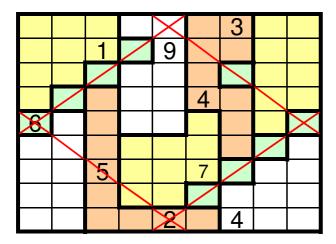
Proof of the Unique Solution of the P Pandemonion submitted to Wikipedia

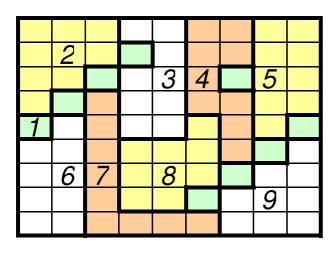
The Puzzle



The Rules

- 1 Every row, column and cluster must contain the numbers 1 to 9.
- 2 The green cluster is in nine parts. It must also contain the numbers 1 to 9.
- 3 The numbers 1 to 9 must appear on the upper red (inverted-V) line.
- 4 The numbers 1 to 9 must appear on the lower red (V) line.

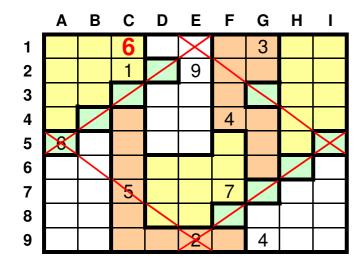
Identifying the clusters



Giving each cluster a number will make the task of explanation much easier.

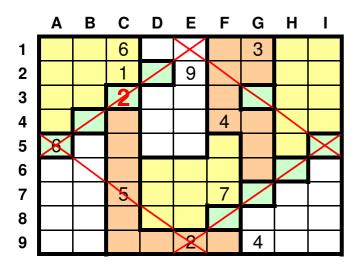
The Proof

Entry Number



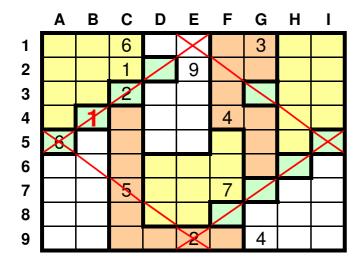
As 6 must appear in cluster 6 anywhere between B7 and B9, the only cell available to 6 in cluster 2 is C1.

Entry Number



2

C3 is the only cell available to 2 to columm C.

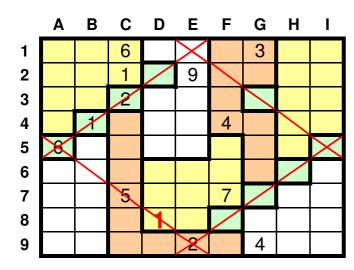


3

Thanks to 1 at C2 the only way to place 1 in both columns A and B is to place it in cell B4.

Entry Number





1 at B4 blocks all other cells on the upper, inverted V. 1 at B4 also blocks all cluster 1 (green) cells on the lower V line and B6. This leaves only cell D8 available to 1 on the lower V line.

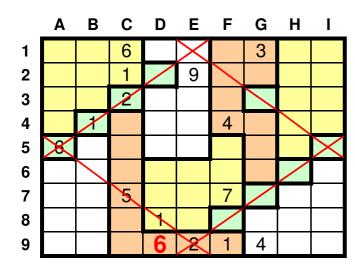
Α В C D Ε F G Н I 6 3 1 9 1 2 3 4 4 **X** 5 6 7 8 4 9

5

1 at C2 blocks all cells in cluster 7 from C4 to C9. 1 at D8 blocks D9, leaving only F9 available to 1 in cluster 7.

Entry Number

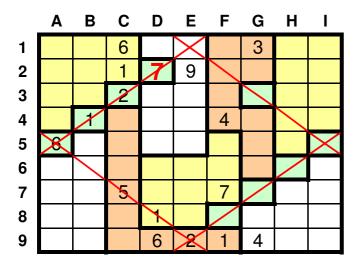
6



6 at C1 blocks all cells in cluster 7 from C4 to C9, leaving only D9 available to 6 in cluster 7.

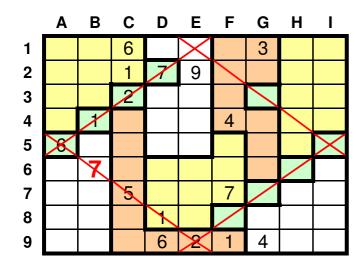
Entry Number

7



Thanks to 7 at F7 we can deduce that, in order for 7 to be placed in both columns D and E it must occupy D2.

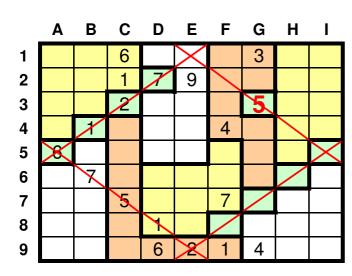
8



Now that 7 occupies cell D2 (a cluster 1 cell), the only cell available to 7 on the lower V line is B6.

Entry Number

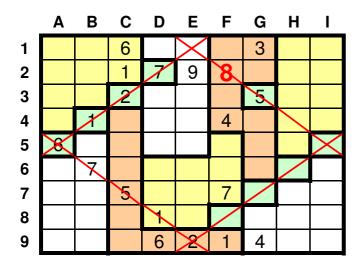
9



5 at C7 blocks all green (cluster 1) cells on the lower V line, leaving only G3, on the upper V line, for 5 in cluster 1.

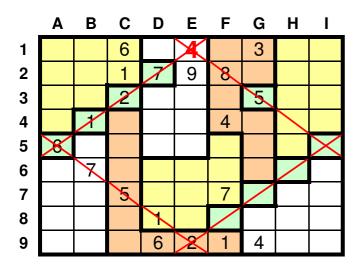
Entry Number

10



The only numbers still available on the upper V line are: 3, 4, 8 an 9. Cell F2 is already blocked by 3 at G1, 4 at F4 and 9 at E2. So, only 8 is still available to cell F2.

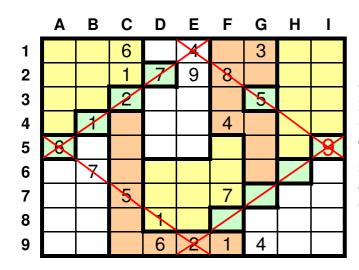
11



Now only 3, 4 and 9 are available on the upper V line. 9 at E2 and 3 at G1 force 4 into cell E1.

Entry Number

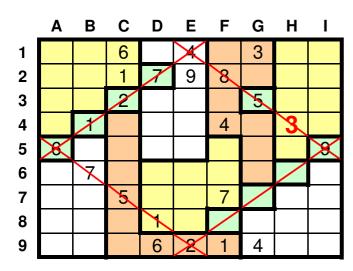
12



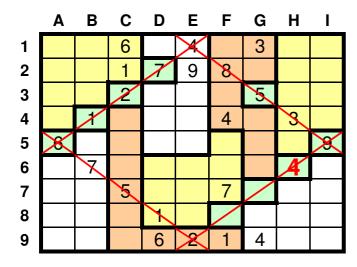
Only 3 and 9 are available on the upper V line. However, 3 cannot occupy cell I5, at the juncture of the upper and lower V lines. The numbers at cluster 1 cells H6 and I5 must be replicated at G8 and G9 (as is the case with cells C1/C2 and A5/B4). So, only 9 is left available to I5.

Entry Number

13



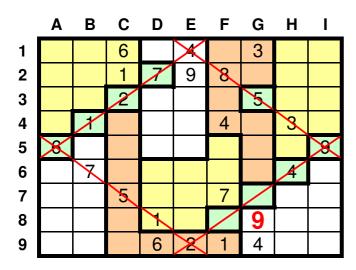
.... so, as the sole survivor so far, 3 must occupy H4 on the upper V line.



14

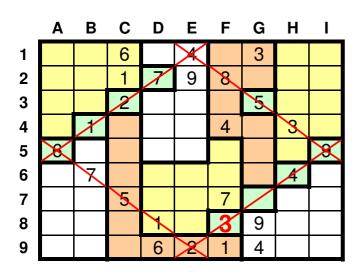
4 at G9 forces placement of 4 into cluster 1 cell H6....

Entry Number 15

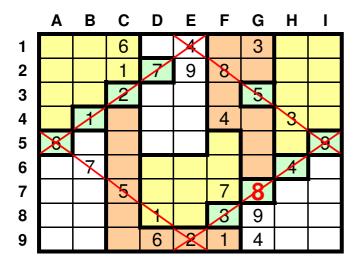


.... and 9 at I5 forces 9 into G8.

Entry Number 16

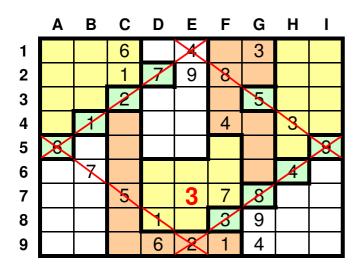


With only two cluster 1 and two lower V cells available - 3 and 8 - 3 at G1 forces placement of 3 at F8



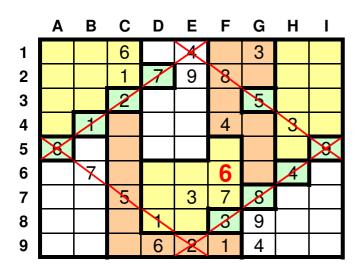
.... leaving 8 at G7 to complete both cluster 1 and the lower V line.

Entry Number 18

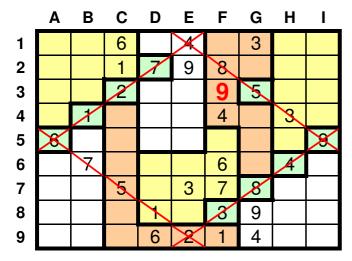


As can be readily seen, cell E7 is already blocked to 1 (D8), 2 (E9), 4 (E1), 5 (C7), 7 F7, 8 (G7) and 9 (E2). What is not so readily-seen is that 6 at D9 forces placement of a 6 at E3 or E4 in cluster 3. This 6 in cluster 3 also blocks cell E7, thus leaving only 3.

Entry Number 19



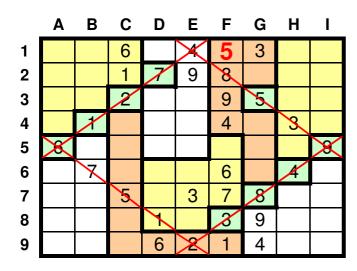
As we established at entry 21 6 must occupy cluster 3 at either E3 or E4. This means that, now, both columns D and E are unavailable to 6 in cluster 8, leaving only column F. What's more 6 at A5 blocks F5, leaving 5 to the only available cell in cluster 8 - F6.



Look around!, all but 9 are blocked to cell F3.

Entry Number

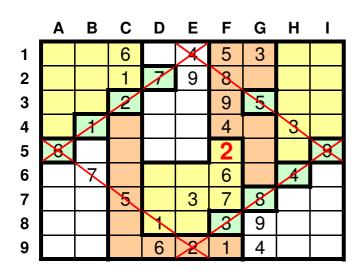
21



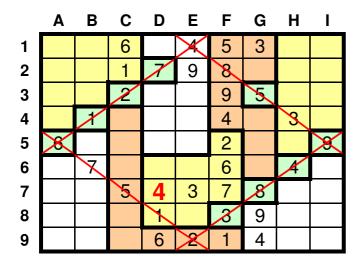
5 at cluster 1 cell G3 blocks all other cells in cluster 4.

Entry Number

22

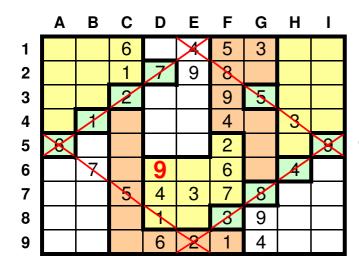


2 is the only number left in column F.



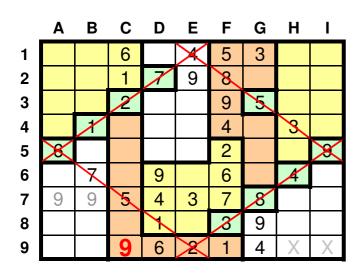
4s at E1 and H6 block all other cells in cluster 8.

Entry Number 24

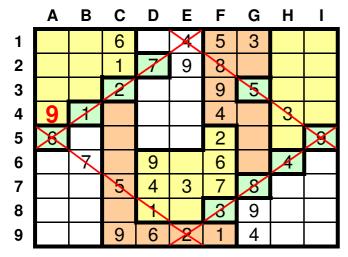


9 at E2 blocks all other cells in cluster 8.

Entry Number 25

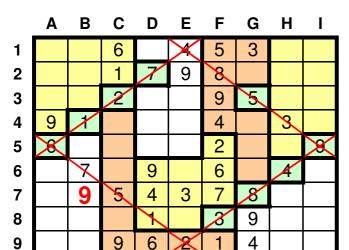


Note that, thanks to 9 at G8 there cannot be a 9 at either H7 or I7. This meanns that, for there to be a 9 in row 7 it must occupy either A7 or B7. This, in turn, excludes a 9 from both A9 and B9 which forces 9 in row 9 into cell C9.



Note that all other vacant cells in row 4 are already blocked by 9s at C9, E2, F3 and I5.

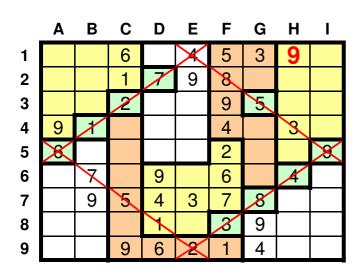
Entry Number



27

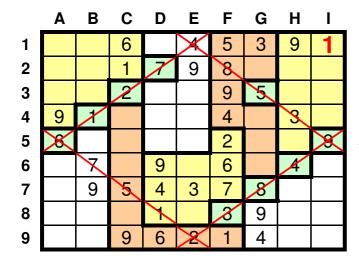
Again, all vacant cells in row 7 are already blocked.

Entry Number



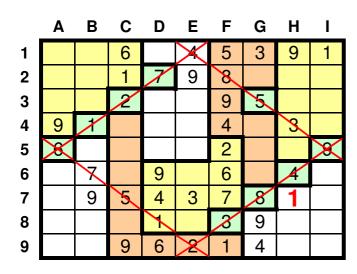
28

9 at H1 is the last of the 9s for the puzzle.



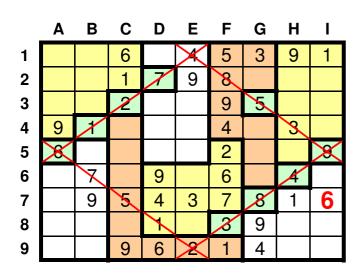
All other cells in row 1 are blocked.

Entry Number 30

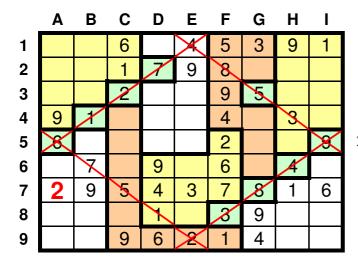


All other cells in cluster 9 are blocked.

Entry Number 31

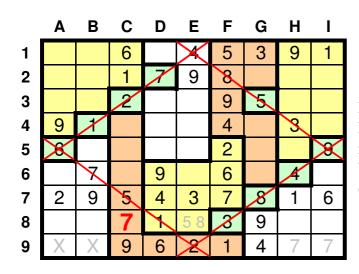


6 at A5 blocks the only other cell in row 7.



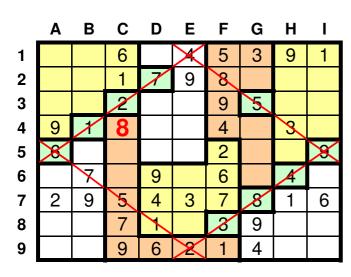
2 at A7 completes row 7.

Entry Number 33

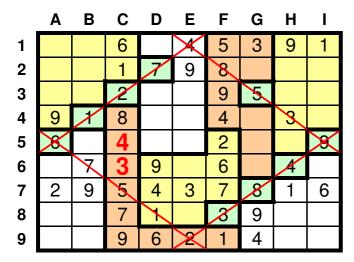


First, note that either 5 or 8 must occupy cell E8. Second, note that 7 at B6 forces a 7 in row 9 to either of cells H9 or I9 in cluster 9. With these matters noted we see that the only cell available to 7 in row 8 is C8.

Entry Number 34

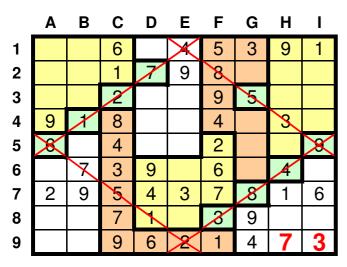


3, 4 and 8 are the only numbers still available in cluster 7 but 3 and 4 are already blocked at C4.



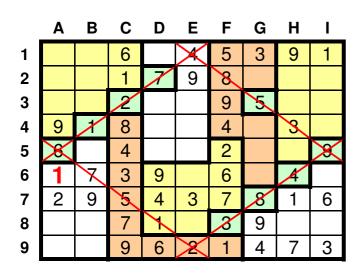
4 at H4 blocks cell C6. So, 4 must occupy C5, while 3 completes cluster 7 at C6.

Entry Number 36



I9 is the only cell available to 3 in cluster 9 and column I while H9 is the only cell available to 7 in cluster 9.

Entry Number 37

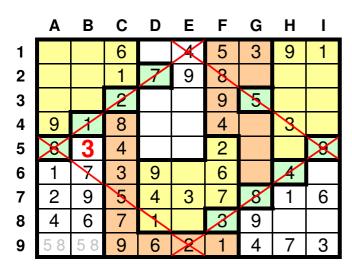


A6 is the only cell available to 1 in cluster 6.

	Α	В	С	D	Е	F	G	Н	I
1			6		\times	5	3	9	1
2			1	7	9	8			
3			2			9	5		
4	9	*	8			4		3	
5	\aleph		4			2			\times
6	1	X	3	9		6		Æ	
7	2	9	75	4	თ	7	8	1	6
8	4	6	7	f		3	9		
9			9	6	X	1	4	7	3

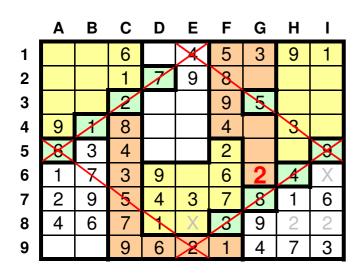
B8 is the only cell available to 6 in cluster 6 which means, in turn that A8 is the only cell available to 4 in cluster 6.

Entry Number 39

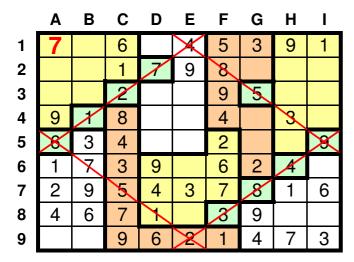


First, note that 5 and 8 must occupy A9 and B9 in some order. This means that the only cell available to 3 in cluster 6 must be B5.

Entry Number 40



First, note that 2 cannot occupy cell E8 which means that it must occupy either H8 or I8 in cluster 9, effectively excluding 2 from I6. This forces 2 in row 6 inot G6.



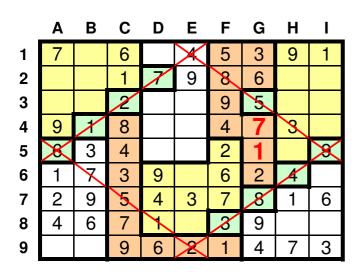
A1 is the only cell available to 7 in row 1.

Entry Number 42

_	Α	В	С	D	Ε	F	G	Н	I
1	7		6		\times	5	3	9	1
2			1	7	9	ø	6		
3			2			9	45		
4	9	1	8			4		3	
5	\aleph	3	4			2			\times
6	1	X	3	9		6	2	A	
7	2	9	155	4	ფ	7	8	1	6
8	4	6	7	f		8	9		
9			9	6	X	1	4	7	3

The only numbers still available in cluster 3 are 1, 6 and 7. Both 1 and 7 are blocked at G2, leaving this cell to 6.

Entry Number 43

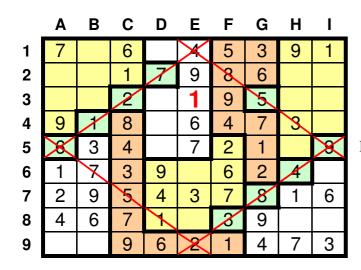


1 at B4 blocks G4, forcing 1 into G5 and leaving 7 at G4 to complete both cluster 4 and column G.

	Α	В	С	D	E	F	G	Н	I
1	7		6		\times	5	3	9	1
2			1	7	9	8	6		
3			2			9	45		
4	9	X	8		6	4	7	95	
5	\aleph	3	4		7	2	1		\times
6	1	X	3	9		6	2	Æ	
7	2	9	R	4	3	7	8	1	6
8	4	6	7	f		3	9		
9			9	6	X	1	4	7	3

The only cell available to 7 in row 5 is E5 while the only cell available to 6 in row 4 is E4.

Entry Number 45



E3 is the only cell available to 1 in cluster 3.

Entry Number 46

	Α	В	С	D	Ε	F	G	Н	I
1	7		6		\times	5	3	9	1
2			1	7	9	ø	6		
3			2		1	9	55	6	7
4	9	1	8		6	4	7	3	
5	\times	3	4		7	2	1		\varkappa
6	1	X	3	9		6	2	A	
7	2	9	75	4	3	7	8	1	6
8	4	6	7	f		8	9		
9			9	6	X	1	4	7	3

Eight 6s and eight 7s have already been located in the puzzle. H3 and I3 are the only cells still available to them.

	Α	В	С	D	Е	F	G	Н	<u> </u>
1	7		6		\times	5	3	9	1
2			1	7	9	8	6		4
3			2		1	9	155	6	7
4	9	X	8		6	4	7	95	
5	\aleph	3	4		7	2	1		\times
6	1	X	3	9		6	2	Æ	
7	2	9	5	4	3	7	8	1	6
8	4	6	7	f		8	9		
9			9	6	X	1	4	7	3

I2 is the only cell available to 4 in cluster 5 and in column I

Entry Number 48

	Α	В	С	D	Ε	F	G	Н	ı
1	7		6		\times	5	3	9	1
2			1	7	9	8	6		4
3		4	2		1	9	55	6	7
4	9	X	8		6	4	7	95	
5	\times	3	4		7	2	1		\times
6	1	X	3	9		6	2	Æ	
7	2	9	155	4	თ	7	8	1	6
8	4	6	7	f		8	9		
9			9	6	X	1	4	7	3

.... which leaves B3 as the only cell available to 4 in cluster 2, row 3, column B and the puzzle.

Entry Number 49

•	Α	В	С	D	Е	F	G	Н	I
1	7		6		\times	5	3	9	1
2	3		1	7	9	Ø	6		4
3		4	2		1	9	55	6	7
4	9	*	8		6	4	7	3	
5	\times	3	4		7	2	1		\times
6	1	X	3	9		6	2	4	
7	2	9	15	4	3	7	8	1	6
8	4	6	7	f		8	9		
9			9	6	X	1	4	7	3

A2 is the last cell available to 3 in row 2 ...

	Α	В	С	D	Е	F	G	Н	ı
1	7		6		\times	5	3	9	1
2	3		1	7	9	8	6		4
3	8	4	2	3	1	9	45	6	7
4	9	X	8		6	4	7	95	
5	\aleph	3	4		7	2	1		\varkappa
6	1	X	3	9		6	2	Æ	
7	2	9	55	4	3	7	8	1	6
8	4	6	7	f		8	9		
9			9	6	X	1	4	7	3

.... which leaves D3 as the only cell available to 3 in cluster 3, row 3, column 3 and the puzzle. This leaves 8 at A3, to complete row 3.

Entry Number 51

•	Α	В	С	D	Е	F	G	Н	I
1	7		6		\times	5	3	9	1
2	3		1	7	9	ø	6		4
3	8	4	2	3	1	9	55	6	7
4	9	1	8		6	4	7	3	
5	\aleph	3	4		7	2	1		\times
6	1	X	3	9		6	2	Æ	
7	2	9	5	4	ფ	7	8	1	6
8	4	6	7	f		8	9		
9	5	8	9	6	X	1	4	7	3

Placement of 8 at A3 allows easy completion of cluster 6, with 5 at A9 and 8 at B9.

Entry Number 52

_	Α	В	С	D	Ε	F	G	Н	I
1	7		6		\times	5	3	9	1
2	3		1	7	9	Ø	6		4
3	8	4	2	3	1	9	5	6	7
4	9	*	8		6	4	7	95	
5	\aleph	3	4		7	2	1	8	\times
6	1	X	3	9		6	2	A	
7	2	9	5	4	3	7	8	1	6
8	4	6	7	f		8	9		
9	5	8	9	6	X	1	4	7	3

H5 is the only cell available to 8 in cluster 5

	Α	В	С	D	E	F	G	Н	I
1	7		6		\times	5	3	9	1
2	3		1	7	9	8	6		4
3	8	4	2	3	1	9	45	6	7
4	9	X	8		6	4	7	95	
5	\aleph	3	4	5	7	2	1	8	\times
6	1	X	3	9		6	2	Æ	
7	2	9	R	4	3	7	8	1	6
8	4	6	7	f		3	9		
9	5	8	9	6	X	1	4	7	3

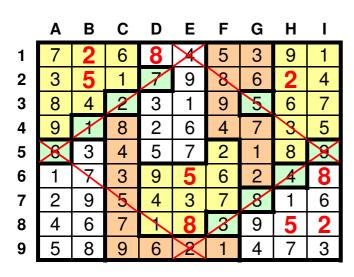
.... which forces 5 into D5, to complete row 5.

Entry Number 54

	Α	В	С	D	Ε	F	G	Н	I
1	7		6		\times	5	3	9	1
2	3		1	7	9	8	6		4
3	8	4	2	3	1	9	5	6	7
4	9	*	8	2	6	4	7	95	5
5	\times	3	4	5	7	2	1	8	\times
6	1	X	3	9		6	2	Æ	
7	2	9	15	4	3	7	8	1	6
8	4	6	7	f		8	9		
9	5	8	9	6	X	1	4	7	3

5 in row 4 is forced into cell I4, leavinf 2 to complete row 4 at D4.

Entry Number 55



Completion of the rest of the P Pandemonion is perfunctory.

The completed P Pandemonion

	Α	В	С	D	E	F	G	Н	I
1	7	2	6	8	\times	5	3	9	1
2	3	5	1	7	9	8	6	2	4
3	8	4	2	3	1	9	45	6	7
4	9	X	8	2	6	4	7	95	5
5	\aleph	3	4	5	7	2	1	8	\times
6	1	X	3	9	5	6	2	Æ	8
7	2	9	75	4	3	7	8	1	6
8	4	6	7	f	8	3	9	5	2
9	5	8	9	6	\times	1	4	7	3