\max} = \max_j(\delta^L_j),\ \eta^L_{\max} = \max_j(\eta^L_j),$$

$$\gamma^U_{\max} = \max_j(\gamma^U_j),\ \delta^U_{\max} = \max_j(\delta^U_j),\ \eta^U_{\max} = \max_j(\eta^U_j);$$

then

$$\tilde{n}^- \preccurlyeq \text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) \preccurlyeq \tilde{n}^+.$$

Proof of Theorem 4 can be easily obtained by using Theorem 2 and 3.

Theorem 5. *(Shift invariant) Let*

$$\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\} = \bigcup_{\substack{[\gamma^L_j,\gamma^U_j]\in\tilde{t}_j,\\ [\delta^L_j,\delta^U_j]\in\tilde{i}_j,\\ [\eta^L_j,\eta^U_j]\in\tilde{f}_j}} \left\{[\gamma^L_j, \gamma^U_j], [\delta^L_j, \delta^U_j], [\eta^L_j, \eta^U_j]\right\}$$

be a collection INHFEs*, and let m be a fuzzy measure on X. Let*

$$\tilde{a} = \{\tilde{t}_{\tilde{a}}, \tilde{i}_{\tilde{a}}, \tilde{f}_{\tilde{a}}\} = \bigcup_{\substack{[\gamma^L_{\tilde{a}},\gamma^U_{\tilde{a}}]\in\tilde{t}_{\tilde{a}},\\ [\delta^L_{\tilde{a}},\delta^U_{\tilde{a}}]\in\tilde{i}_{\tilde{a}},\\ [\eta^L_{\tilde{a}},\eta^U_{\tilde{a}}]\in\tilde{t}_{\tilde{a}}}} \left\{[\gamma^L_{\tilde{a}}, \gamma^U_{\tilde{a}}], [\delta^L_{\tilde{a}}, \delta^U_{\tilde{a}}], [\eta^L_{\tilde{a}}, \eta^U_{\tilde{a}}]\right\}$$

be an INHFE*, then*

$$\text{INHFCI}(\tilde{n}_1 \oplus \tilde{a}, \tilde{n}_2 \oplus \tilde{a}, \ldots, \tilde{n}_n \oplus \tilde{a}) = \text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) \oplus \tilde{a}. \quad (5.9)$$

Theorem 6. *(Homogeneity). Let*

$$\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\} = \bigcup_{\substack{[\gamma^L_j,\gamma^U_j]\in\tilde{t}_j,\\ [\delta^L_j,\delta^U_j]\in\tilde{i}_j,\\ [\eta^L_j,\eta^U_j]\in\tilde{f}_j}} \left\{[\gamma^L_j, \gamma^U_j], [\delta^L_j, \delta^U_j], [\eta^L_j, \eta^U_j]\right\}$$

be a collection INHFEs*, and let m be a fuzzy measure on X. If* $\lambda > 0$*, then*

$$\text{INHFCI}(\lambda\tilde{n}_1, \lambda\tilde{n}_2, \ldots, \lambda\tilde{n}_n) = \lambda\Big(\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n)\Big). \quad (5.10)$$

Proof of Theorem 6 follows directly from Definition 8 and Equation (4.3).

Corollary 1. *Let*

$$\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\}\ (j = 1, 2, \ldots, n) = \bigcup_{\substack{[\gamma^L_j,\gamma^U_j]\in\tilde{t}_j,\\ [\delta^L_j,\delta^U_j]\in\tilde{i}_j,\\ [\eta^L_j,\eta^U_j]\in\tilde{f}_j}} \left\{[\gamma^L_j, \gamma^U_j], [\delta^L_j, \delta^U_j], [\eta^L_j, \eta^U_j]\right\}$$

be a collection INHFEs*, let m be a fuzzy measure on X, and let*

$$\tilde{a} = \{\tilde{t}_{\tilde{a}}, \tilde{i}_{\tilde{a}}, \tilde{f}_{\tilde{a}}\} = \bigcup_{\substack{[\gamma^L_{\tilde{a}},\gamma^U_{\tilde{a}}]\in\tilde{t}_{\tilde{a}},\\ [\delta^L_{\tilde{a}},\delta^U_{\tilde{a}}]\in\tilde{i}_{\tilde{a}},\\ [\eta^L_{\tilde{a}},\eta^U_{\tilde{a}}]\in\tilde{f}_{\tilde{a}}}} \left\{[\gamma^L_{\tilde{a}}, \gamma^U_{\tilde{a}}], [\delta^L_{\tilde{a}}, \delta^U_{\tilde{a}}], [\eta^L_{\tilde{a}}, \eta^U_{\tilde{a}}]\right\}$$

be an INHFE*. If* $\lambda > 0$*, then*

$$\text{INHFCI}(\lambda\tilde{n}_1 \oplus \tilde{a}, \lambda\tilde{n}_2 \oplus \tilde{a}, \ldots, \lambda\tilde{n}_n \oplus \tilde{a}) = \lambda\Big(\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n)\Big) \oplus \tilde{a}. \quad (5.11)$$

The proofs of the next two theorems directly follow from Theorems 5 and 6 and hence omitted.

Theorem 7. *Let* $\tilde{n}_j, \tilde{n}'_j$ $(j = 1, 2, \ldots, n)$ *be two collections of* INHFEs*. If* $\tilde{n}'_j$ $(j = 1, 2, \ldots, n)$ *is a permutation of* $\tilde{n}_j$ $(j = 1, 2, \ldots, n)$*, then*

$$\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n) = \text{INHFCI}(\tilde{n}'_1, \tilde{n}'_2, \ldots, \tilde{n}'_n). \quad (5.12)$$

Theorem 8. *Let*

$$\tilde{n}_j = \{\tilde{t}, \tilde{i}, \tilde{f}\} = \bigcup_{\substack{\tilde{\gamma}_j\in\tilde{t}_j,\\ \tilde{\delta}_j\in\tilde{i}_j,\\ \tilde{\eta}_j\in\tilde{f}_j}} \left\{[\gamma^L_j, \gamma^U_j], [\delta^L_j, \delta^U_j], [\eta^L_j, \eta^U_j]\right\}$$

$$\tilde{n}'_j = \{\tilde{t}', \tilde{i}', \tilde{f}'\} = \bigcup_{\substack{\tilde{\gamma}'_j\in\tilde{t}'_j,\\ \tilde{\delta}'_j\in\tilde{i}'_j,\\ \tilde{\eta}'_j\in\tilde{f}'_j}} \left\{[\gamma'^L_j, \gamma'^U_j], [\delta'^L_j, \delta'^U_j], [\eta'^L_j, \eta'^U_j]\right\}$$

be two collections of INHFEs*, then*

$$\begin{aligned}&\text{INHFCI}(\tilde{n}_1 \oplus \tilde{n}_1^{'}, \tilde{n}_2 \oplus \tilde{n}_2^{'}, \ldots, \tilde{n}_n \oplus \tilde{n}_n^{'})\\ &\quad = \text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n)\\ &\qquad \oplus \text{INHFCI}(\tilde{n}_1^{'}, \tilde{n}_2^{'}, \ldots, \tilde{n}_n^{'}). \end{aligned} \tag{5.13}$$

Theorem 9. *Let*

$$\tilde{n}_j = \{\tilde{t}_j, \tilde{i}_j, \tilde{f}_j\} = \bigcup_{\substack{[\gamma_j^L, \gamma_j^U] \in \tilde{t}_j,\\ [\delta_j^L, \delta_j^U] \in \tilde{i}_j,\\ [\eta_j^L, \eta_j^U] \in \tilde{f}_j}} \left\{[\gamma_j^L, \gamma_j^U], [\delta_j^L, \delta_j^U], [\eta_j^L, \eta_j^U]\right\}$$

be a collection INHFEs*, and let m be a fuzzy measure on X. If $\tilde{i}_j \equiv 0$, for all $j(j = 1, 2, \ldots, n)$, then* INHFCI *operator reduces to interval valued intuitionistic hesitant fuzzy choquet integral* (IVIHFCI) *[31], i.e.,*

$$\begin{aligned}&\text{INHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n)\\ &\quad = \text{IVIHFCI}(\tilde{n}_1, \tilde{n}_2, \ldots, \tilde{n}_n). \end{aligned} \tag{5.14}$$

Remark 4. Theorem 9 asserts that the interval neutrosophic hesitant environment reduces to interval valued intuitionistic hesitant fuzzy environment when the condition $\tilde{i}_j \equiv 0$, for all $j(j = 1, 2, \ldots, n)$ is satisfied, and hence we can conclude that the INHFCI operator is a generalization of the interval valued intuitionistic hesitant fuzzy choquet integral (IVIHFCI) [31].

Remark 5. Theorem 2 shows that the INHFCI with respect to a possibility degree ordering is *Idempotent*, this property can be interpreted as a representation of unanimity among the INHFEs. Theorems 3 and 4 respectively, represent the *Commutativity* and *Boundedness* of the INHFCI with respect to a possibility degree ordering of INHFEs. The *Shift-invariant* and *Homogeneity* properties of the INHFCI represented by Theorems 5 and 6 respectively can be interpreted as the invariant properties of the INHFCI, which allow the translation or dilation of the INHFEs without affecting the relative orderings of aggregated INHFEs by INHFCI. These properties are standard in the literature however, if we restrict the INHFEs to IVIHFEs, then all properties given in this section also hold for the interval valued intuitionistic hesitant fuzzy choquet integral (IVIHFCI) [31].

6. An approach to multicriteria decision making with interval neutrosophic hesitant fuzzy choquet integral operator

Now we are in a position to propose multicriteria decision making (MCDM) based on interval neutrosophic hesitant fuzzy choquet integral (INHFCI). We assume that the evaluation information of the alternatives are given by INHFEs that allows interactions among the criteria.

Let $X = \{x_1, x_2, \ldots, x_m\}$ and let $C = \{c_1, c_2, \ldots, c_n\}$ be a set of criteria. Suppose that the evaluation information of the criteria $c_j \in C_j (j = 1, 2, \ldots, n)$ with respect to the alternative $x_i \in X$ is represented by an INHFE $\tilde{n}_{ij} = \{\tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij}\}$, where $\tilde{t}_{ij}$, $\tilde{i}_{ij}$ and $\tilde{f}_{ij}$ are set of some interval values in [0, 1], denoting the degrees of hesitant truth-membership, indeterminacy-membership and falsity-membership, respectively. The multicriteria decision making approach for obtaining the best alternative with respect to an interval neutrosophic hesitant fuzzy choquet integral involves the following steps.

Step 1 Construct the interval neutrosophic hesitant fuzzy decision matrix $D = (\tilde{n}_{ij})_{m \times n}$, where $\tilde{n}_{ij} = \{\tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij}\}$ is an INHFE, which represents the evaluation information of the criteria $c_j \in C_j$ with respect to the alternative $x_i \in X$.

Step 2 Identify the fuzzy measure $m(A)$ of all the $A \subseteq X$ for each of the criteria c_j $(j = 1, 2, \ldots, n)$ and using Equation (2.9) with parameter λ is determined using Equation (2.10).

Step 3 Utilize the possibility degree ranking approach as mentioned in Definition 15 to reorder the partial evaluation $\tilde{n}_{ij}$ of the alternatives x_i $(i = 1, 2, \ldots, m)$. The PDR given by $P(\tilde{n}_{i(j)} \succcurlyeq \tilde{n}_{i(k)})$ is defined as follows:

$$\begin{aligned} p^i_{jk} &= P(\tilde{n}_{i(j)} \succcurlyeq \tilde{n}_{i(k)})\\ &= \frac{1}{3}\Bigg[\frac{1}{l_j l_k}\sum_{j=1}^{l_j}\sum_{k=1}^{l_k} P([\gamma^L_{i(j)}, \gamma^U_{i(j)}] \succcurlyeq [\gamma^L_{i(k)}, \gamma^U_{i(k)}])\\ &\quad + \frac{1}{p_j p_k}\sum_{j=1}^{p_j}\sum_{k=1}^{p_k} P([\delta^L_{i(k)}, \delta^U_{i(k)}] \succcurlyeq [\delta^L_{i(j)}, \delta^U_{i(j)}])\\ &\quad + \frac{1}{q_j q_k}\sum_{\substack{j=1\\ j \neq k}}^{q_j}\sum_{\substack{k=1\\ j \neq k}}^{q_k} P([\eta^L_{i(k)}, \eta^U_{i(k)}] \succcurlyeq [\eta^L_{i(j)}, \eta^U_{i(j)}])\Bigg], \end{aligned} \tag{6.1}$$

where l_j, p_j, q_j are the numbers of the interval values in $\tilde{\gamma}_{i(j)}, \tilde{\delta}_{i(j)}, \tilde{\eta}_{i(j)}$, and l_k, p_k, q_k are the numbers of the interval values in $\tilde{\gamma}_{i(k)}, \tilde{\delta}_{i(k)}, \tilde{\eta}_{i(k)}$. Now, using Equation (2.2), it is observed that $0 \leq p^i_{jk} \leq 1$, $p^i_{jj} = P(\tilde{n}_{i(j)} = \tilde{n}_{i(k)}) = 0.5$, and $p^i_{jk} + p^i_{kj} = 1$. Also,

$$P([\gamma^L_{i(j)}, \gamma^U_{i(j)}] \succcurlyeq [\gamma^L_{i(k)}, \gamma^U_{i(k)}]) = \max\left\{1 - \max\left(\frac{\gamma^U_{i(k)} - \gamma^L_{i(j)}}{\gamma^U_{i(j)} - \gamma^L_{i(j)} + \gamma^U_{i(k)} - \gamma^L_{i(k)}}, 0\right), 0\right\} \tag{6.2}$$

and,

$$P([\delta^L_{i(k)}, \delta^U_{i(k)}] \succcurlyeq [\delta^L_{i(j)}, \delta^U_{i(j)}]) = \max\left\{1 - \max\left(\frac{\delta^U_{i(j)} - \delta^L_{i(k)}}{\delta^U_{i(j)} - \delta^L_{i(j)} + \delta^U_{i(k)} - \delta^L_{i(k)}}, 0\right), 0\right\}$$

$$P([\eta^L_{i(k)}, \eta^U_{i(k)}] \succcurlyeq [\eta^L_{i(j)}, \eta^U_{i(j)}]) = \max\left\{1 - \max\left(\frac{\eta^U_{i(j)} - \eta^L_{i(k)}}{\eta^U_{i(j)} - \eta^L_{i(j)} + \eta^U_{i(k)} - \eta^L_{i(k)}}, 0\right), 0\right\} \tag{6.3}$$

The matrix of possibility degrees $P^i = (p^i_{jk})_{n\times n}$, $(i = 1, 2, \ldots, m)$ is constructed for each of the alternative x_i $(i = 1, 2, \ldots m)$. Then the ranking of the partial evalution $\tilde{n}_{ij}$ is obtained by using Equation (3.5) on the matrix of possibility degrees P^i $(i = 1, 2, \ldots, m)$

$$r^i_j = \frac{\left(\sum_{k=1}^{n} p^i_{jk} + \frac{n}{2} - 1\right)}{n(n-1)} \tag{6.4}$$

The partial evalution $\tilde{n}_{ij}$ are reordered in a descending order $\tilde{n}_{i(1)} \succcurlyeq \tilde{n}_{i(2)} \succcurlyeq \cdots \succcurlyeq \tilde{n}_{i(n)}$ in accordance with the values of r^i_j $(j = 1, 2, \ldots, n)$, where $\{(1), (2), \ldots, (n)\}$ is a permutation of $\{1, 2, \ldots, n\}$.

Step 4 The overall value $\tilde{n}_i$ of the alternative x_i $(i = 1, 2, \ldots, m)$ can be obtained by the INHFCI operator:

$$\begin{aligned}\tilde{n}_i &= \text{INHFECI}(\tilde{n}_{i1}, \tilde{n}_{i2}, \ldots, \tilde{n}_{in}) \\ &= \bigcup_{\substack{\tilde{\gamma}_{i(j)} \in \tilde{t}_{i(j)},\\ \tilde{\delta}_{i(j)} \in \tilde{i}_{i(j)},\\ \tilde{\eta}_{i(j)} \in \tilde{f}_{i(j)}}} \Bigg\{ \left[1 - \prod_{j=1}^{n} (1 - \gamma^L_{i(j)})^{m(A_{i(j)}) - m(A_{i(j-1)})},\ 1 - \prod_{j=1}^{n} (1 - \gamma^U_{i(j)})^{m(A_{i(j)}) - m(A_{i(j-1)})}\right], \\ &\quad \left[\prod_{j=1}^{n} (\delta^L_{i(j)})^{m(A_{i(j)}) - m(A_{i(j-1)})},\ \prod_{j=1}^{n} (\delta^U_{i(j)})^{m(A_{i(j)}) - m(A_{i(j-1)})}\right] \\ &\quad \left[\prod_{j=1}^{n} (\eta^L_{i(j)})^{m(A_{i(j)}) - m(A_{i(j-1)})},\ \prod_{j=1}^{n} (\eta^U_{i(j)})^{m(A_{i(j)}) - m(A_{i(j-1)})}\right]\Bigg\}\end{aligned} \tag{6.5}$$

where $A_{i(j)} = \{c_{(1)}, c_{(2)}, \ldots, c_{(j)}\}$ and $A_{i(0)} = \emptyset$.

Step 5 Select the best one of $\tilde{n}_i$ $(i = 1, 2, \ldots n)$ using Equation (3.1) of the possibility degree ranking of INHFEs and Equation (3.5) for ranking $\tilde{n}_i$ $(i = 1, 2, \ldots n)$ obtained in **Step 4**.

Step 6 End.

7. Illustrative example

In this section, we present an example to illustrate the proposed decision making method under interval neutrosophic hesitant fuzzy environment. Suppose a manufacturing company wants to recruit a sales executive from a group of four candidates $\mathcal{A}_i$ $(i = 1, 2, 3, 4)$ on the basis of a set of following criteria:

(1) C_1: Management Knowledge
(2) C_2: Communication Skill
(3) C_3: Objection handling Skill
(4) C_4: Ability to Attain Targets,

and the evaluation values are expressed by INHFEs. The evaluation steps of the four alternatives on the basis of above mentioned criteria are as follows:

Step 1 Based on the experts' assesment the interval neutrosophic hesitant fuzzy decision matrix is constructed as shown in Table 1 in the Appendix.

Step 2 Identifying the fuzzy measure $m(C_j)$ for each criteria C_j $(j = 1, 2, 3, 4)$, which represent the importance of each criteria C_j $(j = 1, 2, 3, 4)$. Assuming that according to experts' assesment, fuzzy measure of each criteria are given as:

$m(\emptyset) = 0, m(\{C_1\}) = 0.25, m(\{C_2\}) = 0.18,$

$m(\{C_3\}) = 0.21$ and $m(\{C_4\}) = 0.23$.

Using Equation (2.10), parameter λ is found to be $\lambda = 0.43$. Again using Equation (2.9), we have

$m(\{C_1, C_2\}) = 0.45,\ m(\{C_1, C_3\}) = 0.48,$

$m(\{C_1, C_4\}) = 0.50,\ m(\{C_2, C_3\}) = 0.41,$

$m(\{C_2, C_4\}) = 0.43,\ m(\{C_3, C_4\}) = 0.46,$

$m(\{C_1, C_2, C_3\}) = 0.70,$

$m(\{C_1, C_2, C_4\}) = 0.72,$

$m(\{C_1, C_3, C_4\}) = 0.76,$

$m(\{C_2, C_3, C_4\}) = 0.68,$

$m(\{C_1, C_2, C_3, C_4\}) = 1.$

Step 3 Using the possibility degree ranking approach as mentioned in Definition (15), the INHFEs corresponding to each alternative are rearranged in descending order. For the alternative $\mathcal{A}_1$, we construct the matrix of possibility degree as follows:

$$P^1 = \begin{bmatrix} 0.5000 & 0.7500 & 0.6667 & 0.5000 \\ 0.2500 & 0.5000 & 0.5000 & 0.3333 \\ 0.3333 & 0.5000 & 0.5000 & 0.3333 \\ 0.5000 & 0.6667 & 0.6667 & 0.5000 \end{bmatrix}$$

Again, from Equation (6.4) we have, $r_1^1 = 0.2847$, $r_2^1 = 0.2153$, $r_3^1 = 0.2222$, and $r_4^1 = 0.2778$. Since $r_1^1 > r_4^1 > r_3^1 > r_2^1$, therefore for the alternative $\mathcal{A}_1$ the partial evaluations $\tilde{n}_{1j}\ j = 1, 2, 3, 4$ are reordered as, $\tilde{n}_{11} \succ \tilde{n}_{14} \succ \tilde{n}_{13} \succ \tilde{n}_{12}$.

Then the fuzzy measure $m(A)$ of all the $A \subseteq X$ for each of the criteria $C_j\ (j = 1, 2, 3, 4)$ corresponding to the alternative $\mathcal{A}_1$ are given by

$m(A_{1(1)}) = m(\{C_{(1)}\}) = 0.25,$

$m(A_{1(2)}) = m(\{C_{(1)}, C_{(4)}\}) = 0.50,$

$m(A_{1(3)}) = m(\{C_{(1)}, C_{(4)}, C_{(3)}\}) = 0.76,$

$m(A_{1(4)}) = m(\{C_{(1)}, C_{(4)}, C_{(3)}, C_{(2)}\}) = 1.$

Similarly the partial evaluations $\tilde{n}_{ij}\ (i = 2, 3, 4\ j = 1, 2, 3, 4)$ corresponding to the alternative $\mathcal{A}_i\ (i = 2, 3, 4)$ and the fuzzy measure $m(A)$ of all the $A \subseteq X$ for each of the criteria $C_j\ (j = 1, 2, 3, 4)$ corresponding to the alternative $\mathcal{A}_i\ (i = 2, 3, 4)$ are shown in Table 2 in the Appendix.

Step 4 Using Equation (6.5) the overall value $\tilde{n}_1$ of the alternative $\mathcal{A}_1$ is given as:

$$\tilde{n}_1 = \Big\{\{[0.3764, 0.4767], [0.1181, 0.2204], [0.2449, 0.3464]\}, \{[0.2065, 0.2752], [0.3675, 0.4870], [0.6180, 0.2089]\}\Big\}$$

Similarly, we have

$$\tilde{n}_2 = \Big\{\{[0.4513, 0.5518], [0.1000, 0.2000], [0.2469, 0.3484]\}, \{[0.3249, 0.4040], [0.0000, 0.4455], [0.5153, 0.6064]\}\Big\}$$

$$\tilde{n}_3 = \Big\{\{[0.5771, 0.6776], [0.1690, 0.3009], [0.4296, 0.5300]\}, \{[0.3699, 0.4615], [0.2358, 0.3491], [0.4739, 0.5693]\}\Big\}$$

$$\tilde{n}_4 = \Big\{\{[0.5858, 0.6893], [0.1926, 0.3798], [0.4211, 0.5696]\}, \{[0.4688, 0.5833], [0.3848, 0.5349], [0.4973, 0.6530]\}\Big\}.$$

Step 5 We calculate the possibility degree $P_{ij} = P(\tilde{n}_i \succcurlyeq \tilde{n}_j)\,(i, j = 1, 2, 3, 4)$ by using Equation (3.1) as:

$P_{12} = 0.3780,\ P_{13} = 0.3981,\ P_{14} = 0.3990$

$P_{23} = 0.5449,\ P_{24} = 0.5536,$

$P_{34} = 0.5918.$

Again, from Definition 15, since $P_{ii} = 0.5,\ P_{ij} + P_{ij} = 1\ (i, j = 1, 2, 3, 4)$, so we have

$P_{11} = 0.5000,\ P_{22} = 0.5000,\ P_{33} = 0.5000,$

$P_{44} = 0.5000,\ P_{21} = 0.6220,\ P_{31} = 0.6019,$

$P_{41} = 0.6010,\ P_{32} = 0.4551,\ P_{42} = 0.4464,$

$P_{43} = 0.4082.$

And the matrix of possibility degrees is constructed as:

$$P = \begin{bmatrix} 0.5000 & 0.3780 & 0.3981 & 0.3990 \\ 0.6220 & 0.5000 & 0.5449 & 0.5536 \\ 0.6019 & 0.4551 & 0.5000 & 0.5918 \\ 0.6010 & 0.4464 & 0.4082 & 0.5000 \end{bmatrix}. \tag{7.1}$$

Then the collective overall interval neutrosophic hesitant fuzzy preference values $r_i\ (i = 1, 2, 3, 4)$ are determined by applying Equation (3.5) as follows:

$r_1 = 0.2229, r_2 = 0.2684, r_3 = 0.2624,$

$r_4 = 0.2463.$

Since $r_2 > r_3 > r_4 > r_1$, therefore, the ranking order of the alternatives $A_i\ (i = 1, 2, 3, 4)$ is $\mathcal{A}_2 \succ \mathcal{A}_3 \succ \mathcal{A}_4 \succ \mathcal{A}_1$. Hence the alternative $\mathcal{A}_2$ is the best alternative. That is, $\mathcal{A}_2$ is the most suitable candidate for the job of scales executive.

8. Conclusion

In this paper, we have proposed the interval neutrosophic hesitant fuzzy choquet integral (INHFCI) operator for multi criteria decision making problem under interval neutrosophic hesitant fuzzy environment, and discussed their properties such as idempotency, monotonicity, homogeneity etc. It is shown that the INHFCI generalizes both the interval neutrosophic hesitant fuzzy weighted averaging (INHFWA) and interval neutrosophic hesitant fuzzy ordered weighted (INHFOW) operator. Also, it is shown that the interval valued intuitionistic fuzzy choquet integral (IVIHFCI) operator is a particular case of INHFCI operator. Further, an approach for multicriteria decision making is proposed. Finally, an illustrative example is presented to demonstrate the application of INHFCI in the multicriteria decision making process. It is to be expected that these investigations of generalized choquet integral may open the door for further study in this field.

Acknowledgments

This work is partially supported by the UKIERI Grant 184-15/2017(IC). We acknowledge the anonymous reviewers and the AE for their comments and suggestions in improving the paper to its current form.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] Lotfi and A. Zadeh, Fuzzy sets, *Information and Control* **8**(3) (1965), 338–353.

[2] K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* **20**(1) (1986), 87–96.

[3] K.T. Atanassov, More on intuitionistic fuzzy sets, *Fuzzy sets and Systems* **33**(1) (1989), 37–45.

[4] K. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, *Fuzzy sets and Systems* **31**(3) (1989), 343–349.

[5] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic, *In Philosophy*, American Research Press, 1999, pp. 1–141.

[6] S. Wan and J. Dong, A possibility degree method for interval-valued intuitionistic fuzzy multi-attribute group decision making, *Journal of Computer and System Sciences* **80**(1) (2014), 237–256.

[7] Z.S. Xu, Algorithm for priority of fuzzy complementary judgment matrix, *Journal of Systems Engineering* **16**(4) (2001), 311–314.

[8] Z.S. Xu and Q.L. Da, The uncertain OWA operator, *International Journal of Intelligent Systems* **17**(6) (2002), 569–575.

[9] Z. Xu, Dependent uncertain ordered weighted aggregation operators, *Information Fusion* **9**(2) (2008), 310–316.

[10] Z. Xu, On method for uncertain multiple attribute decision making problems with uncertain multiplicative preference information on alternatives, *Fuzzy Optimization and Decision Making* **4**(2) (2005), 131–139.

[11] H. Wang, F. Smarandache, Y. Zhang and R. Sunderraman, Single valued neutrosophic sets, *Rev Air Force Acad* **17** (2010), 4–10.

[12] H. Wang, F. Smarandache, R. Sunderraman and Y.Q. Zhang, Interval neutrosophic sets and logic: Theory and applications in computing: Theory and applications in computing (Vol-5), *Infinite Study* (2005).

[13] J. Ye, Single valued neutrosophic cross-entropy for multicriteria decision making problems, *Applied Mathematical Modelling* **38**(3) (2014), 1170–1175.

[14] V. Torra and Y. Narukawa, On hesitant fuzzy sets and decision, *In Fuzzy Systems, 2009 FUZZ-IEEE 2009 IEEE International Conference on*, 2009, pp. 1378–1382. IEEE.

[15] V. Torra, Hesitant fuzzy sets, *International Journal of Intelligent Systems* **25**(6) (2010), 529–539.

[16] N. Chen, Z. Xu and M. Xia, Interval-valued hesitant preference relations and their applications to group decision making, *Knowledge-Based Systems* **37** (2013), 528–540.

[17] J. Ye, Multiple-attribute decision-making method under a single-valued neutrosophic hesitant fuzzy environment, *Journal of Intelligent Systems* **24**(1) (2015), 23–36.

[18] J. Ye, Multiple attribute decision-making method based on the possibility degree ranking method and ordered weighted aggregation operators of interval neutrosophic numbers, *Journal of Intelligent & Fuzzy Systems* **28**(3) (2015), 1307–1317.

[19] F. Meng and X. Chen, An approach to interval-valued hesitant fuzzy multi-attribute decision making with incomplete weight information based on hybrid Shapley operators, *Informatica* **25**(4) (2014), 617–642.

[20] Z. Zhang, Multi-criteria decision-making using interval-valued hesitant fuzzy QUALIFLEX methods based on a likelihood-based comparison approach, *Neural Computing and Applications* **28**(7) (2017), 1835–1854.

[21] T. Murofushi and M. Sugeno, A theory of fuzzy measures: Representations, the Choquet integral, and null sets, *Journal of Mathematical Analysis and Applications* **159**(2) (1991), 532–549.

[22] P. Liu and L. Shi, The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making, *Neural Computing and Applications* **26**(2) (2015), 457–471.

[23] G. Choquet, Theory of capacities, *In Annales de l'institut Fourier*, Vol. 5, 1954, pp. 131–295.

[24] M. Sugeno, *Theory of fuzzy integrals and its applications*, Tokyo Institute of Technology, Doctoral Thesis, 1974.

[25] T. Murofushi and M. Sugeno, An interpretation of fuzzy measures and the Choquet integral as an integral with respect to a fuzzy measure, *Fuzzy sets and Systems* **29**(2) (1989), 201–227.

[26] P. Chi and P. Liu, An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set, *Neutrosophic Sets and Systems* **1**(1) (2013), 63–70.

[27] H.Y. Zhang, J.Q. Wang and X.H. Chen, Interval neutrosophic sets and their application in multicriteria decision making problems, *The Scientific World Journal* (2014).

[28] M. Xia and Z. Xu, Hesitant fuzzy information aggregation in decision making, *International Journal of Approximate Reasoning* **52**(3) (2011), 395–407.

[29] N. Chen, Z. Xu and M. Xia, Correlation coefficients of hesitant fuzzy sets and their applications to clustering analysis, *Applied Mathematical Modelling* **37**(4) (2013), 2197–2211.

[30] W.A.N.G. Zhenyuan and G.J. Klir, Fuzzy measure theory, 1992.

[31] D. Joshi and S. Kumar, Interval-valued intuitionistic hesitant fuzzy Choquet integral based TOPSIS method for multi-criteria group decision making, *European Journal of Operational Research* **248**(1) (2016), 183–191.

[32] R. Mesiar and J. Šipǒs, A theory of fuzzy measures: Integration and its additivity, *International Journal of General Systems* **23**(1) (1994), 49–57.

Appendix

The Tables 1 and 2 used in Section 7 are given as follows:

Table 1
Interval neutrosophic hesitant fuzzy decision matrix D

Alternatives	C_1	C_2	C_3	C_4
$\mathcal{A}_1$	{[0.4, 0.5], [0.1, 0.2], [0.2, 0.3]}, {[0.5, 0.6], [0.2, 0.3], [0.3, 0.4]}	{[0.4, 0.5], [0.2, 0.3], [0.3, 0.4]}	{[0.4, 0.5], [0.1, 0.2], [0.3, 0.4]}, {[0.2, 0.3], [0.1, 0.2], [0.5, 0.6]}	{[0.3, 0.4], [0.1, 0.2], [0.2, 0.3]}
$\mathcal{A}_2$	{[0.5, 0.6], [0.1, 0.2], [0.2, 0.3]}, {[0.6, 0.7], [0.2, 0.3], [0.3, 0.4]}	{[0.5, 0.6], [0.1, 0.2], [0.3, 0.4]}	{[0.4, 0.5], [0.1, 0.2], [0.2, 0.3]}, {[0.5, 0.6], [0.0, 0.1], [0.2, 0.3]}	{[0.4, 0.5], [0.1, 0.2], [0.3, 0.4]}
$\mathcal{A}_3$	{[0.6, 0.7], [0.3, 0.4], [0.5, 0.6]}, {[0.5, 0.6], [0.2, 0.3], [0.4, 0.5]}	{[0.6, 0.7], [0.1, 0.2], [0.4, 0.5]}, {[0.4, 0.5], [0.2, 0.3], [0.5, 0.6]}	{[0.6, 0.7], [0.1, 0.3], [0.4, 0.5]}	{[0.5, 0.6], [0.2, 0.3], [0.4, 0.5]}, {[0.4, 0.5], [0.1, 0.2], [0.3, 0.4]}
$\mathcal{A}_4$	{[0.7, 0.8], [0.2, 0.4], [0.6, 0.7]}, {[0.6, 0.7], [0.4, 0.5], [0.5, 0.6]}	{[0.6, 0.7], [0.2, 0.3], [0.4, 0.6]}, {[0.4, 0.6], [0.1, 0.3], [0.2, 0.5]}	{[0.5, 0.6], [0.1, 0.4], [0.3, 0.5]}	{[0.5, 0.6], [0.3, 0.4], [0.4, 0.5]}, {[0.6, 0.7], [0.4, 0.5], [0.5, 0.6]}

Table 2
Reordering of the partial evaluations

Alternatives	Reordering of the partial evaluations	Fuzzy measures
$\mathcal{A}_2$	$\tilde{n}_{23} \succ \tilde{n}_{21} \succ \tilde{n}_{22} \succ \tilde{n}_{24}$	$m(A_{2(1)}) = m(\{C_{(3)}\}) = 0.21,\ m(A_{2(2)}) = m(\{C_{(3)}, C_{(1)}\}) = 0.48,$ $m(A_{2(3)}) = m(\{C_{(3)}, C_{(1)}, C_{(2)}\}) = 0.70,\ m(A_{2(4)}) = m(\{C_{(3)}, C_{(1)}, C_{(2)}, C_{(4)}\}) = 1$
$\mathcal{A}_3$	$\tilde{n}_{33} \succ \tilde{n}_{34} \succ \tilde{n}_{32} \succ \tilde{n}_{31}$	$m(A_{3(1)}) = m(\{C_{(3)}\}) = 0.21,\ m(A_{3(2)}) = m(\{C_{(3)}, C_{(4)}\}) = 0.46,$ $m(A_{3(3)}) = m(\{C_{(3)}, C_{(4)}, C_{(2)}\}) = 0.68,\ m(A_{3(4)}) = m(\{C_{(3)}, C_{(4)}, C_{(2)}, C_{(1)}\}) = 1$
$\mathcal{A}_4$	$\tilde{n}_{42} \succ \tilde{n}_{43} \succ \tilde{n}_{41} \succ \tilde{n}_{44}$	$m(A_{3(1)}) = m(\{C_{(2)}\}) = 0.18,\ m(A_{3(2)}) = m(\{C_{(2)}, C_{(3)}\}) = 0.41,$ $m(A_{3(3)}) = m(\{C_{(2)}, C_{(3)}, C_{(1)}\}) = 0.70,\ m(A_{3(4)}) = m(\{C_{(2)}, C_{(3)}, C_{(1)}, C_{(4)}\}) = 1$