Vulcraft Vibration Research
Composite Joists with Flush Framed Connections
1785 Columbus Avenue, Boston, MA

Prepared for

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1. Introduction

As part of the continual research and development efforts for Vulcraft joist floor systems, a vibration research program was performed for the Fifth and Sixth Floors of the new building at 1785 Columbus Avenue in Boston, Massachusetts. The purpose was to investigate the vibration performance of a composite joist floor system with flush framed connections. An example flush framed connection is in Figure 1.1. Compared with traditional joist seats, flush framed connections result in a shallower floor structure, but may require a slightly heavier girder or Joist Girder. For flush framed connections, the top flange of the girder or the chord of the Joist Girder is connected to the slab resulting in full composite behavior. (When traditional joist seats are used, there is a significant reduction in the full composite behavior response because of the relatively flexible shear connection provided by the joist seats.) Also, the joist to girder or Joist Girder flush framed connection is significantly stiffener than that provided by traditional joist seats and therefore provides greater continuity across the girders. Thus, the resistance to vibrations when flush framed connections are used is generally greater than that for systems with traditional joists seats.

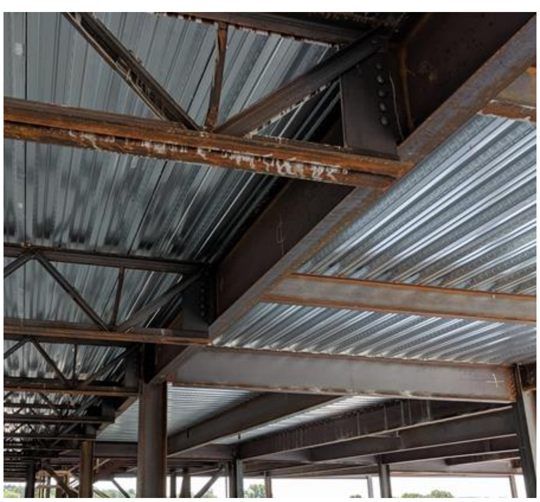
In this research, the dynamic properties and accelerations due to walking are measured to characterize the floor system. Also, measured dynamic properties and vibrations due to walking are compared with predictions from the AISC Design Guide 11 (Murray et al., 2016), hereafter referred to as DG11, to investigate the accuracy of the prediction methods for this type of floor system.

The following drawings were provided by Vulcraft – Verco Group for use in this study.

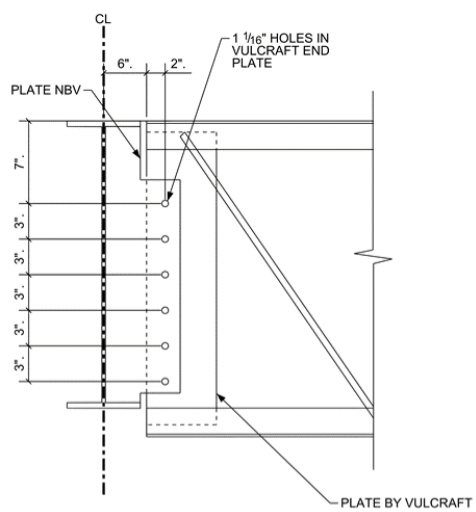
- Architectural drawings, 1/26/2018
- Structural drawings, 1/26/2018
- Vulcraft joist framing plans and details, 11/7/2018 (date on sheet C1)
- Vulcraft joist designs, 6/11/2019

The remainder of this report is organized as follows: (2) Tested Floor Areas, (3) Experimental Techniques, (4) AISC Design Guide 11 Prediction Methods, (5) Results for Fifth Floor, (6) Results for Sixth Floor, and (7) Discussion of Results.

Frequency and acceleration predictions were made using DG11 recommended finite element analysis methods using SAP 2000 (Computers and Structures, 2019) and DG11 manual prediction methods using FloorVibe v3.0 (Structural Engineers, Inc., 2021).



(a) Photograph



(b) Section from Vulcraft Joist Drawings

Figure 1.1 Flush Framed Connections

2. Tested Floor Areas

Areas on the Fifth and Sixth floors of the 1785 Columbus Avenue building that was under construction were tested on September 5, 2019 to determine the dynamic properties of the floors and vibration response due to walking. The floors supported no superimposed mass and were in the bare slab condition as shown in the photograph in Figure 2.1.

The framing plans on the Fifth and Sixth floors are nearly identical in the tested area, the shaded area in Figure 2.2.

Figure 2.3 shows the framing details in the tested area. From the provided structural drawings, the slabs are 6.25 in. thick (total) with 3 in., 20 gage composite deck. The concrete is lightweight (115 pcf) with a minimum compressive strength of 4 ksi. At time of the testing, concrete had been placed from Gridline 4 to the outside edge of floor as shown in Figure 2.3. Cladding had not been installed.

The composite joists are 30 in. deep and are spaced at approximately 10 ft apart. To the north of Gridline F, the joist length is 45 ft 8 in. To the south, the length varies from 41 ft 1 in. to 45 ft 8 in. The joist labels in the plan indicate the joist depth in inches, joist type (CJ indicates composite joist), total uniform load in plf, live load in plf, and Vulcraft joist design designation, J40 for example. Table 2.1 summarizes the top and bottom chord sizes of the joists in the tested area. Top chords are double angles with 3.5 in. or 4 in. legs. Bottom chords are double angles with 4 in. or 5 in. legs. The joists are supported by steel girders with sizes shown in Figure 2.3.

Figure 2.1 Bare Slab Condition

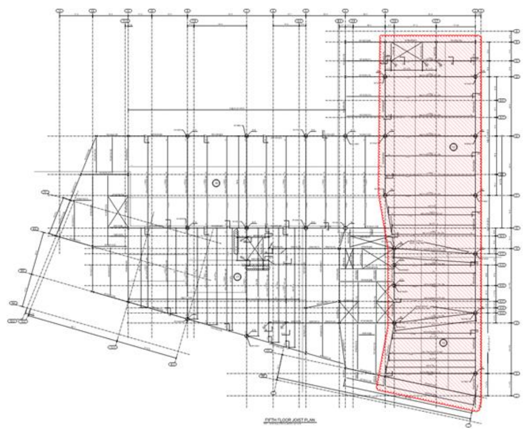


Figure 2.2 Framing Plan with Tested Area Shaded

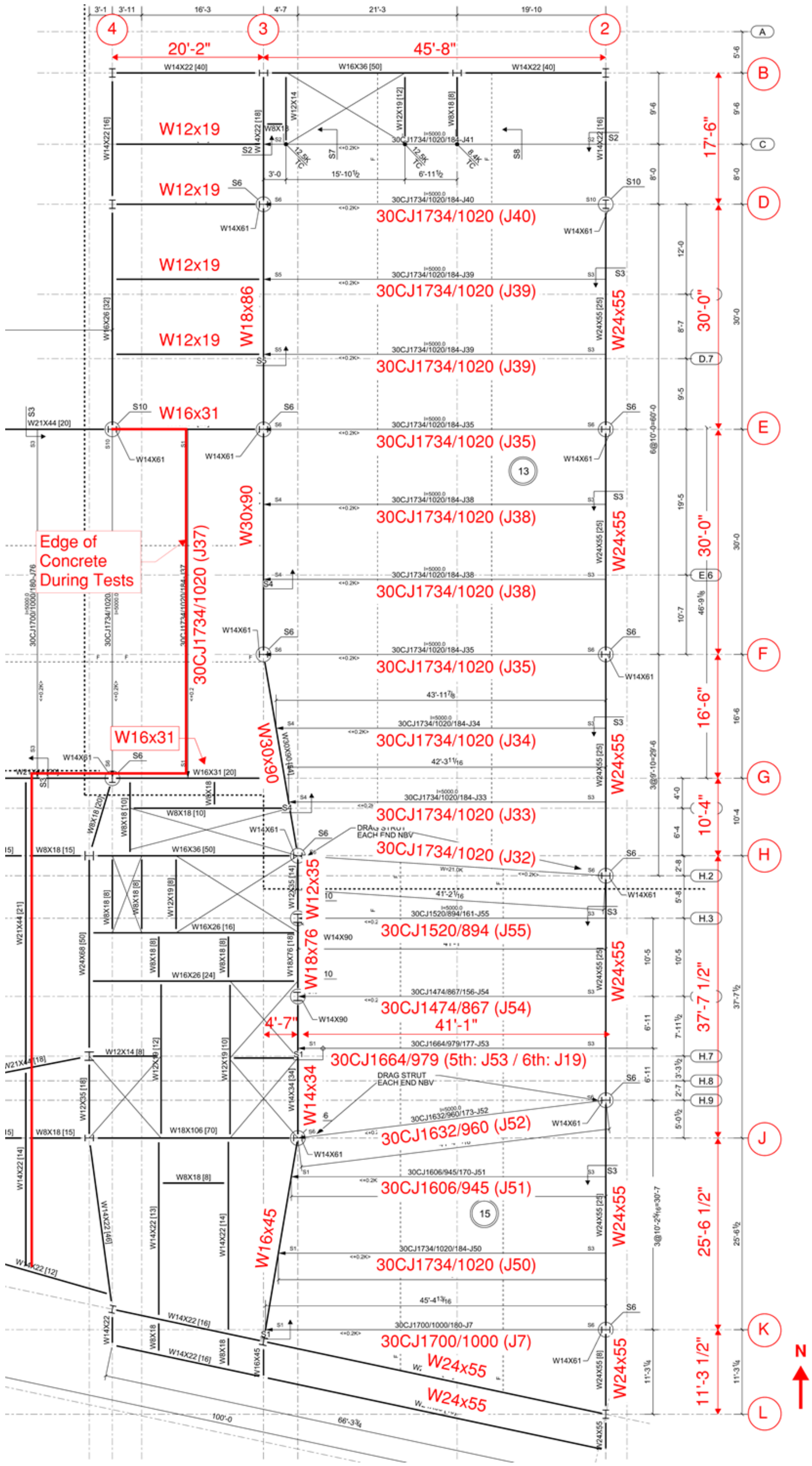


Figure 2.3 Partial Framing Plan

Table 2.1 Summary of Chord Sizes

Mark	Top Chord	Bottom Chord
J7	2L3.5x3.5x0.313	2L5x5x0.438
J32	2L4x4x0.375	2L5x5x0.438
J33	2L3.5x3.5x0.313	2L5x5x0.438
J34	2L3.5x3.5x0.375	2L4x4x0.5
J35, J37, J38, J39	2L3.5x3.5x0.344	2L4x4x0.5
J40	2L3.5x3.5x0.375	2L4x4x0.5
J50	2L3.5x3.5x0.344	2L4x4x0.5
J51	2L3.5x3.5x0.344	2L5x5x0.438
J52	2L4x4x0.375	2L5x5x0.438
J53	2L3.5x3.5x0.375	2L5x5x0.438
J54	2L3.5x3.5x0.313	2L5x5x0.438
J55	2L3.5x3.5x0.344	2L5x5x0.438

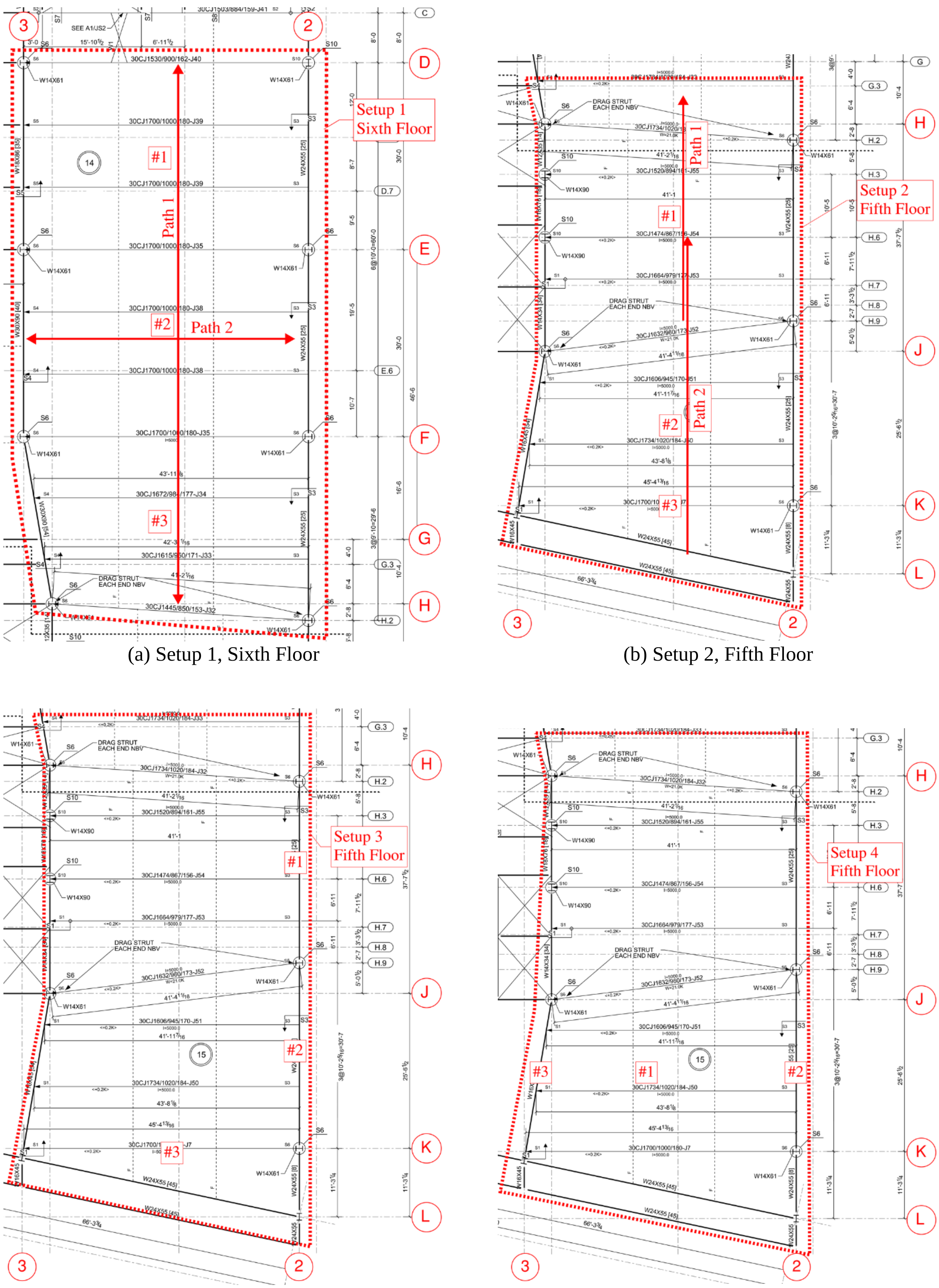
Note: Joist depth is 30.0 in.

3. Experimental Techniques

The study was conducted at the four experimental setups depicted in Figure 3.1. Modal properties from heel-drop excitations and responses to walking were determined for Setup 1 on the Sixth Floor and Setup 2 on the Fifth Floor. Additional modal information was acquired at Setups 3 and 4 on the Fifth Floor. Instrumentation used in this study consisted of a multi-channel spectrum analyzer and three seismic accelerometers at the locations indicated in the figures.

The test designation format is the setup number, heel-drop location, and accelerometer location. For example, S1HD2Loc1 was recorded at Setup 1, the heel-drop was at Location 2, and the accelerometer was at Location 1.

Heel-drop tests, as described in DG11 Chapter 8 were used to determine natural frequencies and approximate mode shapes. During each test, floor accelerations were measured while a measurement team member raised onto the balls of his feet and dropped forcefully onto his heels to apply a force with frequency content up to approximately 20 Hz. Example test results from Setup 1 are in Figure 3.2. Figures 3.2(a) through (c) are the waveforms at Locations 1 through 3. These were Fourier transformed into the spectra in Figure 3.2(d) where peaks indicate natural frequencies. The Fourier transform was also used to generate mode shape indicator plots such as Figure 3.2(e). For example, at 5.69 Hz, Locations 1 and 2 are moving approximately equal distances in opposite directions and Location 3 is not in motion. These results are used to plot an approximate mode shape.



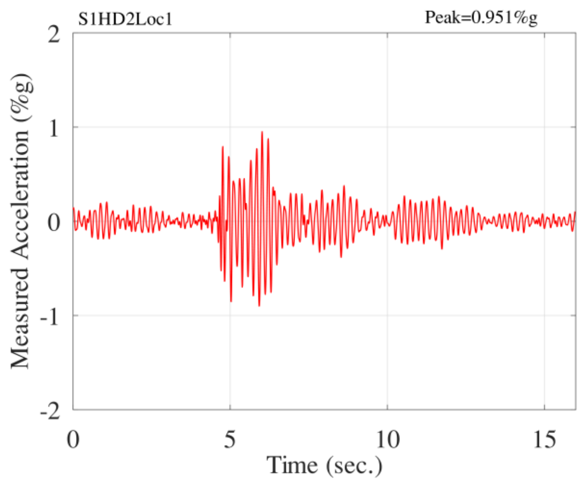
(a) Setup 1, Sixth Floor

(b) Setup 2, Fifth Floor

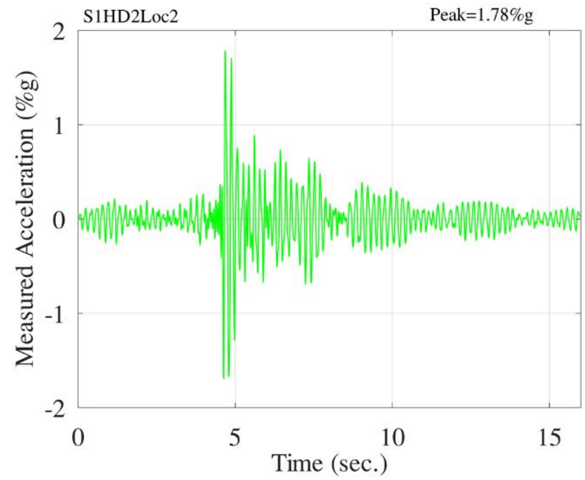
(c) Setup 3, Fifth Floor

(d) Setup 4, Fifth Floor

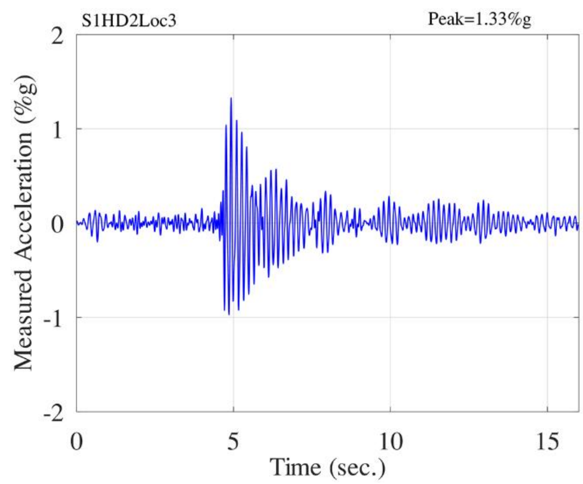
Figure 3.1 Heel-Drop Locations, Accelerometer Locations, and Walking Paths



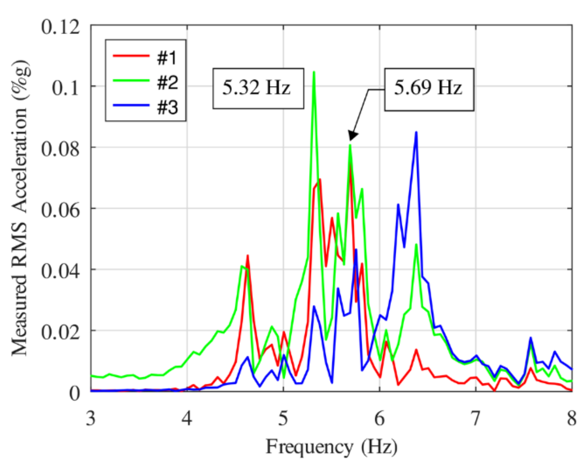
(a)Waveform, Setup 1, Location 1



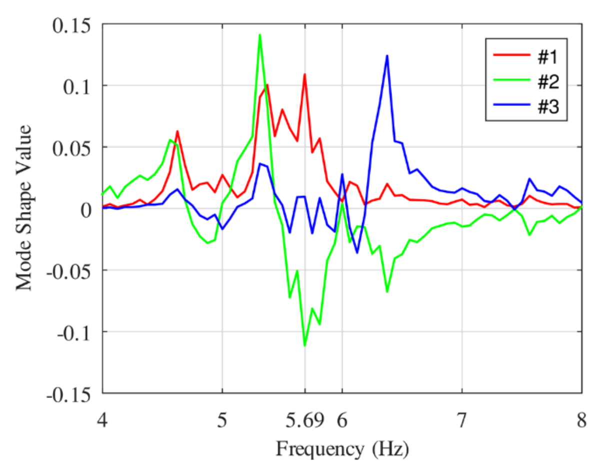
(b)Waveform, Setup 1, Location 2



(c) Waveform, Setup 1, Location 3



(d) Acceleration Spectrum (Locations 1-3)



(e) Mode Shape Indicator Plot

Figure 3.2 Example Heel-Drop Test S1HD2Loc1 Results

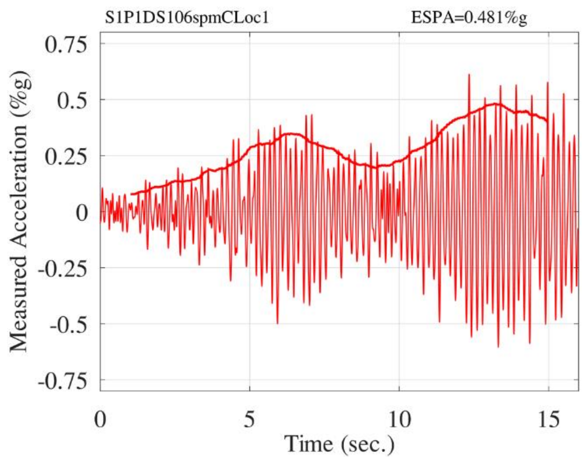
Walking tests were performed using methods from DG11 Chapter 8. Walking paths and accelerometer locations #1 through #3 are shown in Figures 3.1(a) and 3.1(b). Four walkers, weighing 180 lb (initials UJ), 185 lb (BD), 190 lb (TT), and 220 lb (DS), participated in the tests. Multiple tests were performed to assess repeatability. During each walking test, one walker traversed a walking path while listening to a metronome set at a specified step frequency within the normal range of step frequencies: 1.6 Hz (96 steps/min. (spm)) to 2.2 Hz (132 spm). The specified step frequency was an integer division of a responsive natural frequency so that a harmonic frequency matched that natural frequency and caused a resonant response. Each acceleration waveform was post-processed to eliminate high frequency sinusoids and to determine the equivalent sinusoidal peak acceleration (ESPA), which is comparable to sinusoidal peak acceleration tolerance limit in DG11 Chapter 2 for quiet spaces: 0.5%*g*.

The walking test designations are the setup number, path number, walker initials, intended step frequency, test number (A, B, or C), and accelerometer location. For example, "S1P1DS106spmCLoc2" indicates Setup 1, Path 1, the walker was DS, 106 spm, Test C, and the accelerometer was at Location 2.

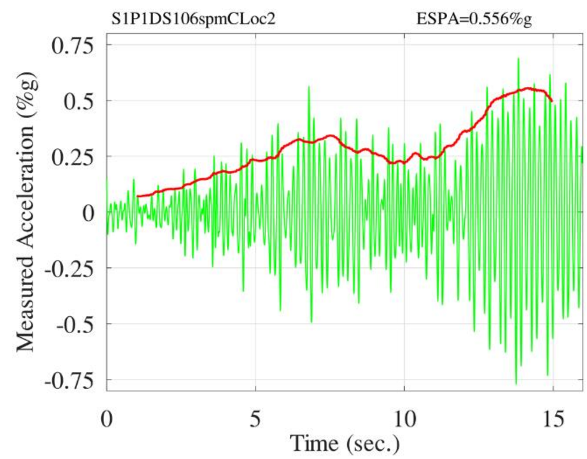
Figure 3.3 is an example walking test result. The dominant natural frequency was 5.32 Hz [Figure 3.2(d)], so the intended step frequency was 5.32 Hz / 3 = 1.77 Hz (106 spm). Figures 3.3(a) through 3.3(c) are the waveforms at Locations 1 through 3. Figures 3(a) and (b) show two resonant build-ups between approximately 2 sec. and 6 sec. and between approximately 10 sec. and 15 sec. In this example, the maximum ESPA was at Location 2, Figure 3.1(a). The spectrum for this location is shown in Figure 3.3(d). The four noted peaks indicate the responses to the first four harmonics of the walking force. The frequency of the first peak, 1.94 Hz, is the step frequency. It was slightly higher than the intended step frequency, 1.77 Hz, so the 5.69 Hz mode (Figure 3.2(d)) was excited rather than the 5.32 Hz mode.

To facilitate comparisons with predictions from DG11, the ESPAs were scaled by the ratio of a reference bodyweight to the actual bodyweight of the walker. Most of the DG11 provisions are based on a 168 lb bodyweight so that was used as the reference. The scaled ESPAs are referred to as $ESPA_{168}$ in the summary tables in Sections 5 and 6.

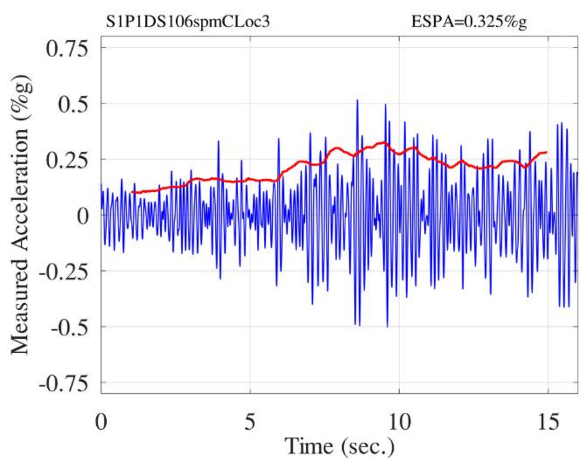
Plots for all measurements are in Appendices A through F.



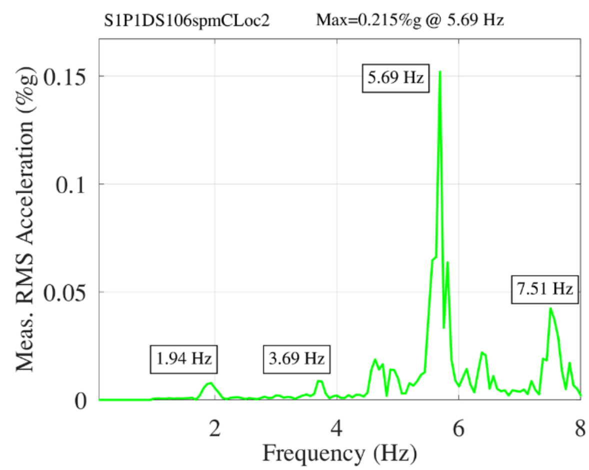
(a) Waveform – Location 1



(b) Waveform – Location 2



(c) Waveform – Location 3



(d) Spectrum – Location 2

Figure 3.3 Example Walking Test S1P1DS106spmCLoc2 Results

4. AISC Design Guide 11 Prediction Methods

4.1 Finite Element Analysis Method

The finite element model in Figure 4.1 was developed in SAP2000 using the methods in DG11 Section 7.2. The slabs were represented by shell elements. Joists and w-shape beams were modeled using frame elements in the plane of the shells. The effective moment of inertia of each joist was computed using DG11 Section 3.5, specifically Equations 3-7, 3-8, and 3-9a, with the actual chord sizes from Table 3.1. Because the top of each girder was connected to the underside of slab, the fully composite transformed moment of inertia of the girders was used in the model. The floors in this study were bare slabs at the time of testing, so the recommended damping ratio, $\beta = 0.01$, in DG11 was used in the analyses.

The natural modes were predicted using eigenvalue analysis and frequency response functions (FRFs) were predicted using the Steady State Analysis feature. Accelerations were predicted at nodes corresponding to accelerometer locations in Figure 3.1.

The accelerations due to walking were predicted using the FRF Method from DG11 Section 7.4.1. With this method, the predicted acceleration is:

$$a_p = 0.09 FRF_{max} e^{-0.075 f_n} Q\rho \qquad \text{(DG11 Eq. 7-1 and 7-2)}$$

where

FRF_{max}	=	maximum magnitude of FRF, %g/lb
f_n	=	natural frequency, Hz
Q	=	bodyweight, lb
ρ	=	resonant build-up factor

The resonant build-up factor was determined from:

$$\rho = 50\beta + 0.25 \text{ if } \beta < 0.01 \qquad \text{(DG11 Eq. 7-3a)}$$

$$\rho = 12.5\beta + 0.625 \text{ if } 0.01 \le \beta < 0.03 \qquad \text{(DG11 Eq. 7-3b)}$$

$$\rho = 1.0 \text{ if } \beta \ge 0.03 \qquad \text{(DG11 Eq. 7-3c)}$$

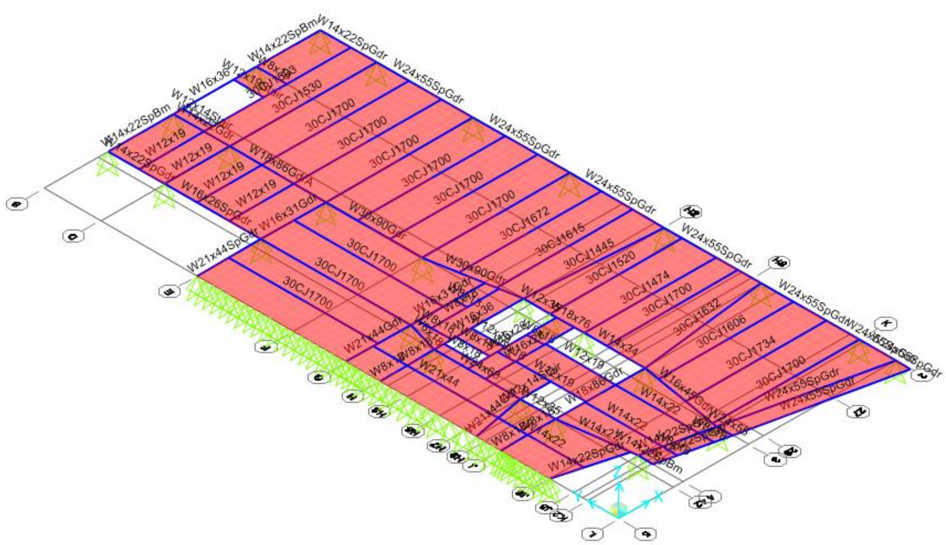


Figure 4.1 Finite Element Model

4.2 Manual Calculation Methods

The manual calculation methods in DG11 Chapters 3 and 4 were used to predict the natural frequencies and accelerations due to walking in the tested area. These calculations directly apply for Setup 1, Location 2 (Figure 3.1(a)). They are approximate for Setup 2, Locations 1 and 2 because the joists are slightly different within the bays, the joist spacings are nonuniform, and because the beams at the edges of the bays are skewed.

Fundamental natural frequencies of joists and girders were predicted using the DG11 Equation 3-1, which applies to simply-supported beams with uniform mass and flexural rigidity. Joist effective moments of inertia were computed using DG11 Section 3.5 for the actual chords from Table 2.1. Because the top of each girder was connected to the underside of the deck, the fully composite girder moment of inertia was used.

$$\begin{aligned} f_{j,g} &= \frac{\pi}{2}\sqrt{\frac{gE_s I_{j,g}}{w_{j,g} L_{j,g}^4}} \\ &= 0.18\sqrt{\frac{g}{\Delta_{j,g}}} \end{aligned} \qquad \text{(DG11 Eq. 3-1)}$$

where

- g = gravitational acceleration, 386 in./sec.^{2}
- E_s = modulus of elasticity of steel, ksi
- I_j = effective moment of inertia of the joist, in.^{4}
- I_g = fully composite transformed moment of inertia of the girder, in.^{4}
- $w_{j,g}$ = lineal weight supported by the joist or girder, kips/in.
- $L_{j,g}$ = length of joist or girder, in.
- $\Delta_{j,g}$ = deflection of a uniformly loaded simple span joist or girder, in.

The bay natural frequencies were computed using the Dunkerley equation from DG11 Equation 3-2, and equivalently, Equation 3-4. Our research indicates this equation underpredicts the natural frequency by 20-30%. However, the natural frequency from the Dunkerley equation is required input for the acceleration equation below.

$$f_n = 0.18\sqrt{\frac{g}{\Delta_j + \Delta_g}} \qquad \text{(DG11 Eq. 3-4)}$$

The bay natural frequencies were also computed using the Minimum Frequency Method from DG11 Equation 6-1 for comparison with measured natural frequencies.

$$f_n = \min(f_j, f_g) \qquad \text{(DG11 Eq. 6-1)}$$

The joist panel effective weights were computed using DG11 Equation 4-2, reproduced below. DG11 Section 4.1.2 (just above Equation 4-5) indicates that the joist panel weight can be increased by 50% where joists are continuous over their supports and an adjacent span length is at least $0.7L_j$. The flush framed connections provide sufficient continuity, but none of the joists in this study had adjacent spans long enough for the 50% increase to apply.

$$W_j = w_j B_j L_j \qquad \text{(DG11 Eq. 4-2)}$$

The girder panel effective weights were also computed using DG11 Equation 4-2.

The combined mode effective panel weight is the flexibility weighted average of the joist panel and girder panel effective weights from DG11 Equation 4-5.

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_g}{\Delta_j + \Delta_g} W_g \qquad \text{(DG11 Eq. 4-5)}$$

The peak accelerations due to walking were predicted using DG11 Chapter 4-1. This equation computes the resonant response of floors with natural frequencies below approximately 9 Hz. It assumes an incomplete resonant build-up and that the affected occupant and walker are not simultaneously at mid-bay (Allen and Murray, 1993). This equation provides extremely accurately predictions of the final evaluation – satisfactory or unsatisfactory – of floor systems supporting quiet environments such as offices and residences.

$$\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta W} \qquad \text{(DG11 Eq. 4-1)}$$

where

f_n = bay natural frequency from the Dunkerley equation, Hz
P_o = static force representing the dynamic load, lb
β = critical damping ratio, 0.01 for the bare slabs in this study

Calculations in this section were accomplished using FloorVibe 3.00; the reports are included in the appendices.

5. Results from Fifth Floor Measurements

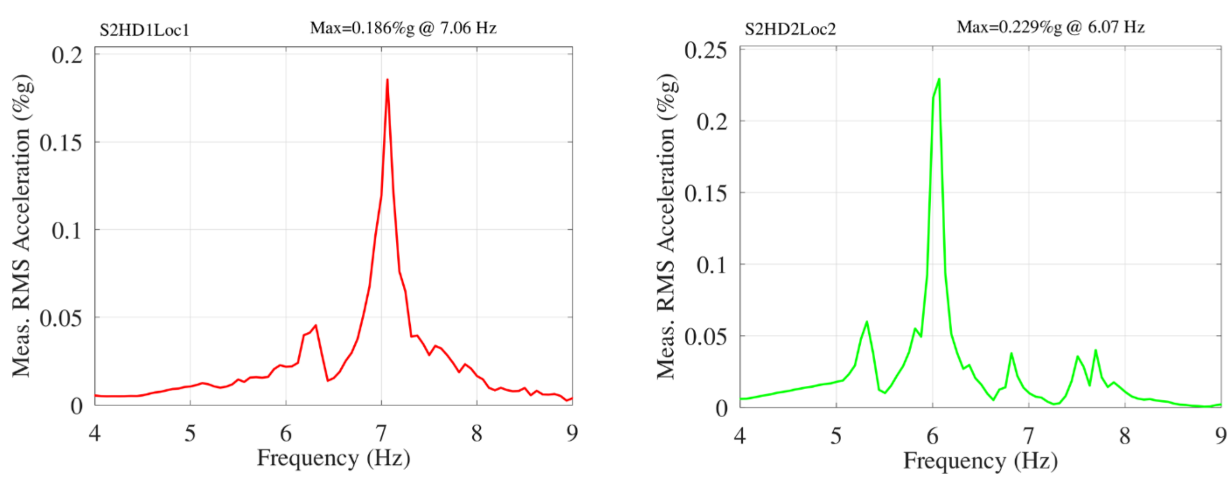
In this section, fifth floor measured natural frequencies, mode shapes, and accelerations due to walking and predictions from finite element analyses and manual calculations are presented for Setup 2. These measurements contain modal and walking data. Setups 3 and 4 measurement results are used to enhance measured mode shapes.

5.1 Natural Frequencies and Mode Shapes

5.1.1 Measured Values

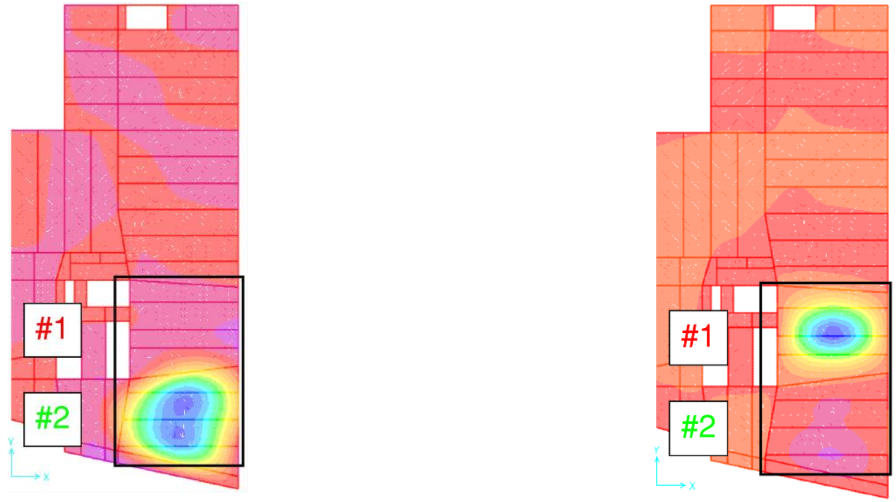
Figure 5.1 shows the heel-drop test results for Setup 2, Locations 1 and 2 (Figure 3.1(b)). At Location 1, the mode at 7.06 Hz was the only mode with significant response. At Location 2, the mode at 6.07 Hz was the only mode with significant response.

The inferred natural mode shapes are shown in Figure 5.2. The black boxes in the figures indicate the extent of the area with mode shape information from the tests in Setups 2, 3, and 4. The mode at 6.07 Hz has motion only in the bay with Location 2, and the mode at 7.06 Hz has motion only in the bay with Location 1. The bay with Location 1 has shorter joists than the bay with Location 2, hence the significantly different natural frequencies in those bays and the lack of interaction between these two bays – the 6.07 Hz mode has little or no motion at Location 1 and the 7.06 Hz mode has little or no motion at Location 2.



(a) Setup 2, Location 1 (b) Setup 2, Location 2

Figure 5.1 Measured Acceleration Due to Heel-Drop – Fifth Floor, Setup 2



(a) Mode at 6.07 Hz

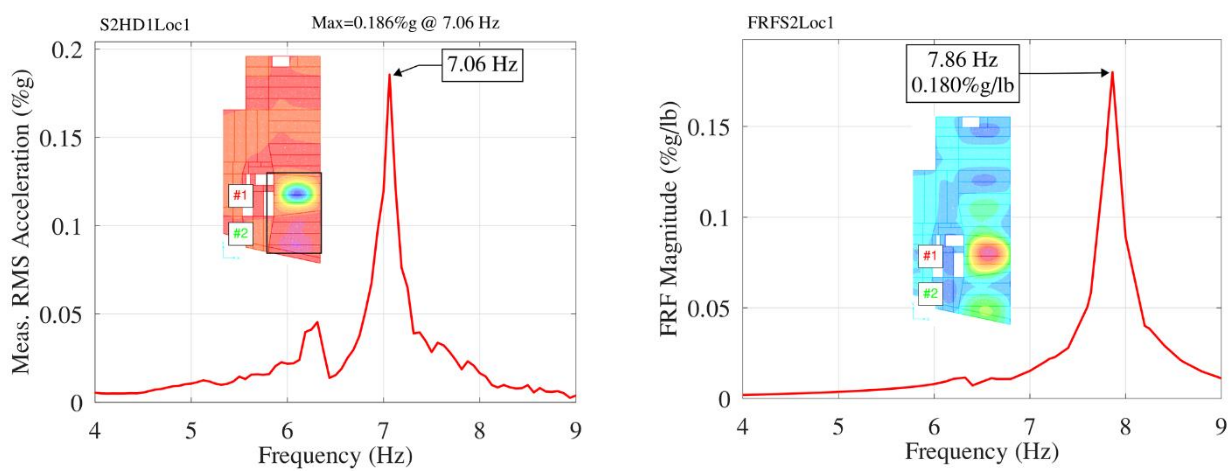
(b) Mode at 7.06 Hz

Figure 5.2 Measured Mode Shapes – Fifth Floor

5.1.2 Comparisons of Measurements and Predictions – Finite Element Analysis

Figure 5.3 shows the measured heel-drop spectrum and predicted FRF magnitude for force and acceleration at Setup 2, Location 1. The measured and predicted natural modes are also included. The measured natural frequency is 7.06 Hz and the corresponding mode shape has much higher values in the bay with Location 1 than in the bay with Location 2. The predicted natural frequency, 7.86 Hz, corresponds to a mode shape that is similar to the measured mode shape. Thus, the measured 7.06 Hz and predicted 7.86 Hz frequencies are comparable. The predicted frequency is 11% higher than the measured value.

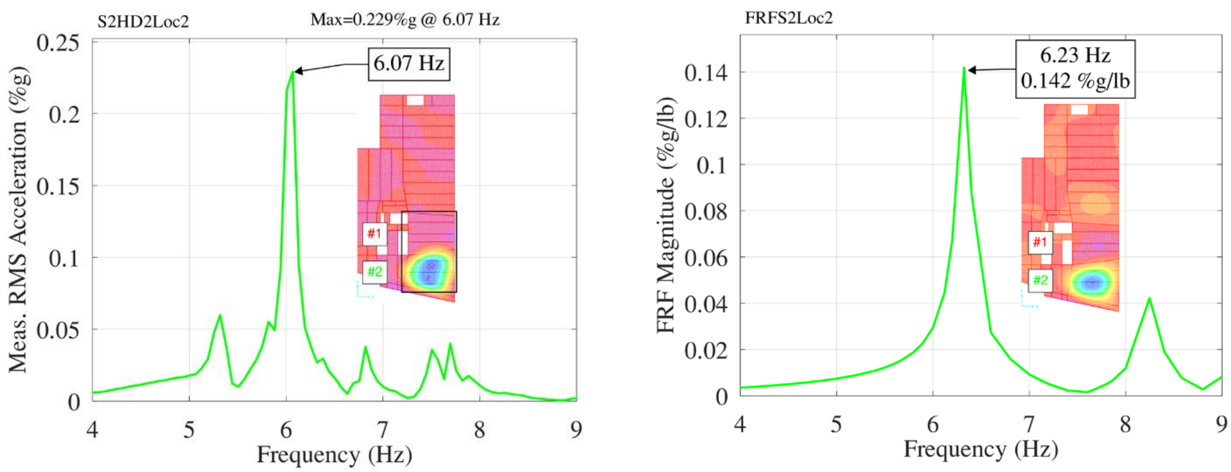
Figure 5.4 shows the measured heel-drop spectrum and predicted FRF magnitude for force and acceleration at Setup 2, Location 2. The measured natural frequency is 6.07 Hz and the corresponding mode shape has much higher values in the bay with Location 2 than in the bay with Location 1. The predicted natural frequency, 6.23 Hz, corresponds to a mode shape that resembles the measured mode shape, so the measured 6.07 Hz and predicted 6.23 Hz frequencies are comparable. The predicted frequency is 2.6% higher than the measured value.



(a) Measured Heel-Drop Acceleration Spectrum

(b) Predicted FRF Magnitude

Figure 5.3 Measured Heel-Drop Acceleration and Predicted FRF Magnitude – Fifth Floor, Setup 2, Location 1



(a) Measured Heel-Drop Acceleration Spectrum

(b) Predicted FRF Magnitude

Figure 5.4 Measured Heel-Drop Acceleration and Predicted FRF Magnitude – Fifth Floor, Setup 2, Location 2

5.1.3 Comparisons of Measurements and Predictions – Manual Calculation Method

The methods in Section 4.2 were used to predict the fundamental natural frequencies of the bays with Setup 2, Locations 1 and 2.

For the bay with Location 1, the measured natural frequency is 7.06 Hz. The three interior girders at the west edge of the bay are much shorter than the exterior girder, so in the frequency prediction, the exterior girder length and size were used. The joists in this bay have variable spacings and loads. Thus, the joists were modeled as uniformly spaced with the load of the joist nearest mid-bay. The predicted joist mode frequency is 7.44 Hz and the girder frequency is 8.60 Hz. The bay frequency from the Dunkerley equation, 5.63 Hz, is 20% lower than the measured frequency. The bay frequency from the Minimum Frequency Method, 7.44 Hz, is 5% higher than the measured frequency.

For the bay with Location 2, the measured natural frequency is 6.07 Hz. The bay with Location 2 has unequal interior and exterior girder lengths and sizes, and the joist lengths and spacings vary. Two analyses were performed: first with the length and size of the interior girder to determine the interior girder frequency and second with the length and size of the exterior girder to determine the exterior girder frequency. In both analyses, the middle joist was used at a uniform spacing. The interior girder frequency is lower, so the results of the first analysis are reported here. The predicted joist mode frequency is 6.78 Hz, the interior girder frequency is 6.75 Hz, and the spandrel girder frequency is 8.05 Hz. The bay frequency from the Dunkerley equation, 4.78 Hz, is 21% lower than the measured frequency. The bay frequency from the Minimum Frequency Method, 6.75 Hz, is 11% higher than the measured frequency.

5.2 Accelerations Due to Walking

5.2.1 Measured Values

Table 5.1 summarizes the walking tests conducted at Setup 2, Path 1 [Figure 3.1(b)]. The acceleration is reported at Location 1, which is along Path 1. Two walkers walked on this path. The average $ESPA_{168}$ was 0.483%g. The average $ESPA_{168}$ was less than the tolerance limit for a quiet office, 0.5%g. The maximum $ESPA_{168}$, 0.690%g, exceeded the limit. When office furniture, partitions, and ceilings are added, the damping would increase significantly, and will result in accelerations below the limit.

Table 5.2 summarizes the walking tests conducted at Setup 2, Path 2. The acceleration is reported at Location 2, which is along Path 2. Three walkers walked on this path. The average and maximum $ESPA_{168}$, 0.295%g and 0.322%g, were far below the 0.5%g recommended tolerance limit for quiet offices. Even in the tested bare slab condition, the vibration levels in this bay are satisfactory. When office furniture, partitions, and ceilings are added, the mass and damping will increase significantly, further reducing the accelerations.

Table 5.1 Summary of Walking Tests – Fifth Floor, Setup 2, ESPA at Location 1

Test Designation	Bodyweight (lb)	ESPA (%g)	$ESPA_{168}$ (%g)
S2P1DS106spmALoc1	220	0.684	0522
S2P1DS106spmBLoc1	220	0.635	0.485
S2P1DS106spmCLoc1	220	0.903	0.690
S2P1UJ106spmALoc1	180	0.431	0.402
S2P1UJ106spmBLoc1	180	0.439	0.409
S2P1UJ106spmCLoc1	180	0.414	0.387
		Average =	0.483
		Maximum =	0.690

Table 5.2 Summary of Walking Tests – Fifth Floor, Setup 2, ESPA at Location 2

Test Designation	Bodyweight (lb)	ESPA (%g)	$ESPA_{168}$ (%g)
S2P2BD121spmALoc2	185	0.322	0.293
S2P2BD121spmBLoc2	185	0.233	0.211
S2P2BD121spmCLoc2	185	0.361	0.328
S2P2DS121spmALoc2	220	0.386	0.295
S2P2DS121spmBLoc2	220	0.422	0.322
S2P2UJ121spmALoc2	180	0.327	0.305
S2P2UJ121spmBLoc2	180	0.327	0.305
S2P2UJ121spmCLoc2	180	0.320	0.299
		Average =	0.295
		Maximum =	0.328

5.2.2 Comparisons of Measurements and Predictions – Finite Element Analysis

The method from Section 4.1 was used to predict acceleration due to walking through the middle of the bay with Setup 2 Location 1, Figure 3.1(b). The predicted FRF is shown in Figure 5.3(b). Using the maximum FRF magnitude and a damping ratio of 0.01, the predicted sinusoidal peak acceleration is 1.13%g. This value is 1.6 times the maximum measured $ESPA_{168}$.

The same method was used to predict acceleration due to walking through the middle of the bay with Location 2. The predicted FRF is shown in Figure 5.4(b). Using the maximum FRF magnitude and a damping ratio of 0.01, the predicted sinusoidal peak acceleration is 1.01%g. This value is 3.1 times the maximum measured $ESPA_{168}$.

5.2.3 Comparisons of Measurements and Predictions – Manual Calculation Method

The method from Section 4.2 was used to predict acceleration due to walking through the middle of the bay with Setup 2 Location 1. The details of the analysis of this irregular bay are in Section 5.1.3. The predicted sinusoidal peak acceleration is 1.51%g. This value is 2.2 times the maximum $ESPA_{168}$.

The same method was used to predict acceleration due to walking through the middle of the bay with Location 2. The predicted sinusoidal peak acceleration is 1.85%g, which is 5.6 times the maximum $ESPA_{168}$.

6. Results from Sixth Floor Measurements

In this section, measured natural frequencies, mode shapes, and accelerations due to walking and predictions from finite element analyses and manual calculations are presented for Setup 1 (Figure 3.1(a)).

6.1 Natural Frequencies and Mode Shapes

6.1.1 Measured Values

Figure 6.1 shows test results for a heel-drop at Setup 1, Location 2 with measured accelerations at Locations 1, 2 and 3 as shown in Figure 3.1(a). This heel-drop location was chosen because the walking tests described below were conducted through this bay. This test indicates natural modes at 4.63 Hz, 5.32 Hz, 5.69 Hz, and 6.38 Hz. At Location 2, the mode at 5.32 Hz provided the highest response; the mode at 5.69 Hz also provided a high response.

The inferred natural mode shapes are shown in Figure 5.2. The red boxes in the figures indicate the extent of the area with mode shape information. (The testing program provides no information on the mode shape outside that area.)

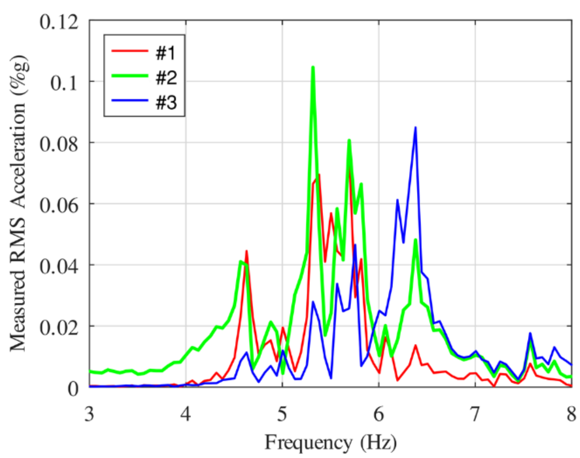
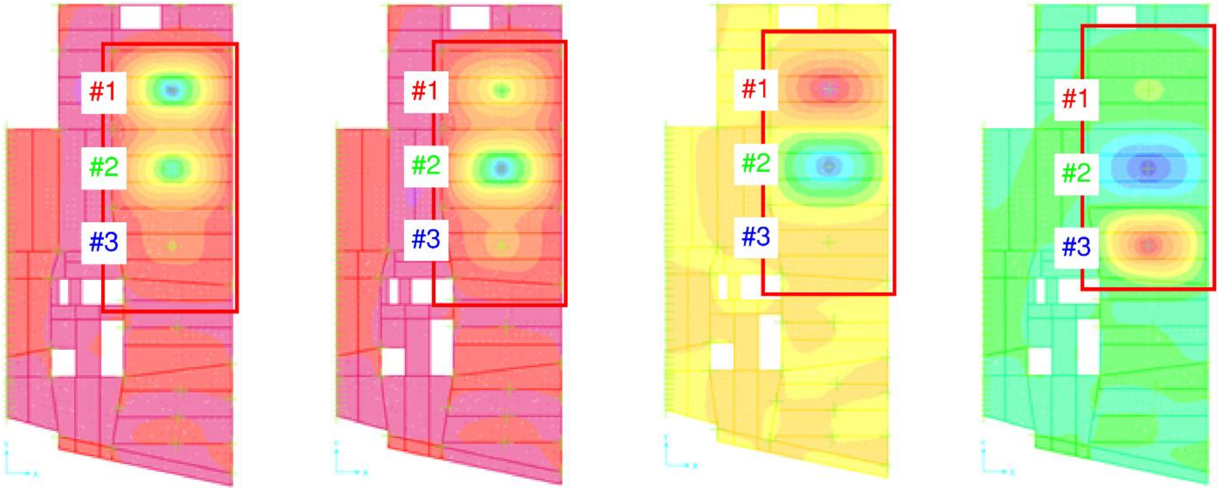


Figure 6.1 Measured Accelerations Sixth Floor, Setup 1, Locations 1, 2 and 3 Due to Heel-Drop –at Location 2.



(a) Mode at 4.63 Hz (b) Mode at 5.32 Hz (c) Mode at 5.69 Hz (d) Mode at 6.38 Hz

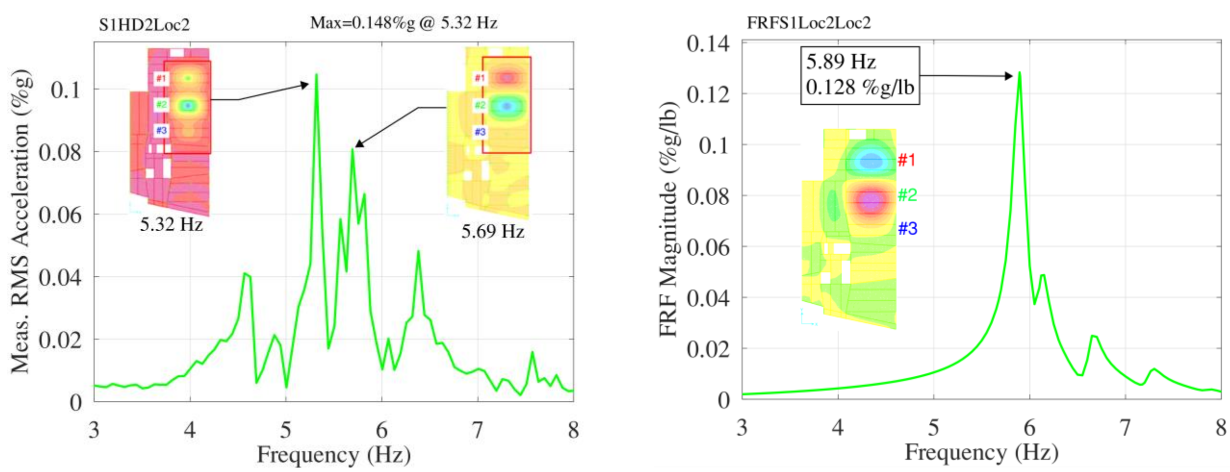
Figure 6.2 Measured Mode Shapes – Sixth Floor, Setup 1

6.1.2 Comparisons of Measurements and Predictions – Finite Element Analysis

In this section, the measured heel-drop spectrum and predicted FRF magnitude are compared to investigate the accuracy of the prediction method. For both, the force and response are at Setup 1, Location 2, which was chosen because the walking measurements focused on that location.

The measured heel-drop spectrum in Figure 6.3(a) indicates responsive natural modes at 5.32 Hz and 5.69 Hz. The 5.32 Hz peak is due to a mode shape with motion in the same direction at Setup 1, Locations 1 and 2, and slight motion at Location 3. The 5.69 Hz peak is due to a mode with opposite direction motion at Locations 1 and 2 and very slight motion at Location 3.

The predicted FRF indicates a responsive mode at 5.89 Hz. This mode has opposite direction motion at Locations 1 and 2, which is the same as the shape for the measured 5.69 Hz mode, so these modes are comparable. The predicted frequency is about 3.5% higher than the measured value.



(a) Measured Heel-Drop Acceleration Spectrum (b) Predicted FRF Magnitude

Figure 6.3 Measured Heel-Drop Acceleration and Predicted FRF Magnitude – Sixth Floor, Setup 1, Location 2

6.1.3 Comparisons of Measurements and Predictions – Manual Calculation Method

The fundamental natural frequency of the bay with Location 2 was predicted by methods in Section 4.2 assuming a damping ratio of 0.01 and fully composite girder moment of inertia, and using the effective joist moment of inertia.

The predicted joist mode frequency is 6.05 Hz, the interior girder frequency is 11.6 Hz, and the spandrel girder frequency is 8.24 Hz

The bay frequency from the Dunkerley equation is 4.88 Hz. This value is 8.3% lower than the measured 5.32 Hz frequency and 14% lower than the measured 5.69 Hz natural frequency.

The bay frequency from the Minimum Frequency Method, 6.05 Hz, is 14% higher than the measured 5.32 Hz frequency and 6.3% higher than the 5.69 Hz natural frequency.

At this location, the most responsive measured frequency at 5.32 Hz is approximately half-way between the under-prediction from the Dunkerley equation and the over-prediction from the Minimum Frequency Method.

6.2 Accelerations Due to Walking

6.2.1 Measured Values

Table 6.1 summarizes the walking tests conducted at Setup 1 (Figure 3.1(a)). Two walkers walked along Path 1 and three walkers walked along Path 2. The average $ESPA_{168}$ from all tests was only 0.247%g. The average for walking along Path 1 was slightly higher than the average for Path 2. Path 1 is longer than Path 2, so there were more opportunities to generate a series of several footsteps at the resonant frequency.

The average $ESPA_{168}$ was less than half the tolerance limit for a quiet office or residence, 0.5%g. Also, the maximum $ESPA_{168}$, 0.425%g, was below the limit. Even in the tested bare slab condition, the vibration levels in this bay would be satisfactory. When office furniture, partitions, and ceilings are added, the damping would increase significantly, further reducing the accelerations.

Table 6.1 Summary of Walking Tests – Sixth Floor, Setup 1, Paths 1 and 2, ESPA at Location 2

Test Designation	Bodyweight (lb)	ESPA (%g)	$ESPA_{168}$ (%g)	
S1P1DS106spmALoc2	220	0.331	0.253	Path 1 Average 0.279 %g
S1P1DS106spmBLoc2	220	0.351	0.268	
S1P1DS106spmCLoc2	220	0.556	0.425	
S1P1TT106spmALoc2	190	0.300	0.265	
S1P1TT106spmBLoc2	190	0.259	0.229	
S1P1TT106spmCLoc2	190	0.269	0.238	
S1P2DS106spmALoc2	220	0.348	0.266	Path 2 Average 0.225 %g
S1P2DS106spmBLoc2	220	0.368	0.281	
S1P2DS106spmCLoc2	220	0.324	0.248	
S1P2TT106spmALoc2	190	0.223	0.197	
S1P2TT106spmBLoc2	190	0.241	0.213	
S1P2TT106spmCLoc2	190	0.318	0.281	
S1P2UJ106spmALoc2	180	0.168	0.156	
S1P2UJ106spmBLoc2	180	0.248	0.231	
S1P2UJ106spmCLoc2	180	0.164	0.153	
		Average =	0.247	
		Maximum =	0.425	

6.2.2 Comparisons of Measurements and Predictions – Finite Element Analysis

The method from Section 4.1 was used to predict acceleration at Location 2 due to walking through the middle of the bay with Setup 1. The predicted FRF is shown in Figure 6.3(b). Using the maximum FRF magnitude and a damping ratio of 0.01, the predicted sinusoidal peak acceleration is 0.935%g. This value is 2.2 times the maximum measured $ESPA_{168}$.

6.2.3 Comparisons of Measurements and Predictions – Manual Calculation Method

The method from Section 4.2 was used to predict acceleration at Location 2 due to walking through the middle of the bay with Setup 1. The predicted sinusoidal peak acceleration is 1.60%g. This value is 3.8 times the maximum $ESPA_{168}$.

7. Discussion of Results

7.1 Natural Frequencies

Table 7.1 summarizes the measured and predicted natural frequencies, and the measured-to-predicted (M / P) frequency ratios.

The finite element analysis method using the effective moment of inertia of the joists and fully composite moment inertia of the girder slightly over-predicted the natural frequency at each of the three locations. The average measured-to-predicted ratio was 0.946. Predictions within 5-10% are typical for the finite element analysis method used herein.

The manual calculation method with the Dunkerley equation under-predicted the natural frequency at each of the three locations. The average measured-to-predicted ratio was 1.20. This finding is consistent with our experience on previous projects.

The manual Minimum Frequency Method over-predicted the natural frequency at each of the three locations. The average measured-to-predicted ratio was 0.909, a result that is typical – within 5-10% – for the minimum frequency method.

Conclusions

- The finite element analysis method slightly over-predicted the natural frequencies. The average prediction was approximately 5% higher than the measured value.
- The finite element analysis method predicted mode shapes that agree with three of the four measured mode shapes. It did not predict a mode resembling the 5.32 Hz mode shape at Setup 1 on the Sixth Floor; the cause is unknown.
- The manual calculation method with the Dunkerley equation resulted in significant under-predictions of the natural frequencies. However, it is noted that the frequency from the Dunkerley equation must be used in DG11 Equation 4-1.
- The manual Minimum Frequency Method slightly over-predicted the natural frequencies, with the average prediction within approximately 10% of the measured value.

Table 7.1 Comparisons of Measured and Predicted Natural Frequencies

	Measured (Hz)	Finite Element Analysis		Manual (Dunkerley)		Manual (min(f_j, f_g))	
		Predicted (Hz)	M / P[1]	Predicted (Hz)	M / P[1]	Predicted (Hz)	M / P[1]
Setup 1, Location 2 Sixth Floor E-2/F-3	5.32, 5.69	5.89	0.966	4.88	1.09	6.05	0.879
Setup 2, Location 1 Fifth Floor H.2-2/J-2.9[2]	7.06	7.86	0.898	5.63	1.25	7.44	0.949
Setup 2, Location 2 Fifth Floor H.9-2/CC-3[2]	6.07	6.23	0.974	4.78	1.27	6.75	0.899
		Average:	0.946	Average:	1.20	Average:	0.909

Notes: 1. M/P = Measured/Predicted; 2. irregular bay – analysis details in Section 5.1.3

7.2 Acceleration Due to Walking

Table 7.2 summarizes the maximum measured $ESPA_{168}$ values and predictions by the finite element analysis and manual calculation methods. Each bay had a natural frequency below 9 Hz, so each bay was subject to resonant responses due to walking.

For the Finite Element Analysis method, the measured-to-predicted ratios ranged from 0.325 to 0.611 and averaged 0.464. The average prediction is slightly over double the maximum measured acceleration.

For the Manual Calculations Method, the measured-to-predicted ratios ranged from 0.177 to 0.457 and averaged 0.300. The average prediction is slightly over triple the maximum measured acceleration.

Interestingly, the Fifth Floor Setup 2, Location 1 [Figure 3.1(b)] has openings along the plan-west side of the bay, so the joists in this bay have no potential for continuity with adjacent framing. The maximum measured $ESPA_{168}$ are much higher in this bay than at Locations 2 and 3 in adjacent two bays. Likewise, the predicted acceleration from the finite element method is slightly higher than in the adjacent two bays

To determine whether the over-predictions are typical for each method, the measured-to-predicted acceleration ratios are compared to those from previous experimental programs. Figures 7.1 and 7.2 are plots of these ratios for approximately two dozen bays from the writer's research and the three bays reported herein. The figures indicate that the predictions for the present bays fall toward the lower end of the normal scatter for the prediction methods. For both methods, most of the underpredictions with M/P ratios below 0.5 were for bare slabs. This is probably because the extent of the vibrating area is much larger for bare slab floors due to the lack of nonstructural components, resulting in effective masses that are larger than predicted; however, the cause is not known with certainty.

Table 7.2 Comparisons of Measured and Predicted Accelerations Due to Walking

	Max. Measured	Finite Element Analysis		Manual Calculations	
	$ESPA_{168}$ (%g)	Acceleration (%g)	M / P	Acceleration (%g)	M / P
Setup 1, Location 2 Sixth Floor E-2/F-3	0.425	0.935	0.455	1.60	0.266
Setup 2, Location 1 Fifth Floor H.2-2/J-2.9*	0.690	1.13	0.611	1.51	0.457
Setup 2, Location 2 Fifth Floor H.9-2/CC-3*	0.328	1.01	0.325	1.85	0.177
		Average:	0.464	Average:	0.300

*Irregular bay – analysis details in Section 5.1.3

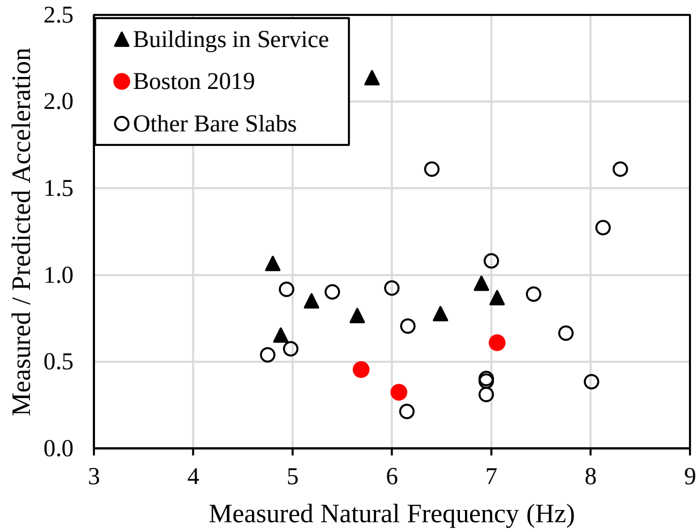


Figure 7.1 Measured-to-Predicted Ratios – Finite Element Analysis

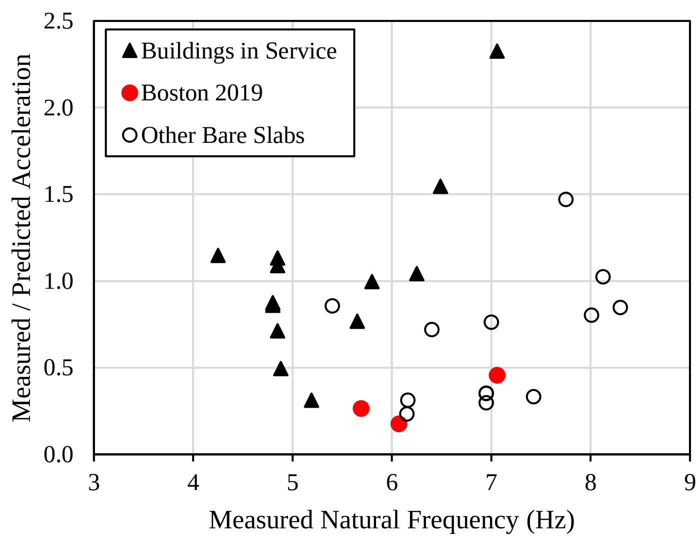


Figure 7.2 Measured-to-Predicted Ratios – Manual Calculation Method

8. Summary and Conclusions

In this study, a floor system with Vulcraft composite joists using flush framed connections is analyzed using combined experimental measurements and analytical predictions. The purpose was to quantify the vibration characteristics of the floor system and to investigate the accuracy and applicability of the vibration design methods in the AISC Design Guide 11.

The finite element analysis method accurately predicted the responsive natural frequencies and mode shapes in the tested bays. On average, the predicted natural frequencies were 5% higher than measured. Four responsive modes were identified from the measurements and the corresponding mode shapes were approximated. The finite element analysis method correctly predicted the shape of three of these four modes; it did not predict a mode resembling one measured mode.

The Minimum Natural Frequency manual calculation method fairly accurately predicted the responsive natural frequencies in the tested bays. On average, the predicted natural frequencies were 10% higher than measured.

Accelerations due to walking were measured in three bays. Even though the floor was in the bare slab condition without nonstructural components that add damping and mass, the vast majority of accelerations were far below the recommended tolerance limit for quiet office spaces.

The finite element analysis method over-predicted the acceleration due to walking by a factor of approximately two. The manual calculation method over-predicted the acceleration due to walking by a factor of approximately three. Based on previous experience, over-prediction of acceleration is fairly typical for bare slab floors. This is probably due to assumptions in the extent of vibrating mass outside the bay with the walker. With a bare slab floor without nonstructural components, bays outside the area with the walker vibrate also, increasing the effective mass beyond what would be predicted. After nonstructural components are added, vibration is likely to be limited to only the bay with the walker and a small surrounding area, resulting in more accurate effective mass predictions.

In conclusion, it is clear from this study that the tested floors are highly resistant to walking-induced vibrations and will perform well in service. It is also concluded that the acceleration prediction methods used in AISC Design Guide 11 can be confidently used for vibration analysis of composite joists with flush framed connections to supporting girders.

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References

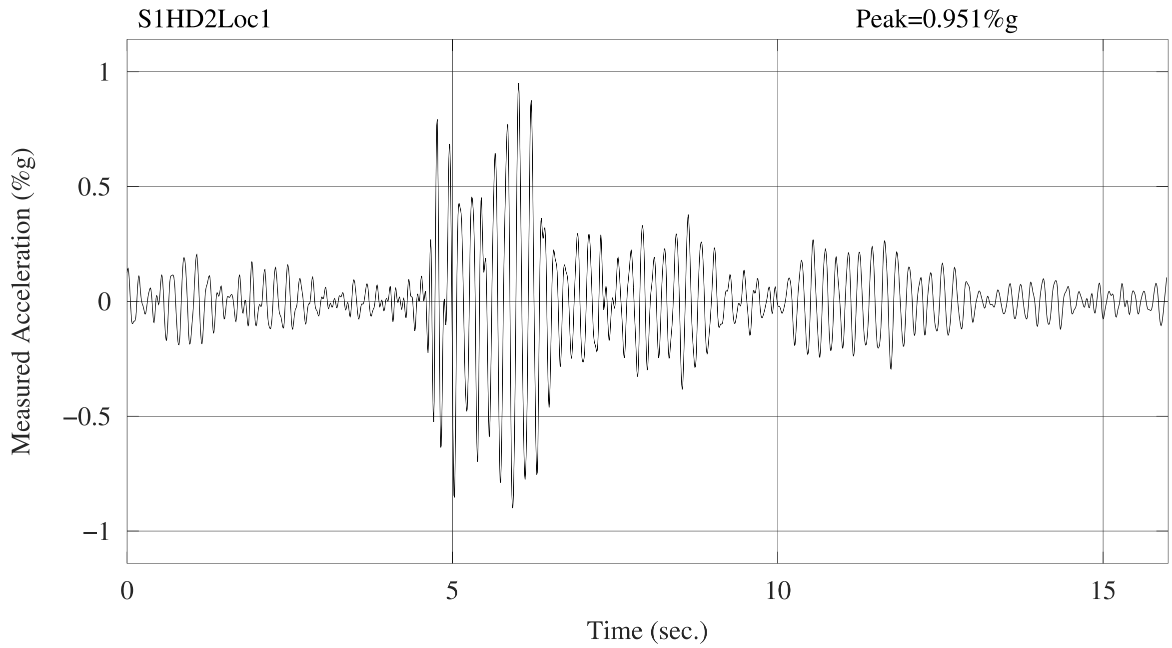
Allen, D.E. and Murray, T.M. (1993), “Design Criterion for Vibrations Due to Walking,” *Engineering Journal*, 4th Qtr, pp. 117-129.

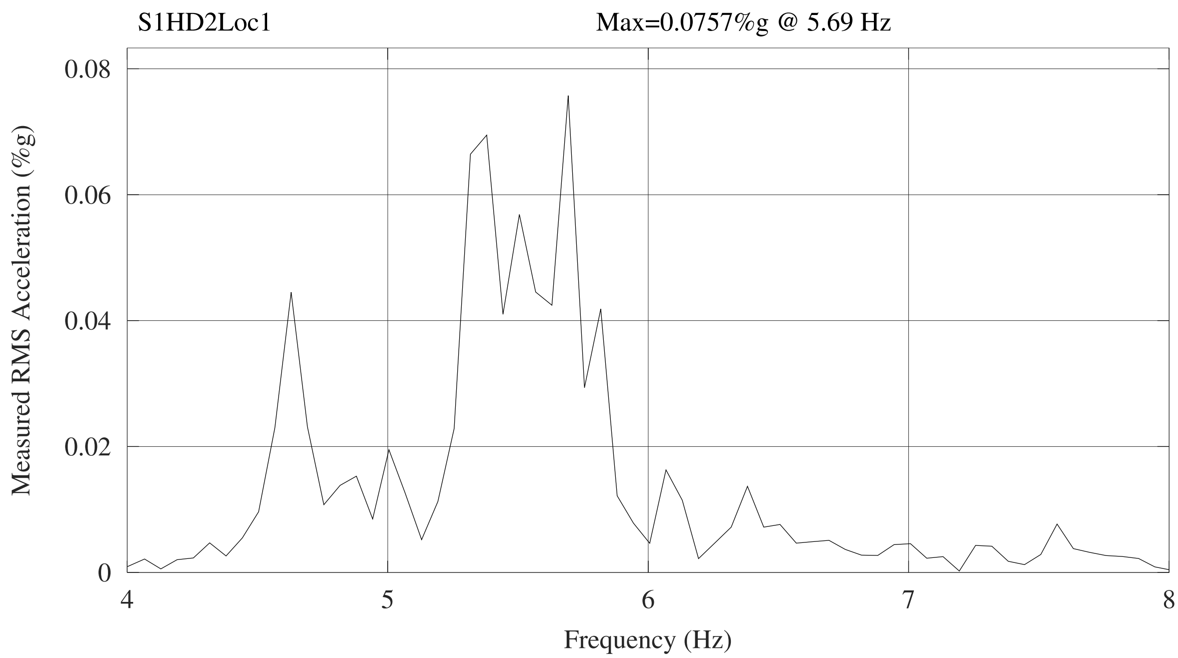
Computers and Structures, Inc. (2019), *SAP2000*, Computer Software, Version 21.0.2.

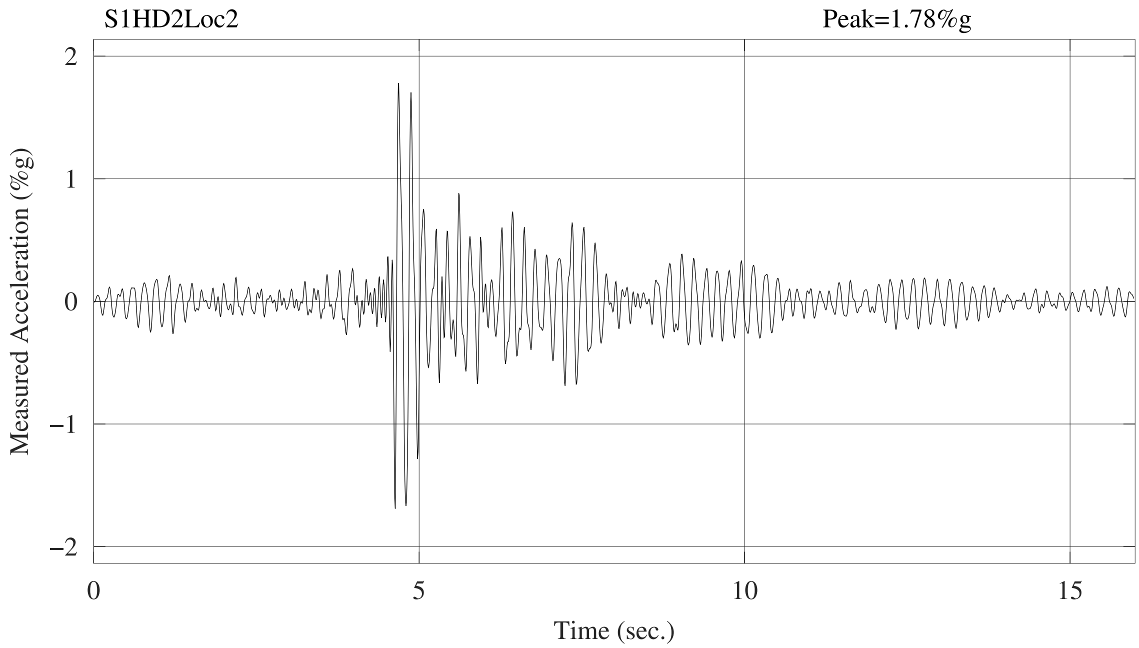
Murray, T.M., Allen, D.E., Ungar, E.E. and Davis, D.B. (2016), *Vibrations of Steel-Framed Structural Systems Due to Human Activity*, Design Guide 11, Second Edition, American Institute of Steel Construction, Chicago, IL.

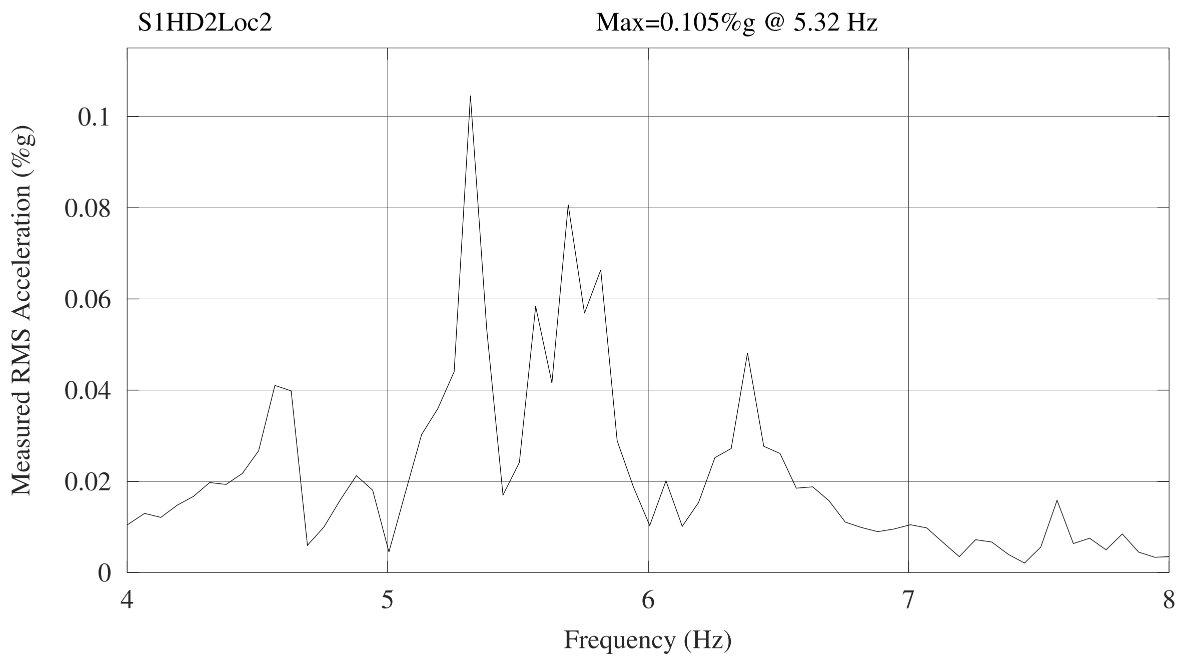
Structural Engineers, Inc. (2021), *FloorVibe*, Computer Software, Version 3.1.

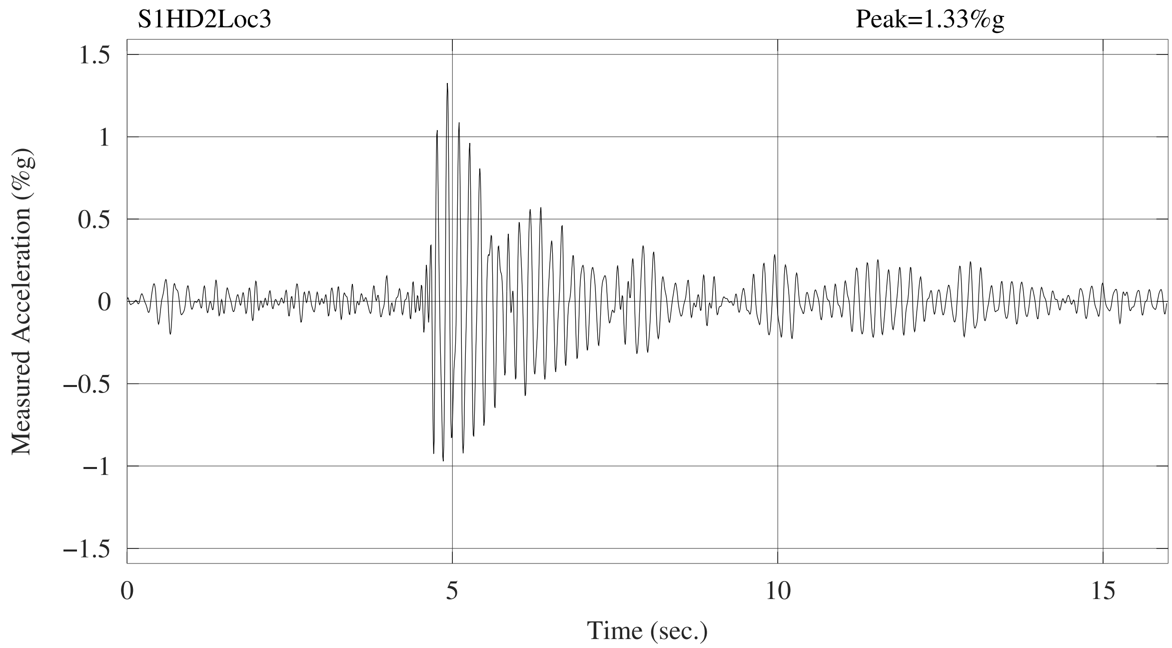
Appendix A - Heel-Drop Test Results for Setup 1

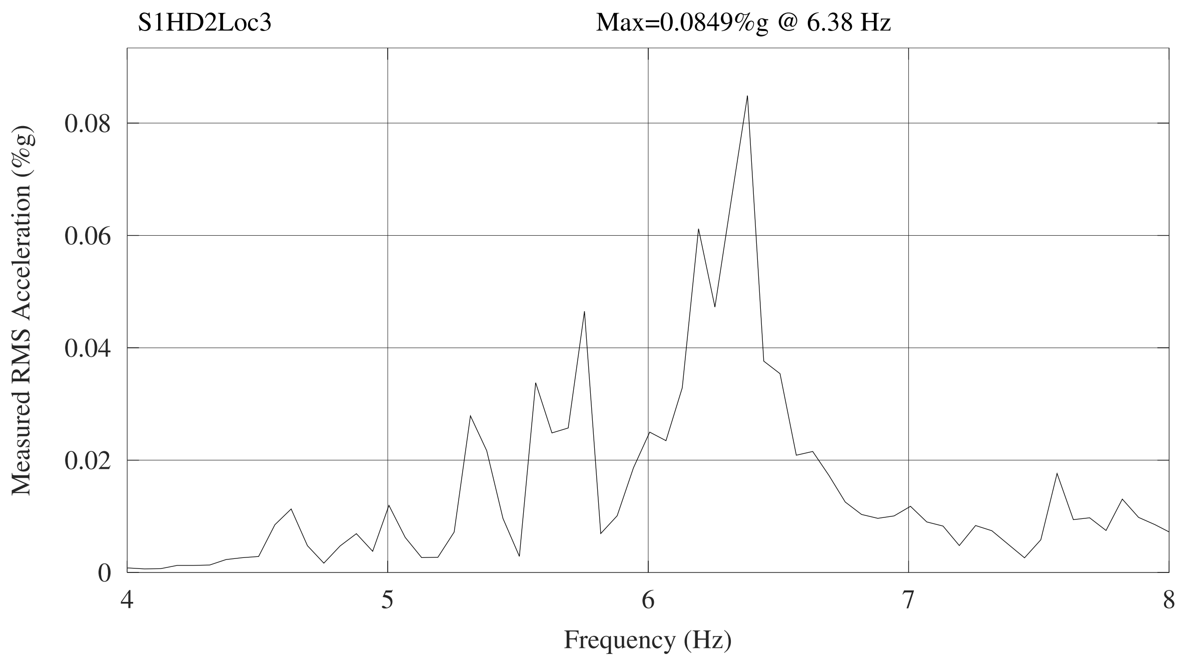




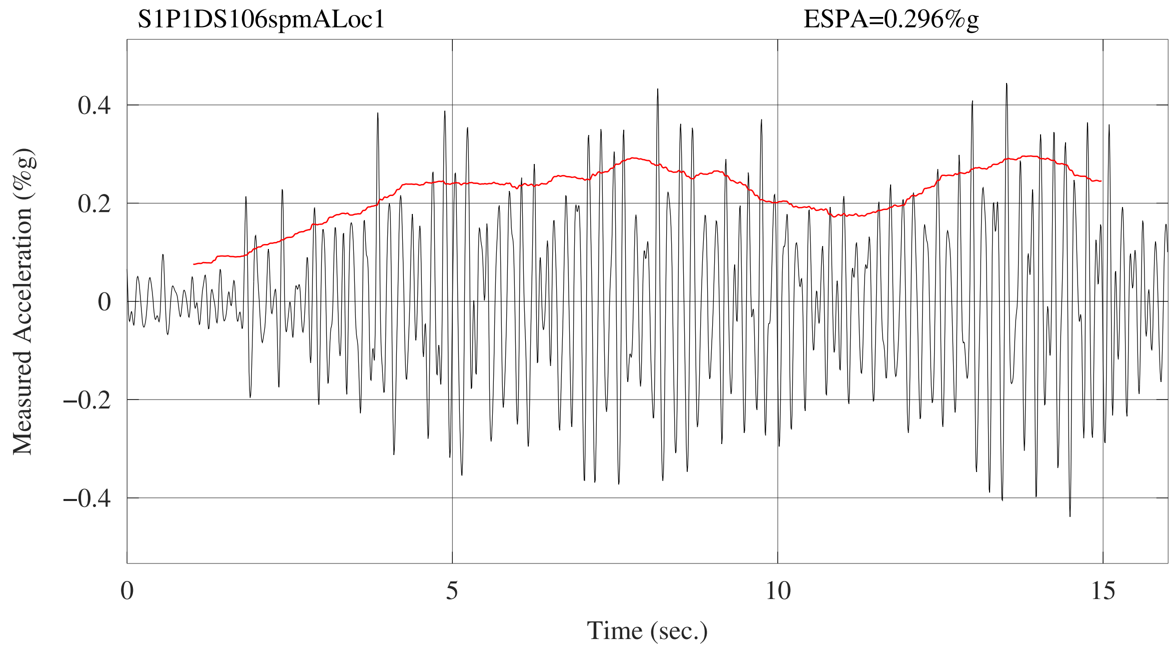

S1HD2Loc2
Peak=1.78%g
Measured Acceleration (%g)
2
1
0
−1
−2
0
5
10
15
Time (sec.)

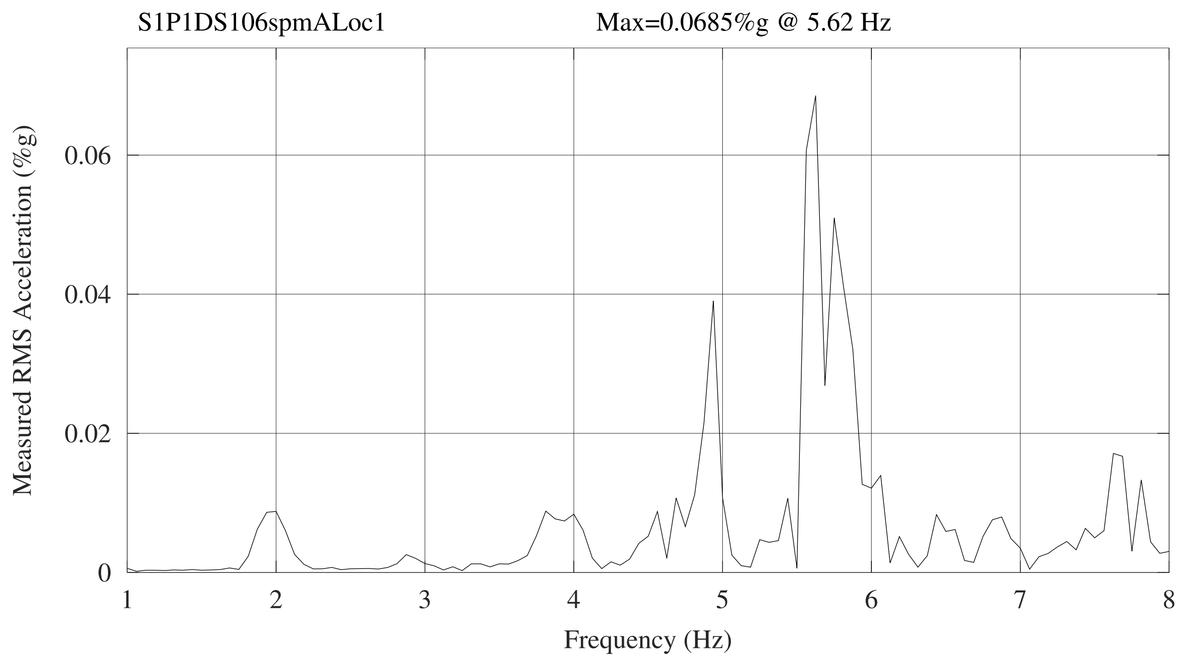

S1HD2Loc2
Max=0.105%g @ 5.32 Hz
Measured RMS Acceleration (%g)
0.1
0.08
0.06
0.04
0.02
0
4
5
6
7
8
Frequency (Hz)

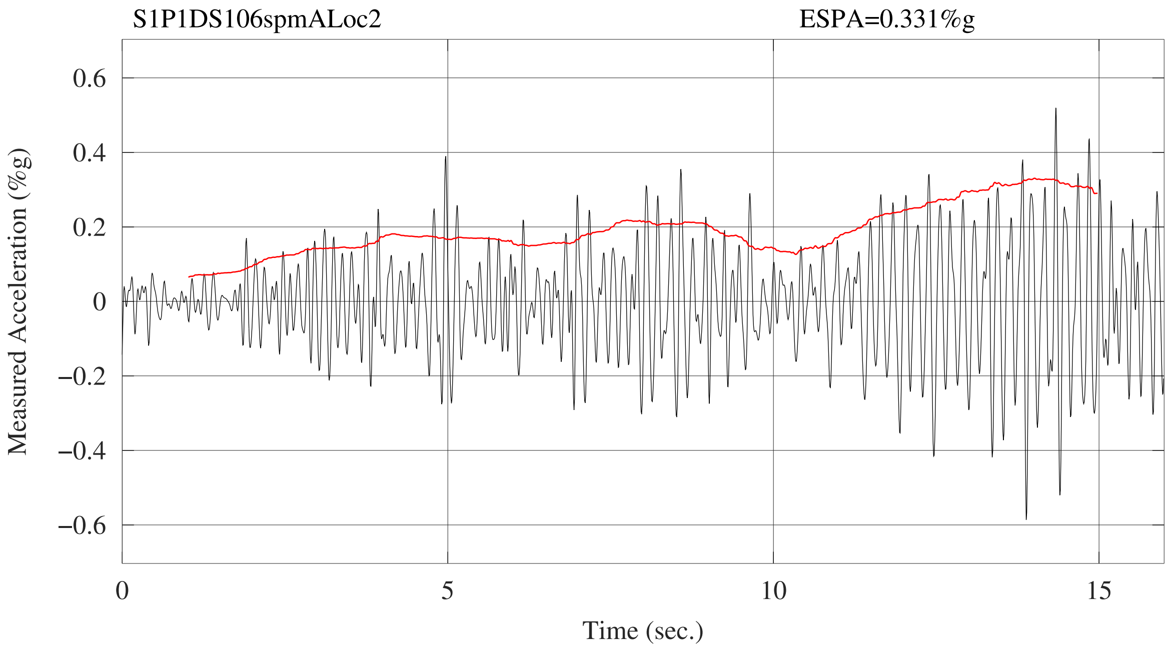

S1HD2Loc3
Peak=1.33%g
Measured Acceleration (%g)
1.5
1
0.5
0
−0.5
−1
−1.5
0
5
10
15
Time (sec.)


S1HD2Loc3
Max=0.0849%g @ 6.38 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
4
5
6
7
8
Frequency (Hz)

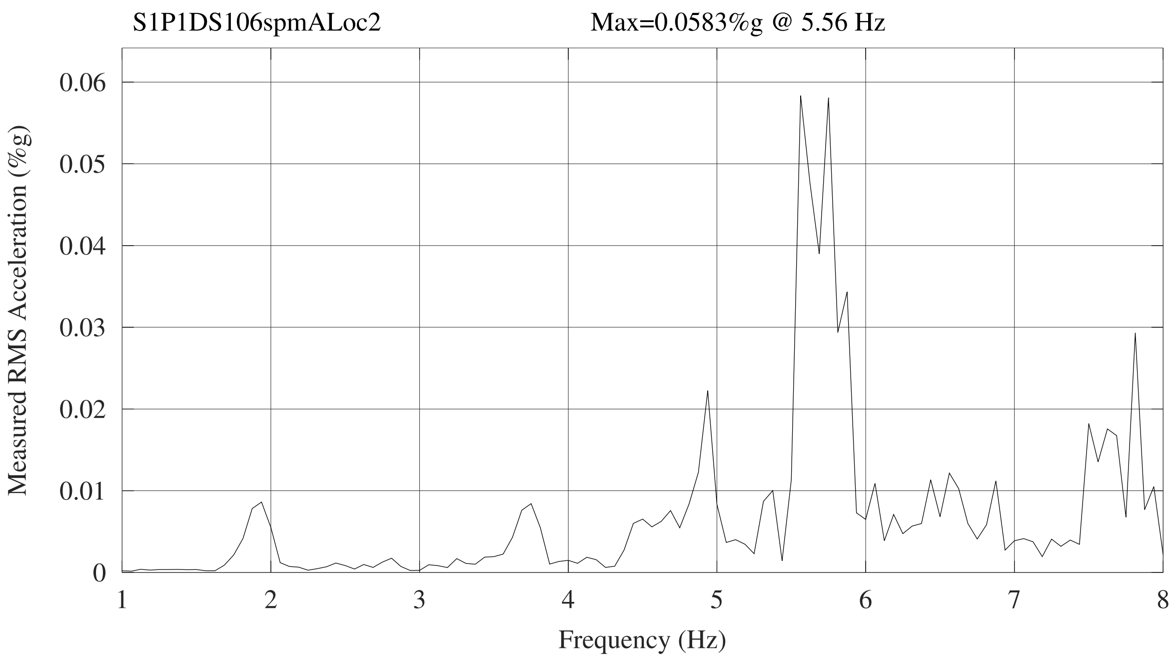
Appendix B - Walking Test Results for Setup 1



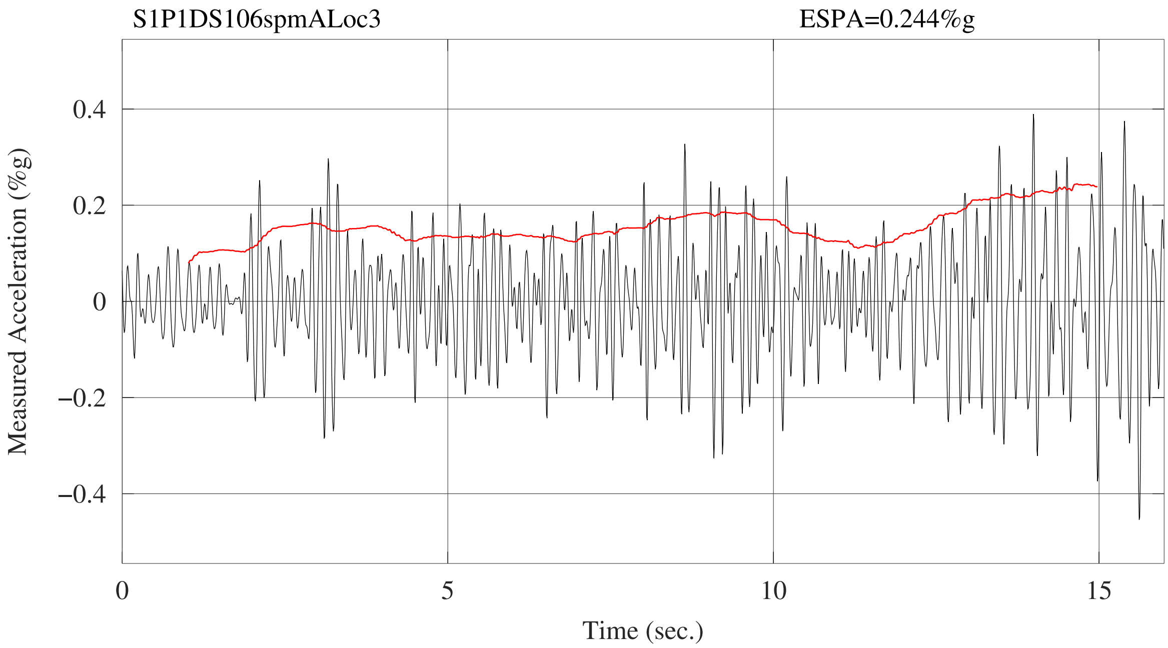


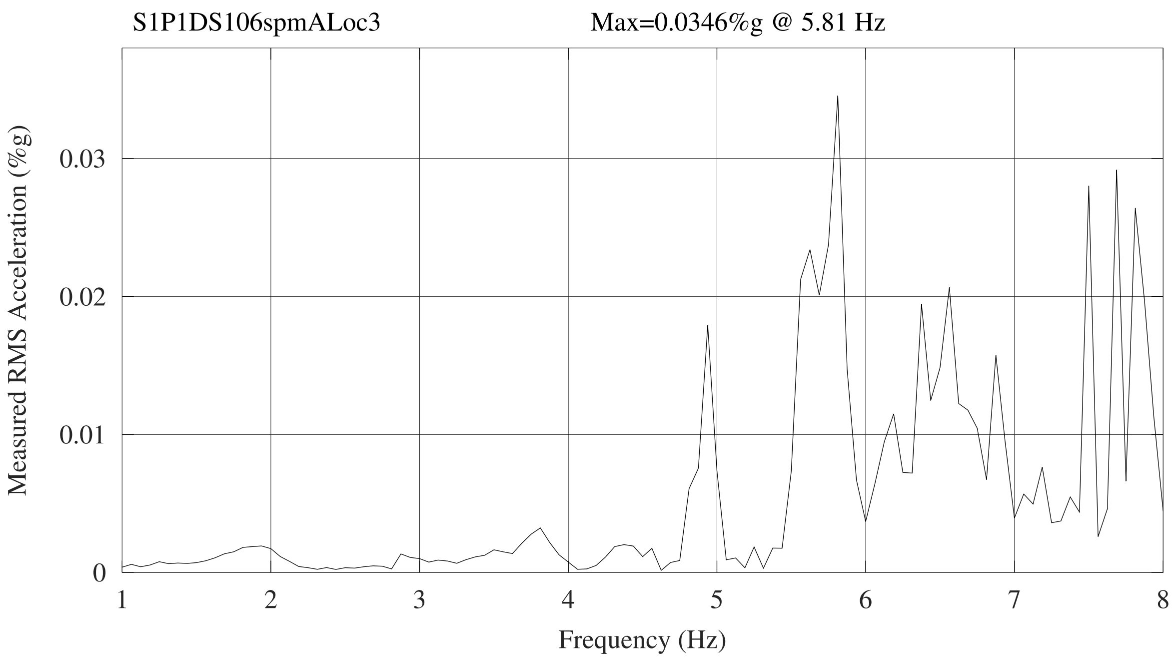


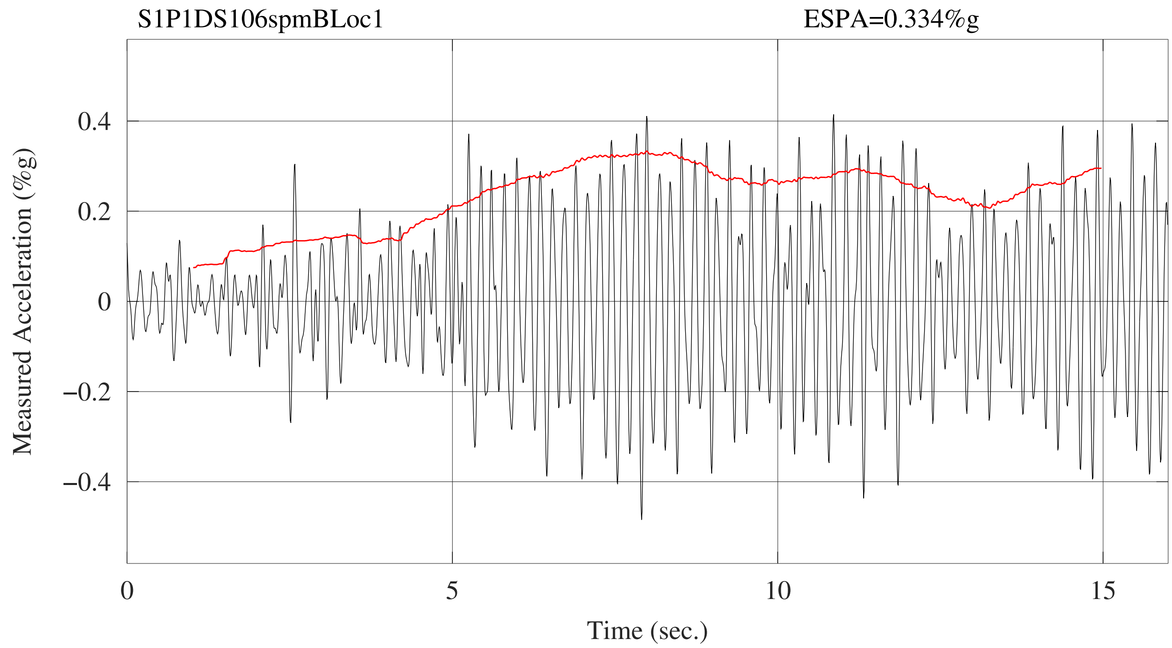
S1P1DS106spmALoc2
ESPA=0.331%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

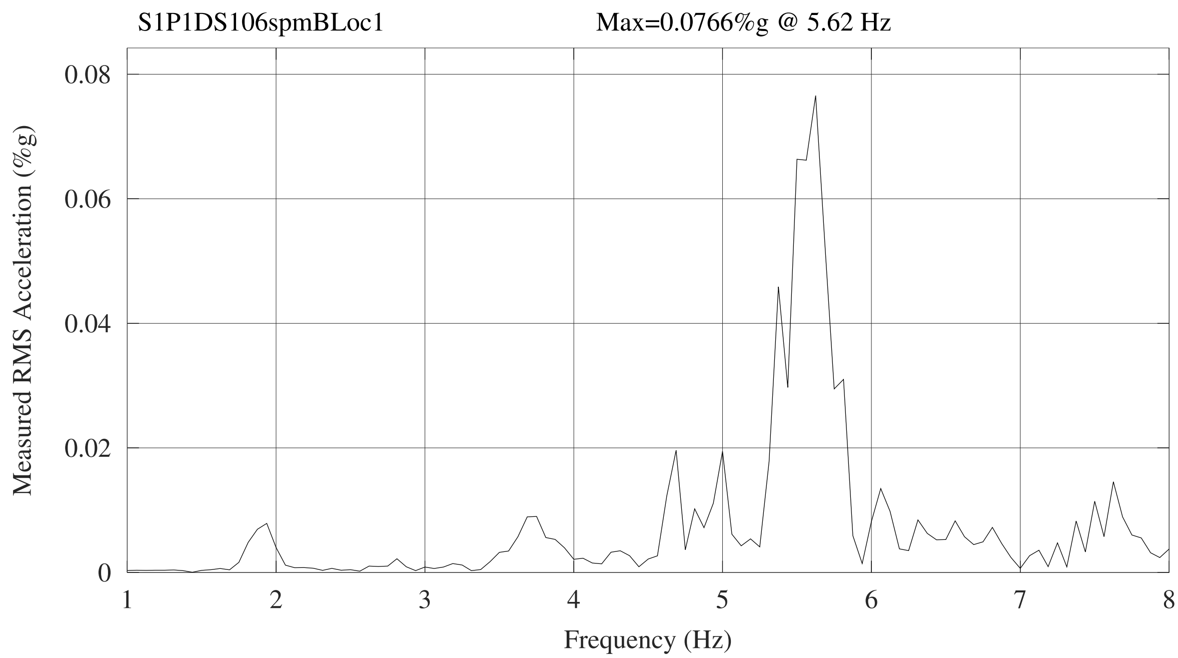


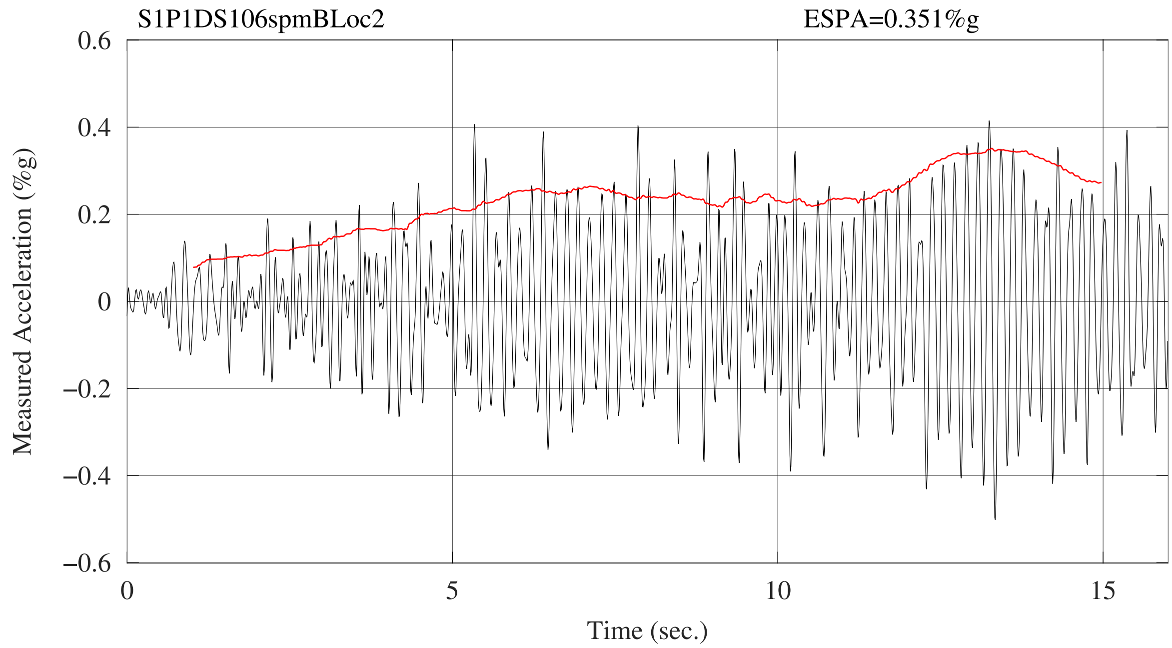
S1P1DS106spmALoc2
Max=0.0583%g @ 5.56 Hz
Measured RMS Acceleration (%g)
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S1P1DS106spmALoc3
ESPA=0.244%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

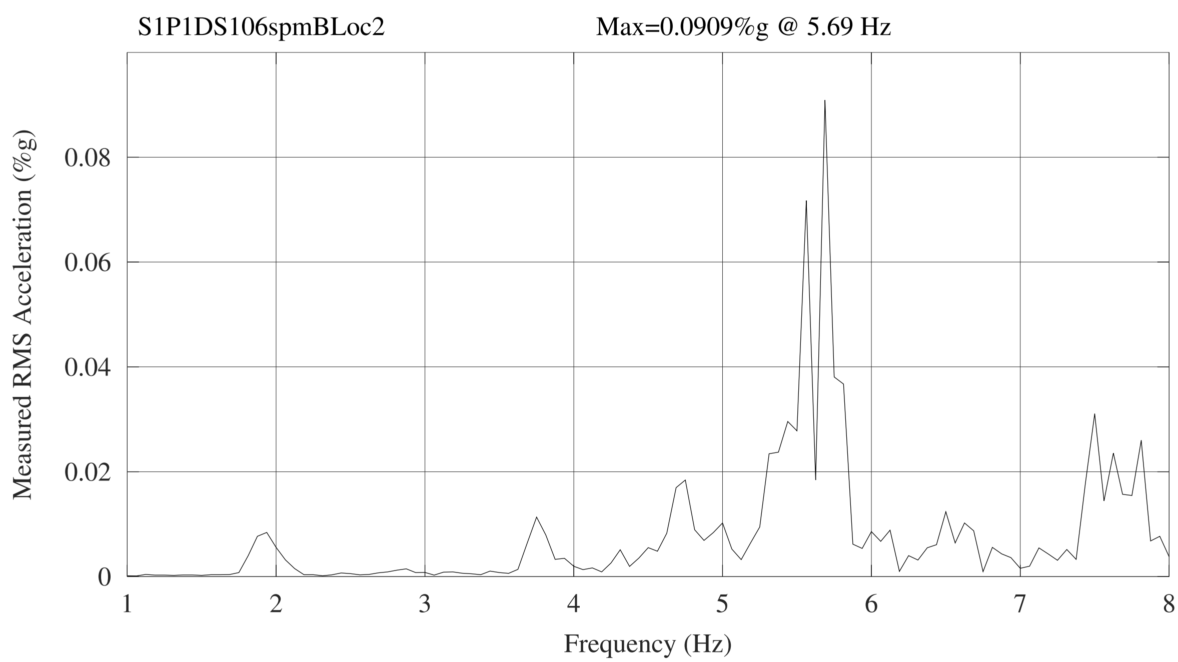

S1P1DS106spmALoc3
Max=0.0346%g @ 5.81 Hz
Measured RMS Acceleration (%g)
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S1P1DS106spmBLoc1
ESPA=0.334%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

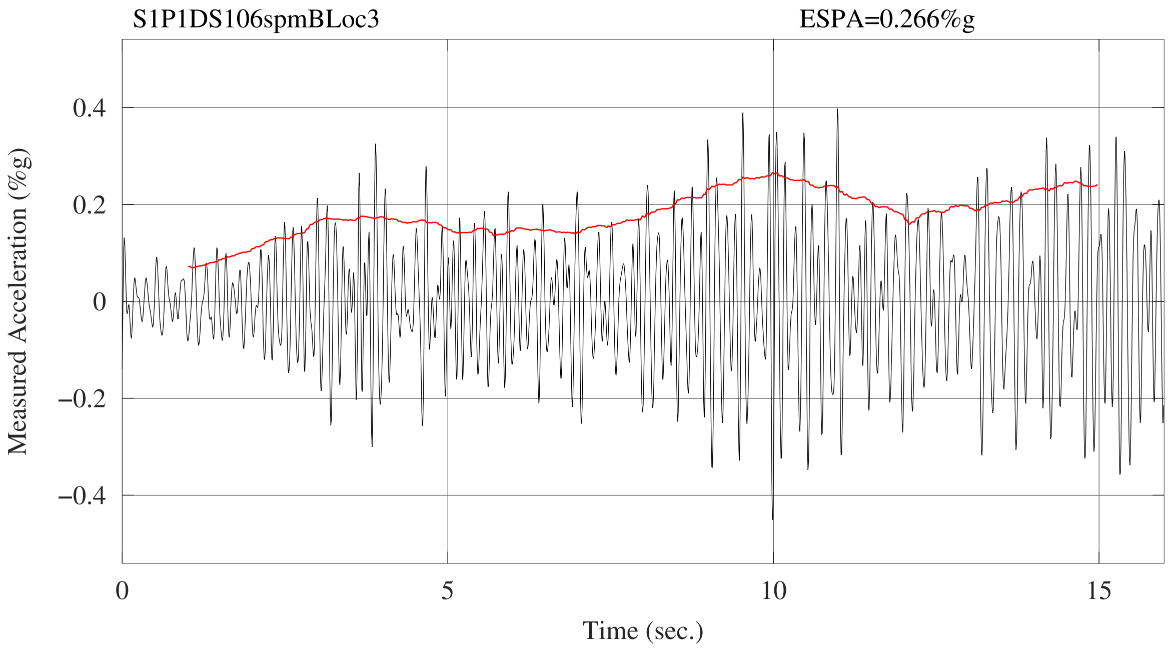

S1P1DS106spmBLoc1
Max=0.0766%g @ 5.62 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

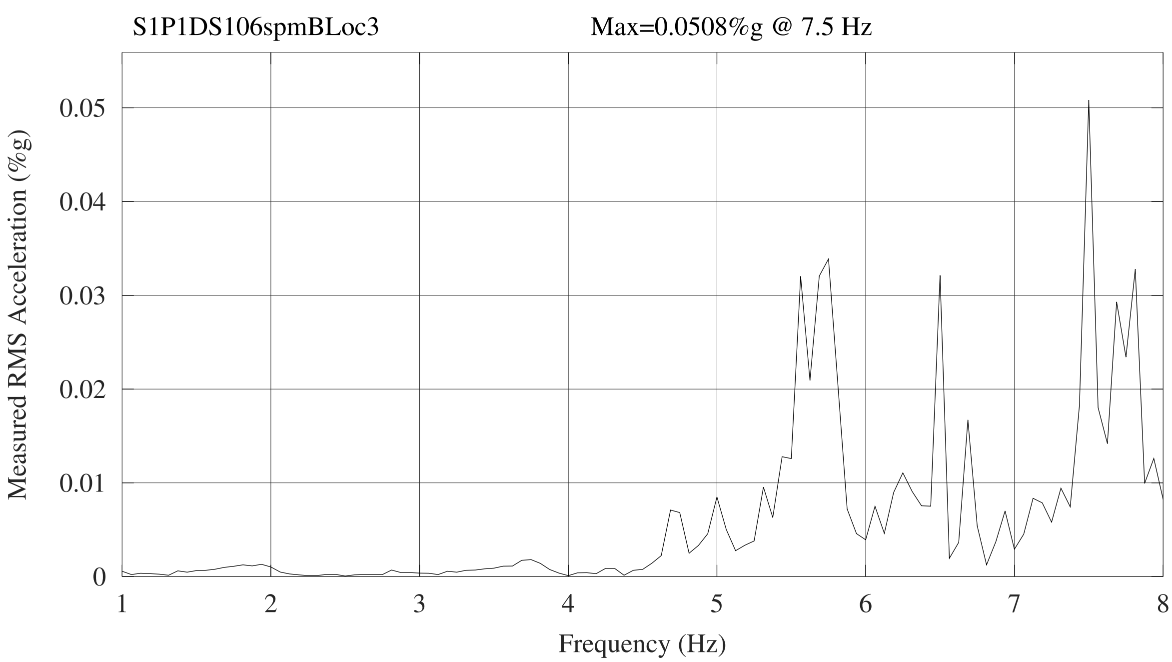


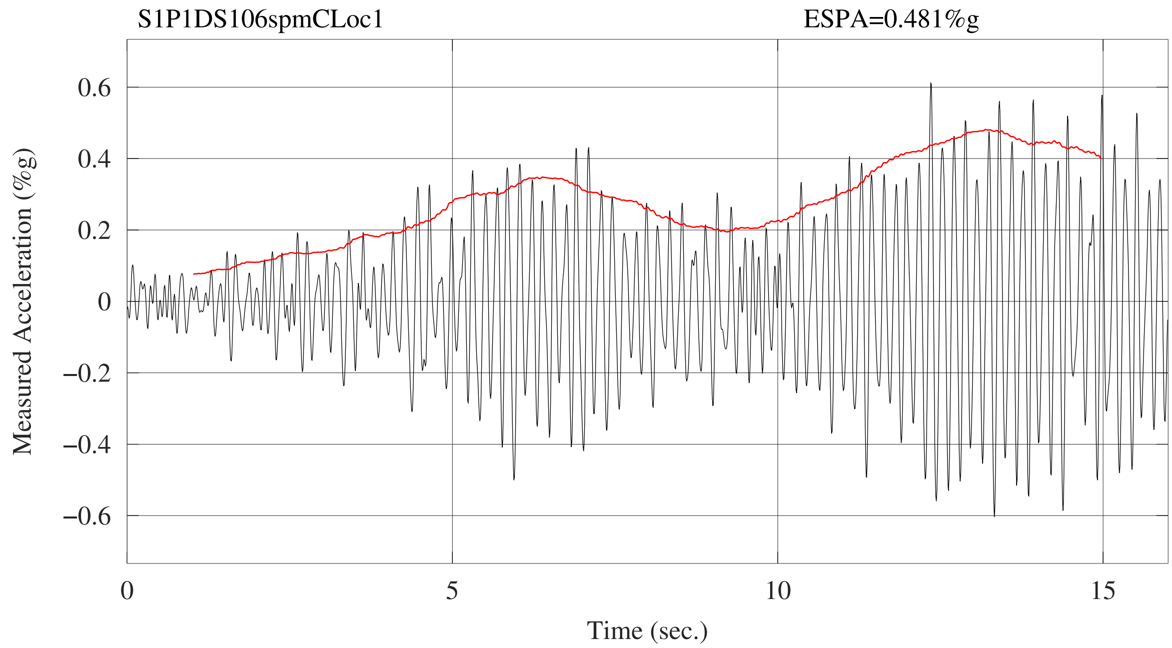
S1P1DS106spmBLoc2
ESPA=0.351%g
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

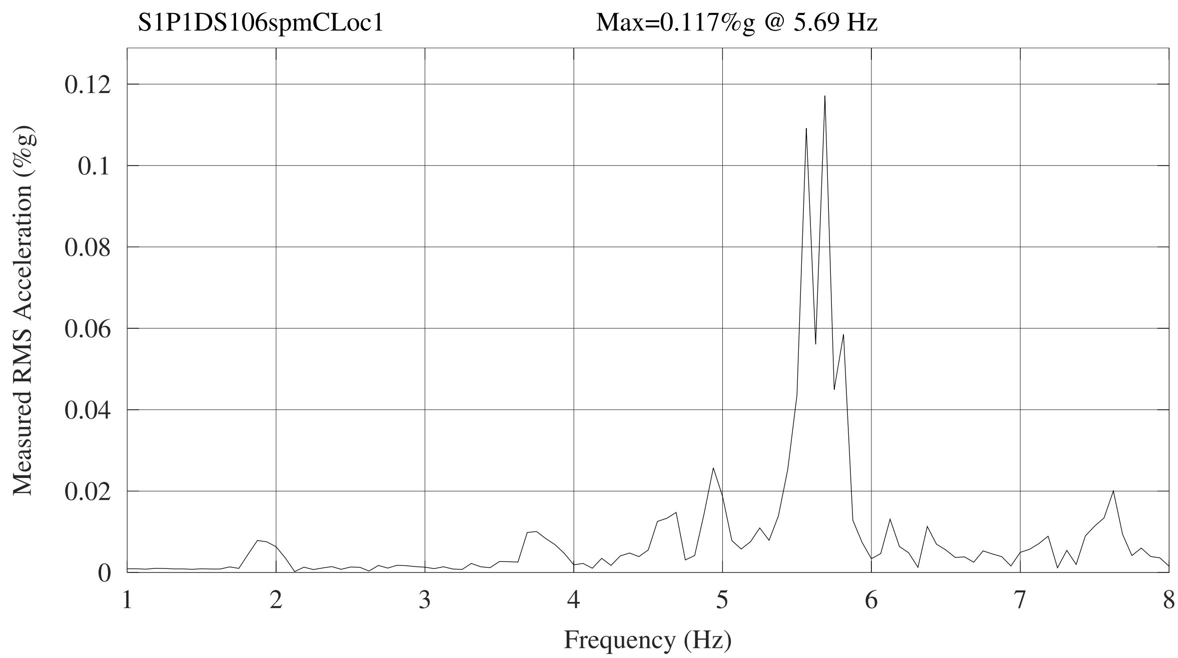


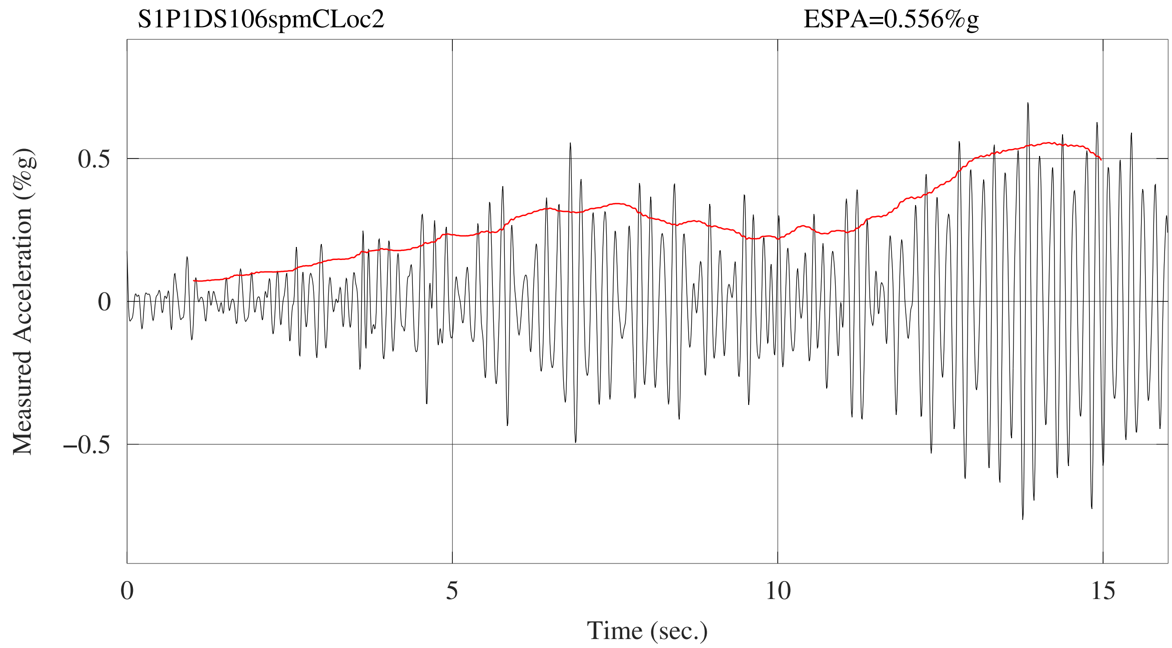
S1P1DS106spmBLoc2
Max=0.0909%g @ 5.69 Hz
0.08
0.06
0.04
0.02
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)

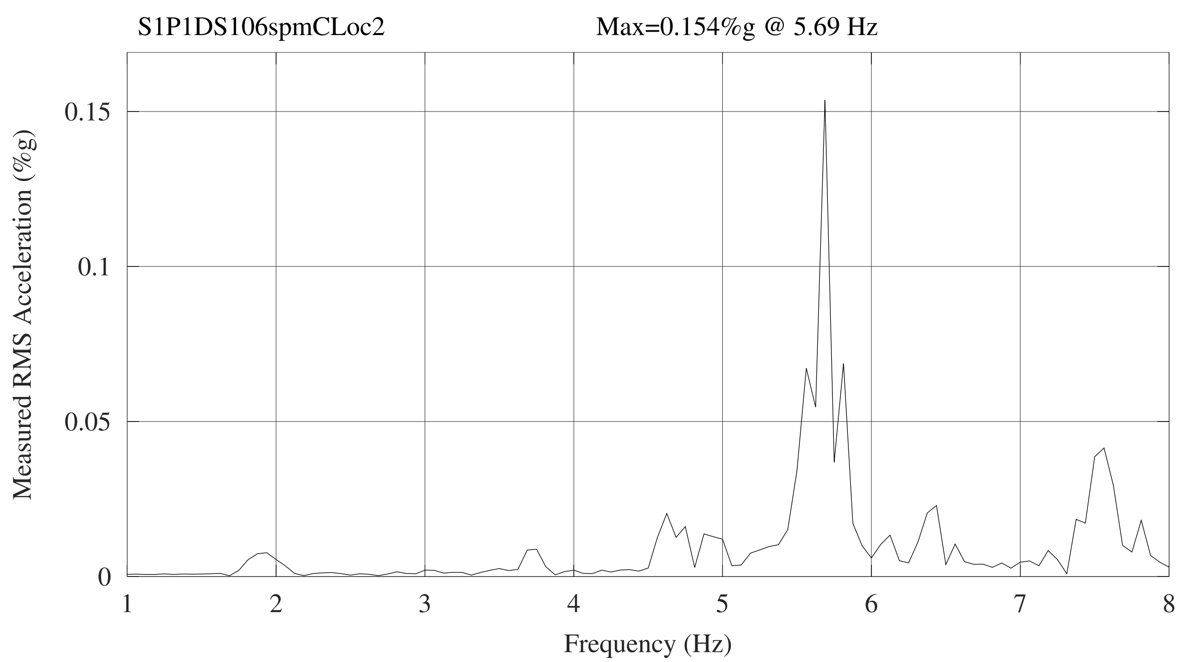

S1P1DS106spmBLoc3
ESPA=0.266%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

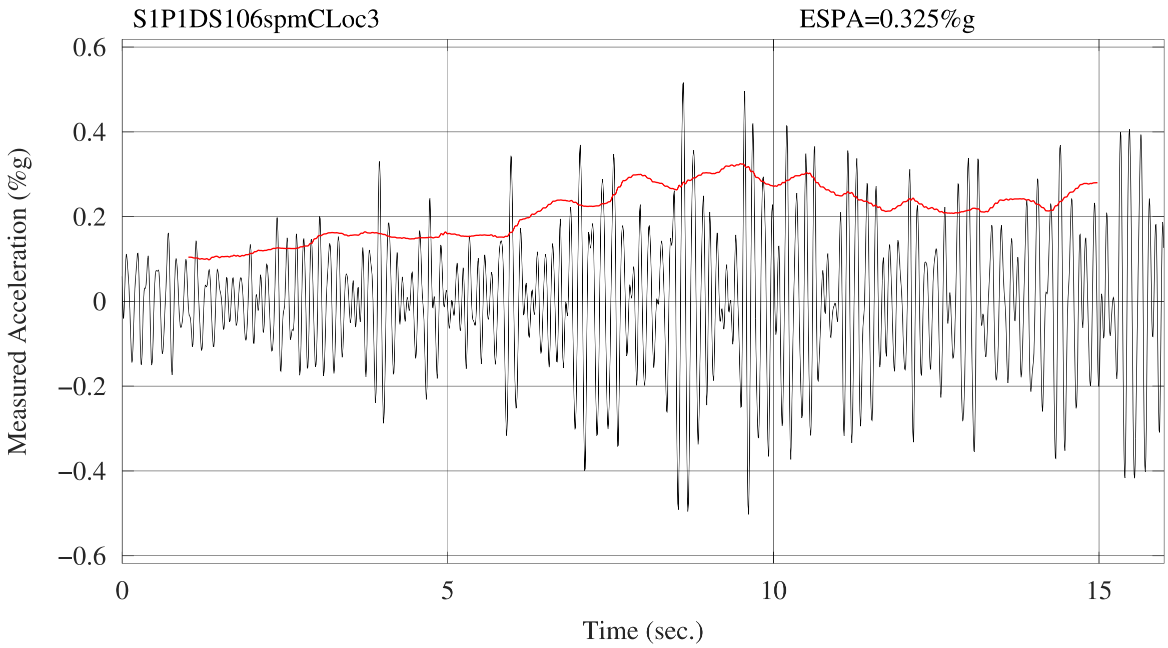

S1P1DS106spmBLoc3
Max=0.0508%g @ 7.5 Hz
Measured RMS Acceleration (%g)
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

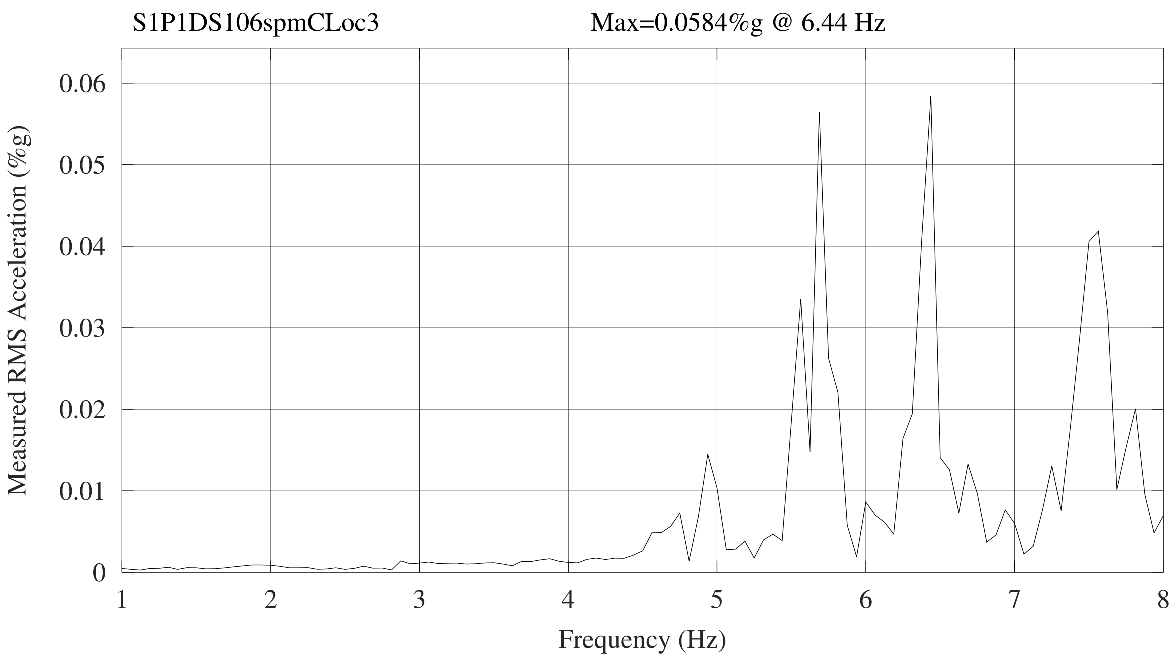

S1P1DS106spmCLoc1
ESPA=0.481%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

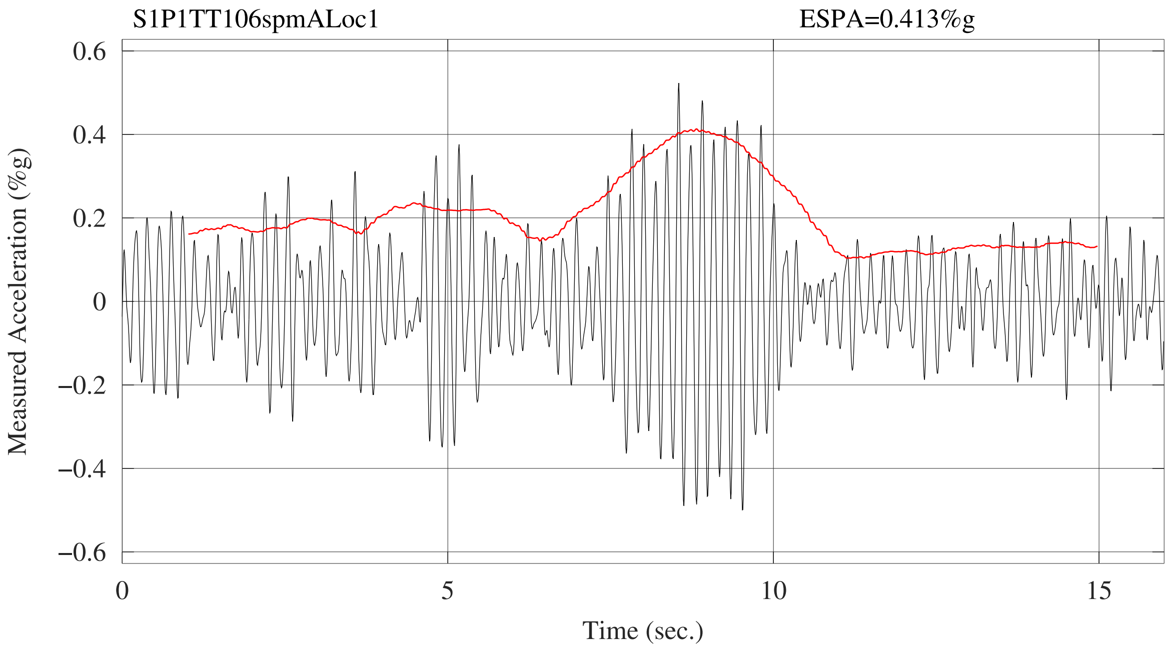

S1P1DS106spmCLoc1
Max=0.117%g @ 5.69 Hz
Measured RMS Acceleration (%g)
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

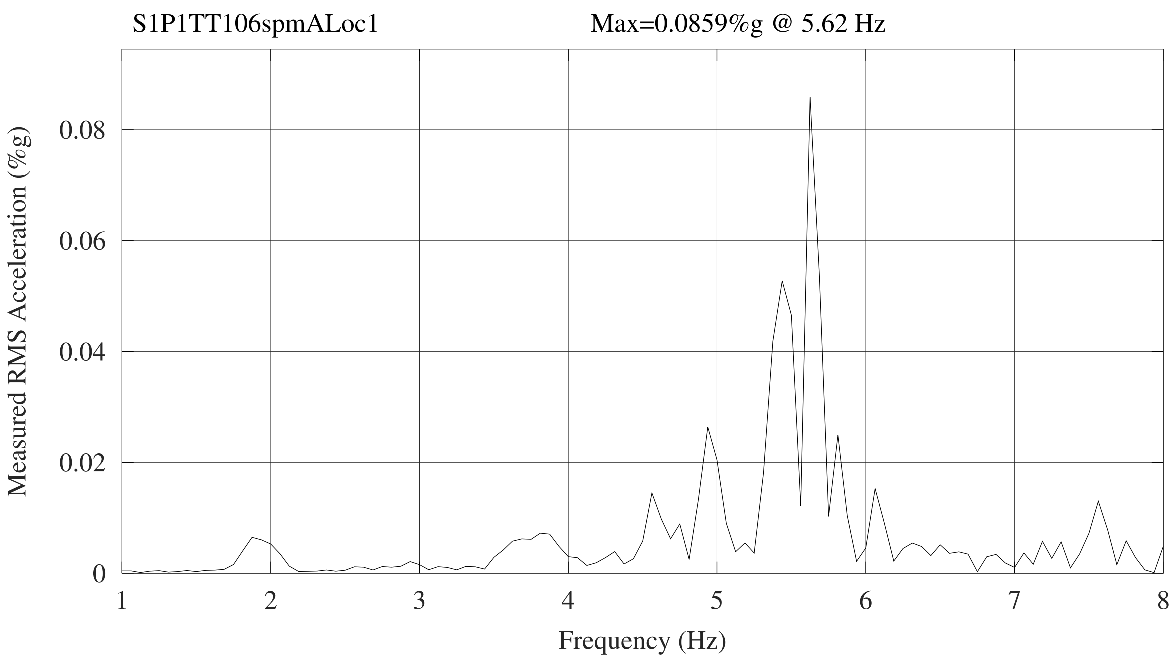

S1P1DS106spmCLoc2
ESPA=0.556%g
Measured Acceleration (%g)
0.5
0
−0.5
0
5
10
15
Time (sec.)

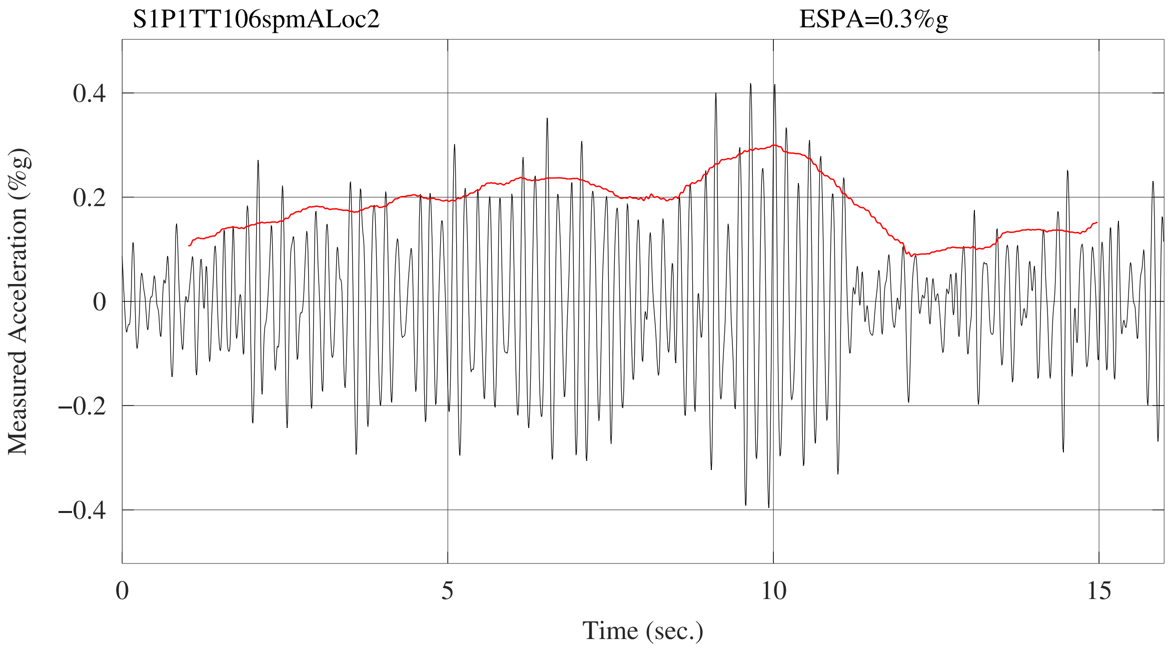

S1P1DS106spmCLoc2
Max=0.154%g @ 5.69 Hz
Measured RMS Acceleration (%g)
0.15
0.1
0.05
0
1
2
3
4
5
6
7
8
Frequency (Hz)

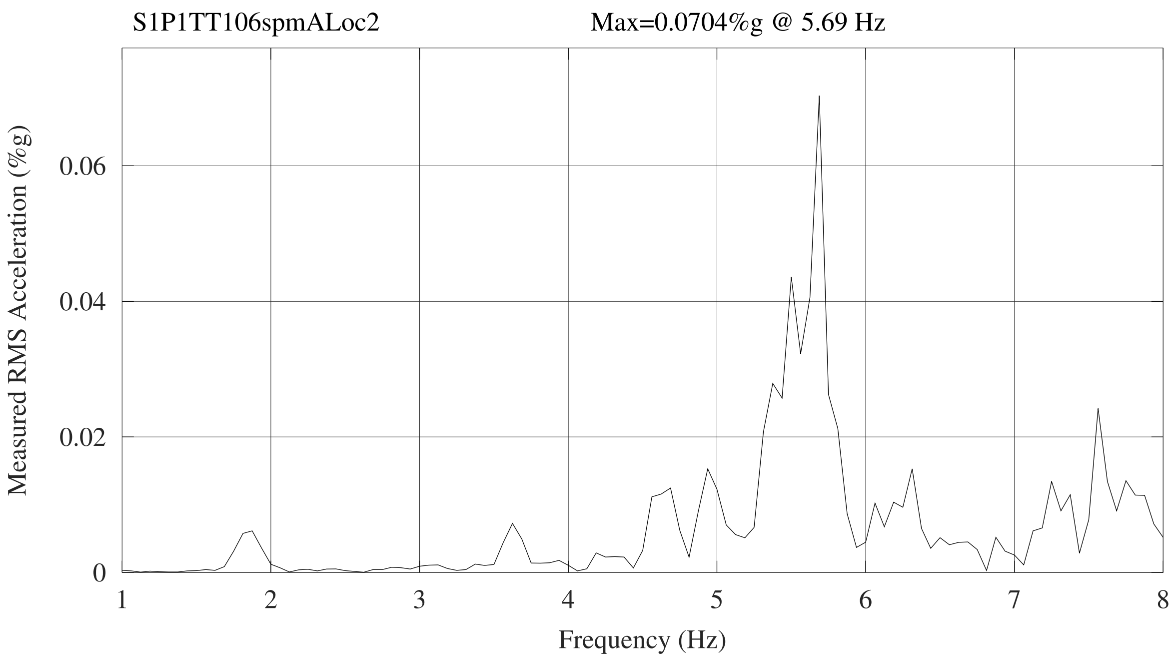

S1P1DS106spmCLoc3
ESPA=0.325%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

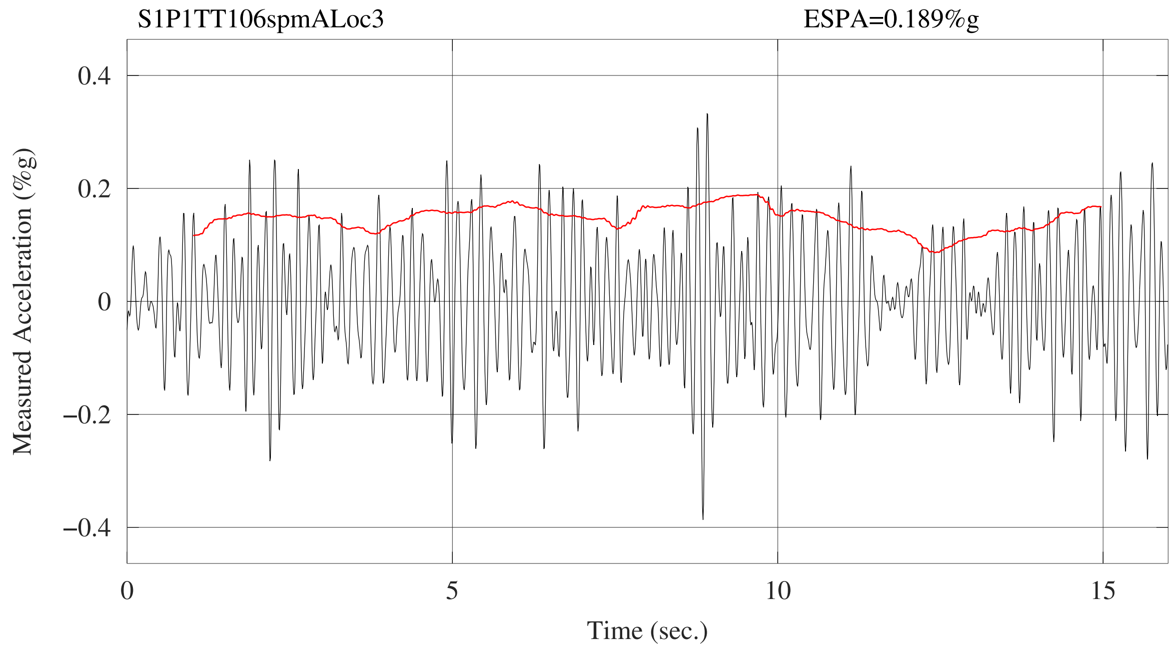

S1P1DS106spmCLoc3
Max=0.0584%g @ 6.44 Hz
Measured RMS Acceleration (%g)
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

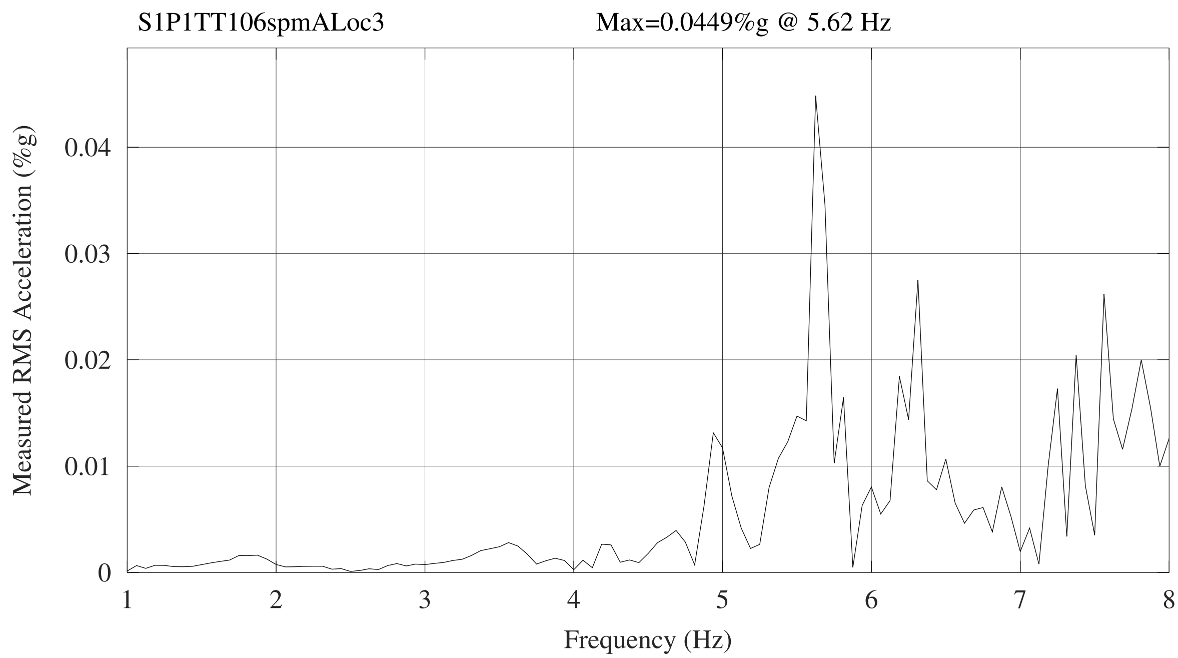

S1P1TT106spmALoc1
ESPA=0.413%g
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

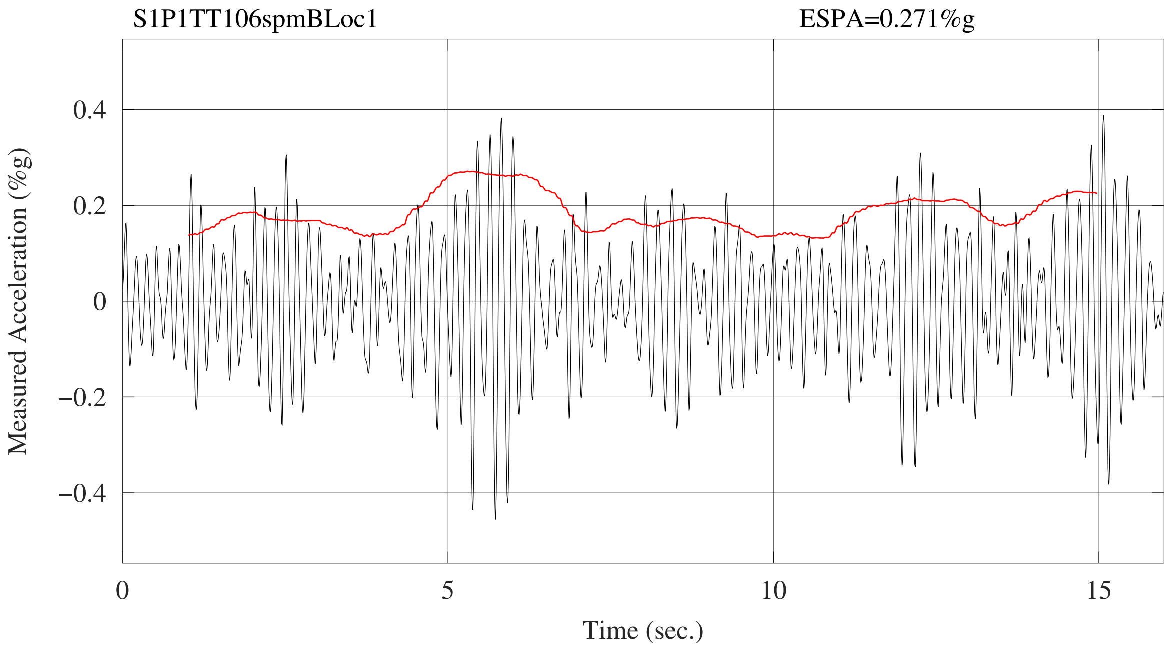

S1P1TT106spmALoc1
Max=0.0859%g @ 5.62 Hz
0.08
0.06
0.04
0.02
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)

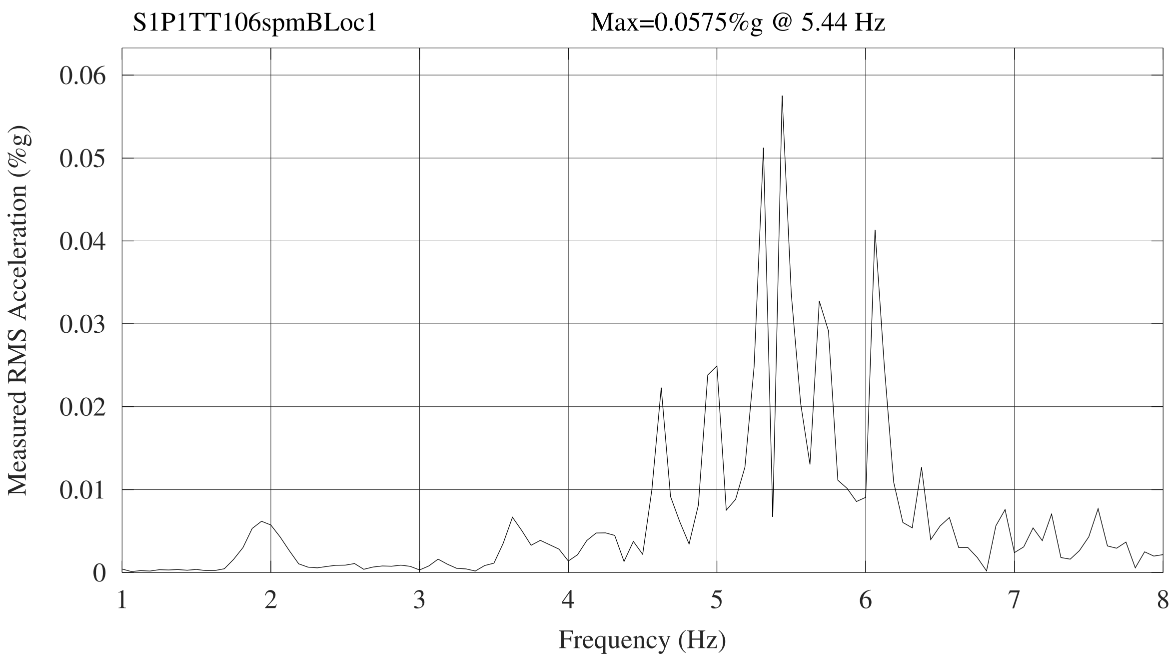

S1P1TT106spmALoc2
ESPA=0.3%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

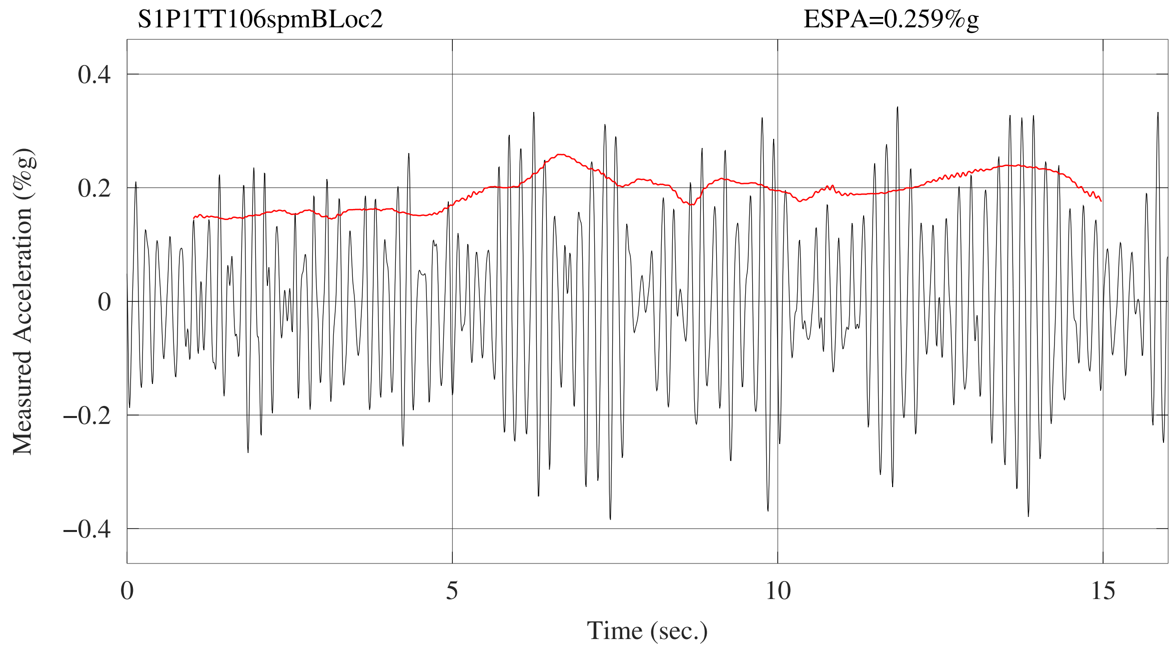

S1P1TT106spmALoc2
Max=0.0704%g @ 5.69 Hz
Measured RMS Acceleration (%g)
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

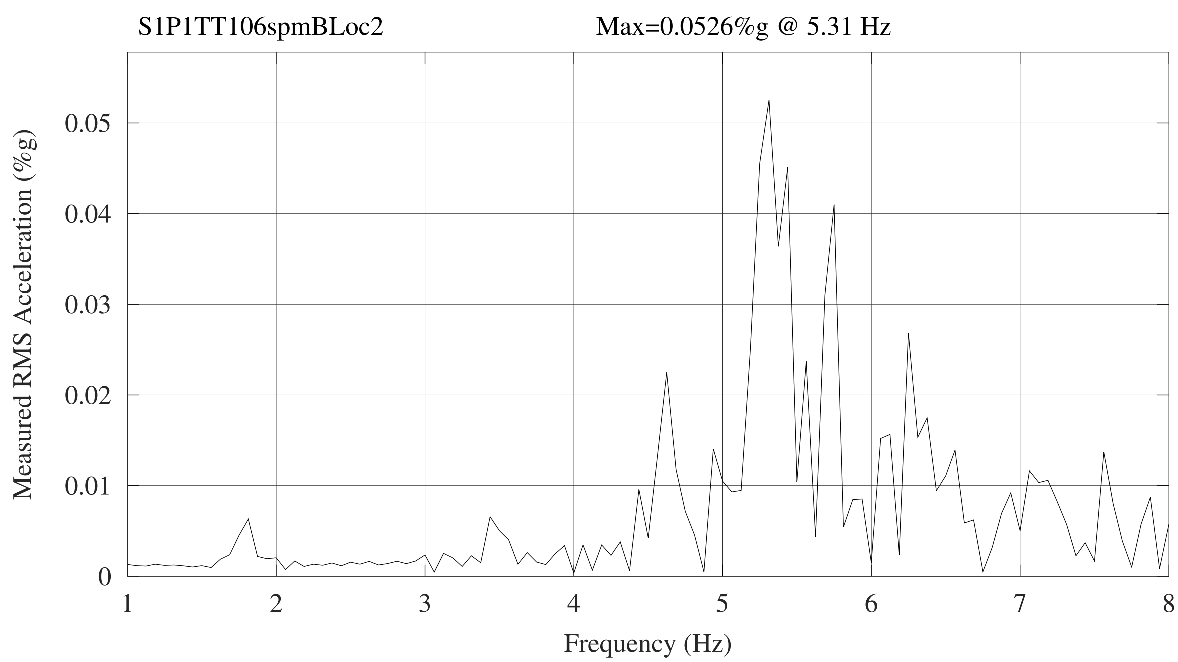

S1P1TT106spmALoc3
ESPA=0.189%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

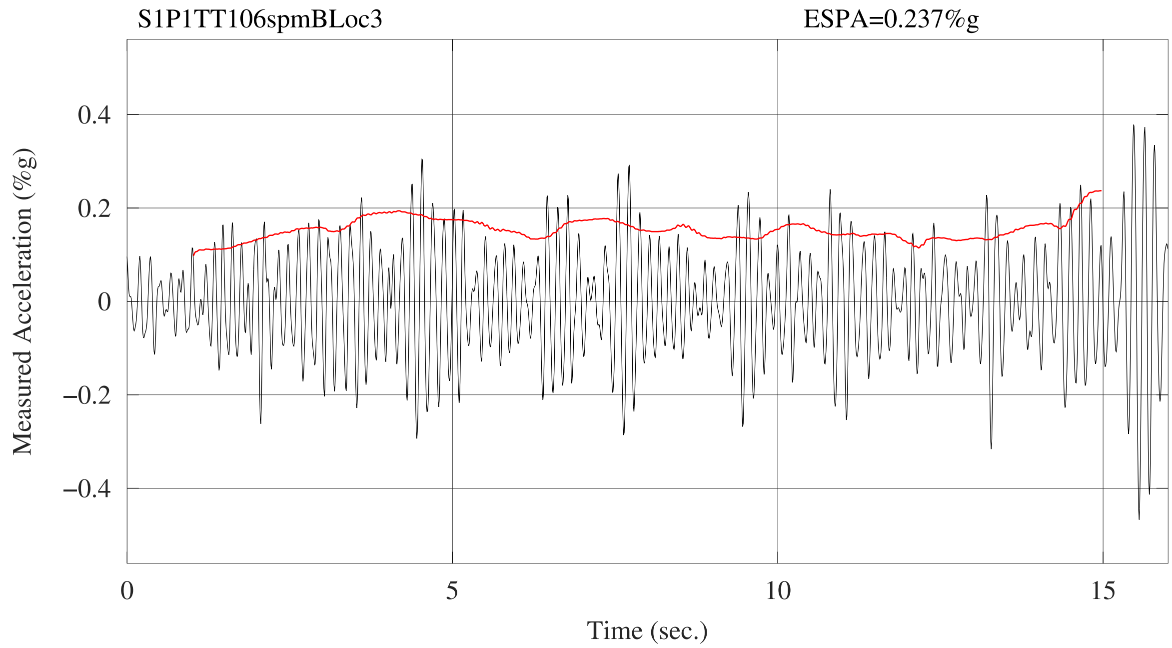

S1P1TT106spmALoc3
Max=0.0449%g @ 5.62 Hz
Measured RMS Acceleration (%g)
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

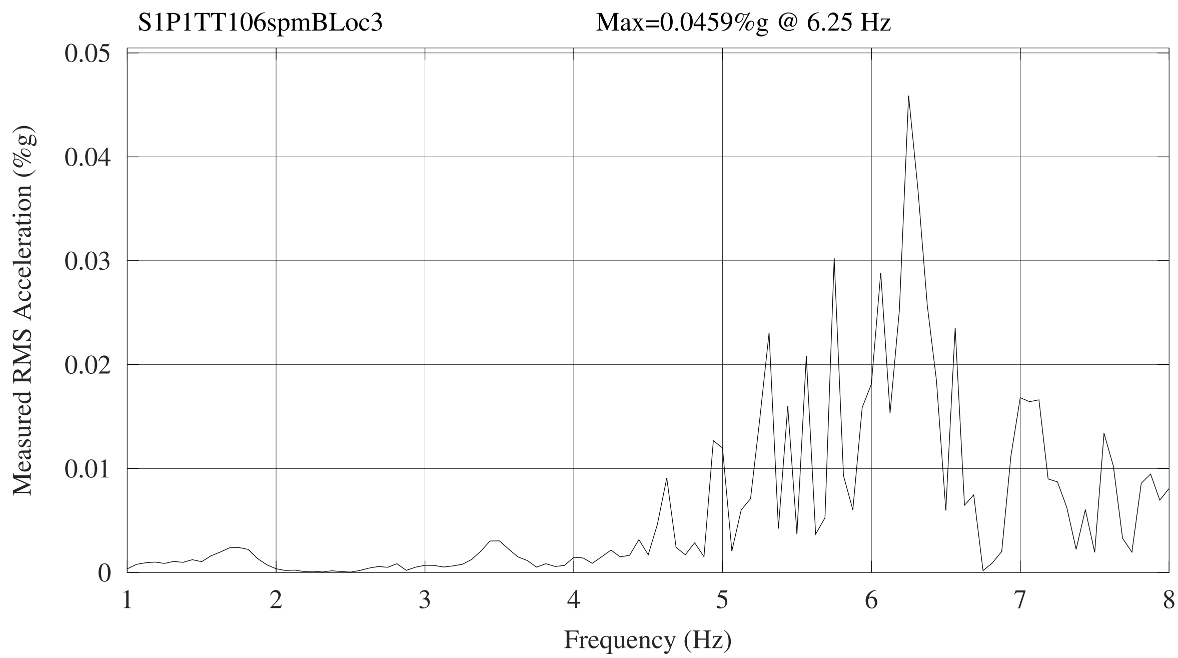

S1P1TT106spmBLoc1
ESPA=0.271%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

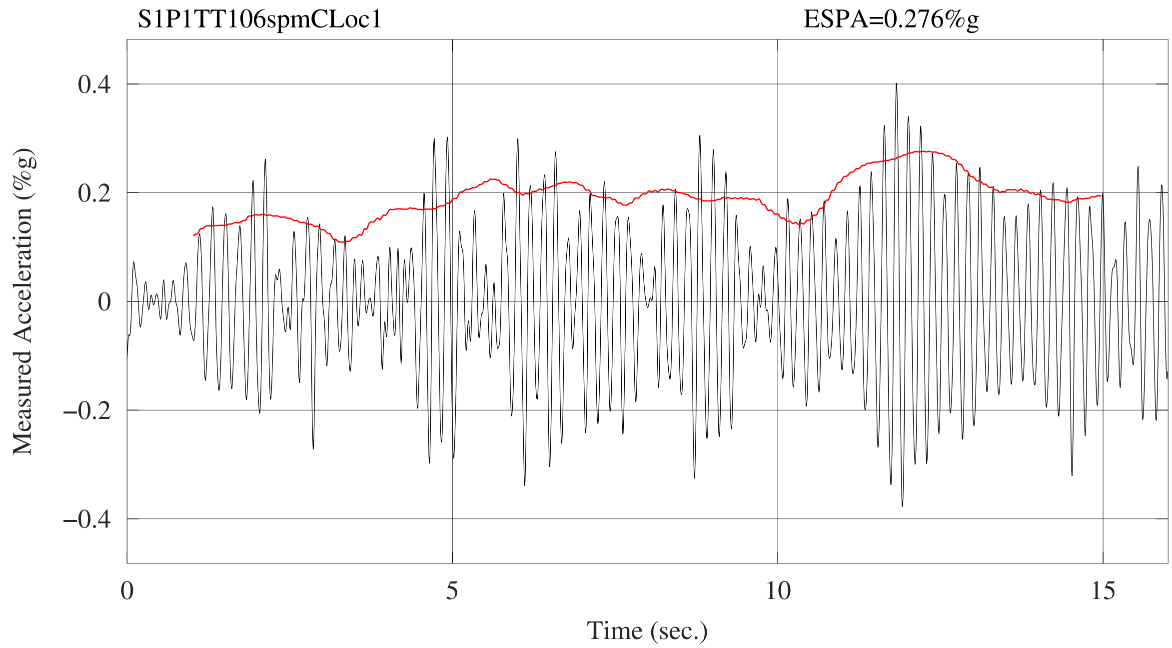

S1P1TT106spmBLoc1
Max=0.0575%g @ 5.44 Hz
Measured RMS Acceleration (%g)
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S1P1TT106spmBLoc2
ESPA=0.259%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

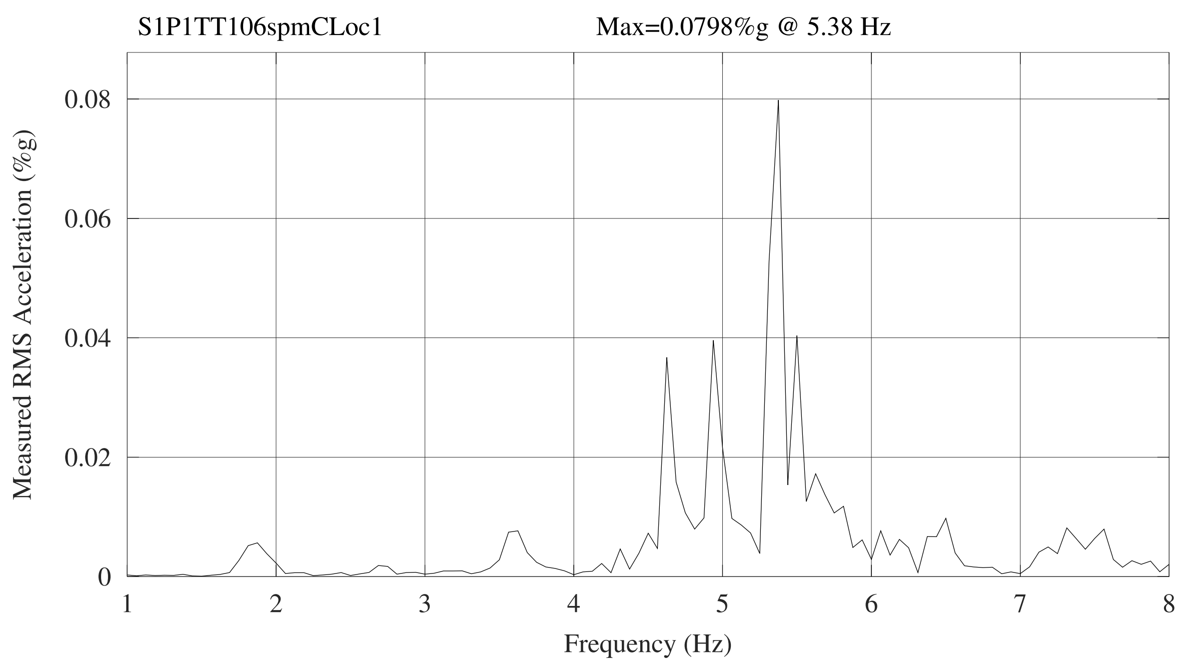

S1P1TT106spmBLoc2
Max=0.0526%g @ 5.31 Hz
Measured RMS Acceleration (%g)
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S1P1TT106spmBLoc3
ESPA=0.237%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

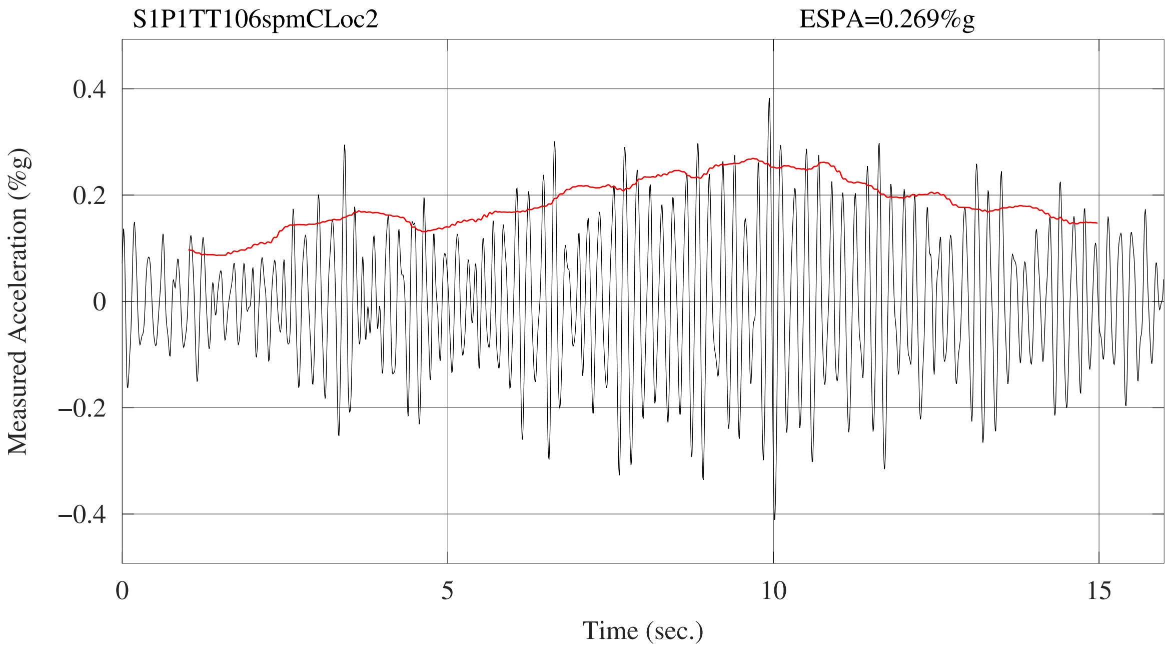

S1P1TT106spmBLoc3
Max=0.0459%g @ 6.25 Hz
Measured RMS Acceleration (%g)
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

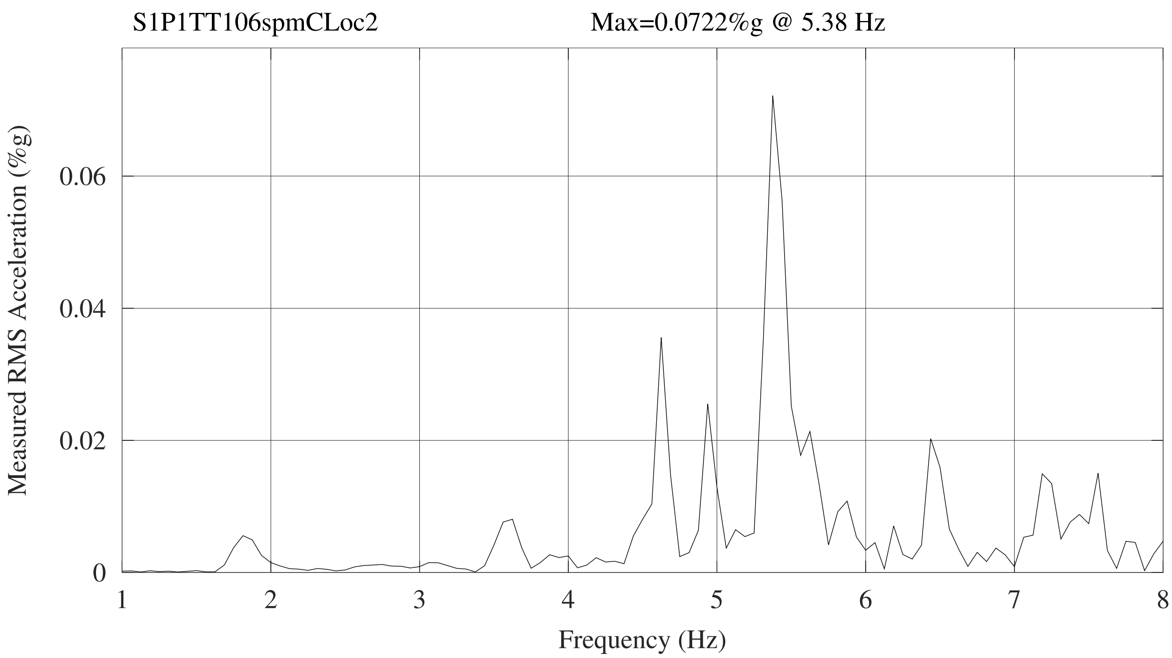


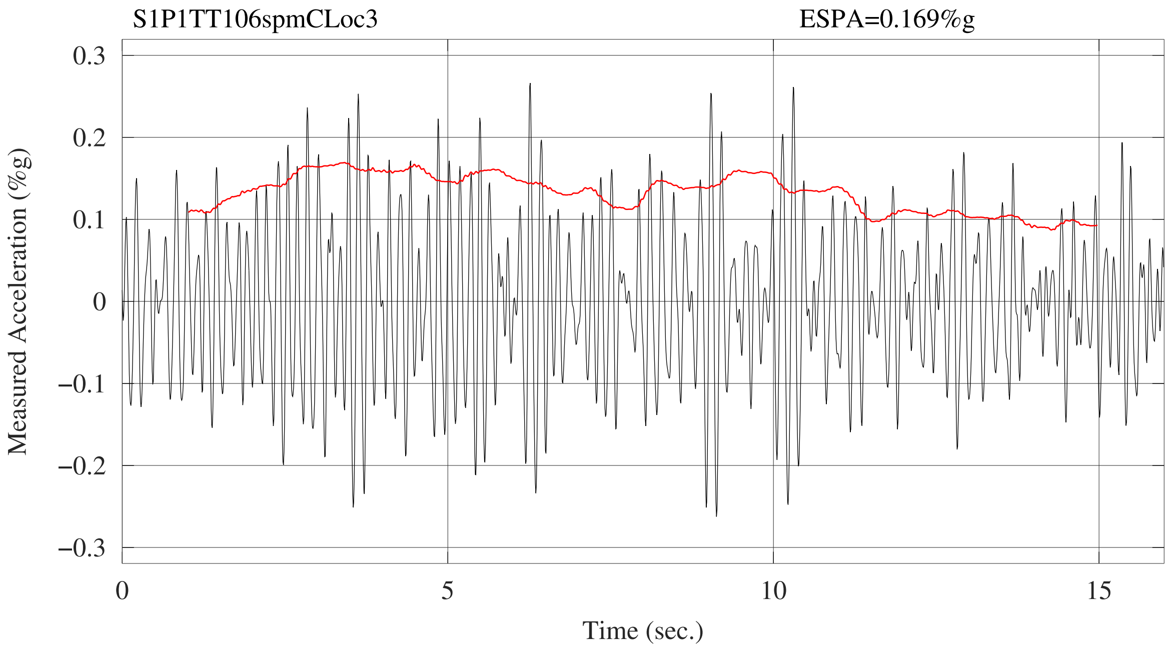
S1P1TT106spmCLoc1
ESPA=0.276%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

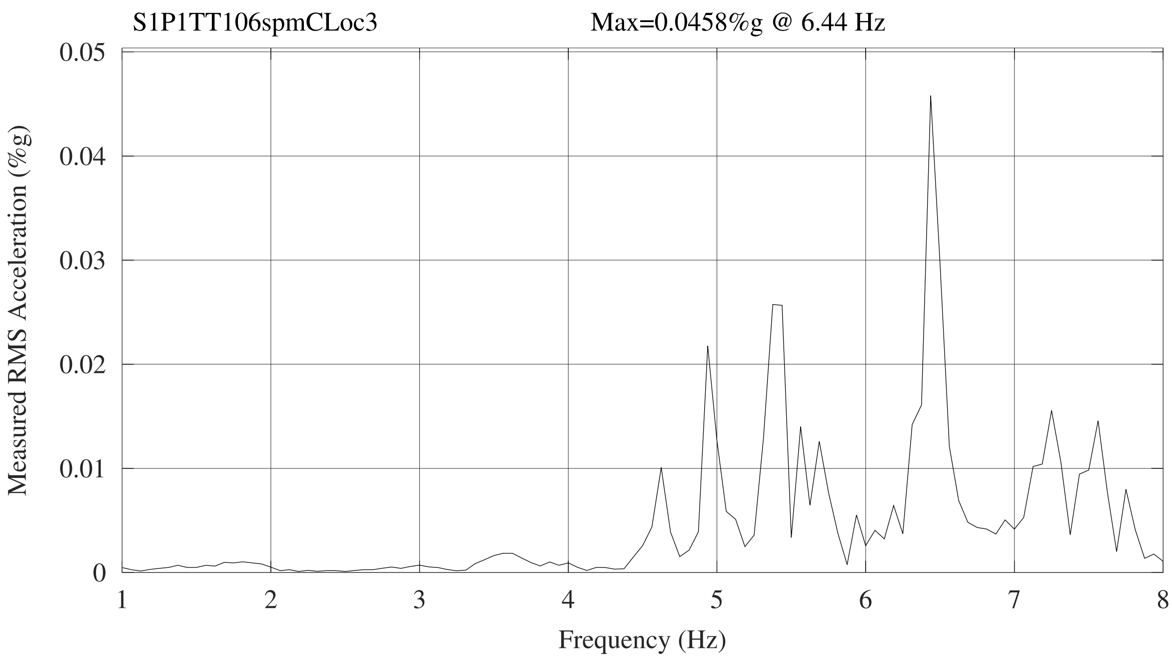


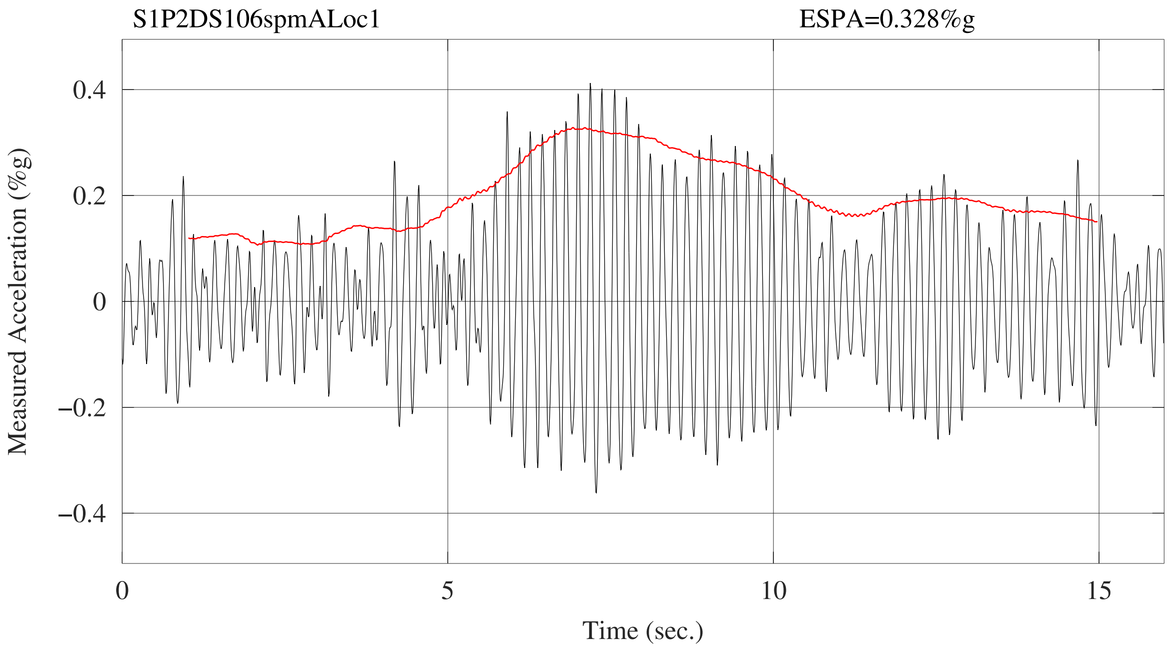
S1P1TT106spmCLoc1
Max=0.0798%g @ 5.38 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

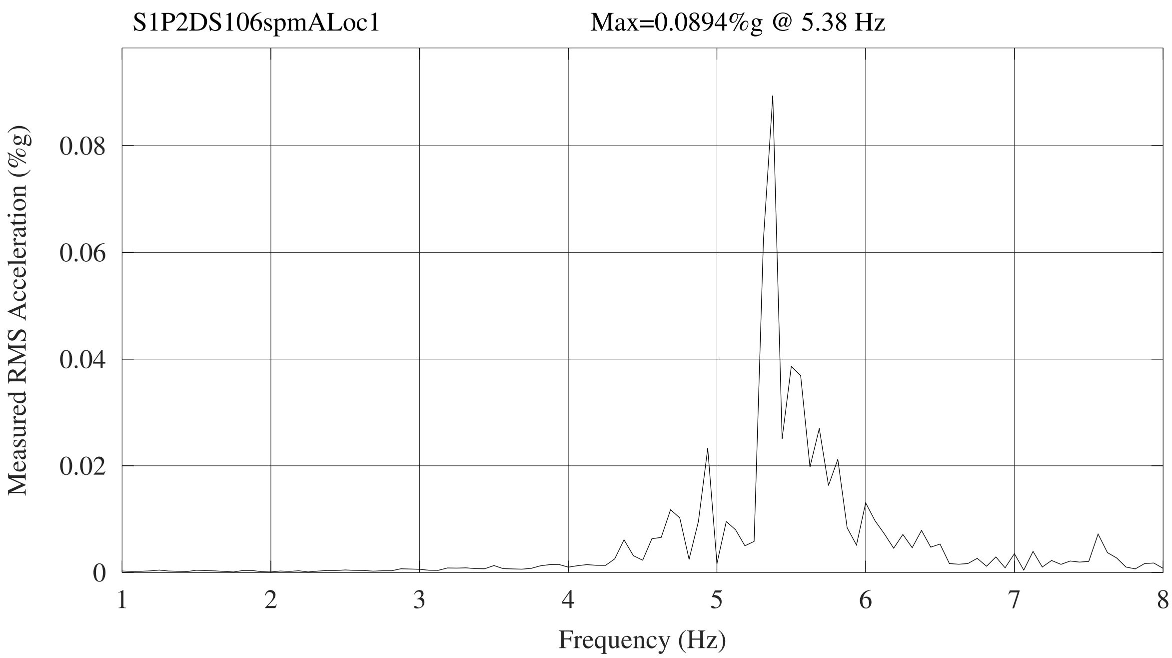

S1P1TT106spmCLoc2
ESPA=0.269%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

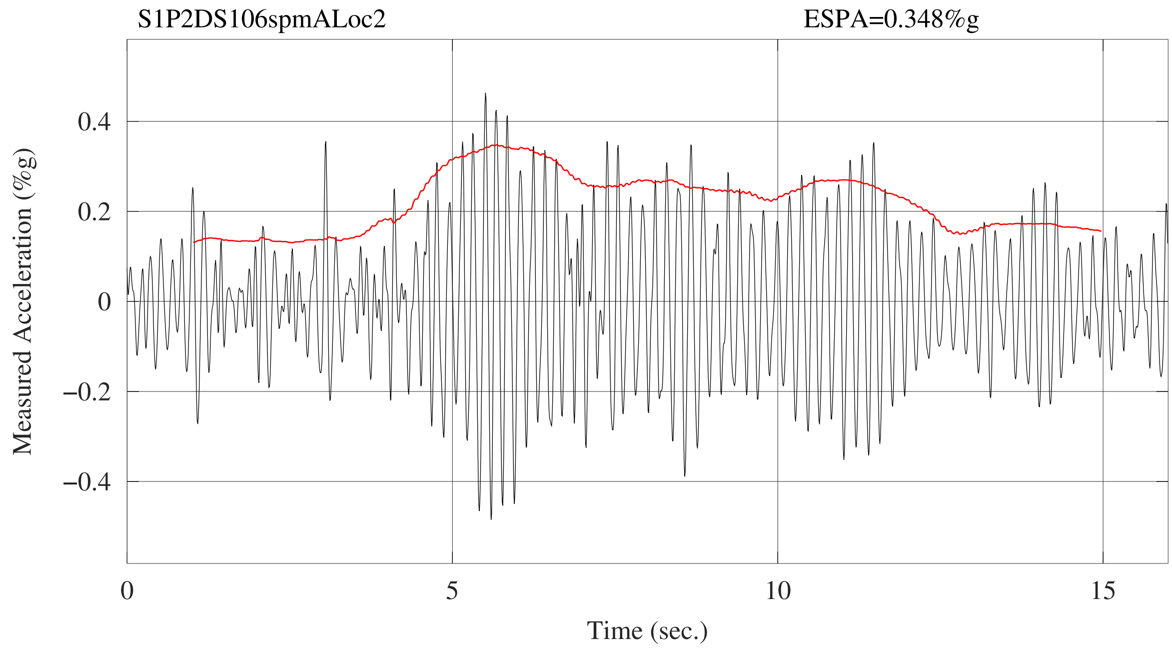

S1P1TT106spmCLoc2
Max=0.0722%g @ 5.38 Hz
Measured RMS Acceleration (%g)
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

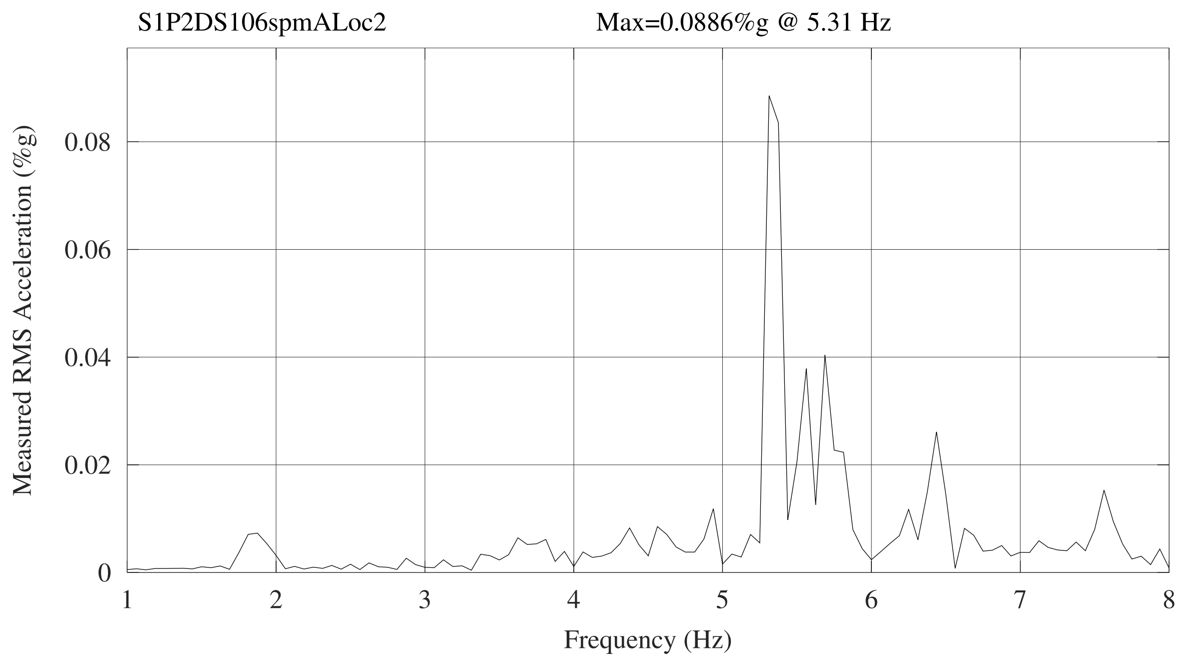

S1P1TT106spmCLoc3
ESPA=0.169%g
Measured Acceleration (%g)
Time (sec.)

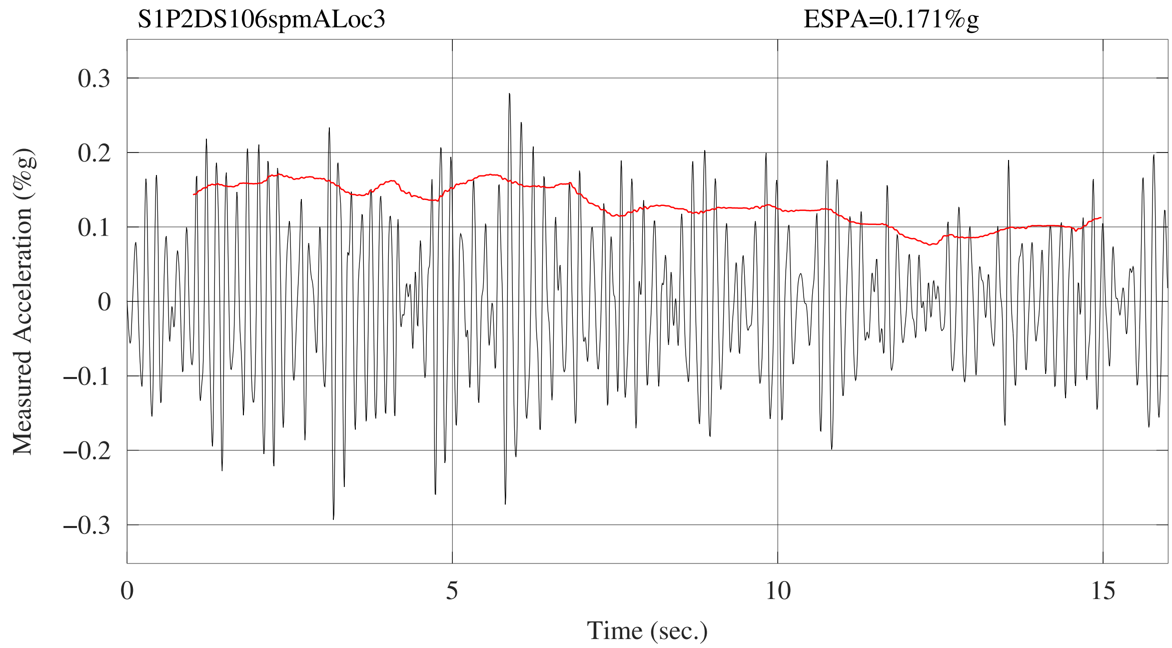

S1P1TT106spmCLoc3
Max=0.0458%g @ 6.44 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)

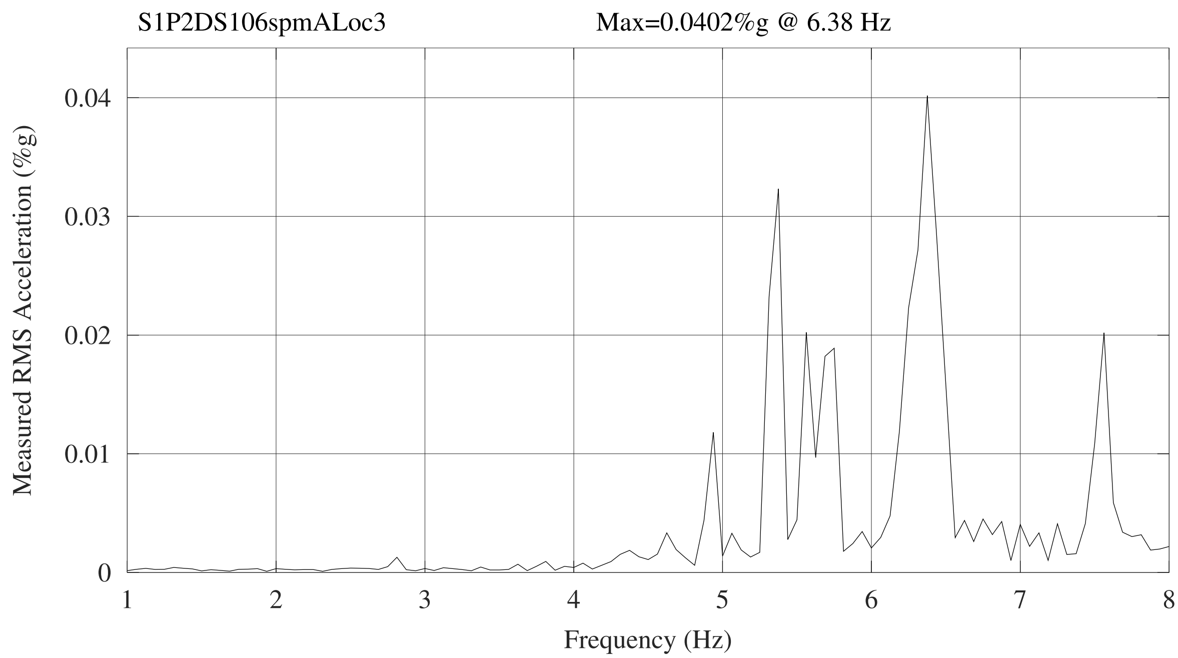

S1P2DS106spmALoc1
ESPA=0.328%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

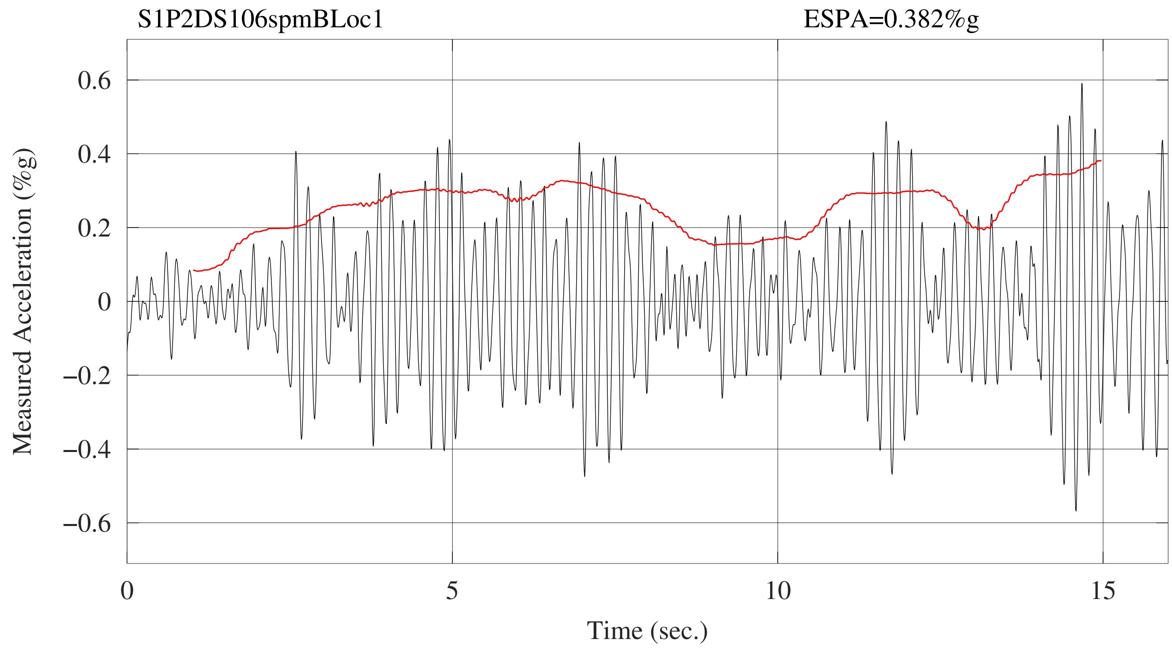

S1P2DS106spmALoc1
Max=0.0894%g @ 5.38 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

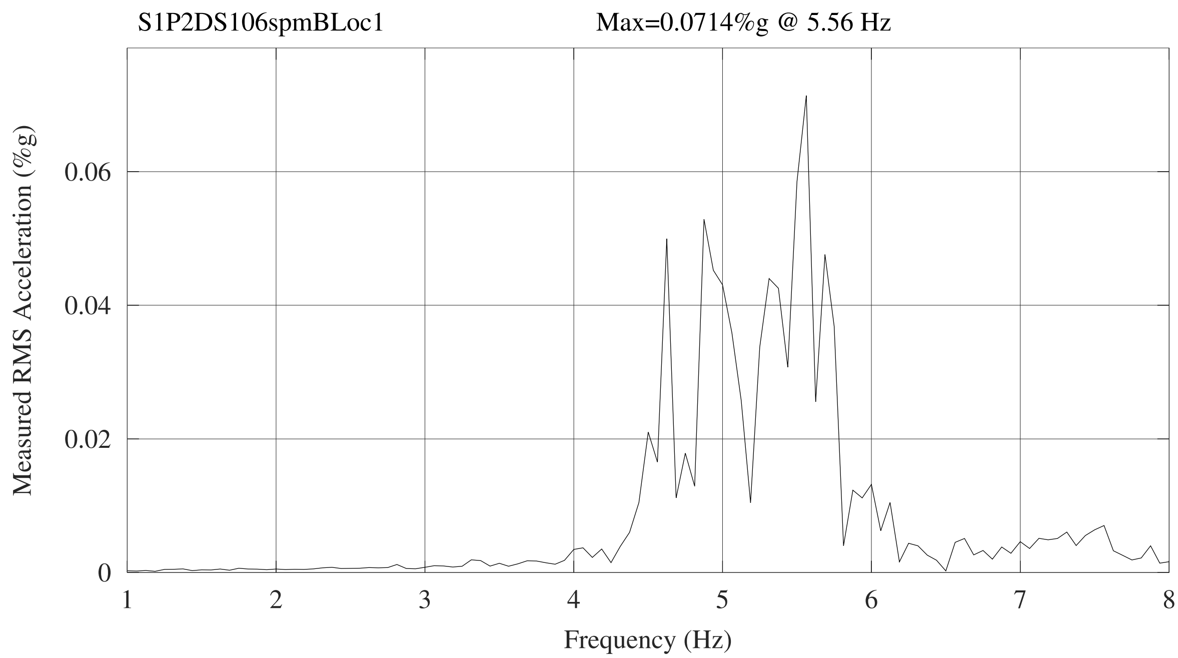

S1P2DS106spmALoc2
ESPA=0.348%g
Measured Acceleration (%g)
Time (sec.)

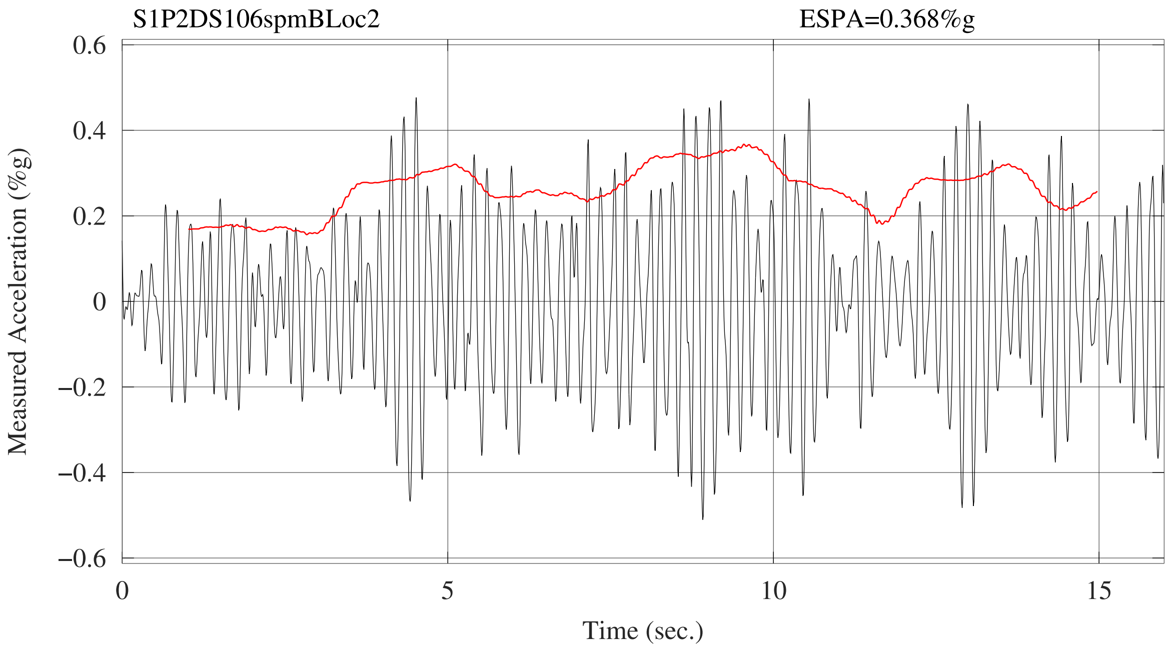

S1P2DS106spmALoc2
Max=0.0886%g @ 5.31 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)

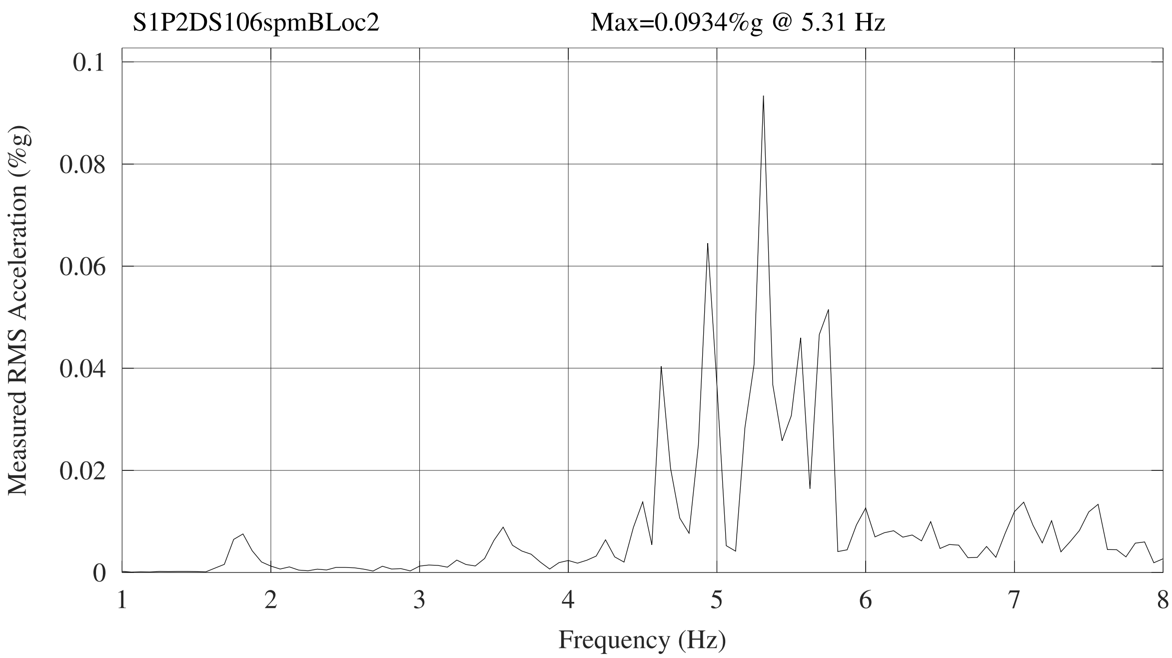

S1P2DS106spmALoc3
ESPA=0.171%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

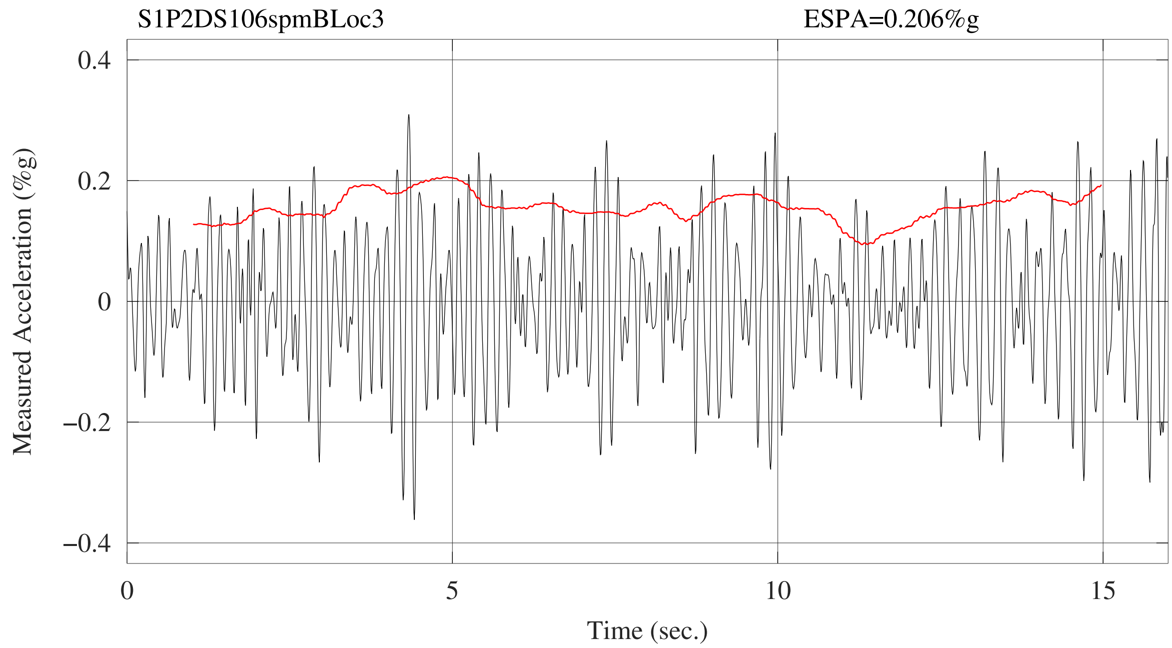

S1P2DS106spmALoc3
Max=0.0402%g @ 6.38 Hz
Measured RMS Acceleration (%g)
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

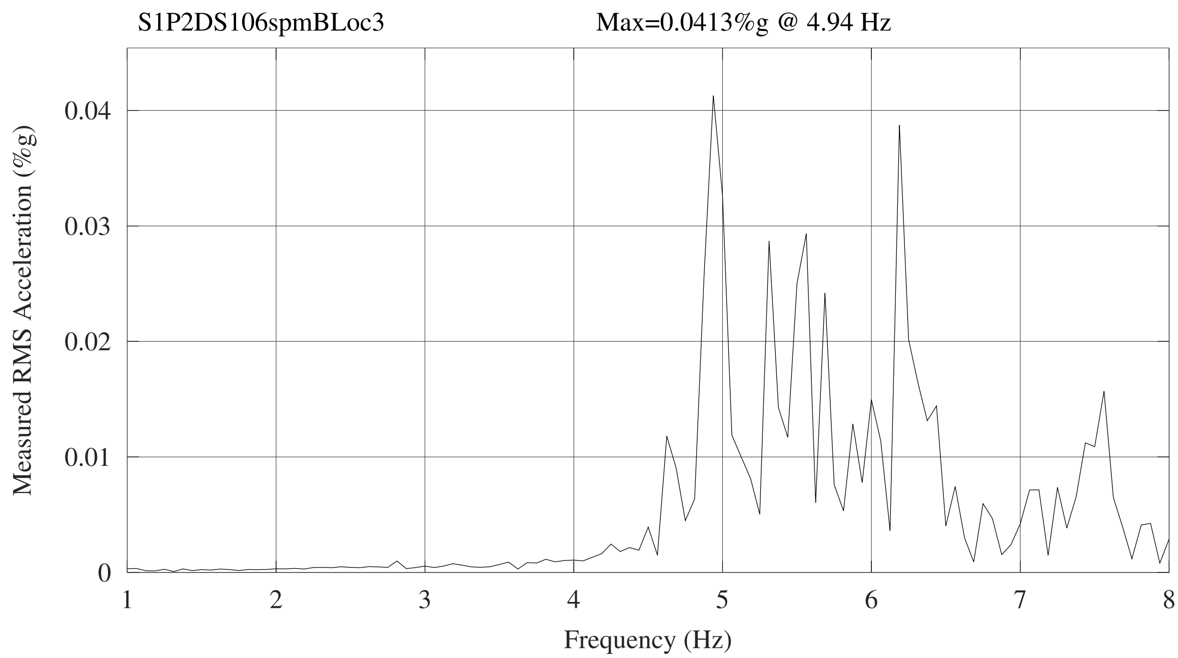

S1P2DS106spmBLoc1
ESPA=0.382%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

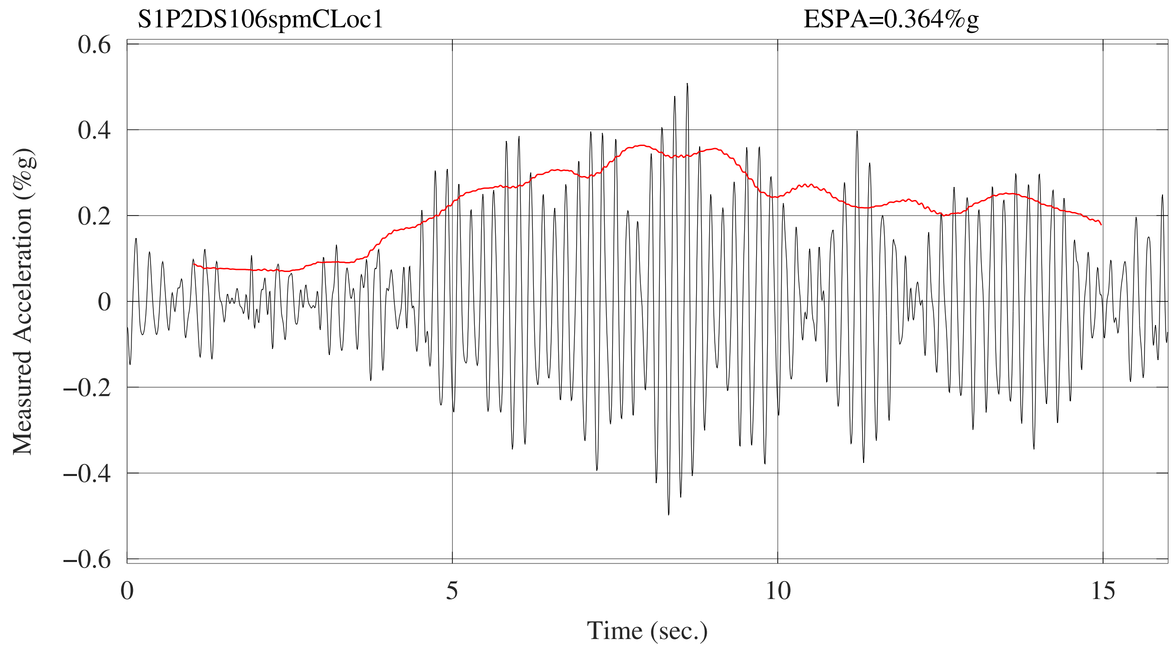

S1P2DS106spmBLoc1
Max=0.0714%g @ 5.56 Hz
Measured RMS Acceleration (%g)
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

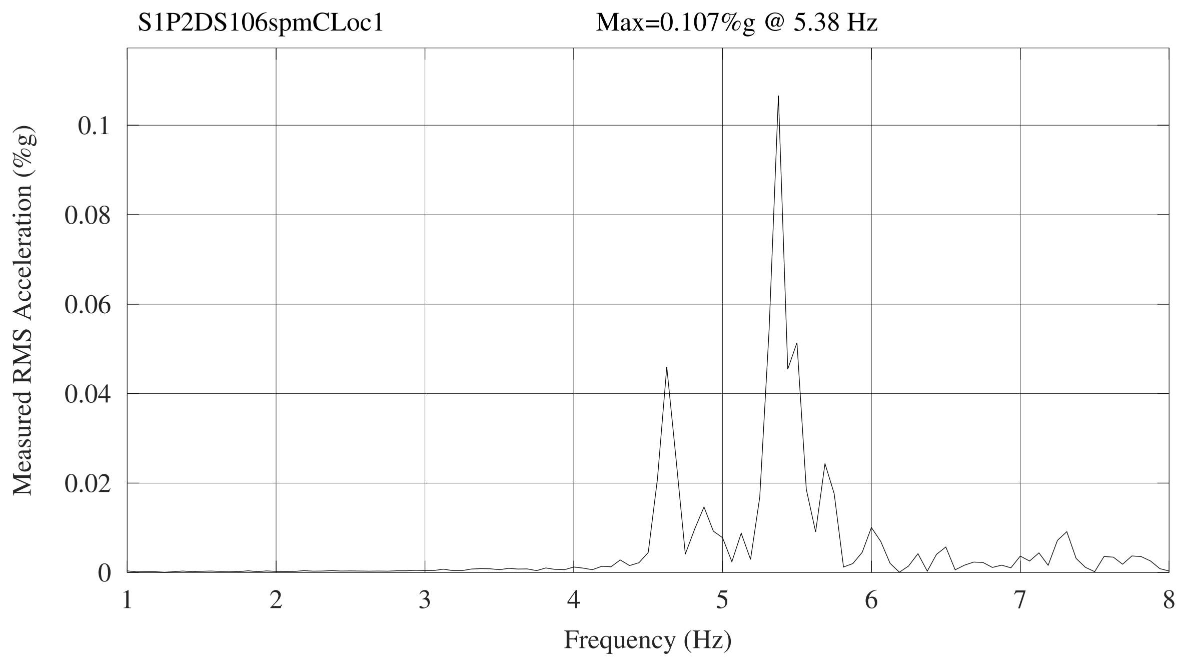

S1P2DS106spmBLoc2
ESPA=0.368%g
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

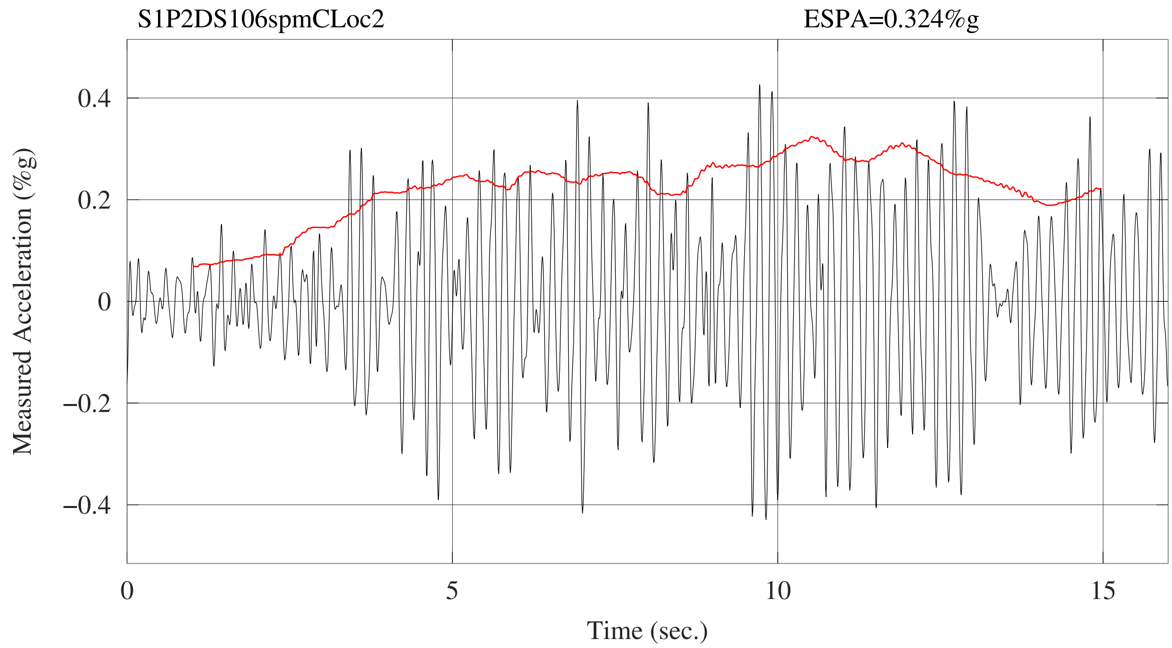

S1P2DS106spmBLoc2
Max=0.0934%g @ 5.31 Hz
0.1
0.08
0.06
0.04
0.02
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)

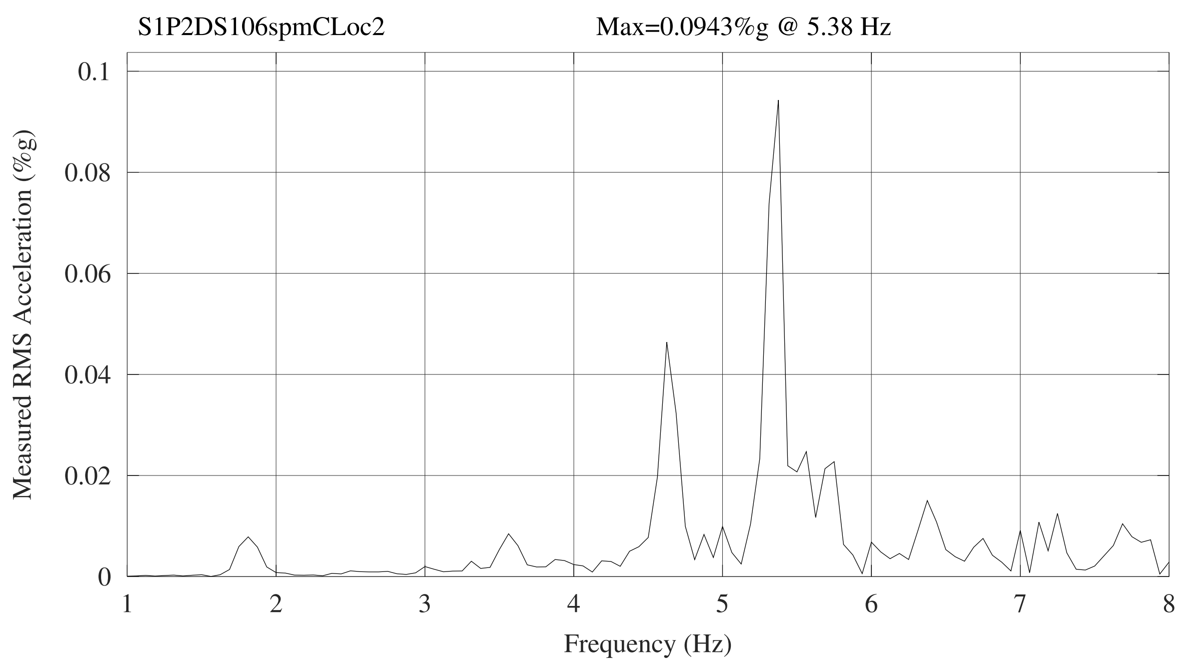

S1P2DS106spmBLoc3
ESPA=0.206%g
0.4
0.2
0
−0.2
−0.4
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

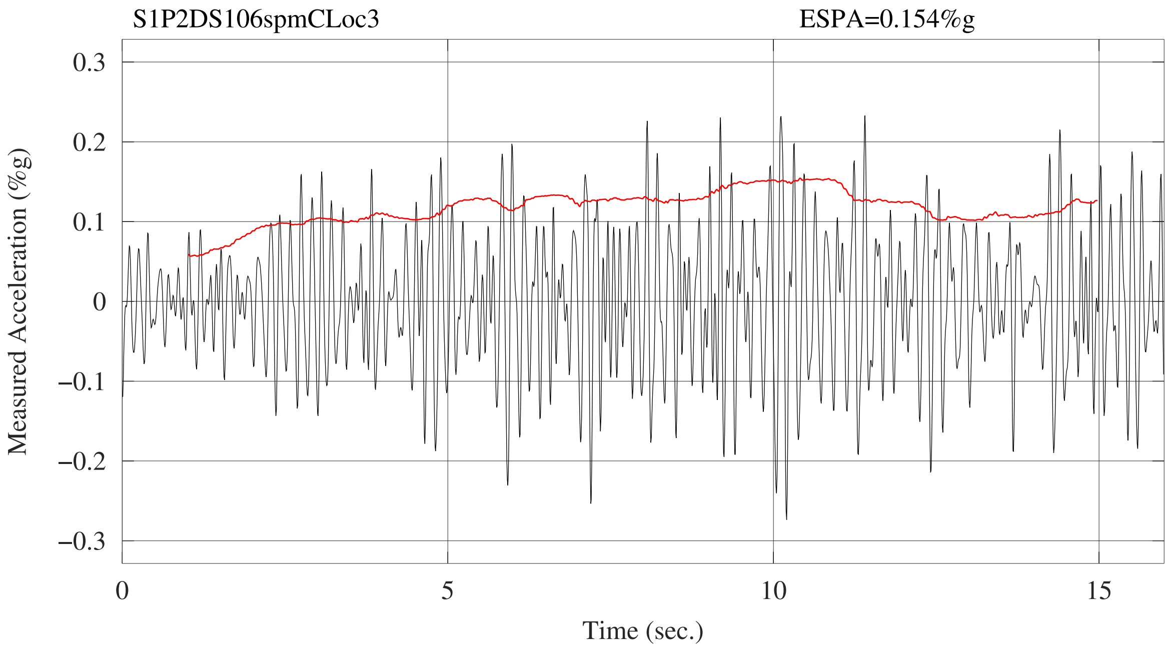

S1P2DS106spmBLoc3
Max=0.0413%g @ 4.94 Hz
0.04
0.03
0.02
0.01
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)

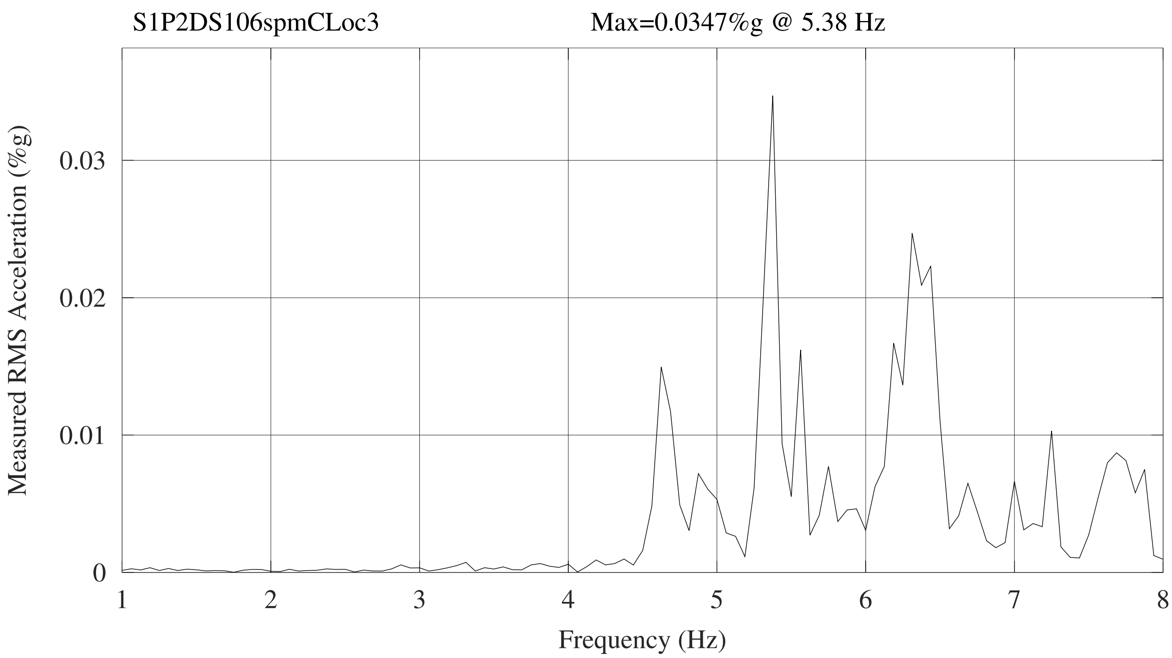

S1P2DS106spmCLoc1
ESPA=0.364%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

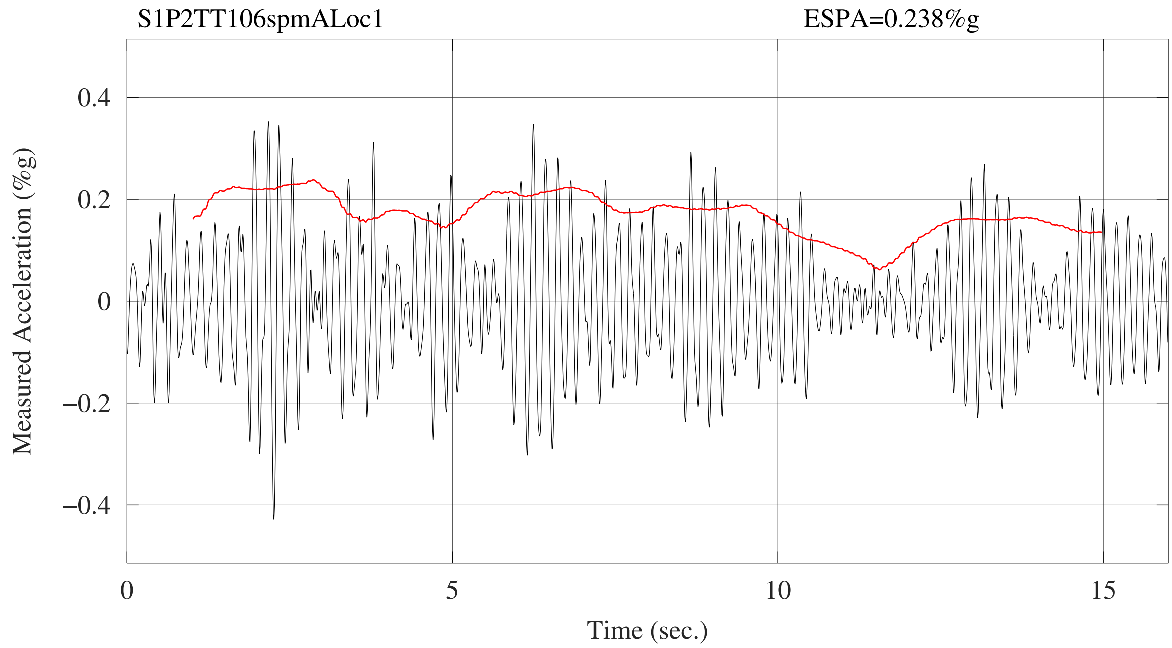

S1P2DS106spmCLoc1
Max=0.107%g @ 5.38 Hz
Measured RMS Acceleration (%g)
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

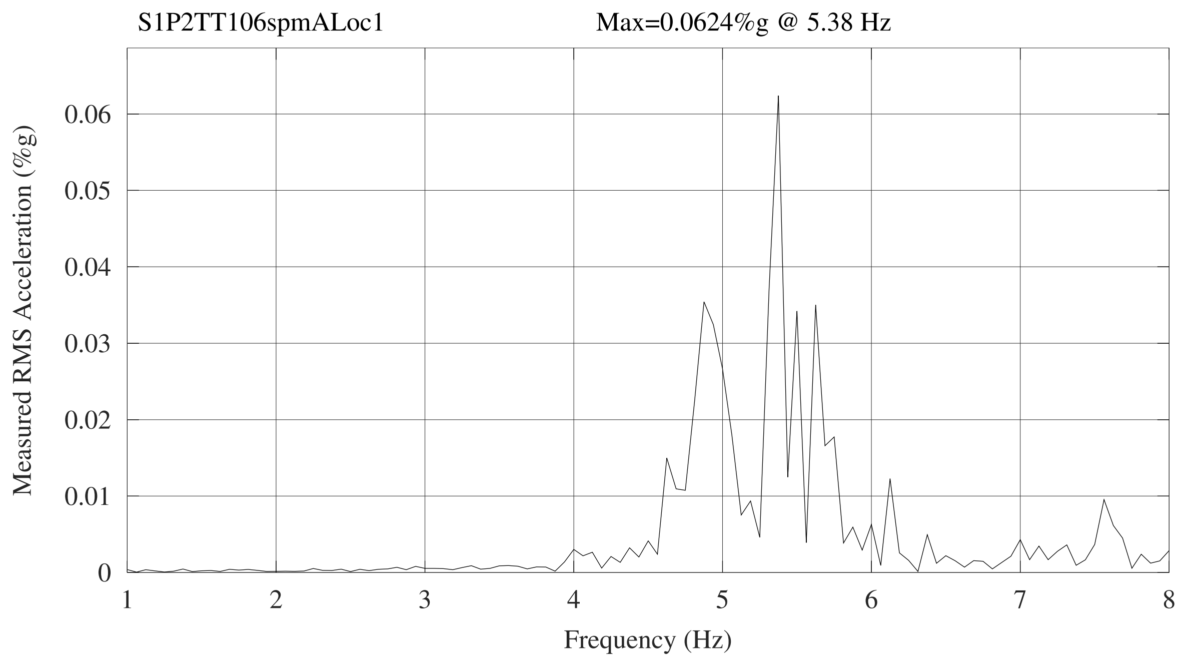

S1P2DS106spmCLoc2
ESPA=0.324%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

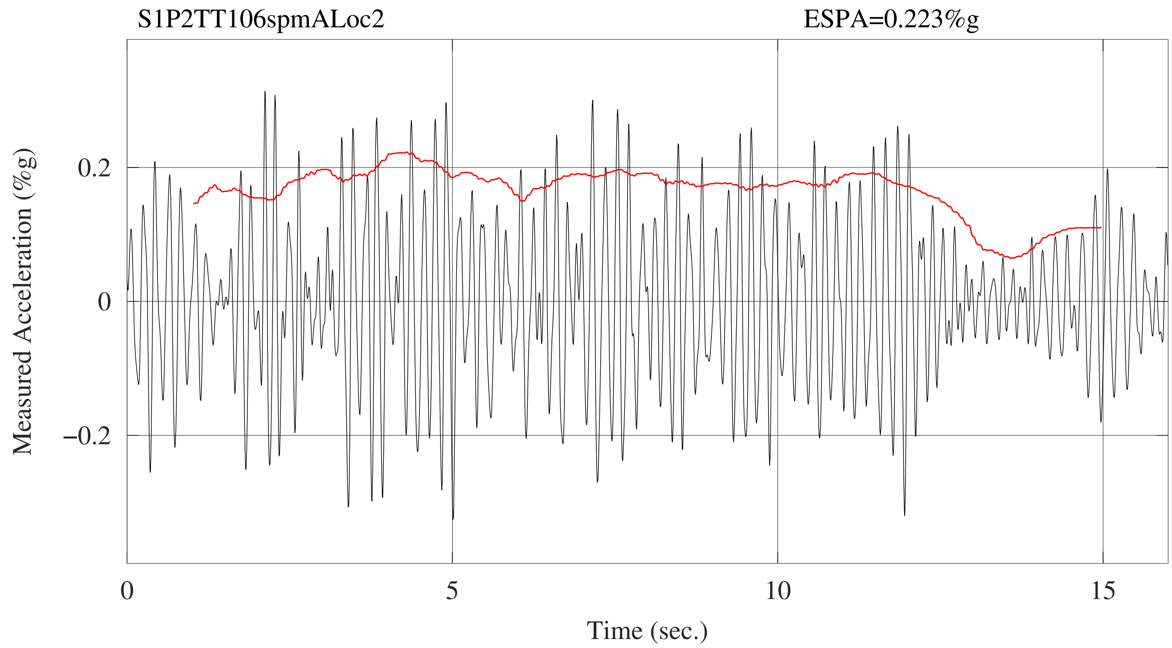

S1P2DS106spmCLoc2
Max=0.0943%g @ 5.38 Hz
Measured RMS Acceleration (%g)
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

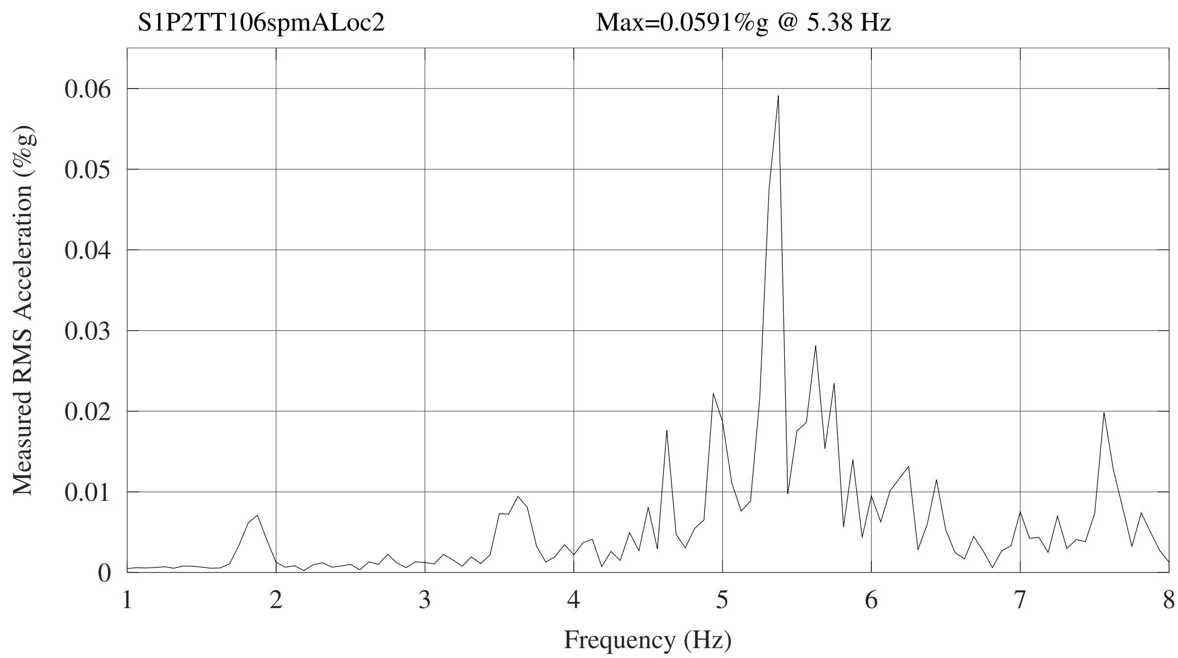

S1P2DS106spmCLoc3
ESPA=0.154%g
Measured Acceleration (%g)
Time (sec.)

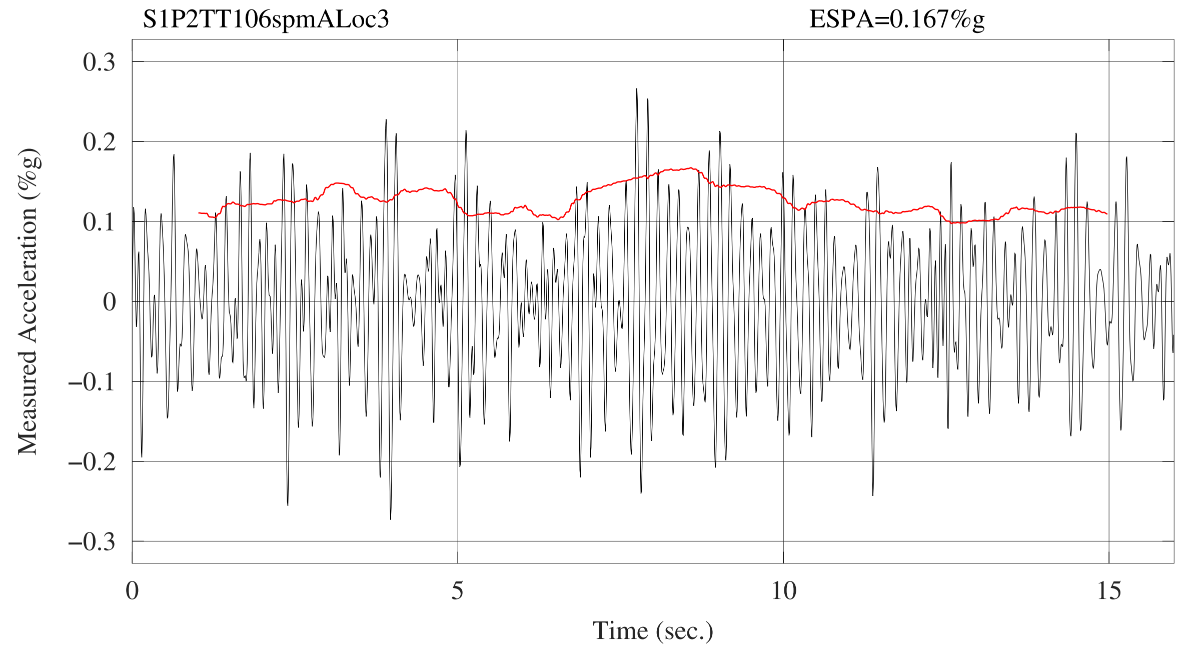

S1P2DS106spmCLoc3
Max=0.0347%g @ 5.38 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)

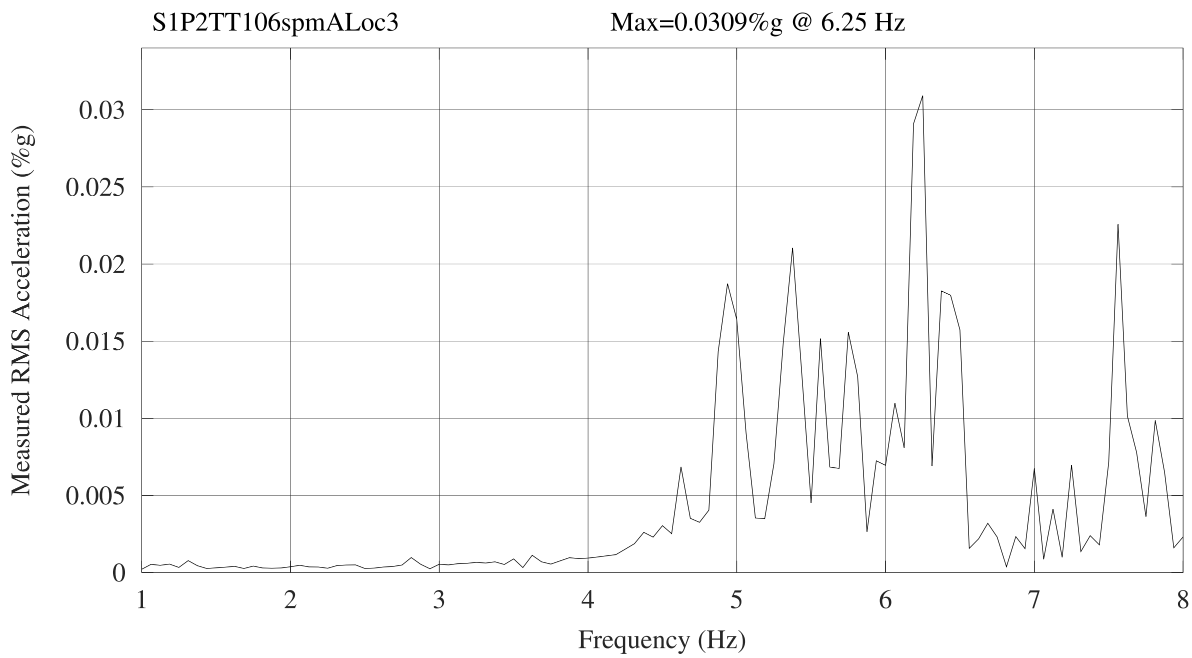

S1P2TT106spmALoc1
ESPA=0.238%g
Measured Acceleration (%g)
0.4
0.2
0
–0.2
–0.4
0
5
10
15
Time (sec.)

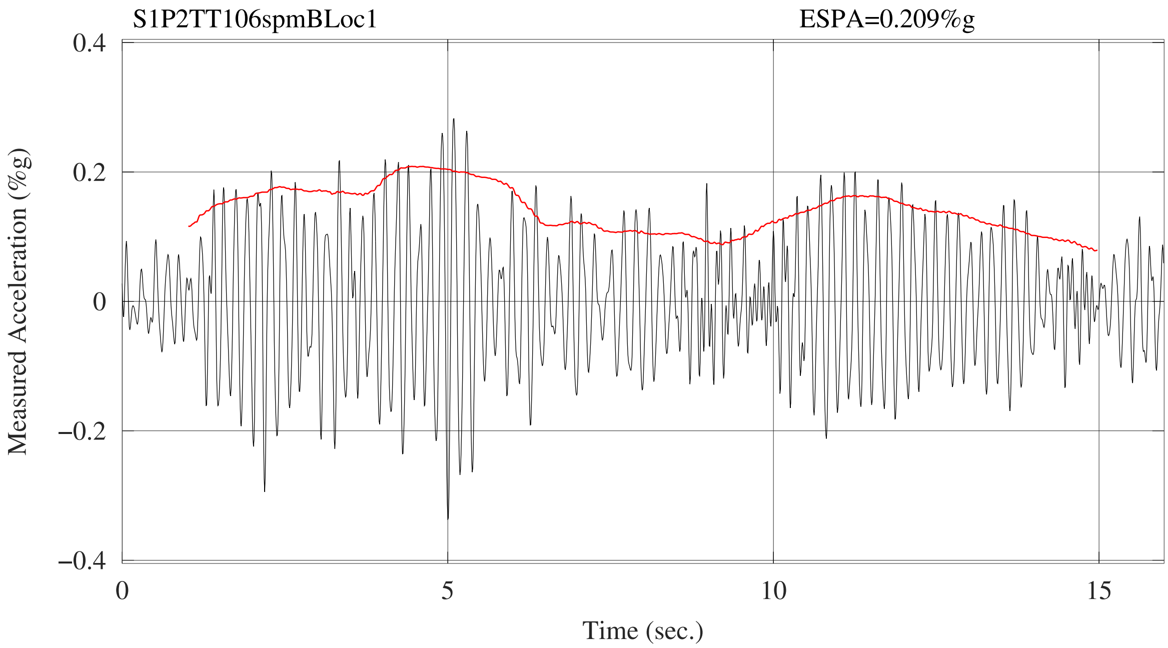

S1P2TT106spmALoc1
Max=0.0624%g @ 5.38 Hz
Measured RMS Acceleration (%g)
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

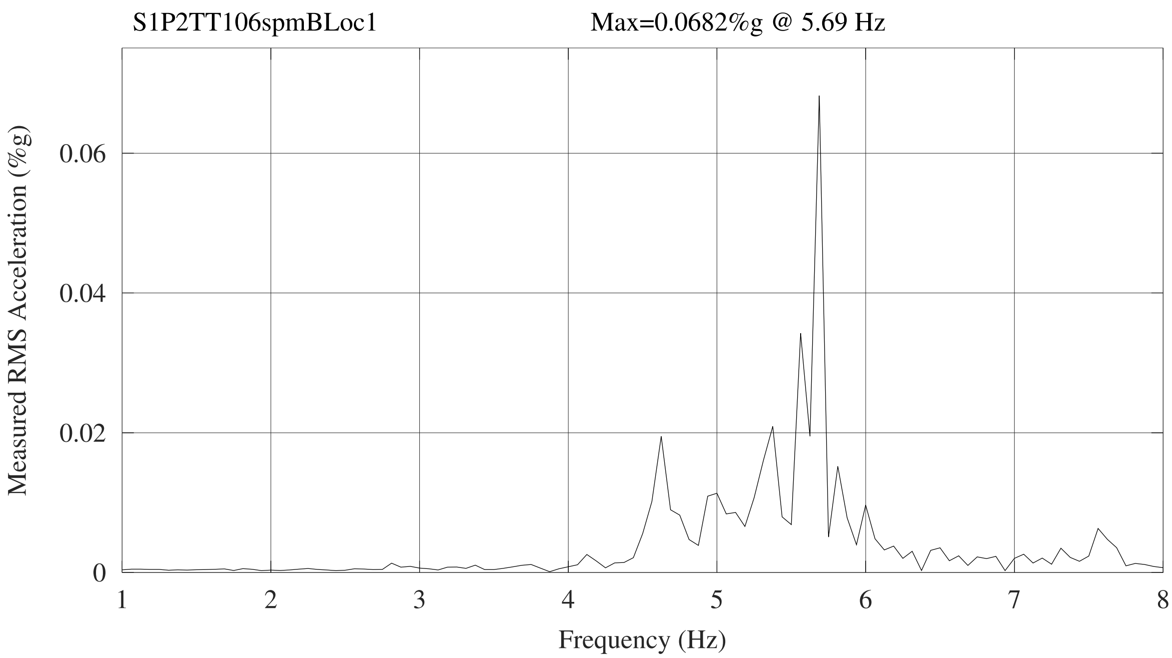

S1P2TT106spmALoc2
ESPA=0.223%g
Measured Acceleration (%g)
0.2
0
−0.2
0
5
10
15
Time (sec.)

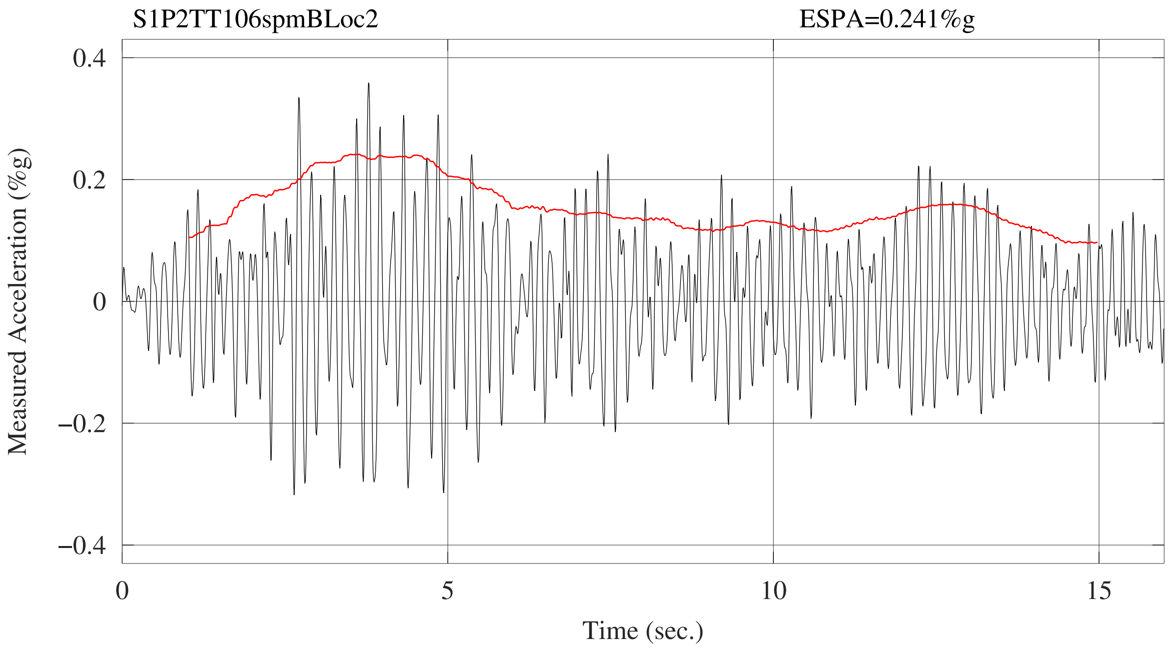

S1P2TT106spmALoc2
Max=0.0591%g @ 5.38 Hz
Measured RMS Acceleration (%g)
0.06
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

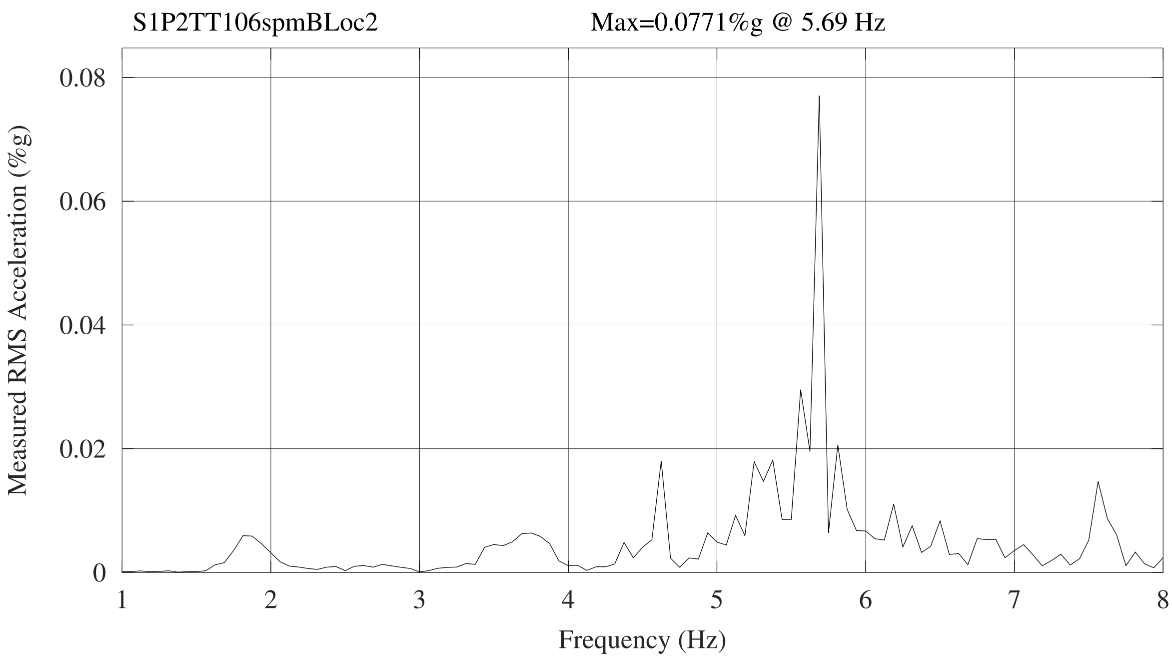

S1P2TT106spmALoc3
ESPA=0.167%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

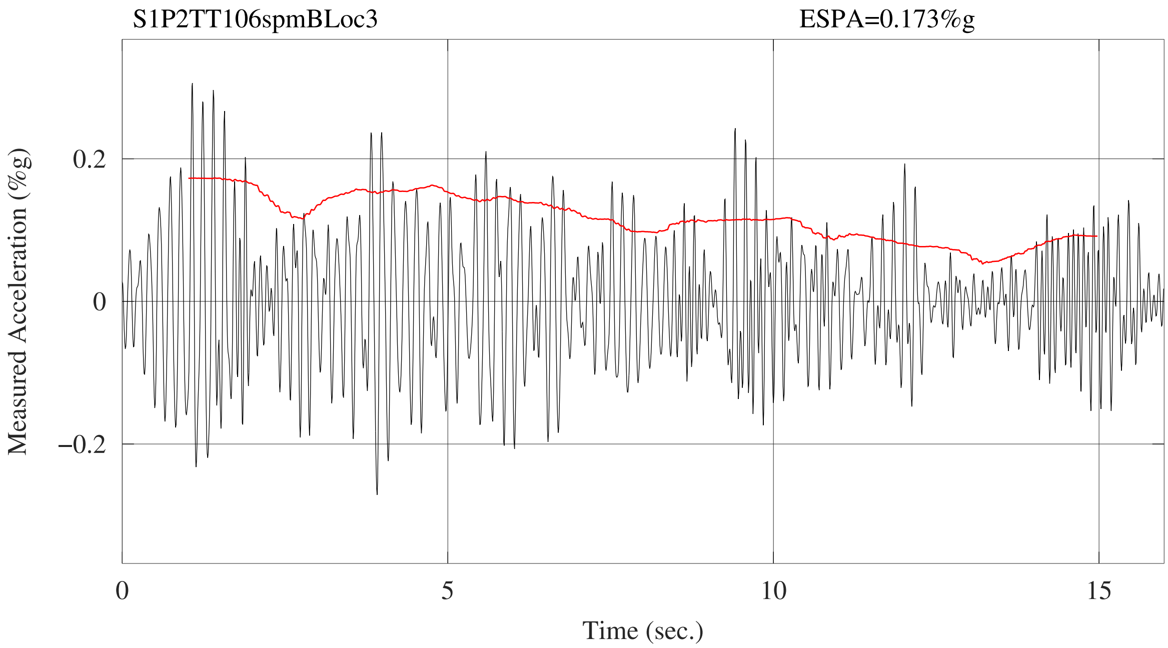

S1P2TT106spmALoc3
Max=0.0309%g @ 6.25 Hz
Measured RMS Acceleration (%g)
0.03
0.025
0.02
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S1P2TT106spmBLoc1
ESPA=0.209%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

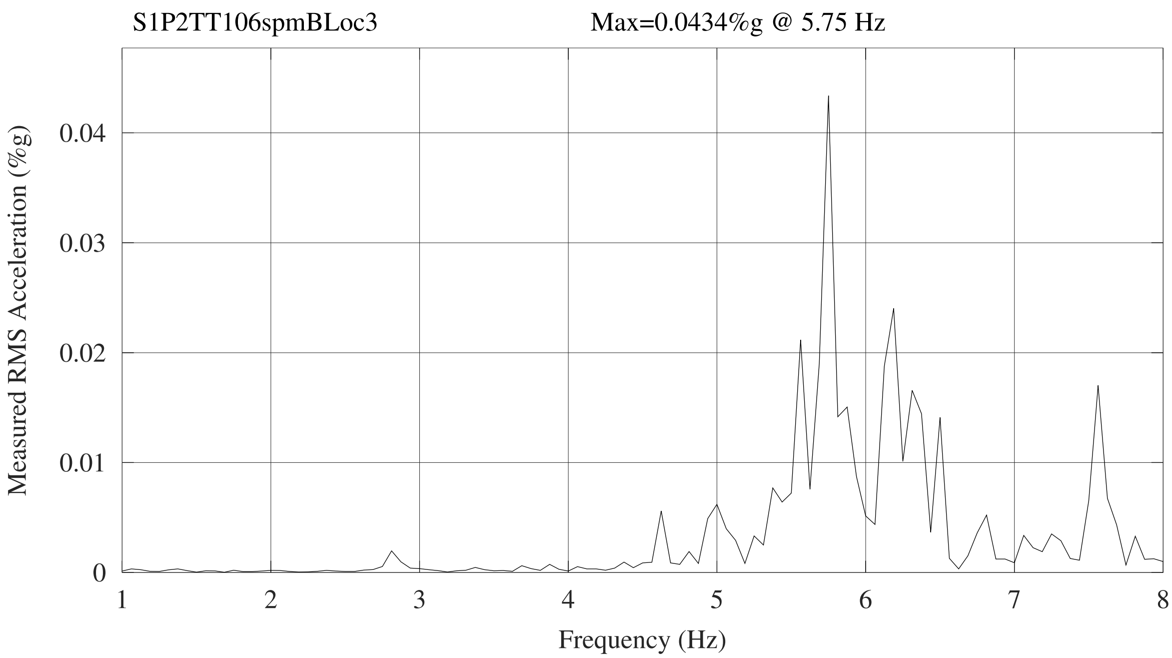

S1P2TT106spmBLoc1
Max=0.0682%g @ 5.69 Hz
Measured RMS Acceleration (%g)
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S1P2TT106spmBLoc2
ESPA=0.241%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

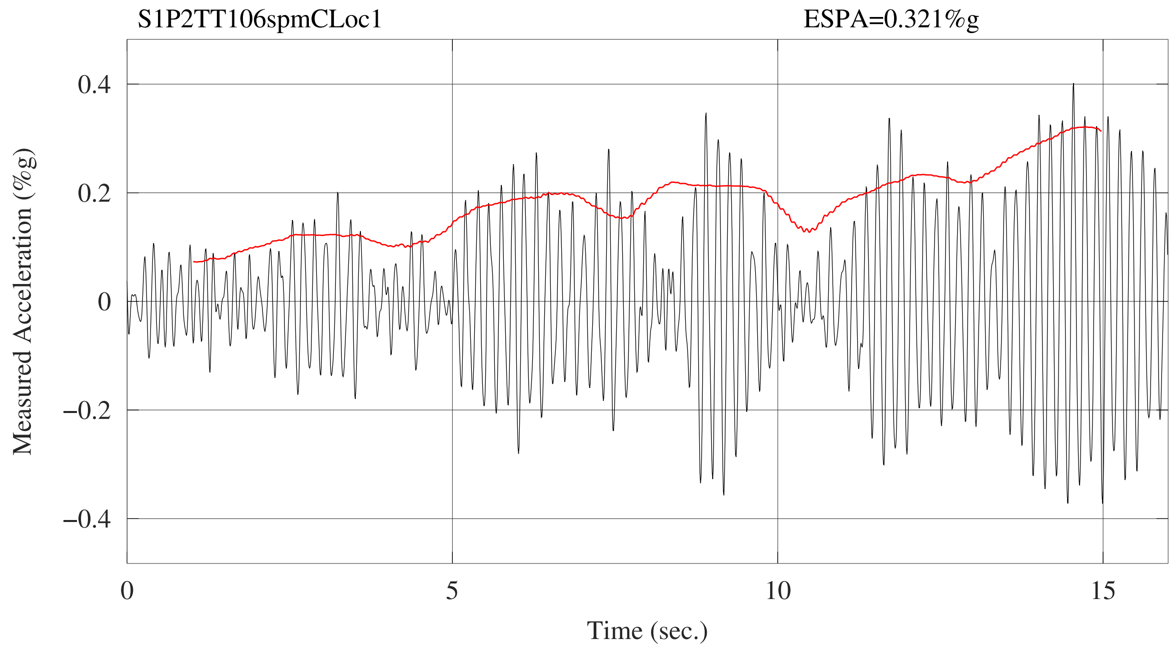

S1P2TT106spmBLoc2
Max=0.0771%g @ 5.69 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)



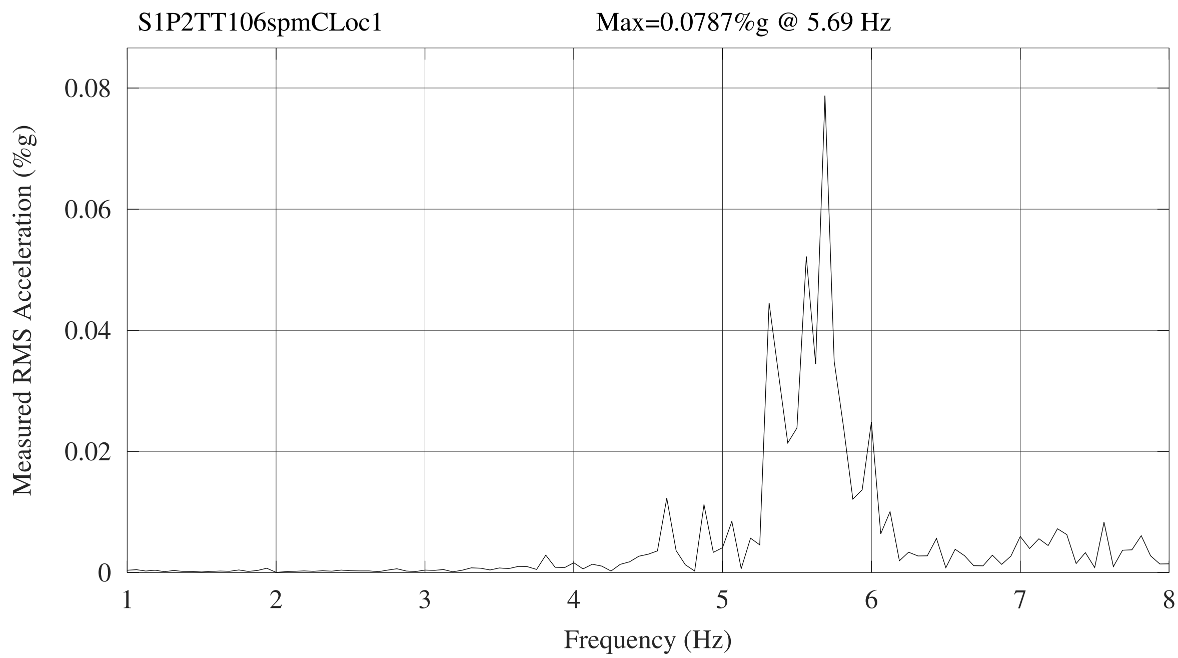
S1P2TT106spmBLoc3
ESPA=0.173%g
Measured Acceleration (%g)
0.2
0
−0.2
0
5
10
15
Time (sec.)



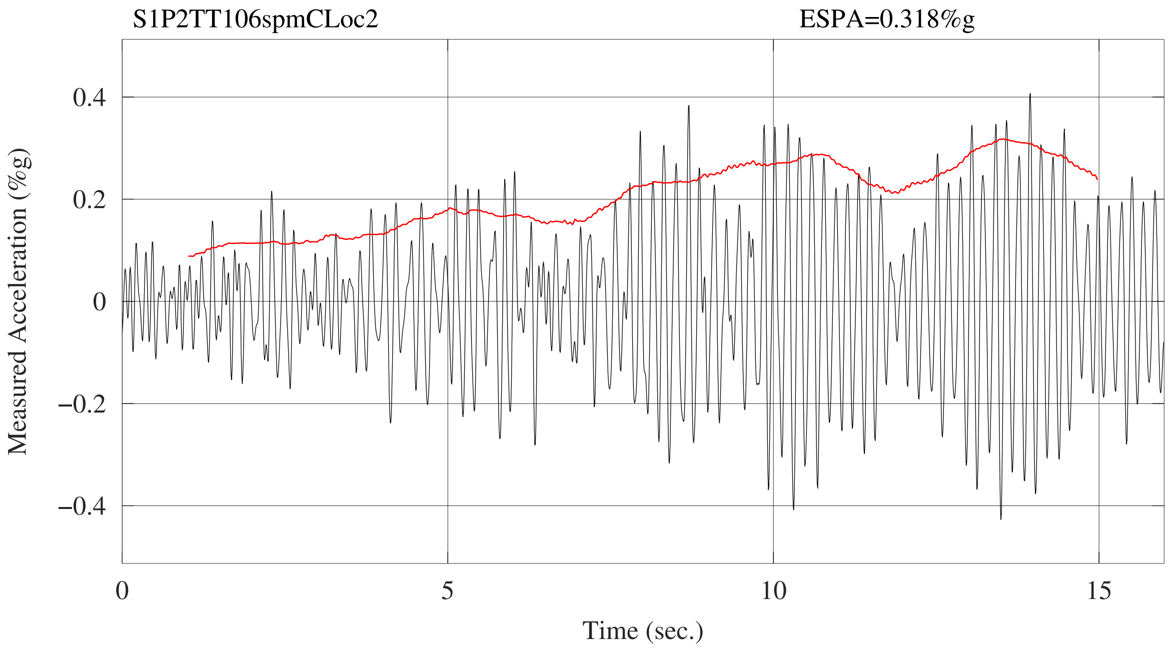
S1P2TT106spmBLoc3
Max=0.0434%g @ 5.75 Hz
Measured RMS Acceleration (%g)
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)



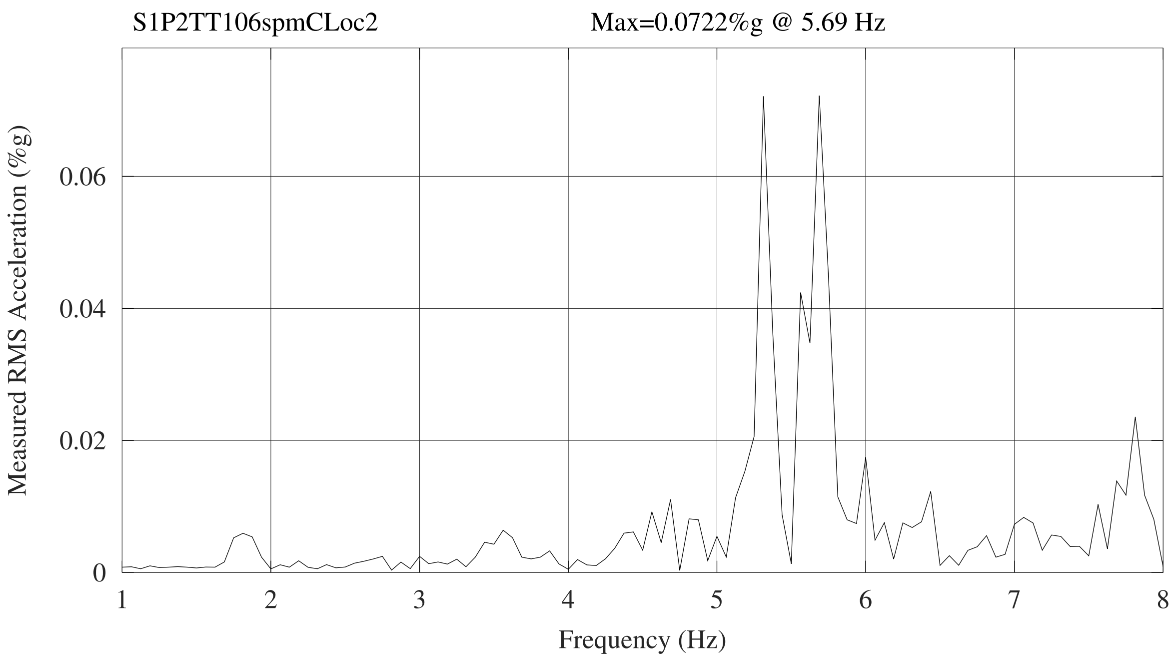
S1P2TT106spmCLoc1
ESPA=0.321%g
Measured Acceleration (%g)
Time (sec.)



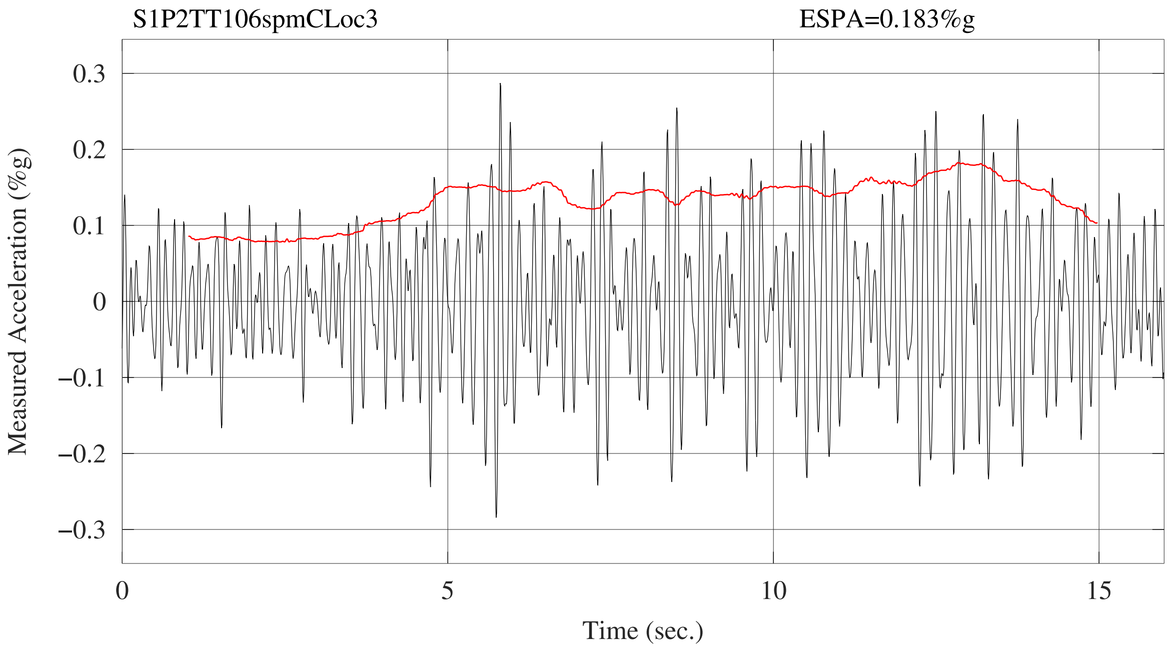
S1P2TT106spmCLoc1
Max=0.0787%g @ 5.69 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)



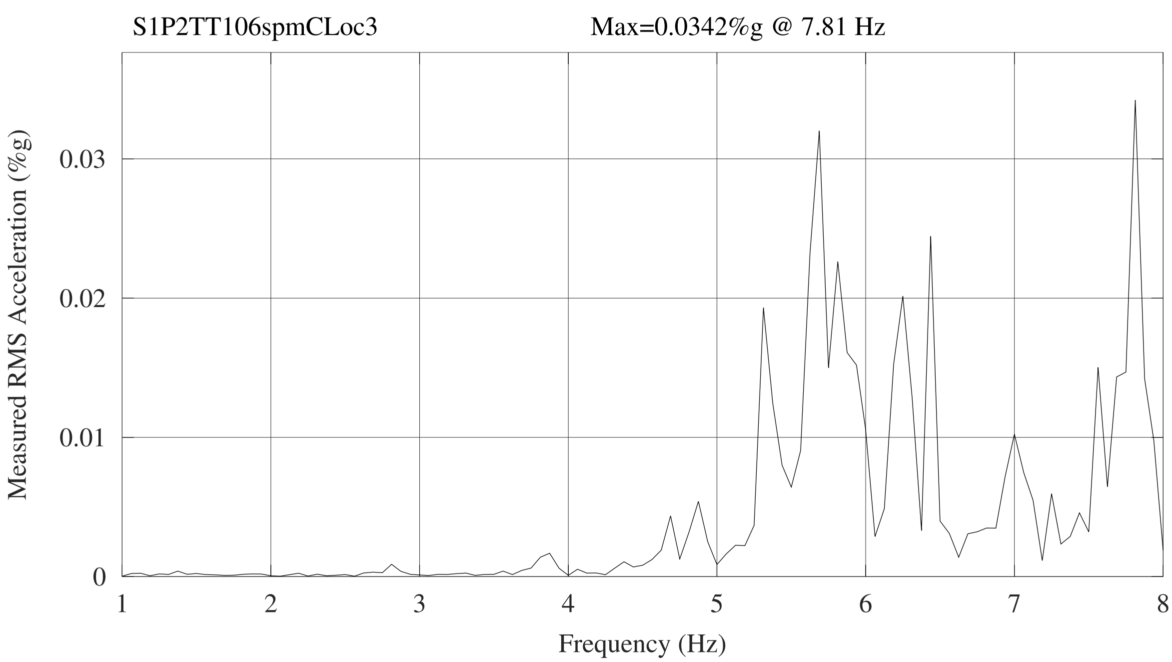
S1P2TT106spmCLoc2
ESPA=0.318%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)



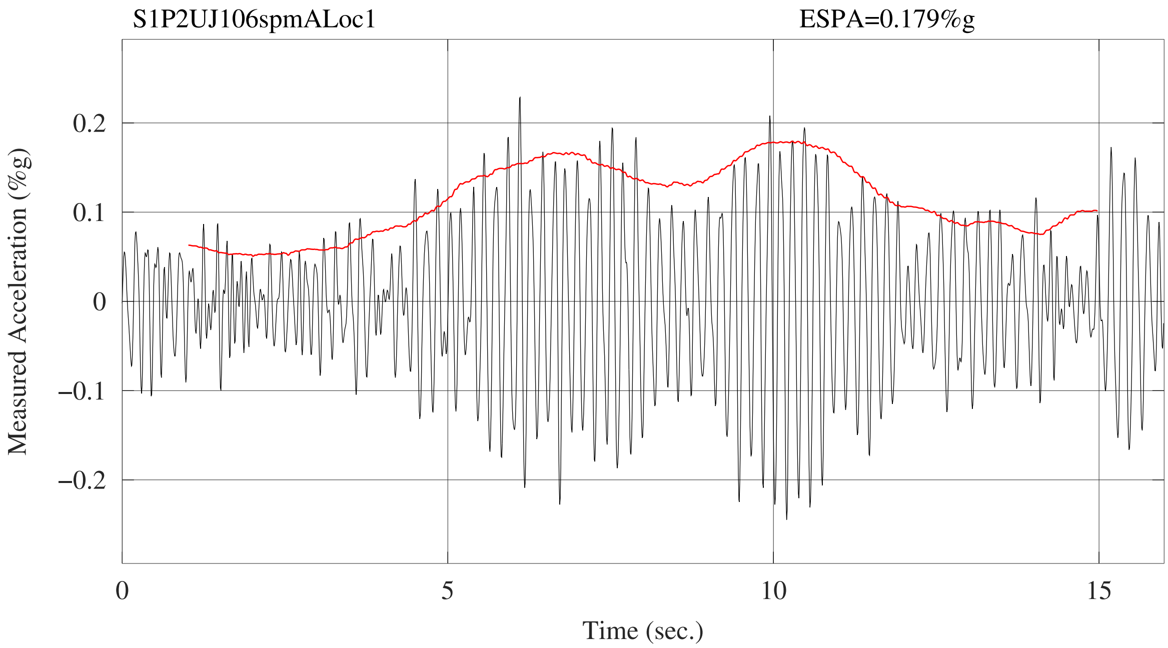
S1P2TT106spmCLoc2
Max=0.0722%g @ 5.69 Hz
Measured RMS Acceleration (%g)
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

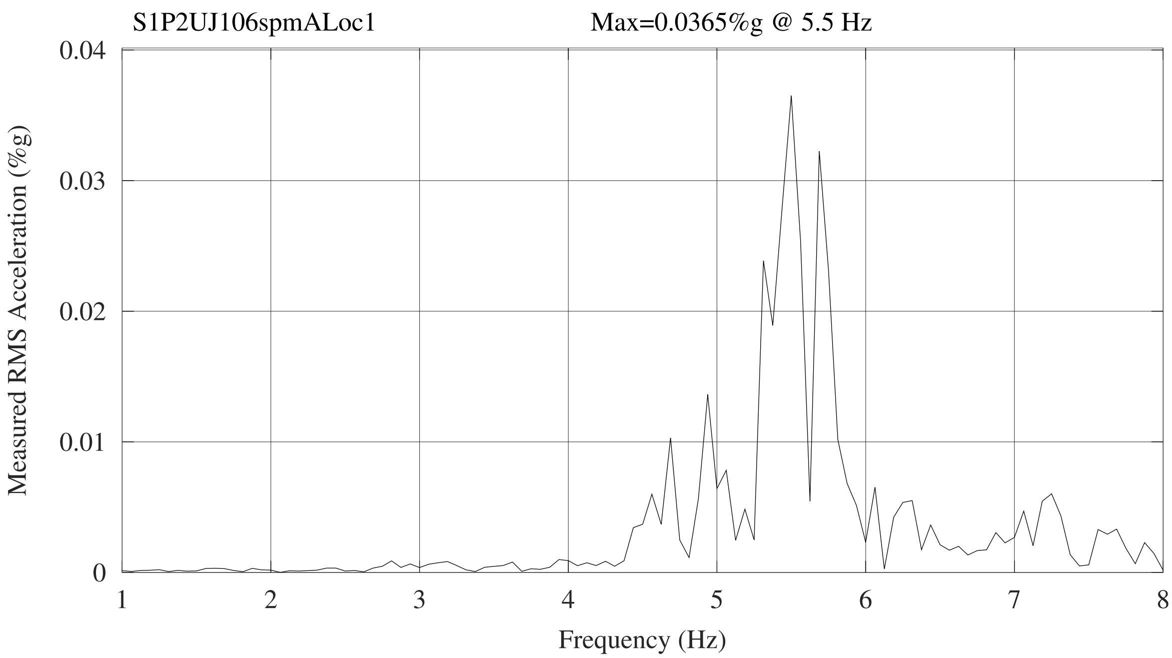


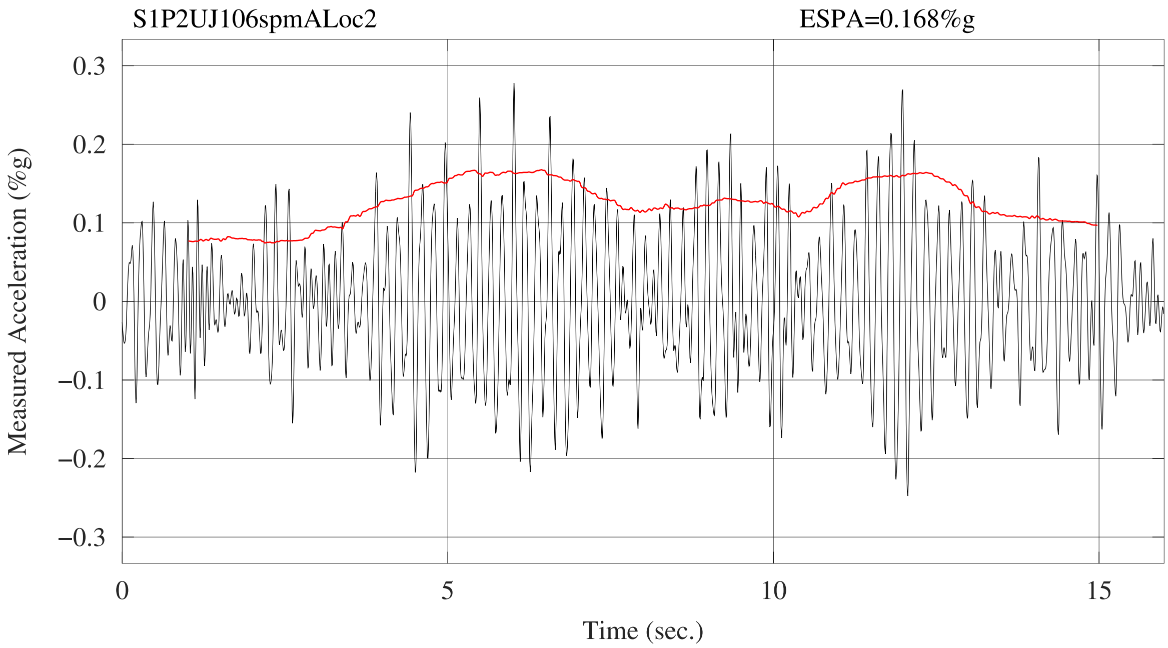
S1P2TT106spmCLoc3
ESPA=0.183%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

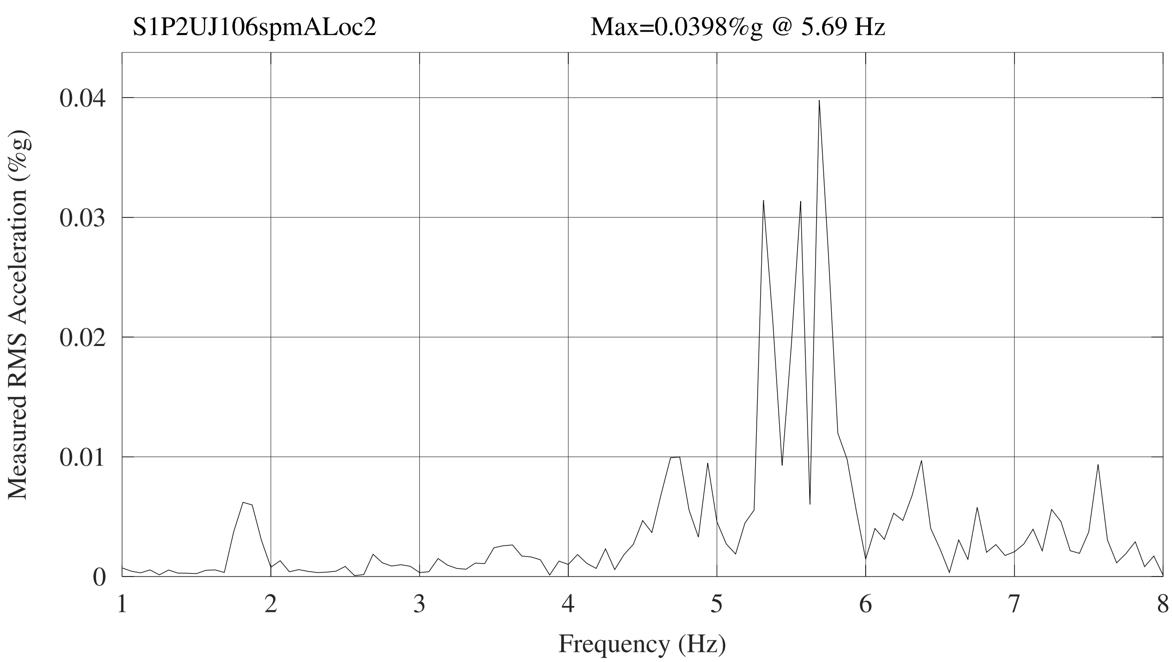


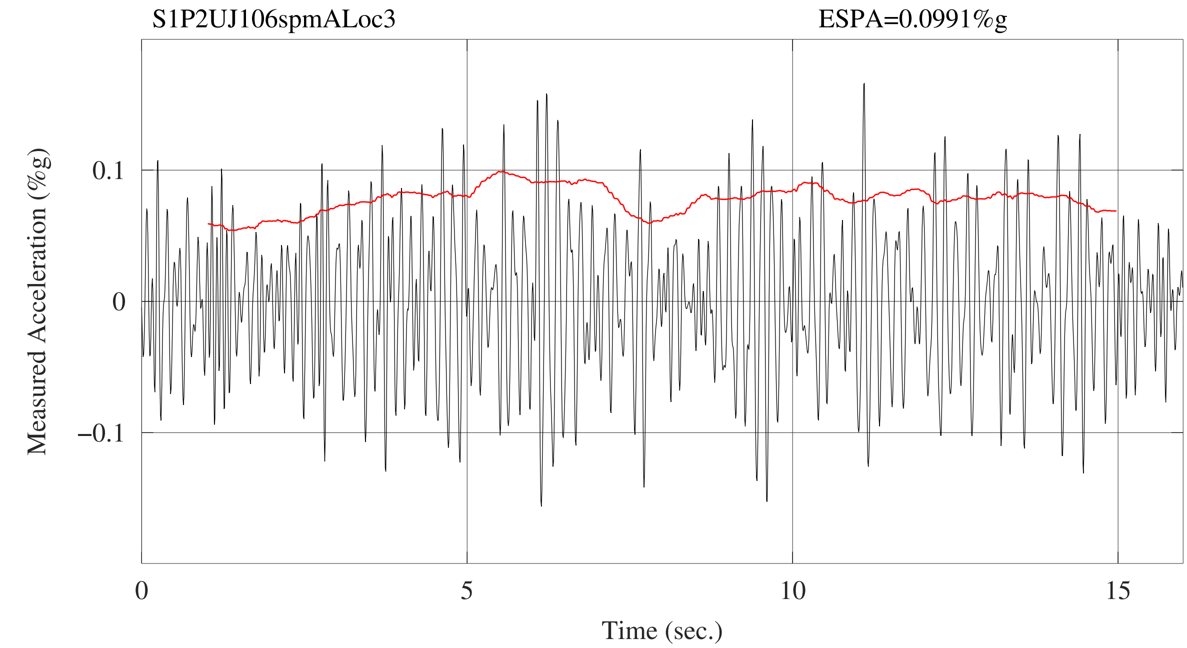
S1P2TT106spmCLoc3
Max=0.0342%g @ 7.81 Hz
Measured RMS Acceleration (%g)
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

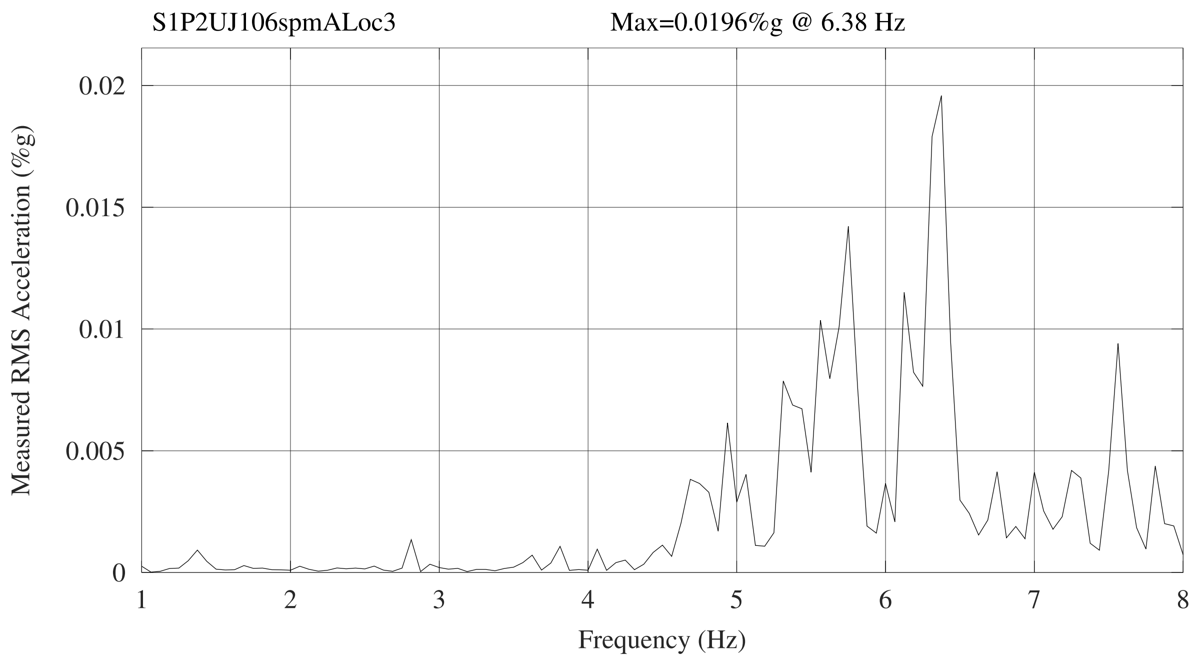

S1P2UJ106spmALoc1
ESPA=0.179%g
Measured Acceleration (%g)
Time (sec.)

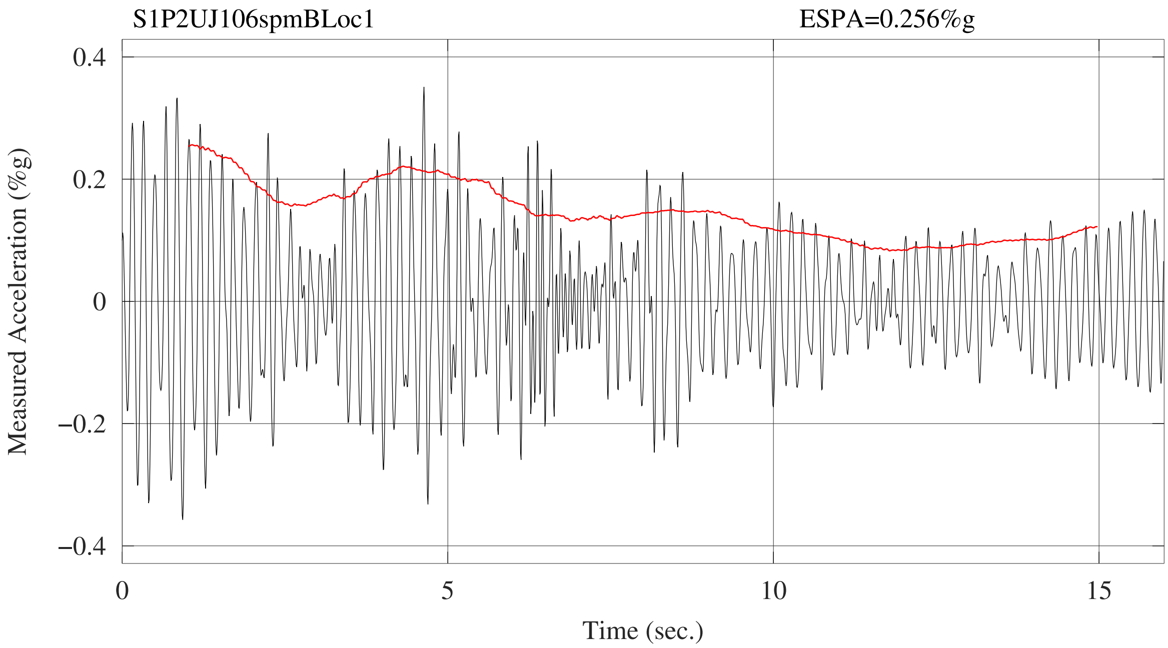

S1P2UJ106spmALoc1
Max=0.0365%g @ 5.5 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)


S1P2UJ106spmALoc2
ESPA=0.168%g
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

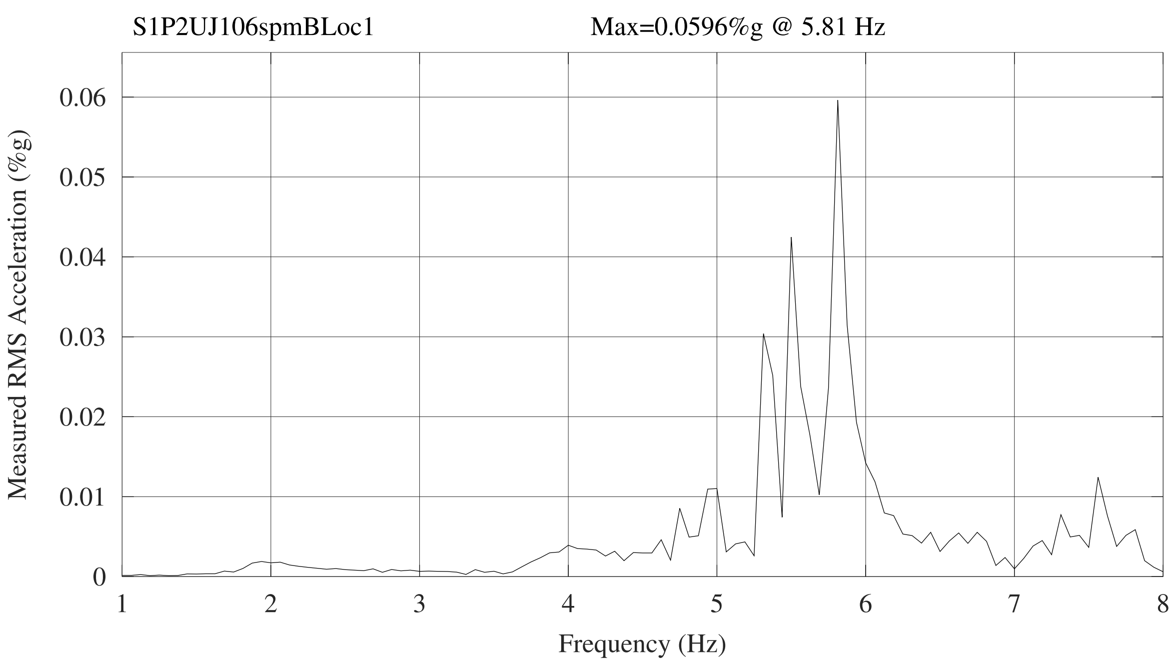

S1P2UJ106spmALoc2
Max=0.0398%g @ 5.69 Hz
0.04
0.03
0.02
0.01
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)


S1P2UJ106spmALoc3
ESPA=0.0991%g
Measured Acceleration (%g)
0.1
0
−0.1
0
5
10
15
Time (sec.)

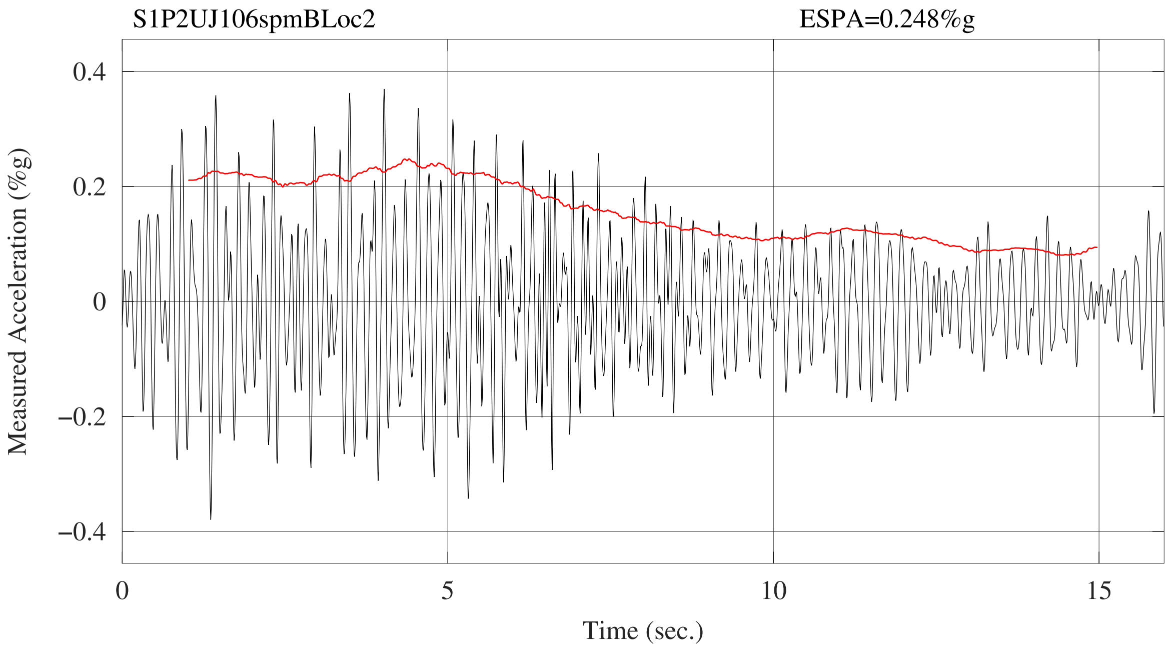

S1P2UJ106spmALoc3
Max=0.0196%g @ 6.38 Hz
Measured RMS Acceleration (%g)
0.02
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)

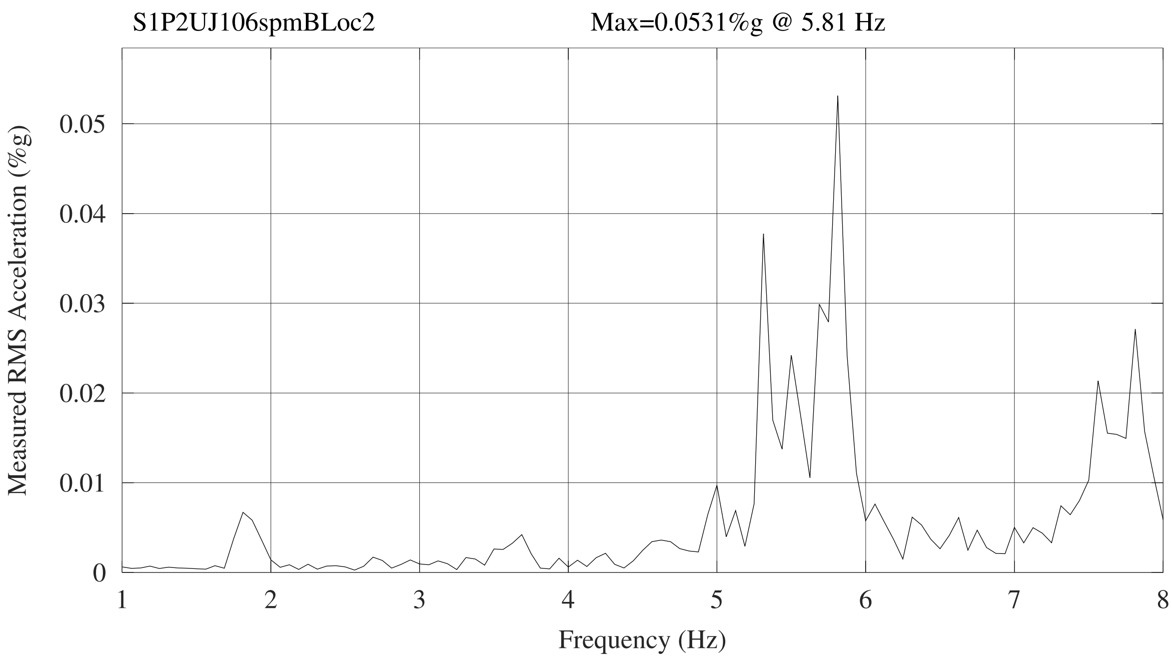


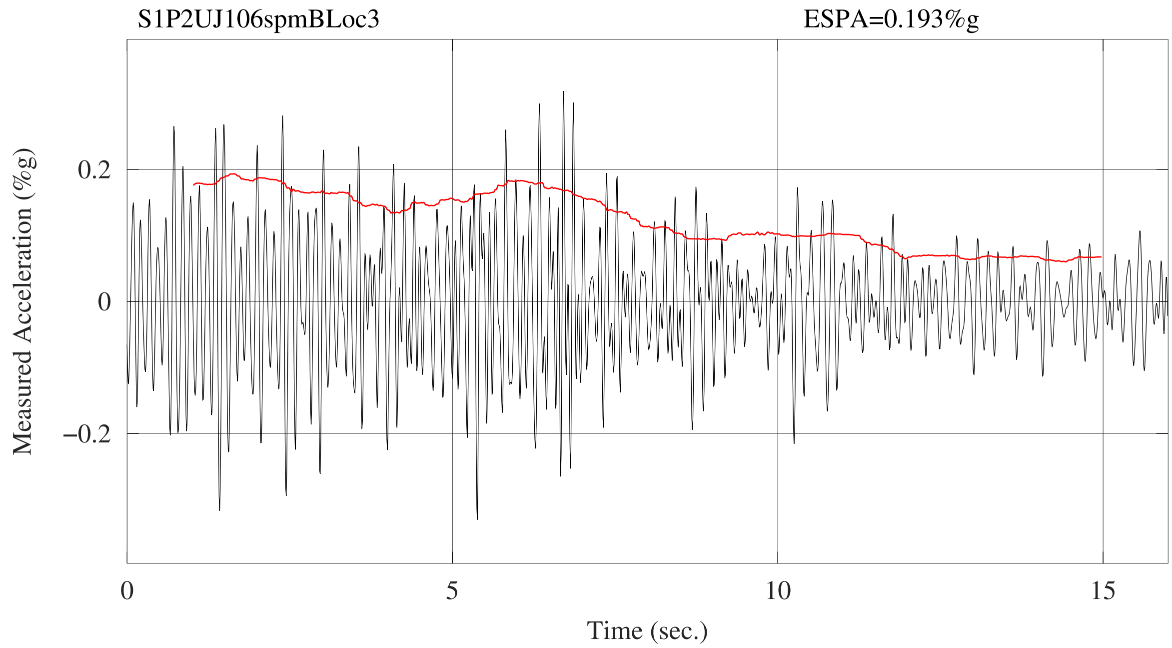
S1P2UJ106spmBLoc1
ESPA=0.256%g
0.4
0.2
0
−0.2
−0.4
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

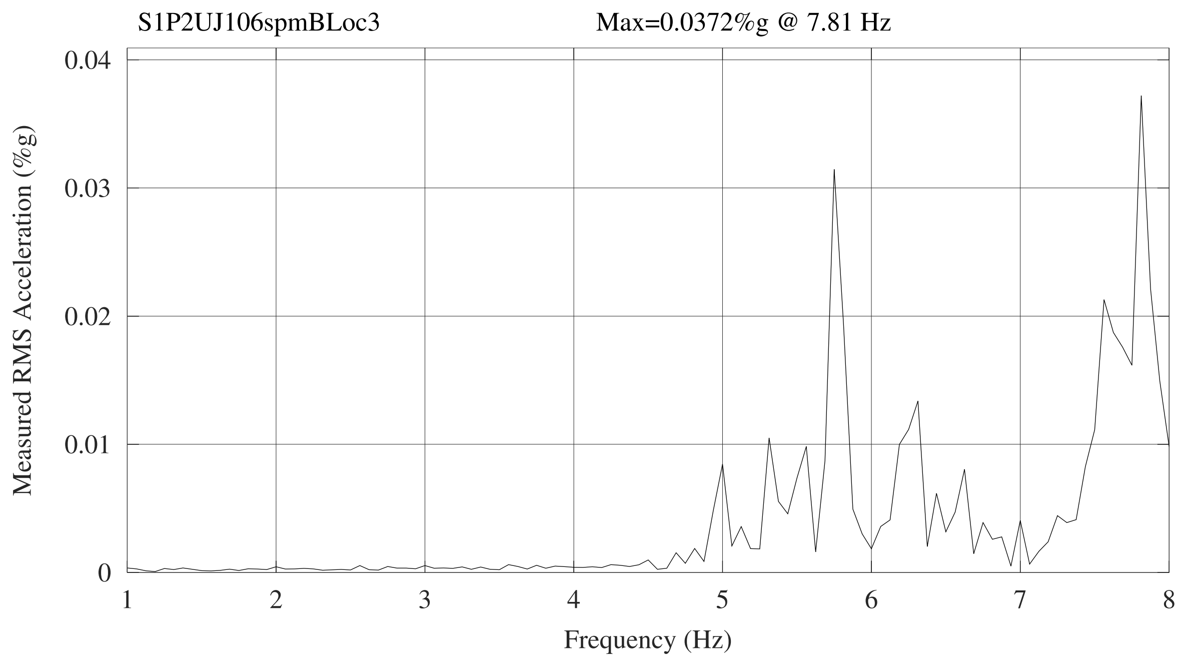


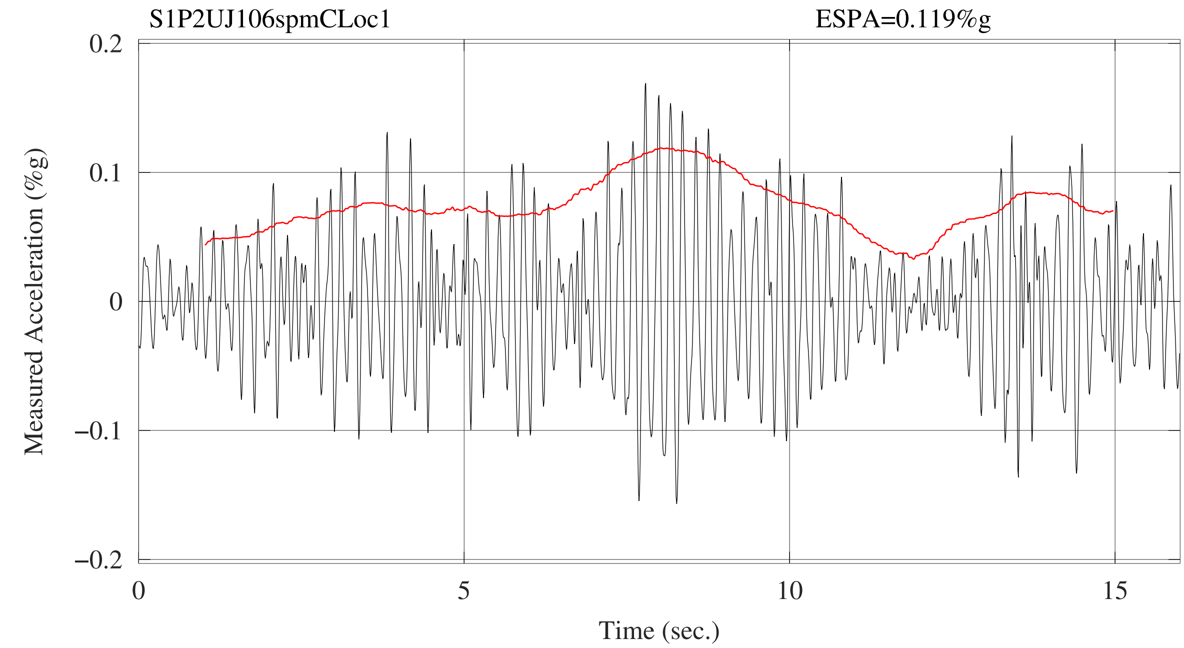
S1P2UJ106spmBLoc1
Max=0.0596%g @ 5.81 Hz
0.06
0.05
0.04
0.03
0.02
0.01
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)

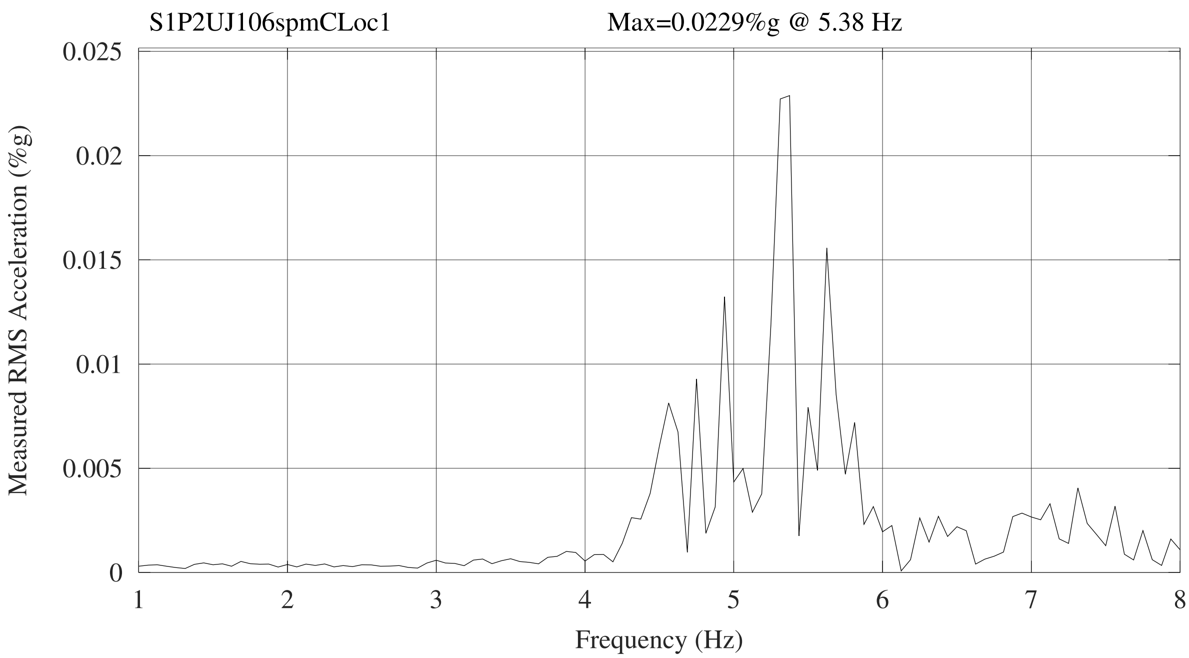

S1P2UJ106spmBLoc2
ESPA=0.248%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

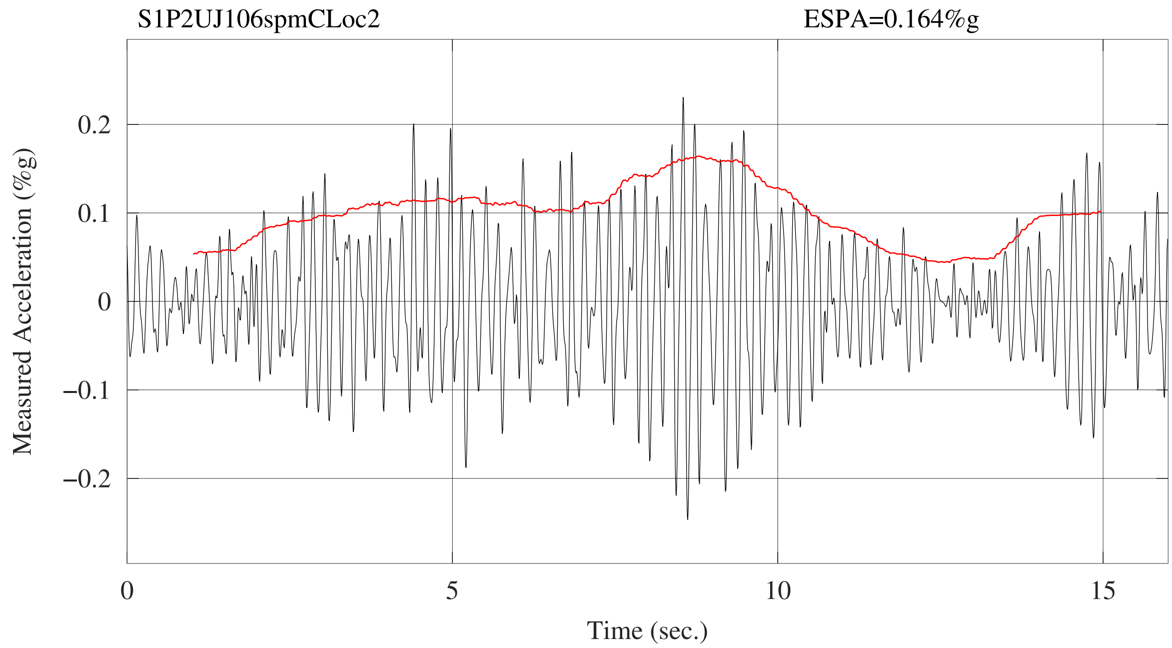

S1P2UJ106spmBLoc2
Max=0.0531%g @ 5.81 Hz
Measured RMS Acceleration (%g)
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

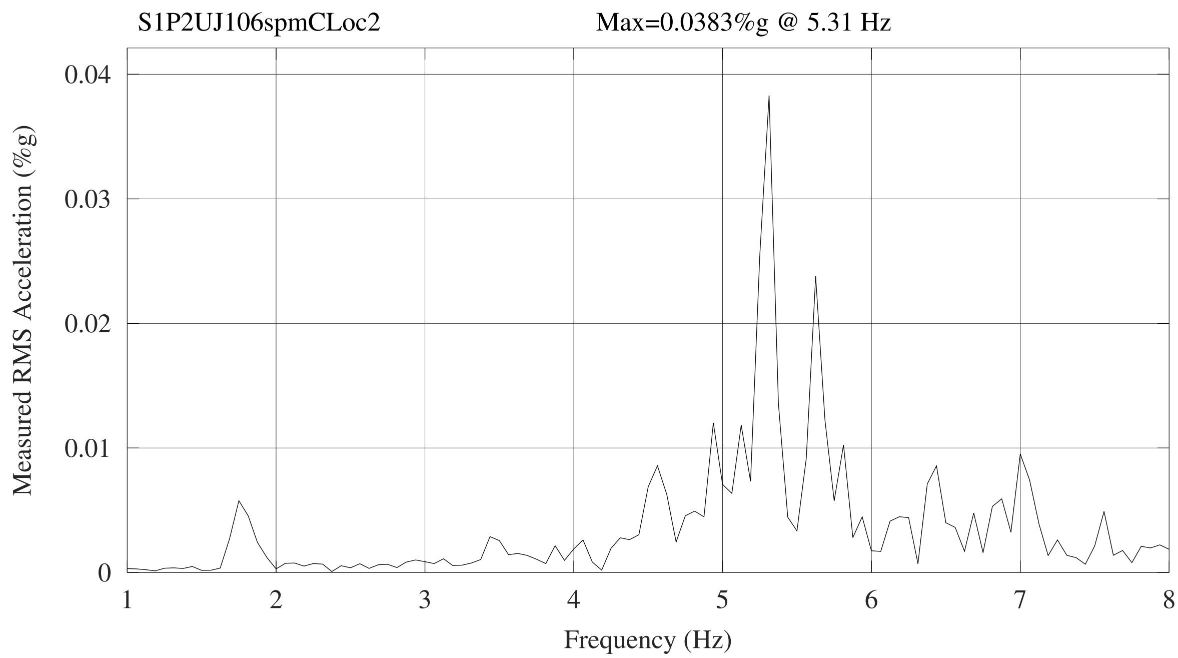

S1P2UJ106spmBLoc3
ESPA=0.193%g
Measured Acceleration (%g)
0.2
0
−0.2
0
5
10
15
Time (sec.)

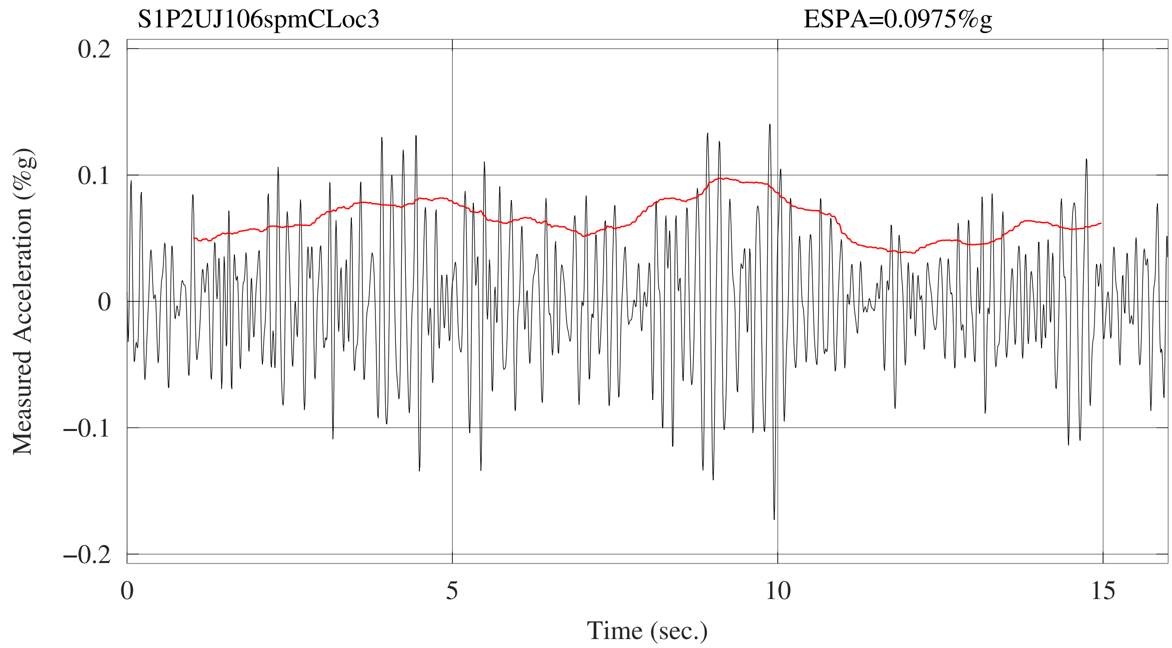

S1P2UJ106spmBLoc3
Max=0.0372%g @ 7.81 Hz
Measured RMS Acceleration (%g)
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

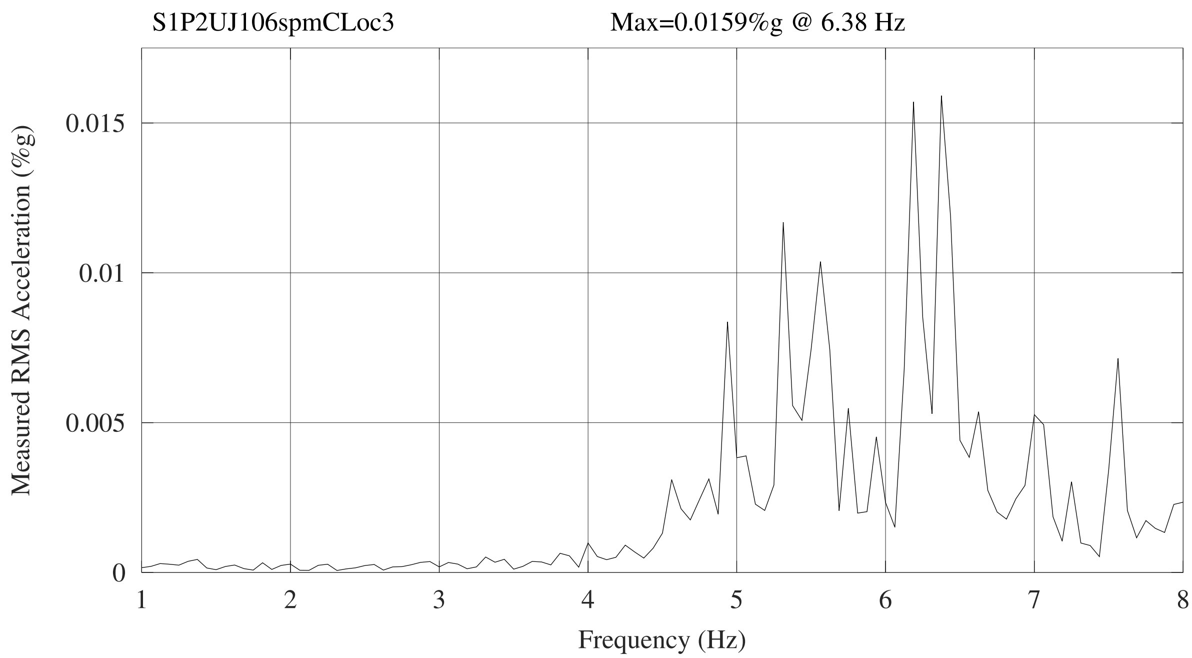

S1P2UJ106spmCLoc1
ESPA=0.119%g
Measured Acceleration (%g)
Time (sec.)

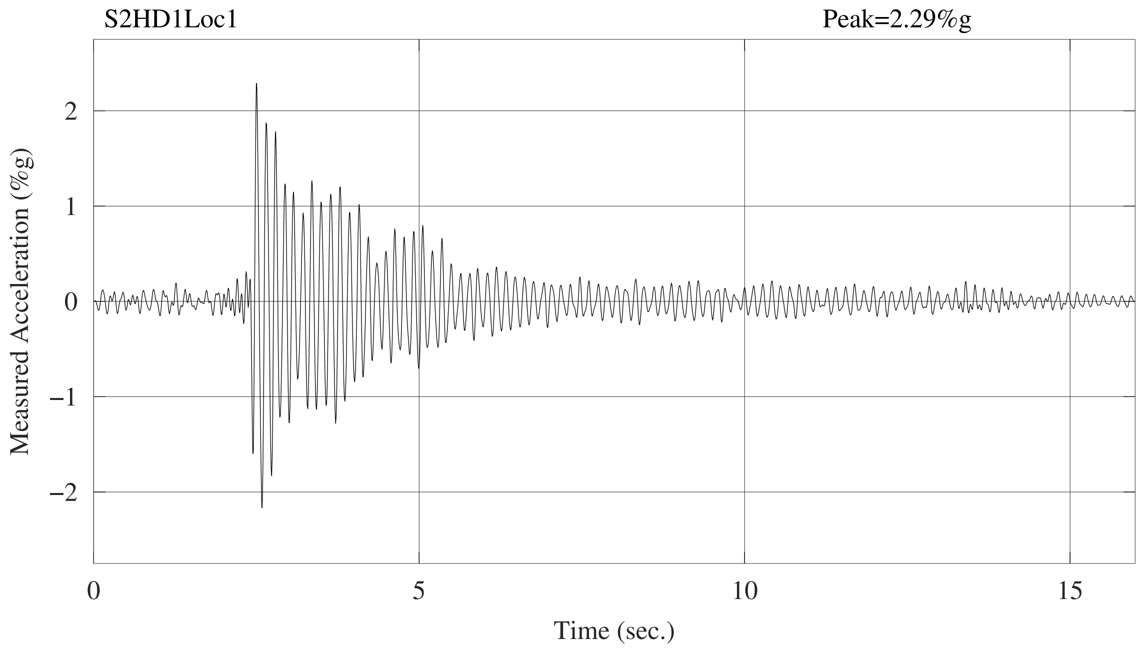

S1P2UJ106spmCLoc1
Max=0.0229%g @ 5.38 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)

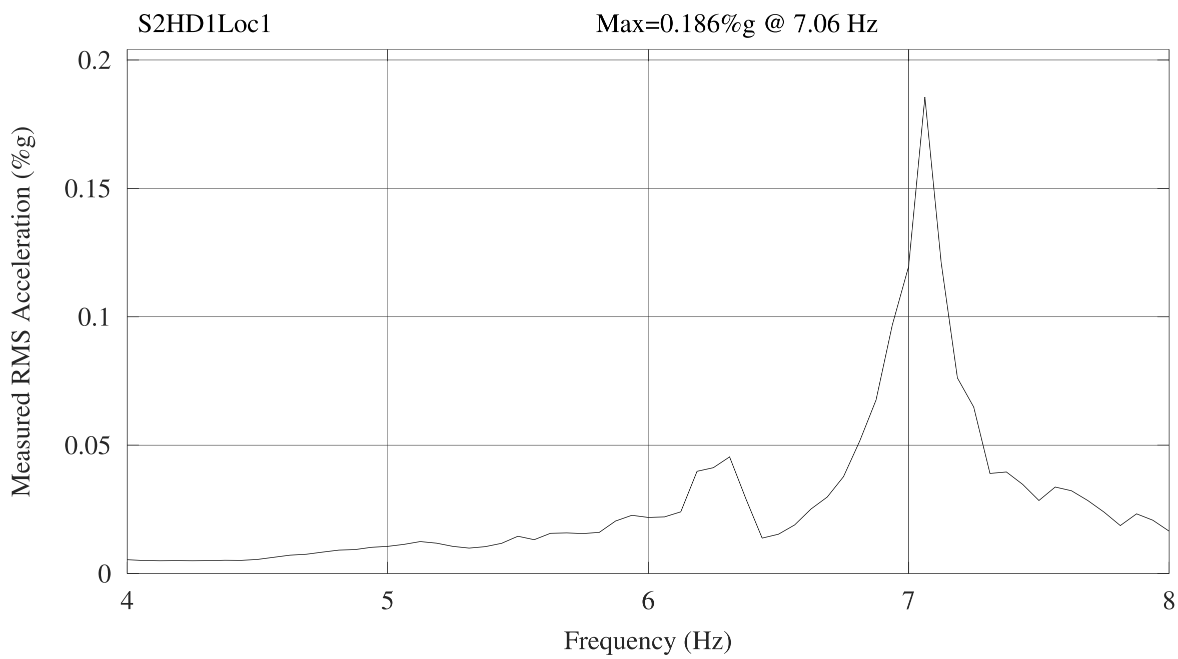

S1P2UJ106spmCLoc2
ESPA=0.164%g
Measured Acceleration (%g)
0.2
0.1
0
−0.1
−0.2
0
5
10
15
Time (sec.)

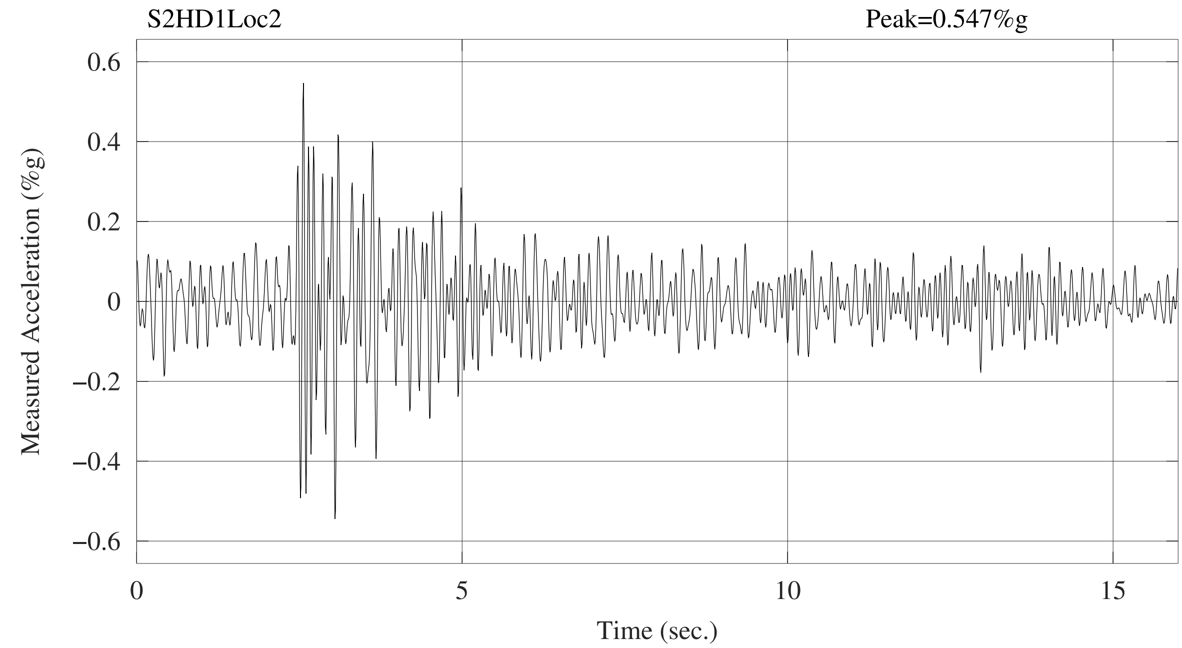

S1P2UJ106spmCLoc2
Max=0.0383%g @ 5.31 Hz
Measured RMS Acceleration (%g)
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

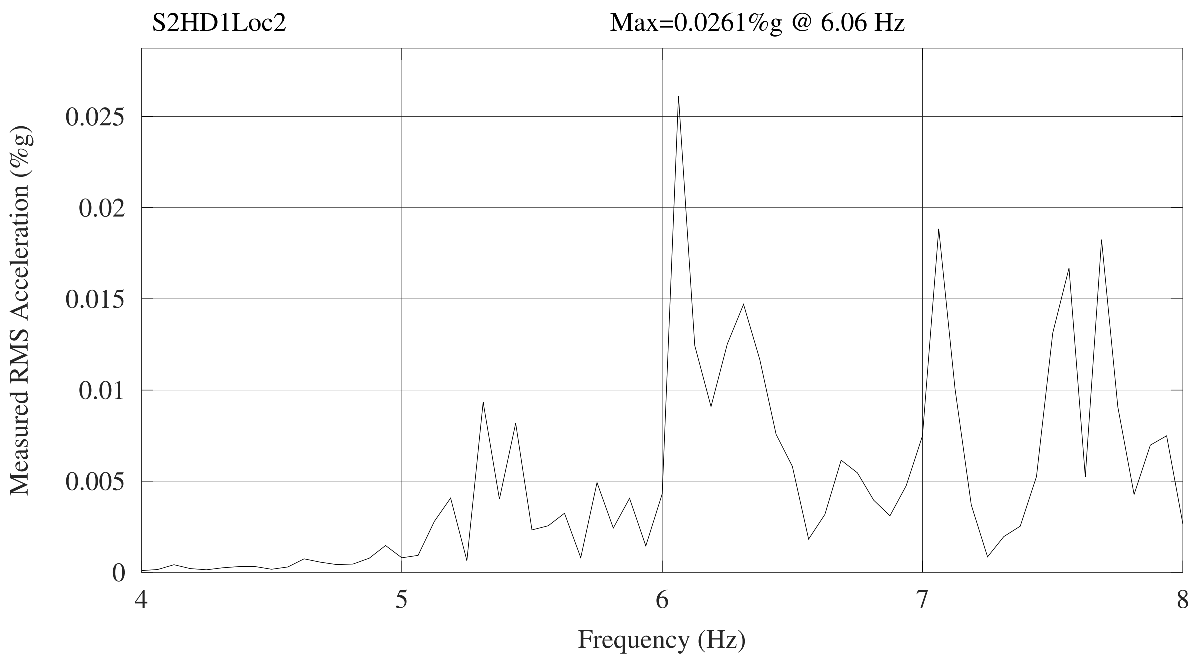

S1P2UJ106spmCLoc3
ESPA=0.0975%g
Measured Acceleration (%g)
0.2
0.1
0
−0.1
−0.2
0
5
10
15
Time (sec.)

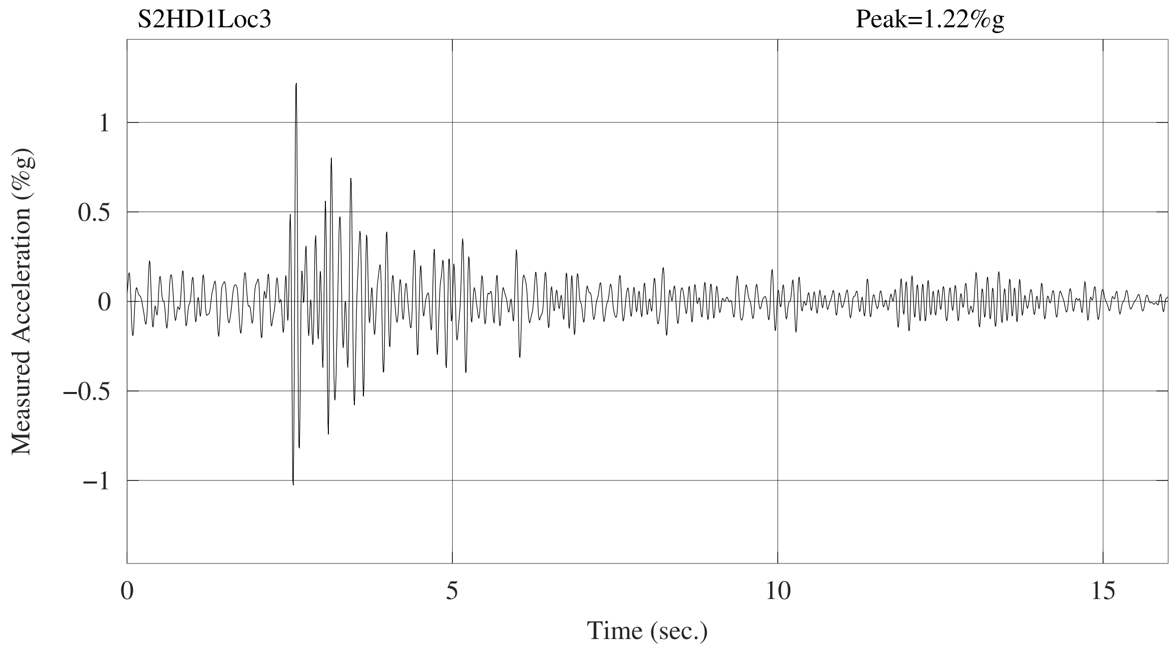

S1P2UJ106spmCLoc3
Max=0.0159%g @ 6.38 Hz
Measured RMS Acceleration (%g)
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)

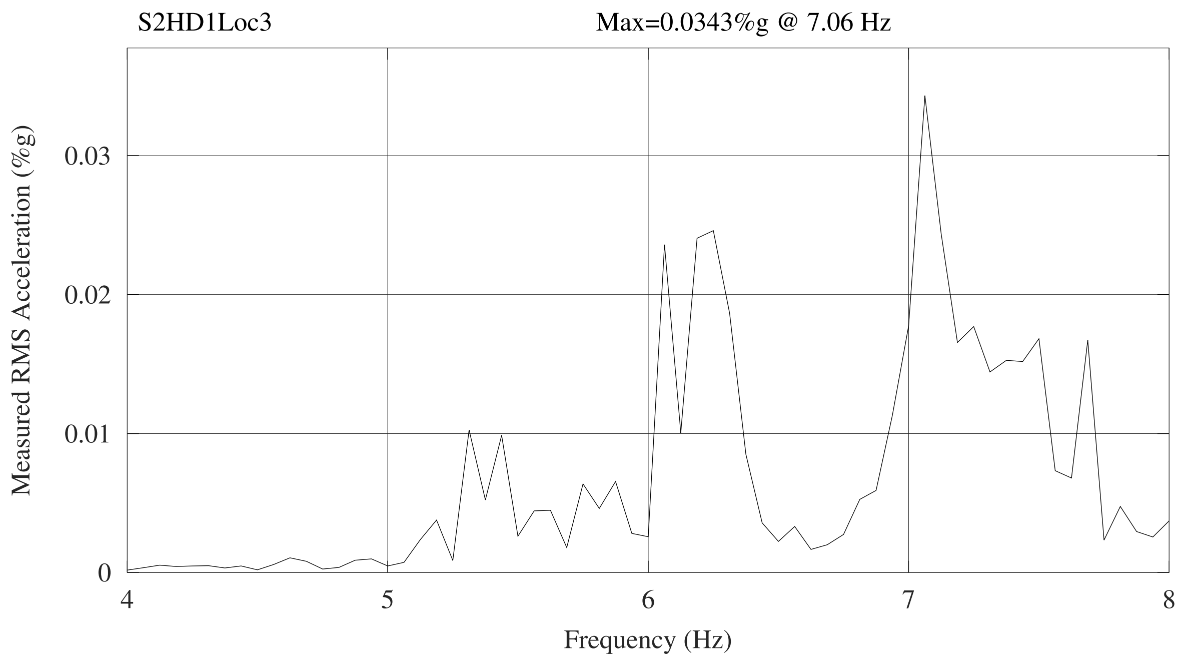

S2HD1Loc1
Peak=2.29%g
Measured Acceleration (%g)
2
1
0
−1
−2
0
5
10
15
Time (sec.)

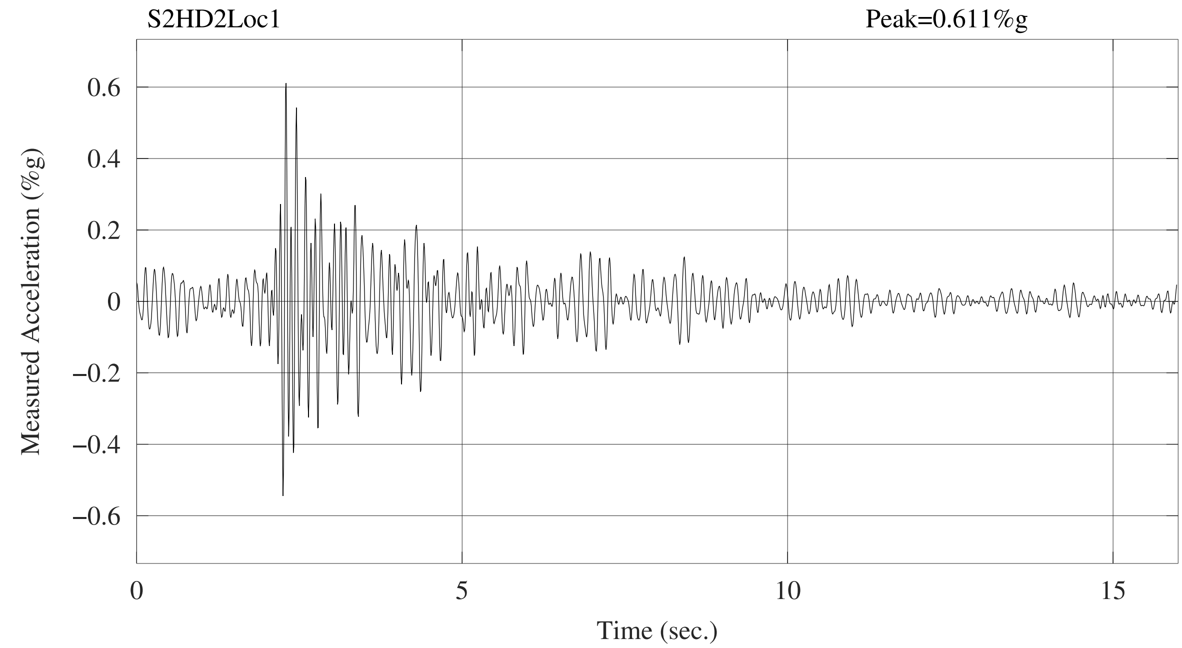

S2HD1Loc1
Max=0.186%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.2
0.15
0.1
0.05
0
4
5
6
7
8
Frequency (Hz)

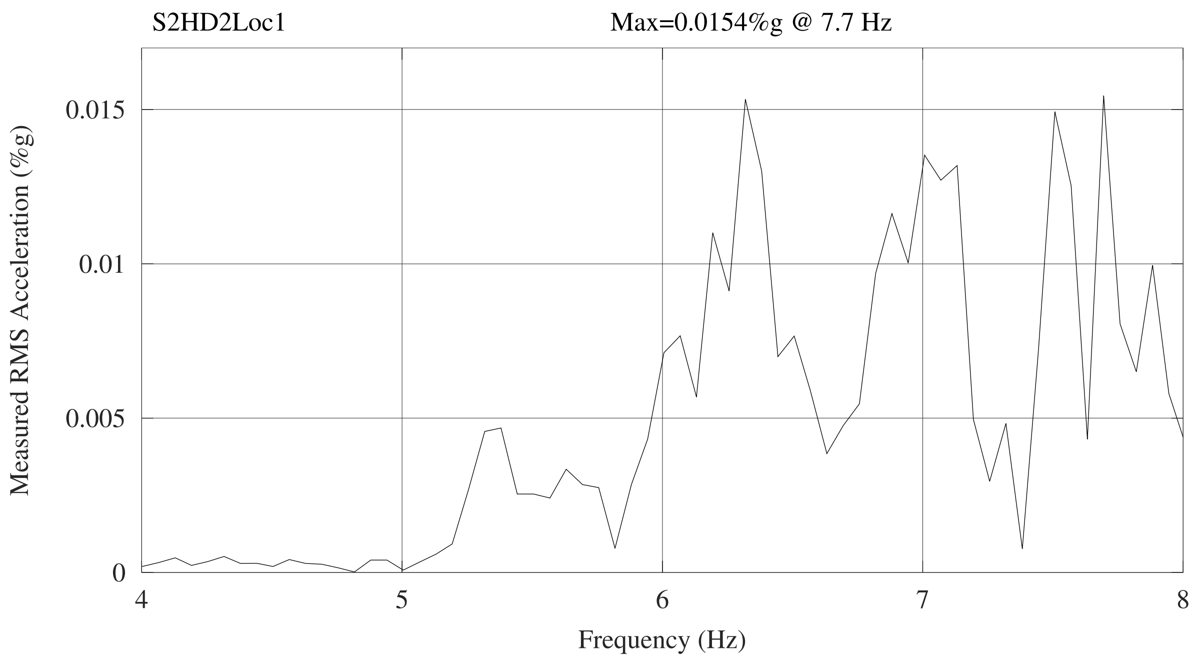

S2HD1Loc2
Peak=0.547%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

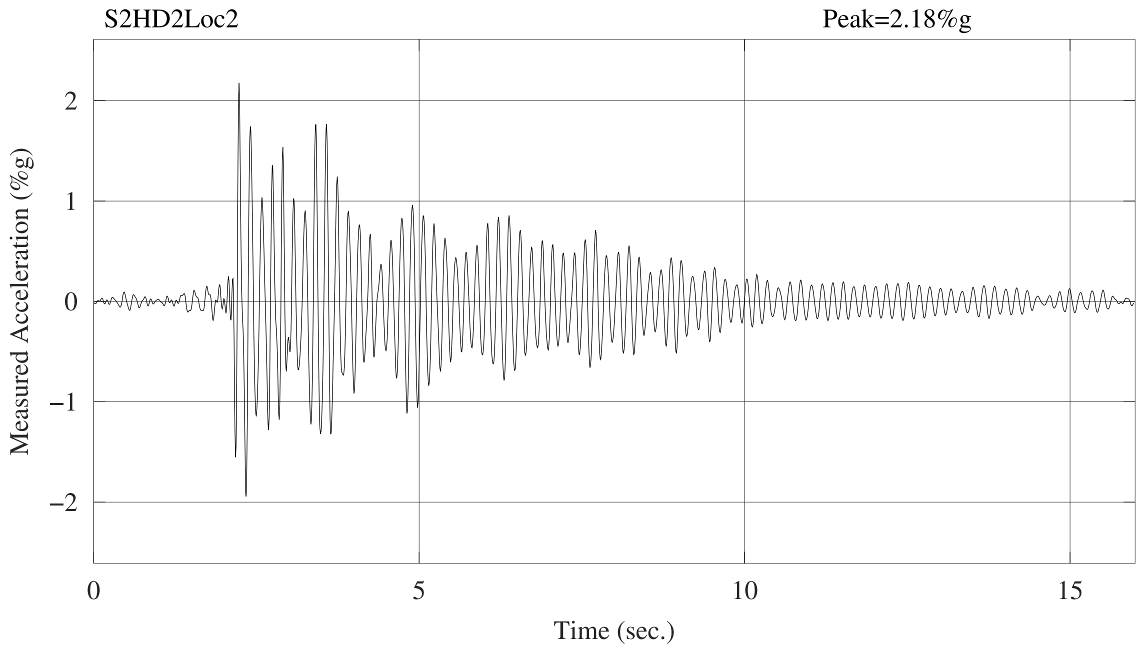

S2HD1Loc2
Max=0.0261%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.025
0.02
0.015
0.01
0.005
0
4
5
6
7
8
Frequency (Hz)

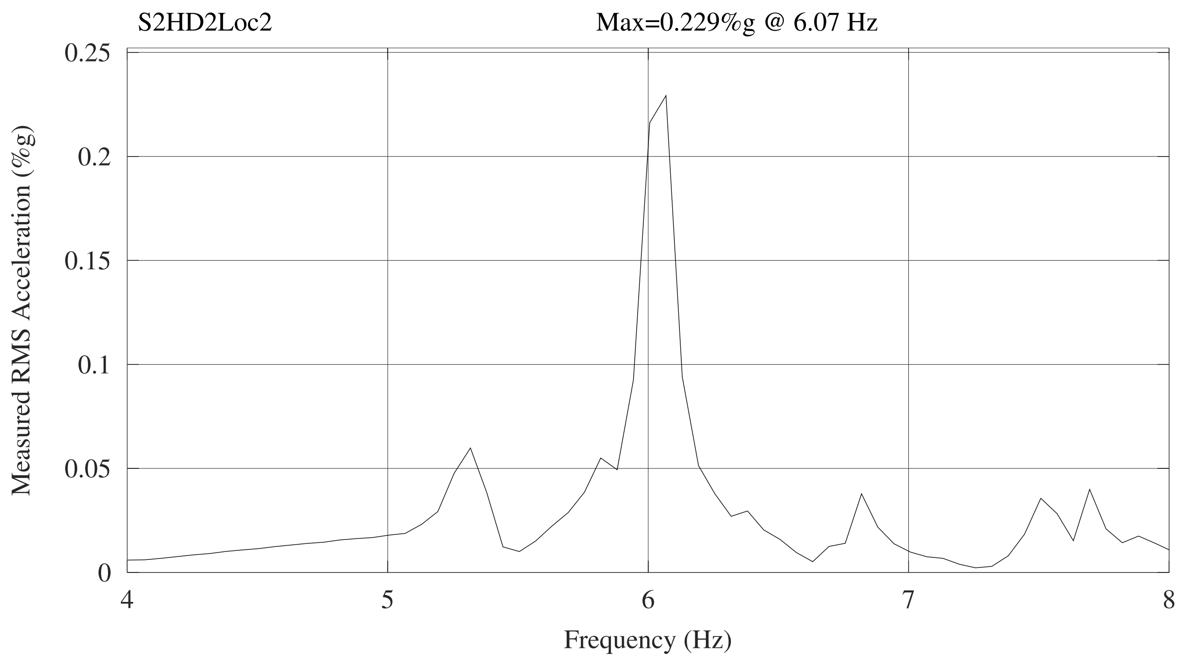

S2HD1Loc3
Peak=1.22%g
Measured Acceleration (%g)
1
0.5
0
−0.5
−1
0
5
10
15
Time (sec.)

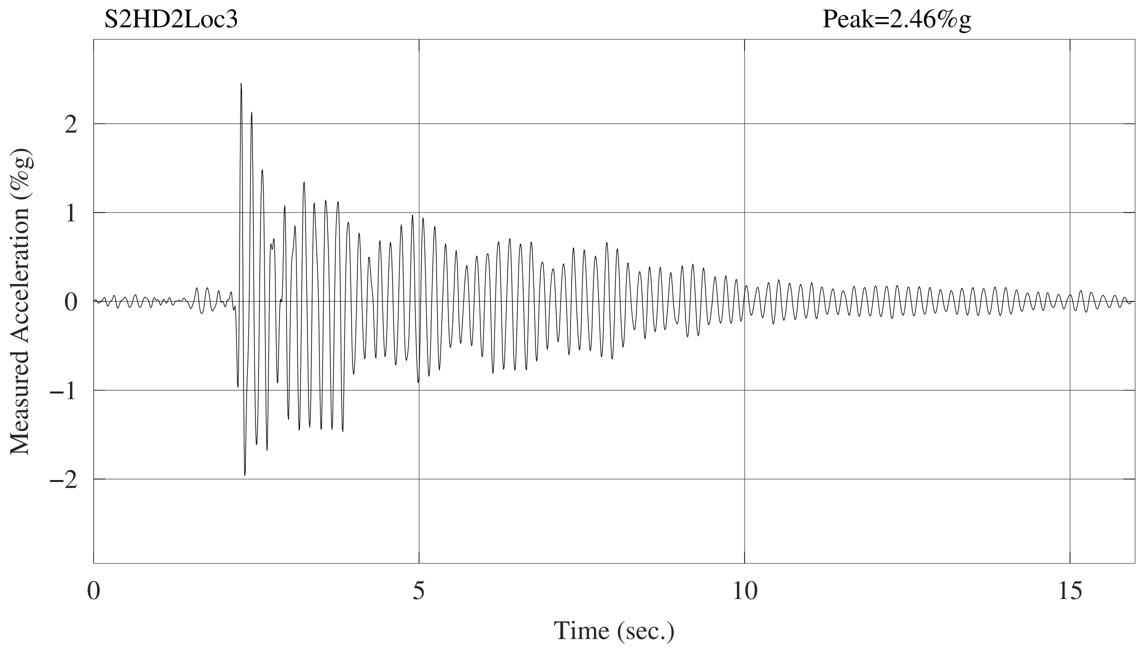

S2HD1Loc3
Max=0.0343%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.03
0.02
0.01
0
4
5
6
7
8
Frequency (Hz)

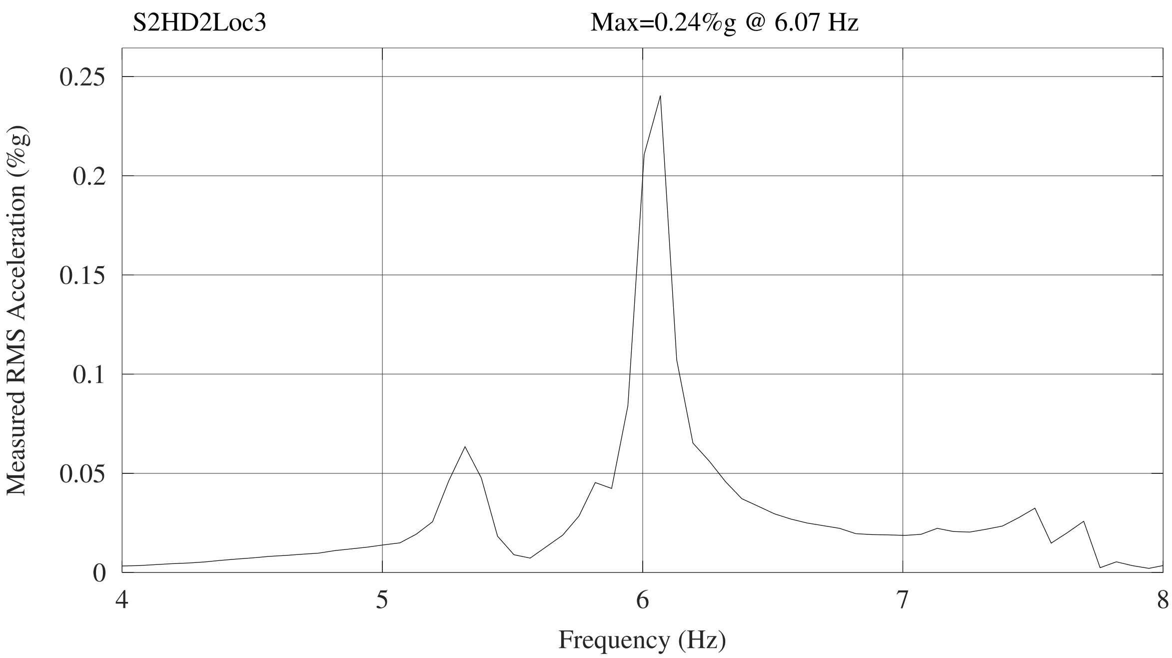

S2HD2Loc1
Peak=0.611%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

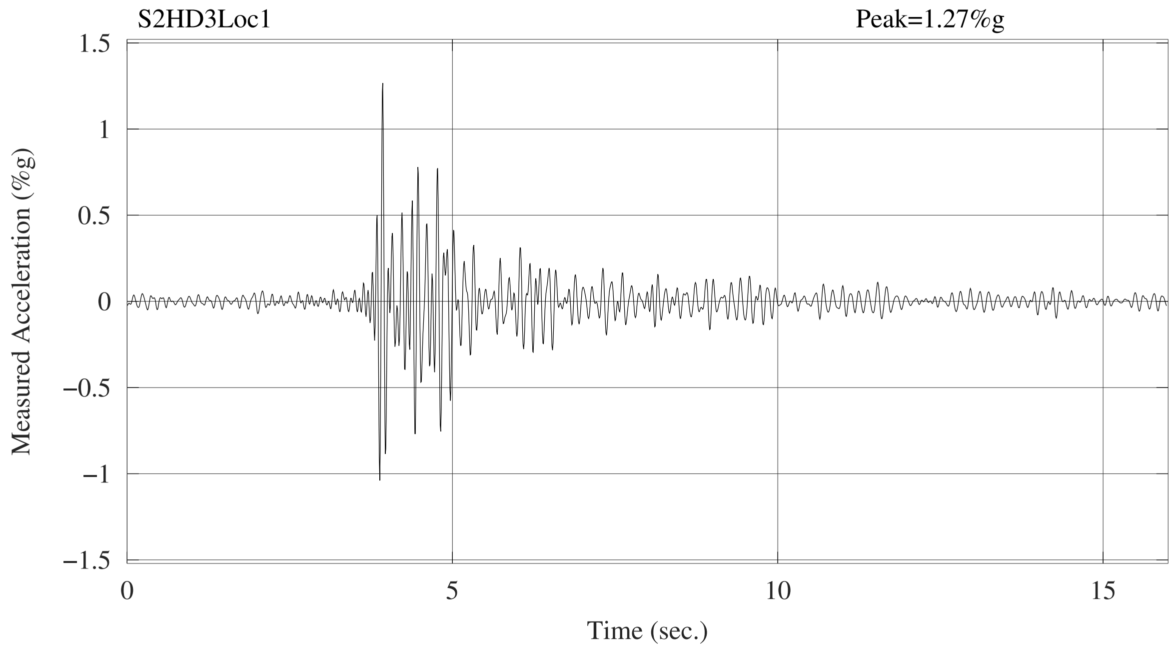

S2HD2Loc1
Max=0.0154%g @ 7.7 Hz
Measured RMS Acceleration (%g)
0.015
0.01
0.005
0
4
5
6
7
8
Frequency (Hz)

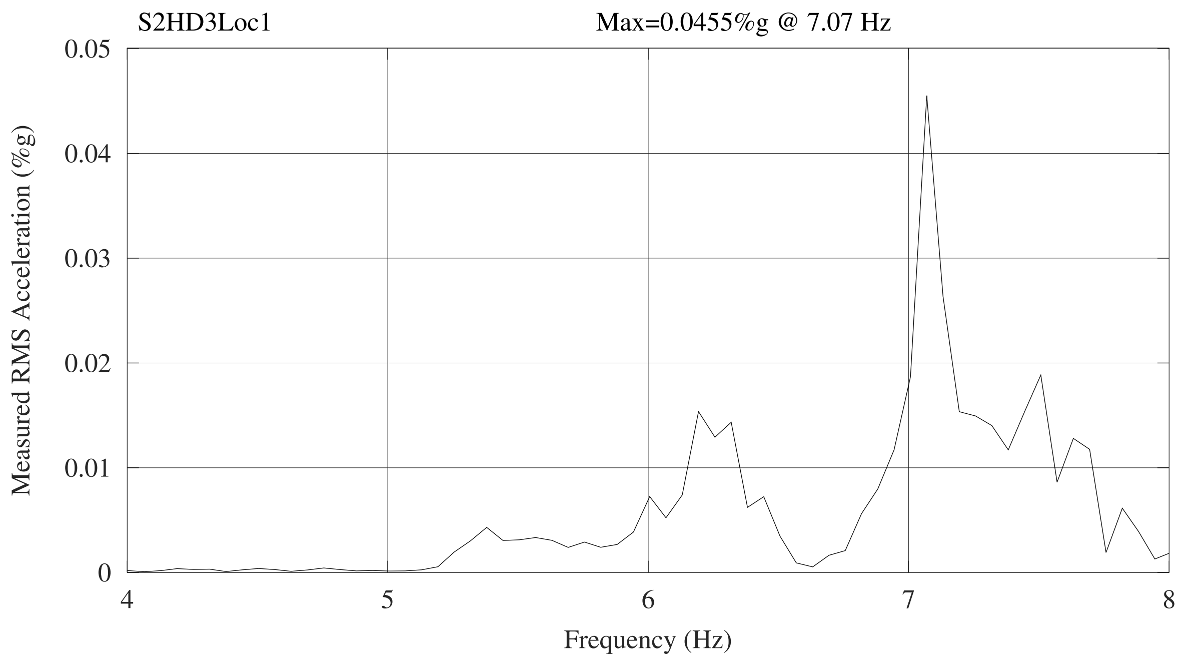

S2HD2Loc2
Peak=2.18%g
Measured Acceleration (%g)
2
1
0
−1
−2
0
5
10
15
Time (sec.)

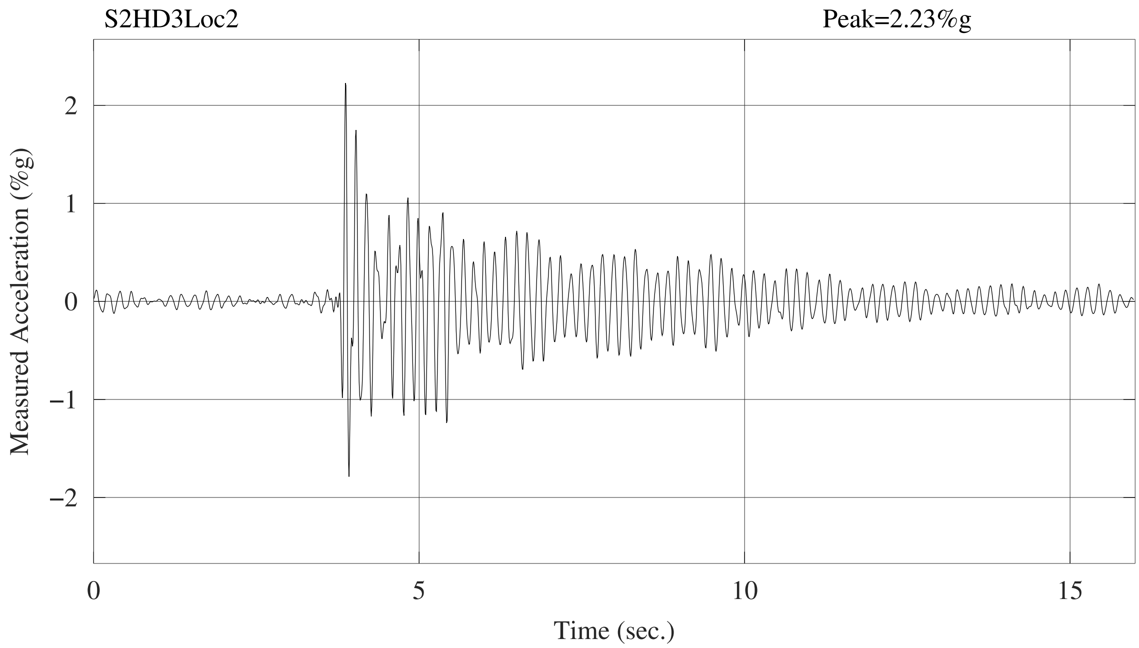

S2HD2Loc2
Max=0.229%g @ 6.07 Hz
Measured RMS Acceleration (%g)
0.25
0.2
0.15
0.1
0.05
0
4
5
6
7
8
Frequency (Hz)

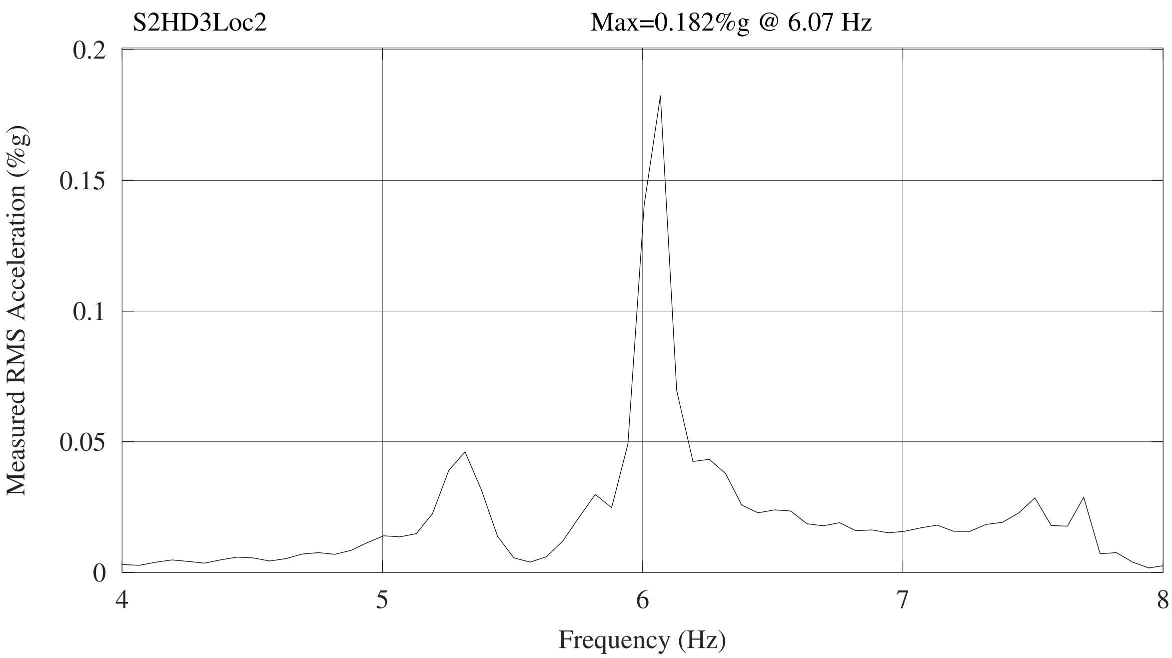

S2HD2Loc3
Peak=2.46%g
Measured Acceleration (%g)
2
1
0
−1
−2
0
5
10
15
Time (sec.)

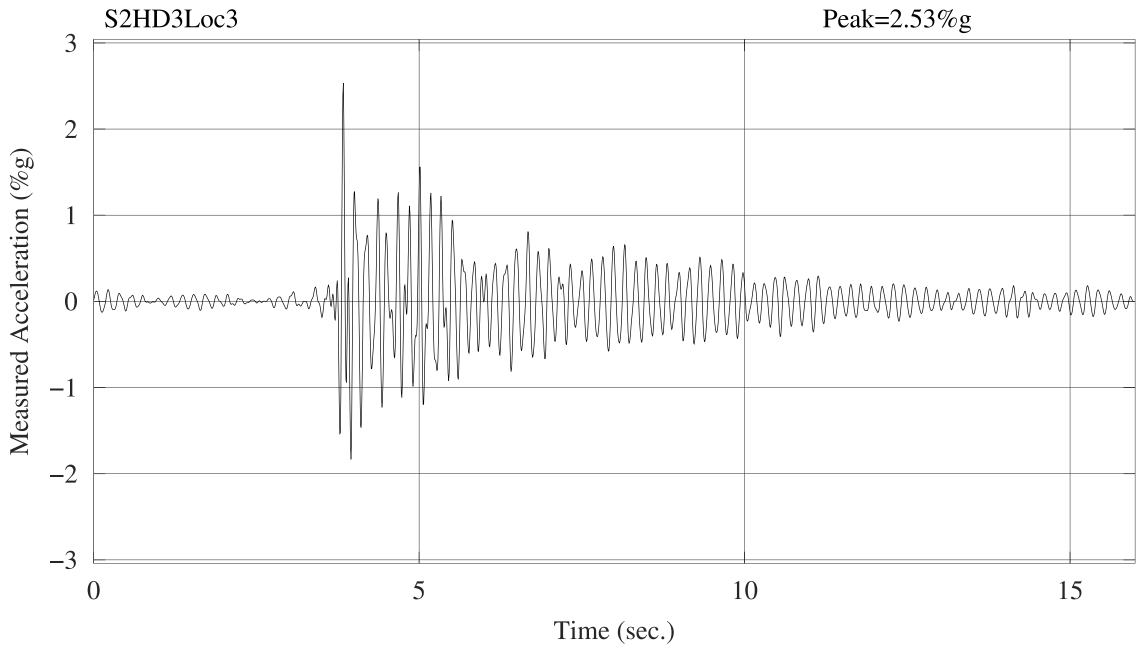

S2HD2Loc3
Max=0.24%g @ 6.07 Hz
Measured RMS Acceleration (%g)
0.25
0.2
0.15
0.1
0.05
0
4
5
6
7
8
Frequency (Hz)


S2HD3Loc1
Peak=1.27%g
Measured Acceleration (%g)
1.5
1
0.5
0
−0.5
−1
−1.5
0
5
10
15
Time (sec.)

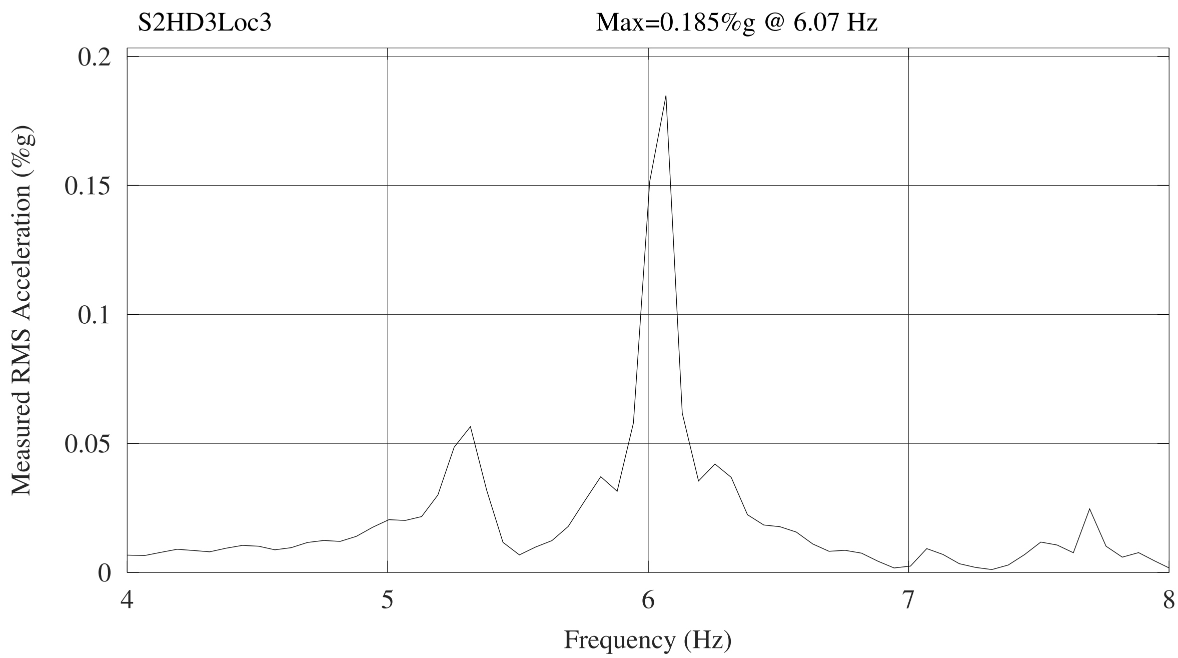

S2HD3Loc1
Max=0.0455%g @ 7.07 Hz
Measured RMS Acceleration (%g)
0.05
0.04
0.03
0.02
0.01
0
4
5
6
7
8
Frequency (Hz)


S2HD3Loc2
Peak=2.23%g
Measured Acceleration (%g)
2
1
0
−1
−2
0
5
10
15
Time (sec.)


S2HD3Loc2
Max=0.182%g @ 6.07 Hz
Measured RMS Acceleration (%g)
0.2
0.15
0.1
0.05
0
4
5
6
7
8
Frequency (Hz)

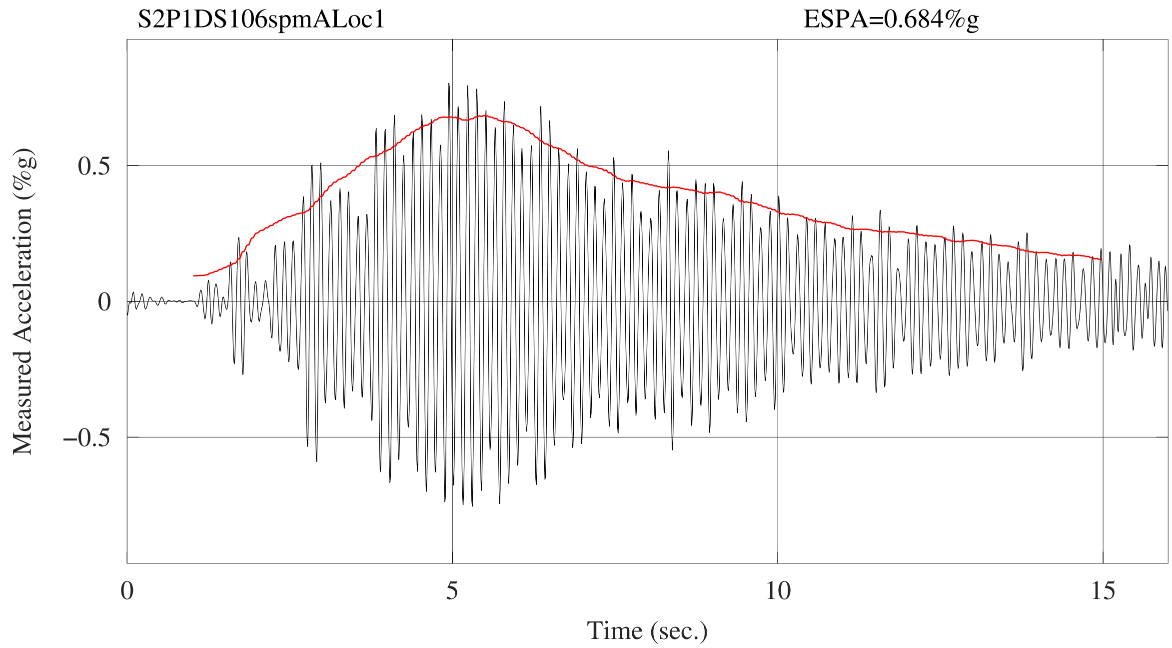


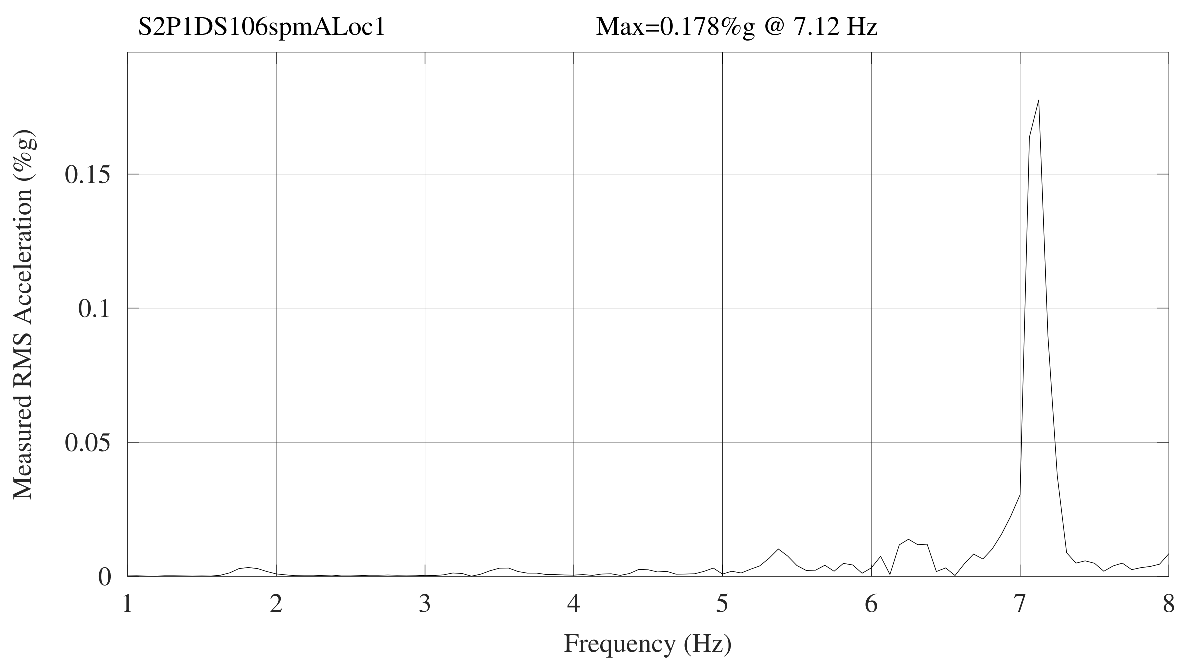
S2HD3Loc3
Peak=2.53%g
Measured Acceleration (%g)
Time (sec.)

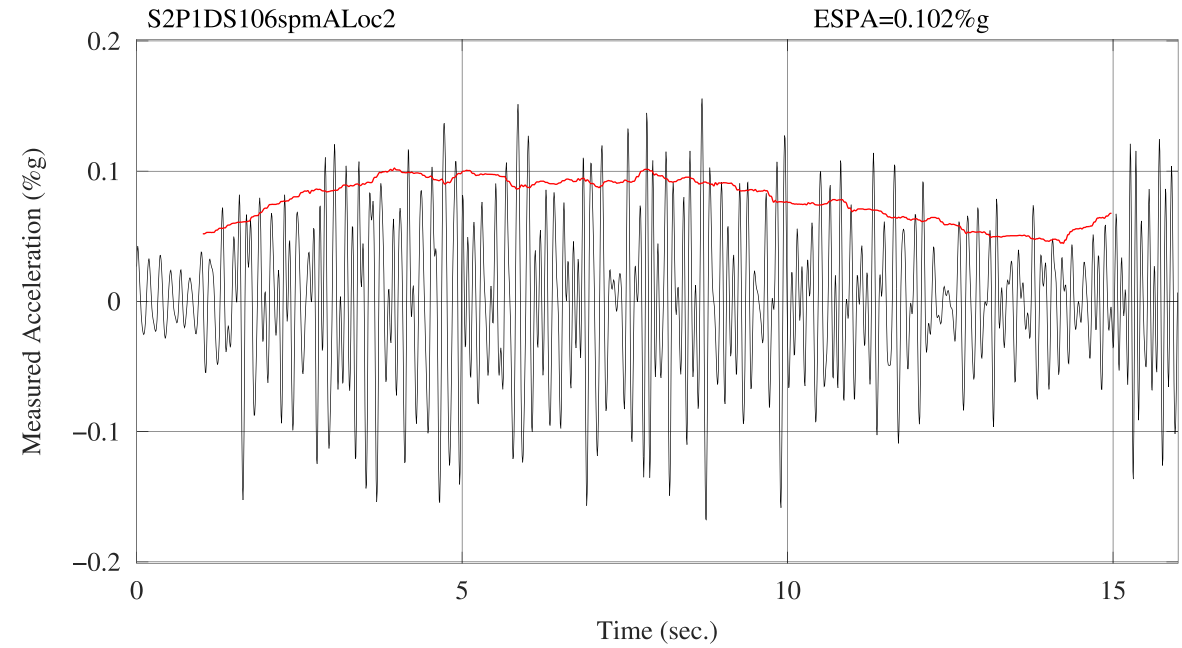


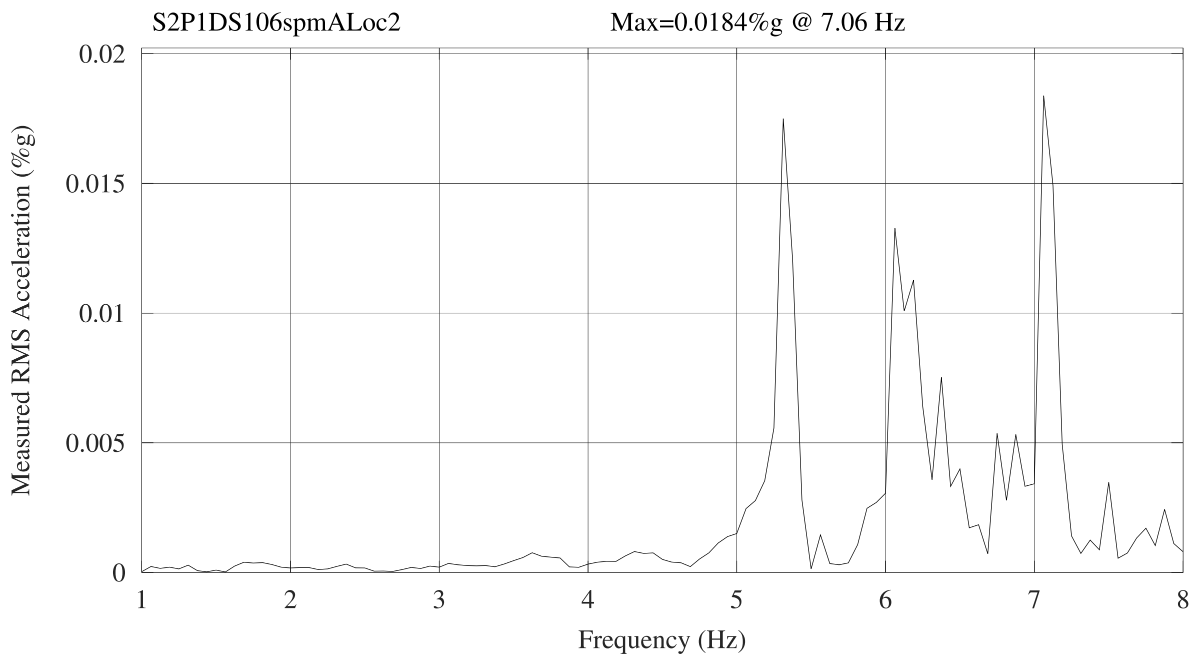
S2HD3Loc3
Max=0.185%g @ 6.07 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)

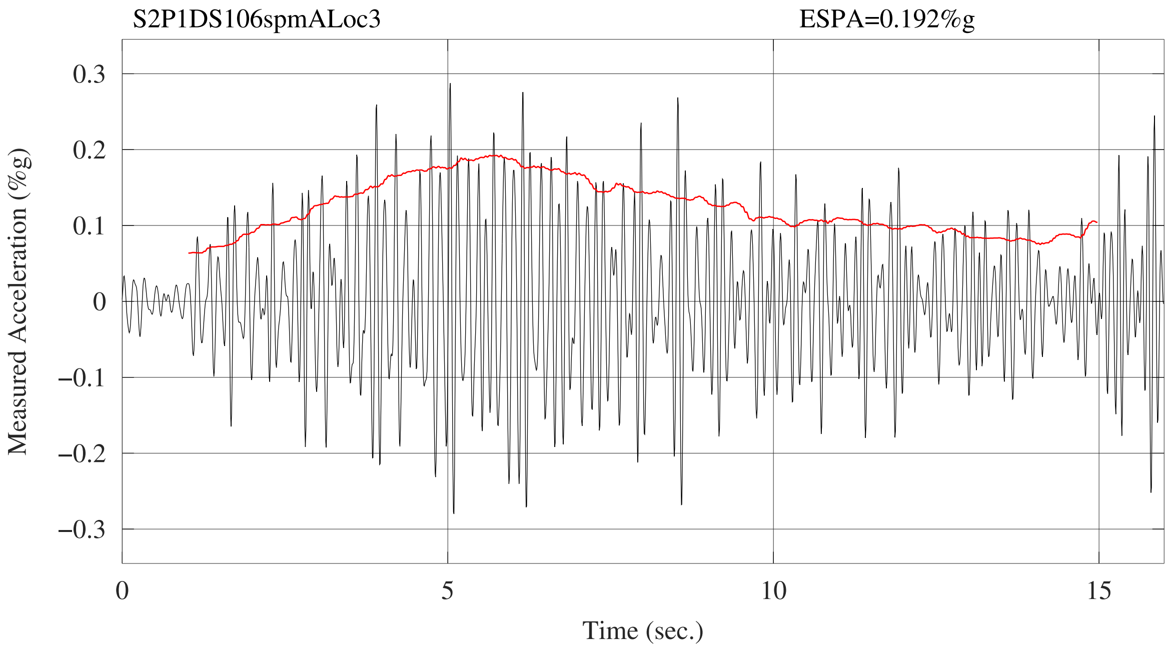
Appendix D - Walking Test Results for Setup 2

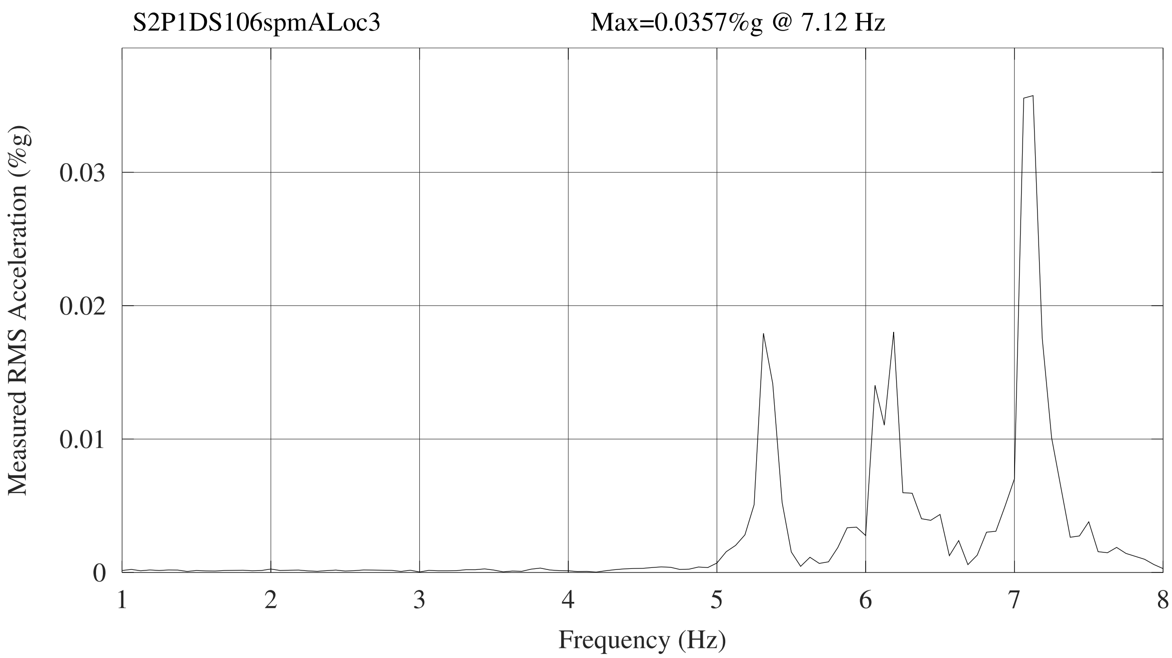


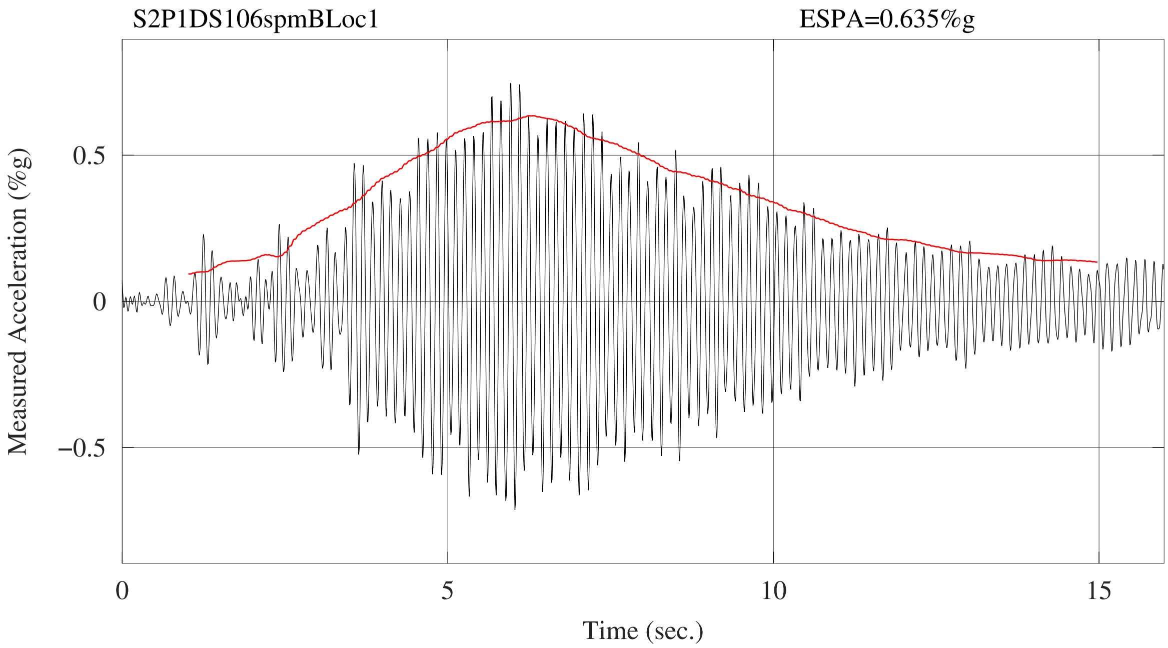


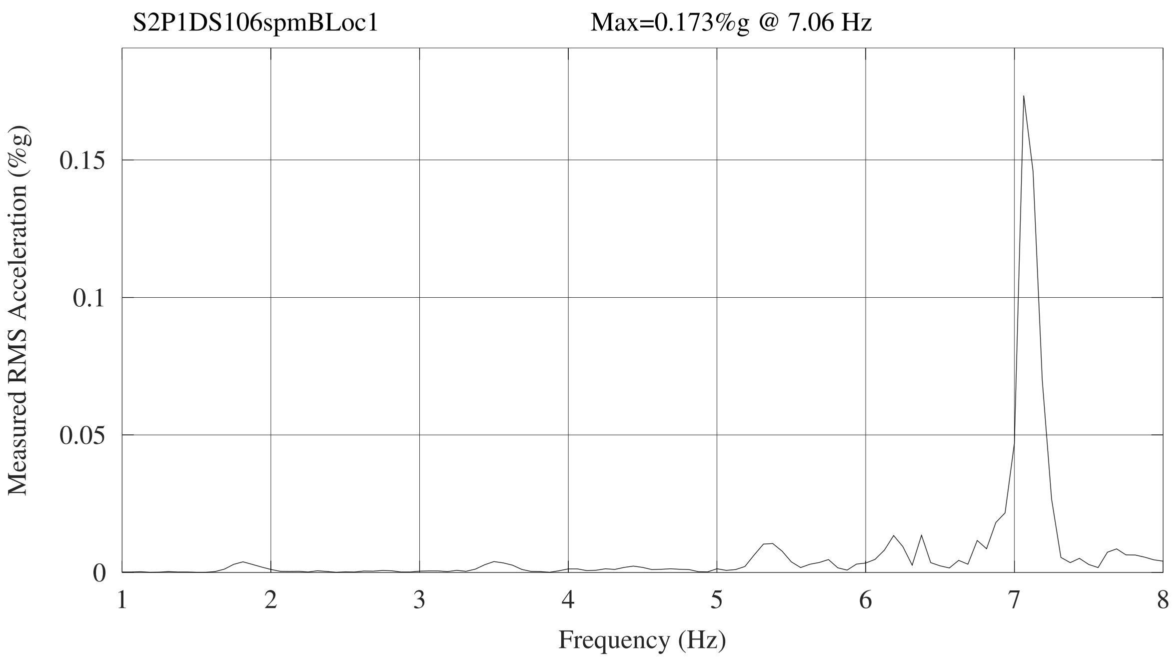

S2P1DS106spmALoc2
ESPA=0.102%g
Measured Acceleration (%g)
Time (sec.)
0.2
0.1
0
−0.1
−0.2
0
5
10
15

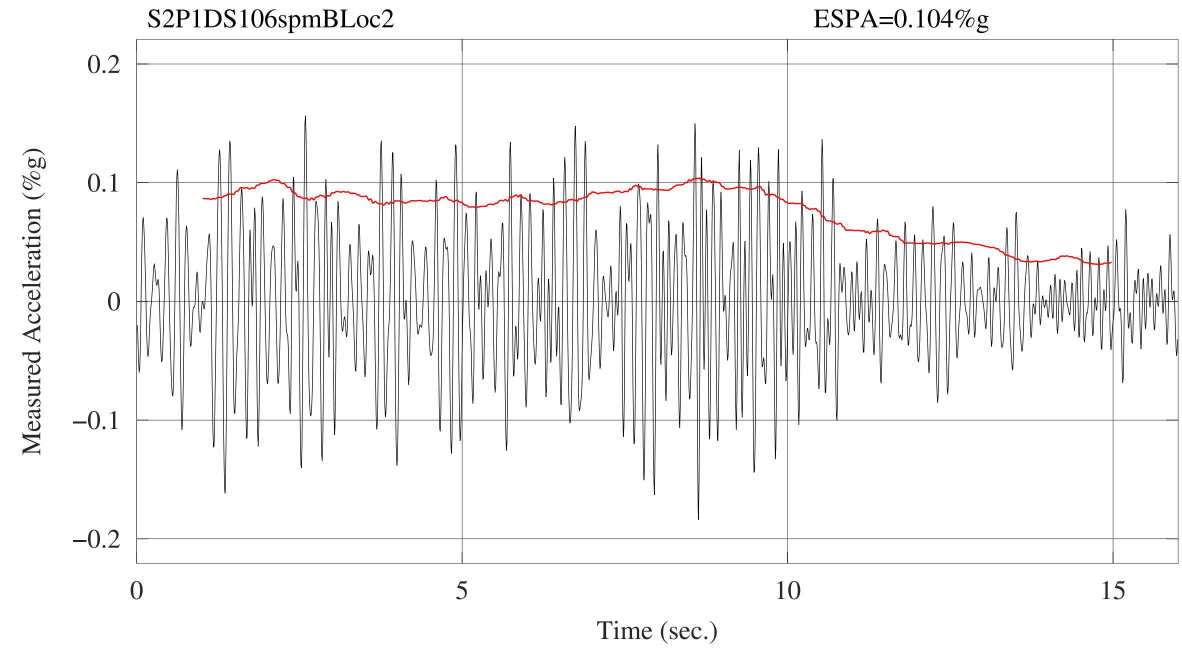

S2P1DS106spmALoc2
Max=0.0184%g @ 7.06 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)
0.02
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8

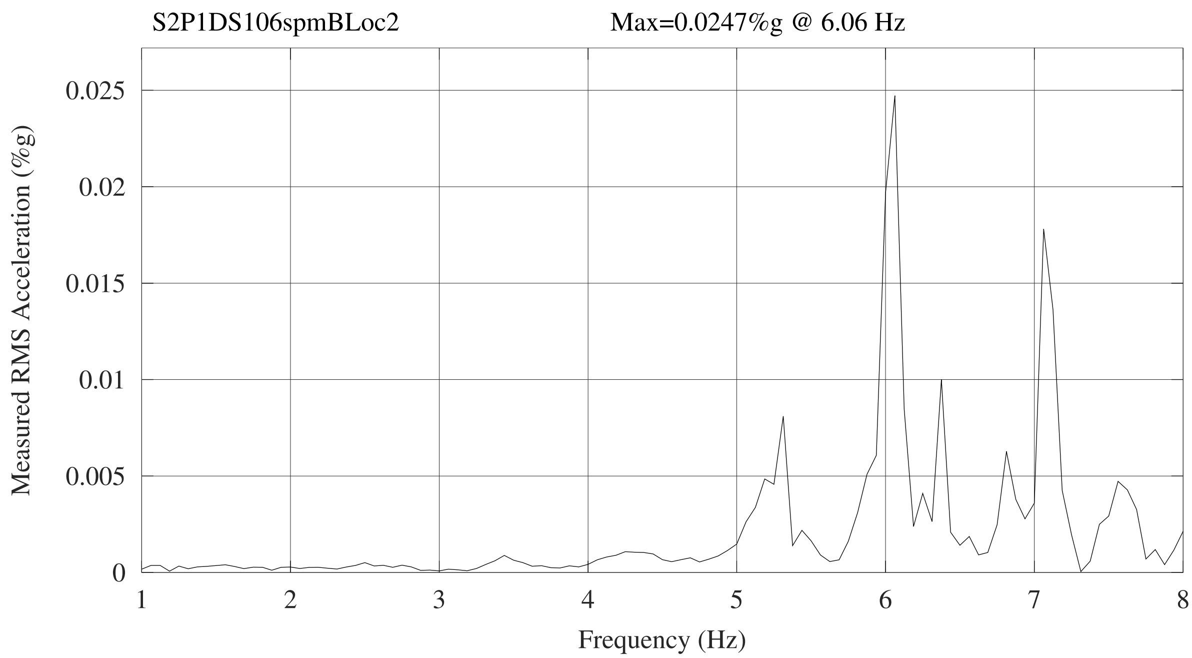

S2P1DS106spmALoc3
ESPA=0.192%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

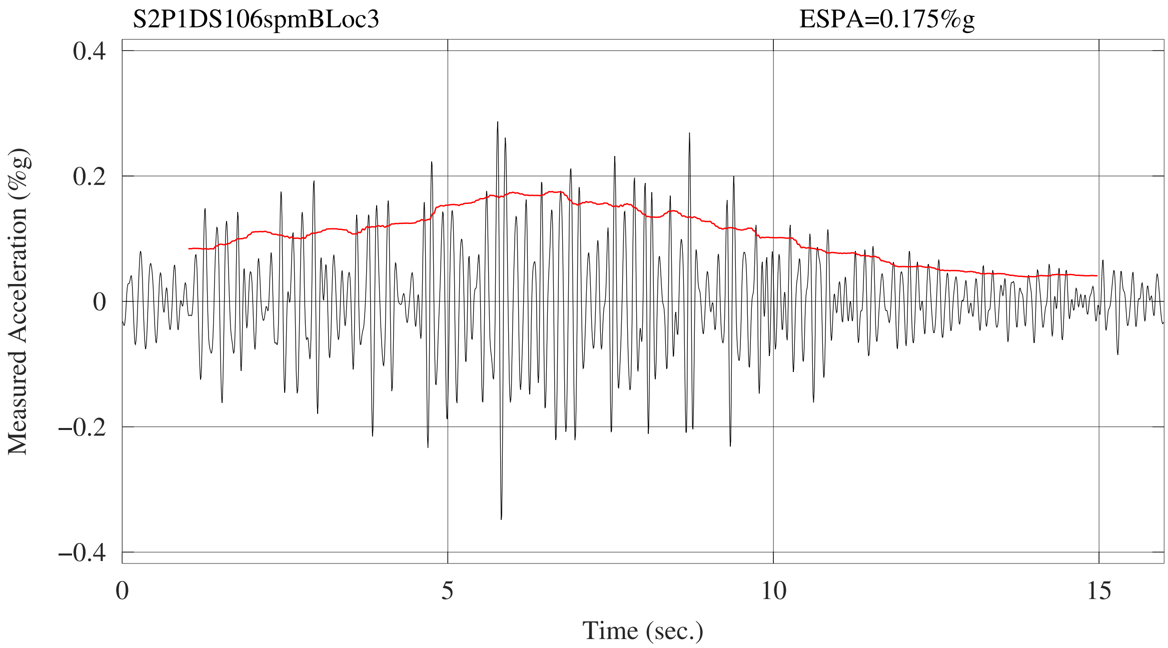

S2P1DS106spmALoc3
Max=0.0357%g @ 7.12 Hz
Measured RMS Acceleration (%g)
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

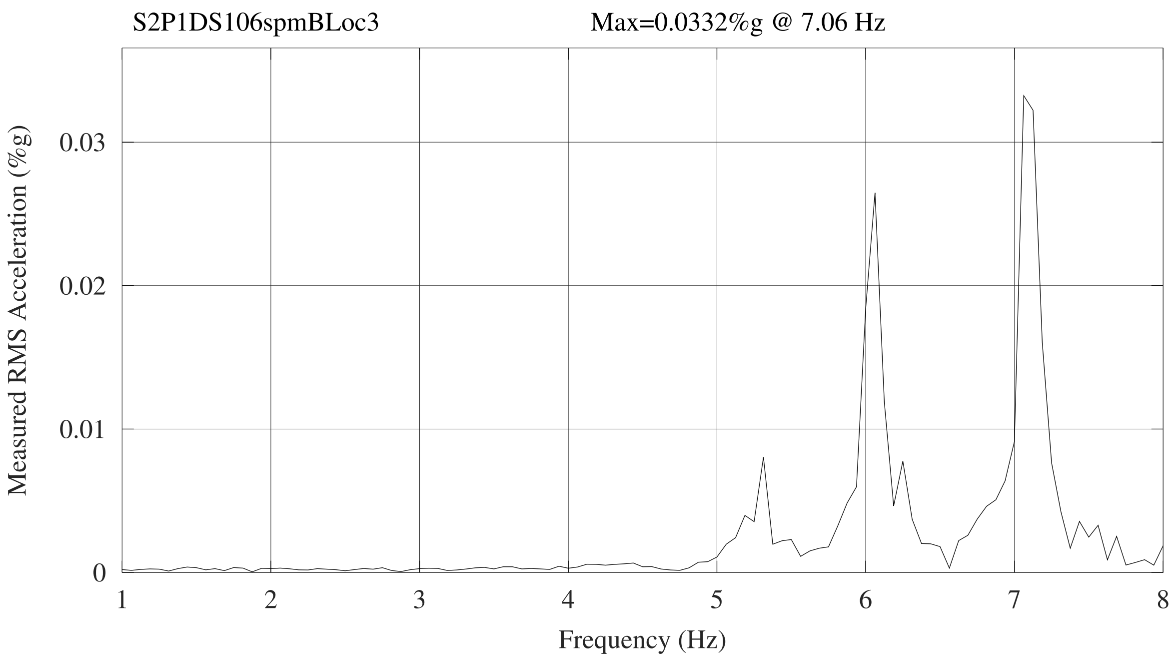

S2P1DS106spmBLoc1
ESPA=0.635%g
Measured Acceleration (%g)
0.5
0
−0.5
0
5
10
15
Time (sec.)

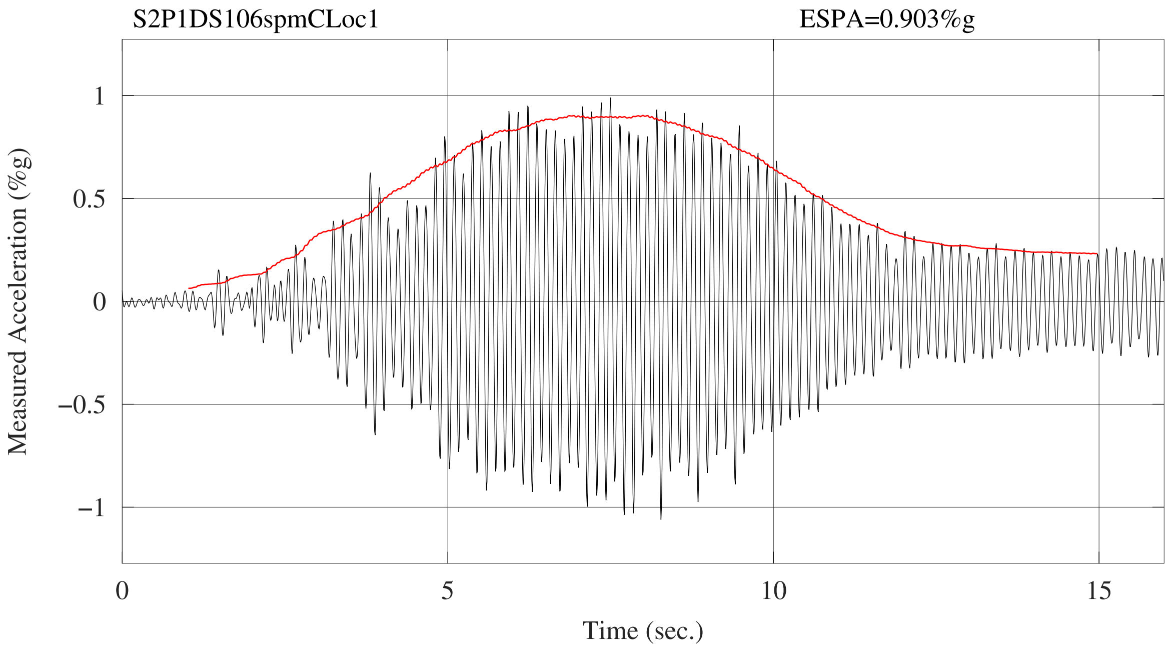

S2P1DS106spmBLoc1
Max=0.173%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.15
0.1
0.05
0
1
2
3
4
5
6
7
8
Frequency (Hz)

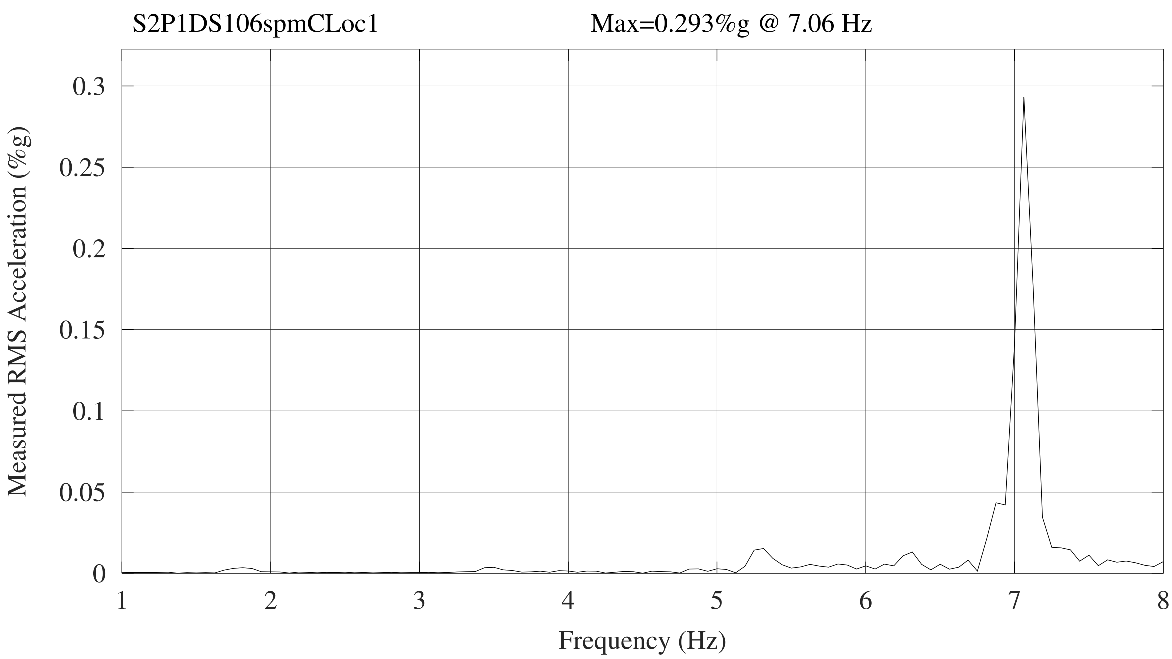

S2P1DS106spmBLoc2
ESPA=0.104%g
0.2
0.1
0
−0.1
−0.2
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

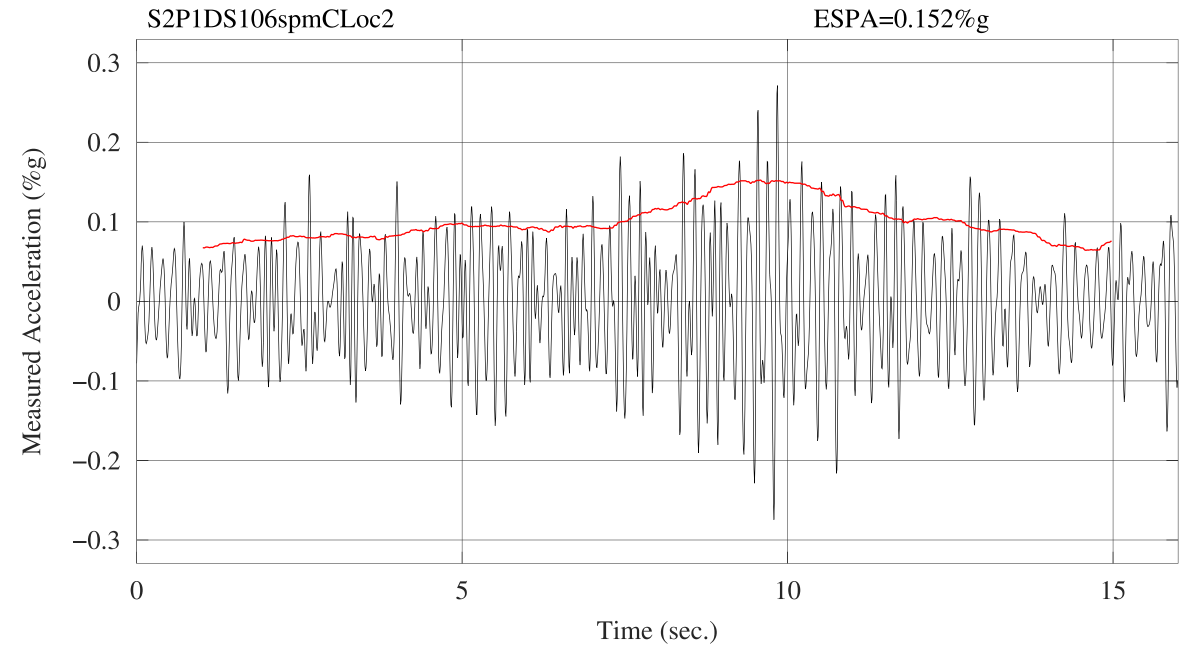

S2P1DS106spmBLoc2
Max=0.0247%g @ 6.06 Hz
0.025
0.02
0.015
0.01
0.005
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)

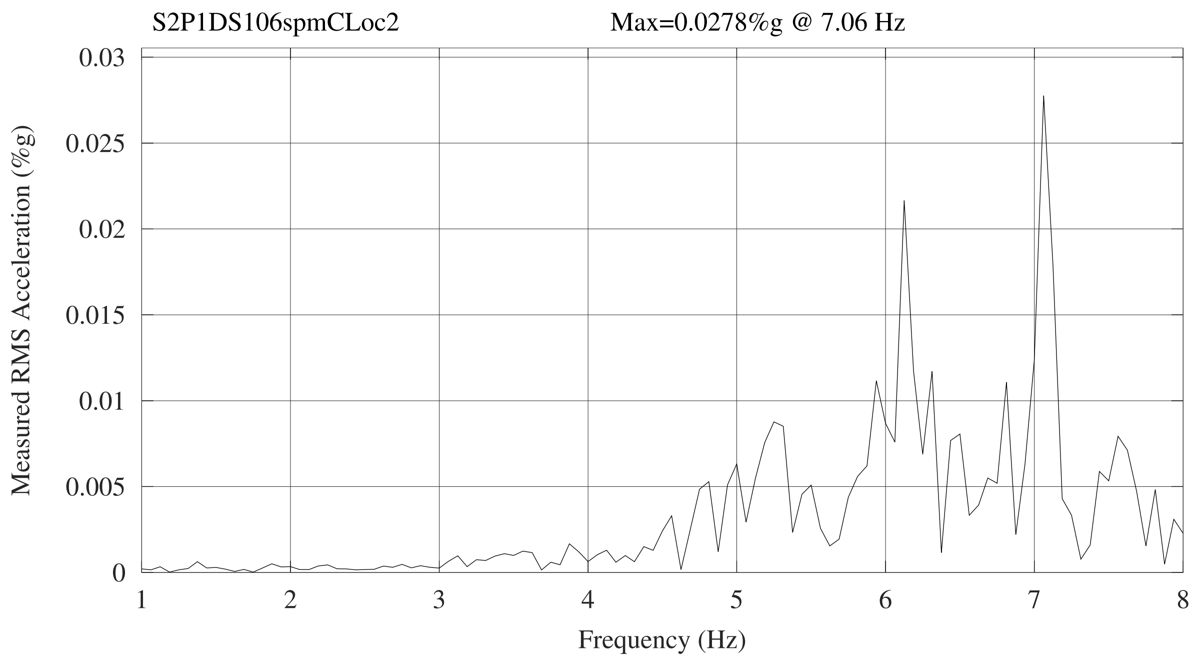

S2P1DS106spmBLoc3
ESPA=0.175%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

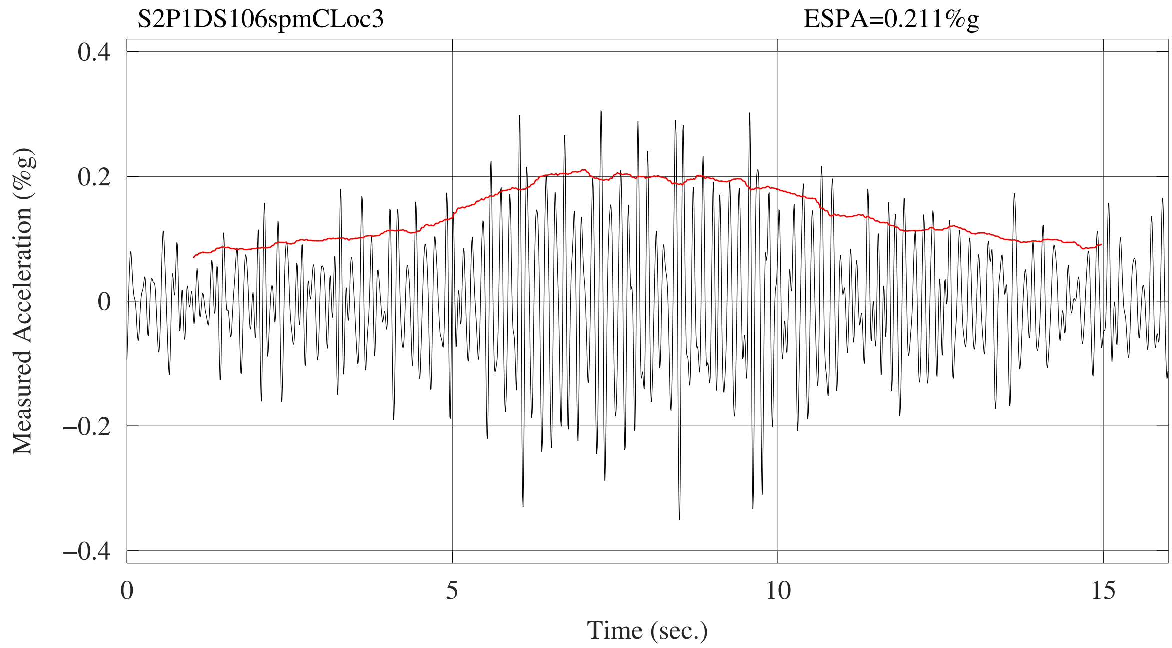

S2P1DS106spmBLoc3
Max=0.0332%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

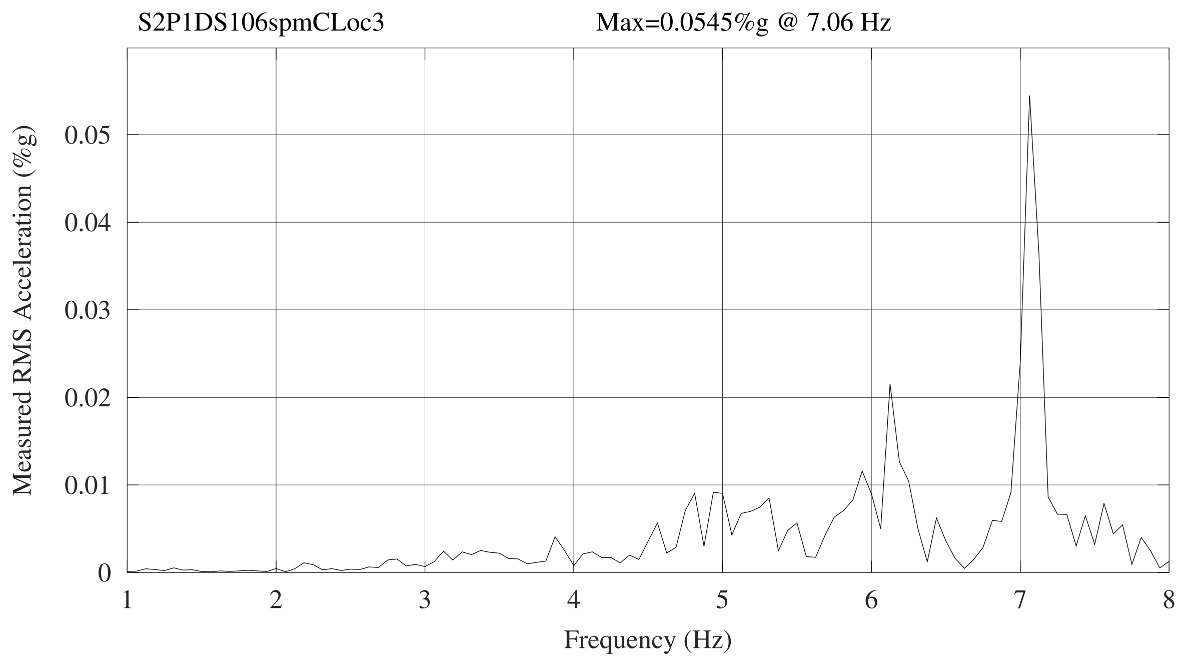

S2P1DS106spmCLoc1
ESPA=0.903%g
Measured Acceleration (%g)
1
0.5
0
−0.5
−1
0
5
10
15
Time (sec.)

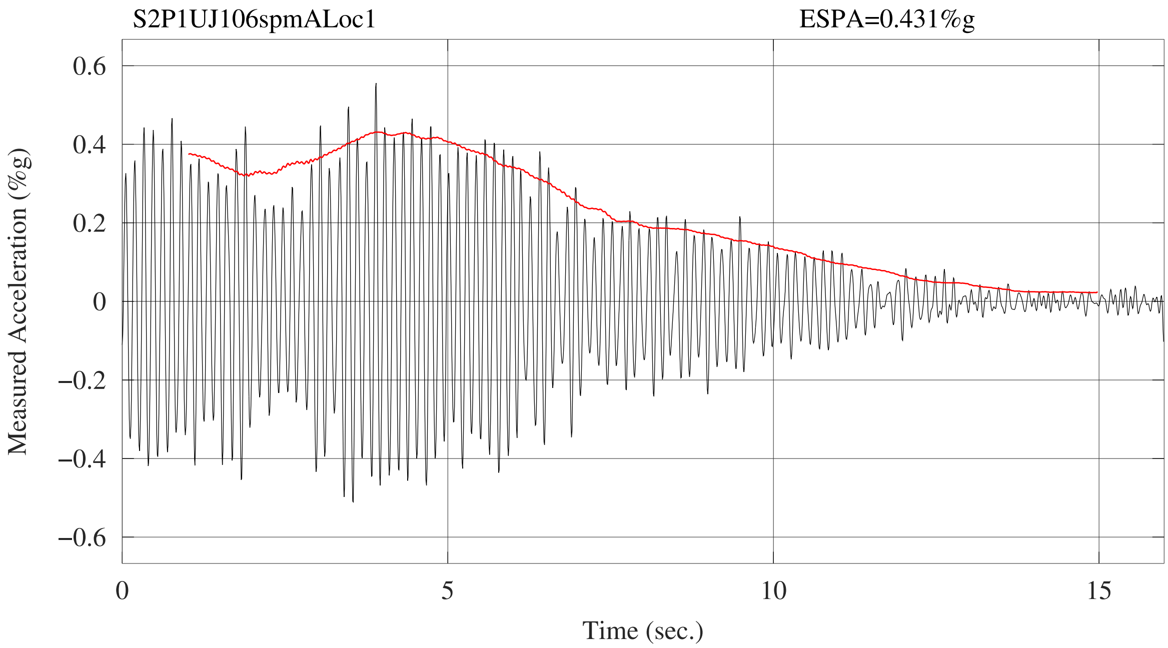

S2P1DS106spmCLoc1
Max=0.293%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
7
8
Frequency (Hz)

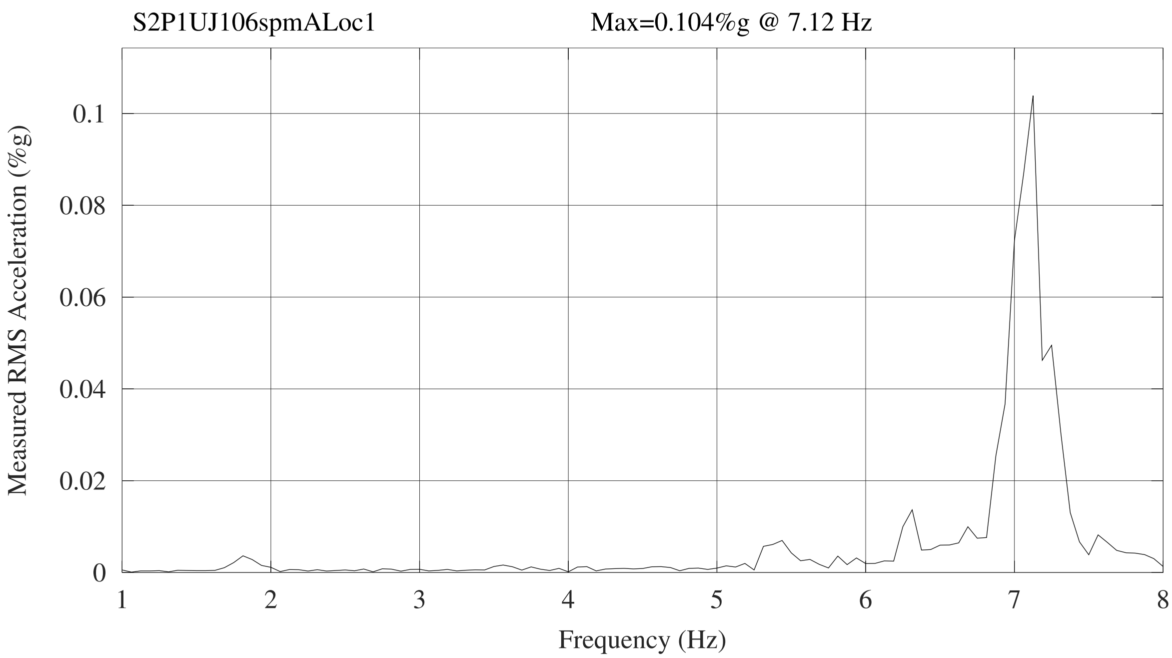

S2P1DS106spmCLoc2
ESPA=0.152%g
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

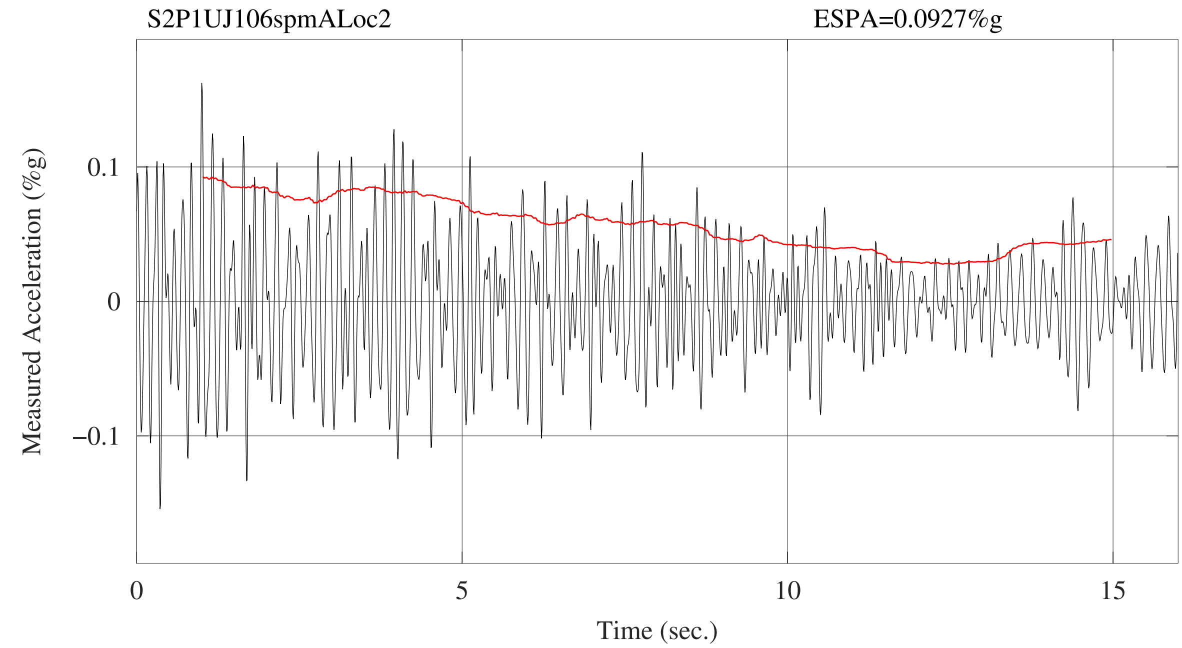

S2P1DS106spmCLoc2
Max=0.0278%g @ 7.06 Hz
0.03
0.025
0.02
0.015
0.01
0.005
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)

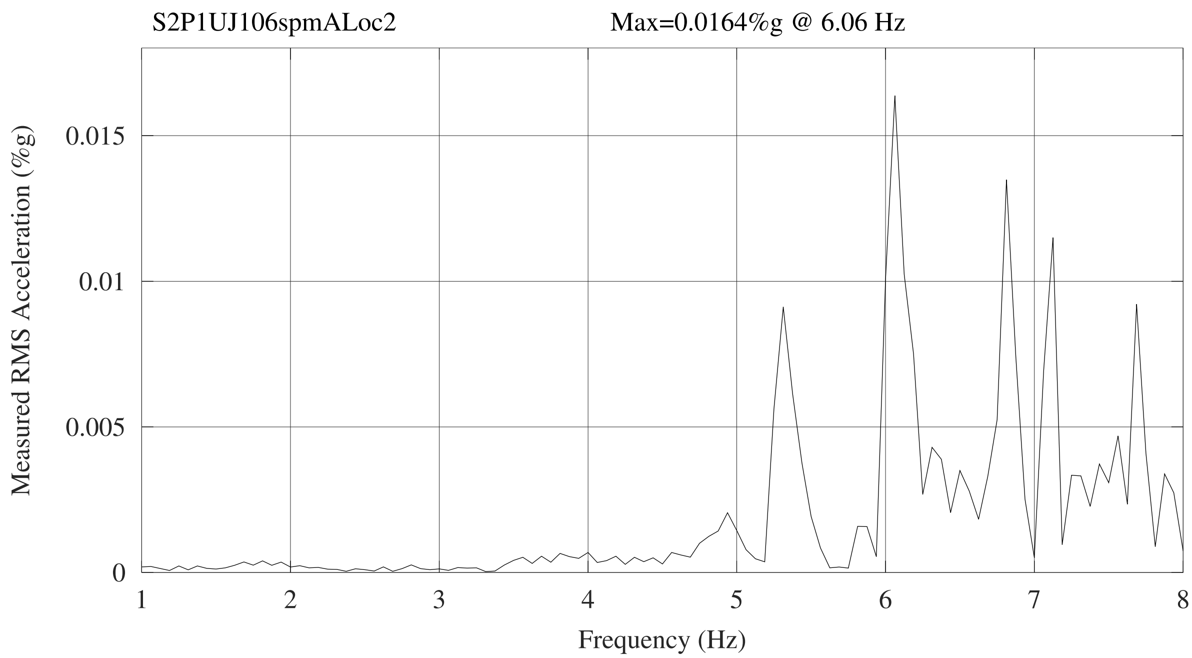

S2P1DS106spmCLoc3
ESPA=0.211%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

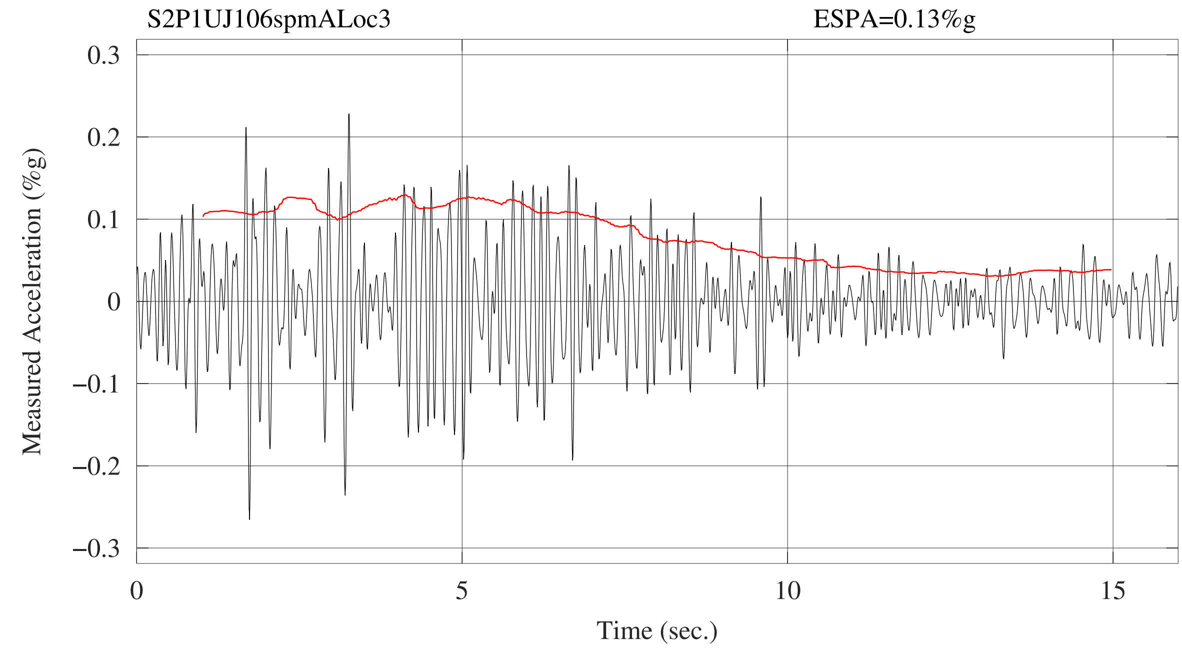

S2P1DS106spmCLoc3
Max=0.0545%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

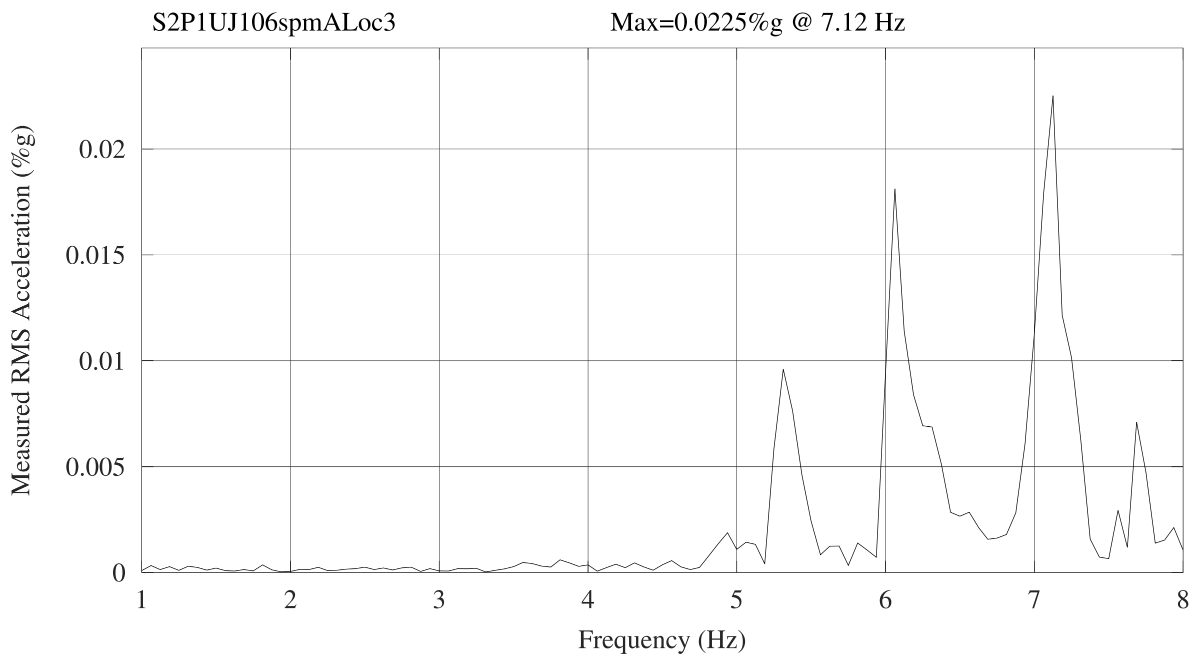

S2P1UJ106spmALoc1
ESPA=0.431%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

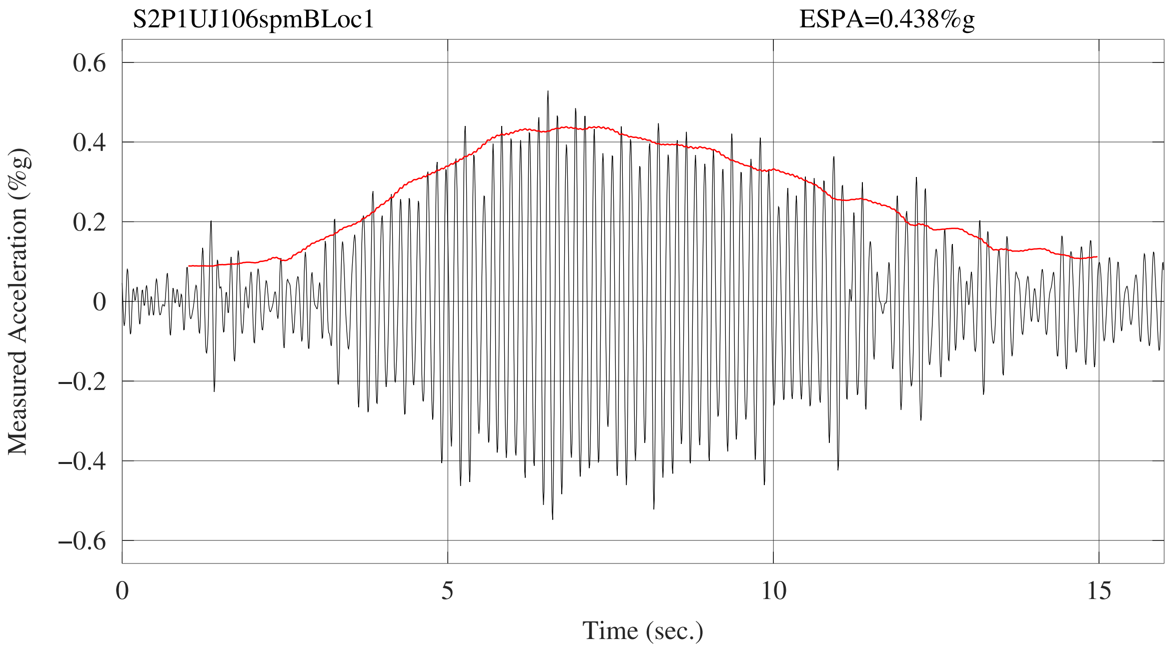

S2P1UJ106spmALoc1
Max=0.104%g @ 7.12 Hz
Measured RMS Acceleration (%g)
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

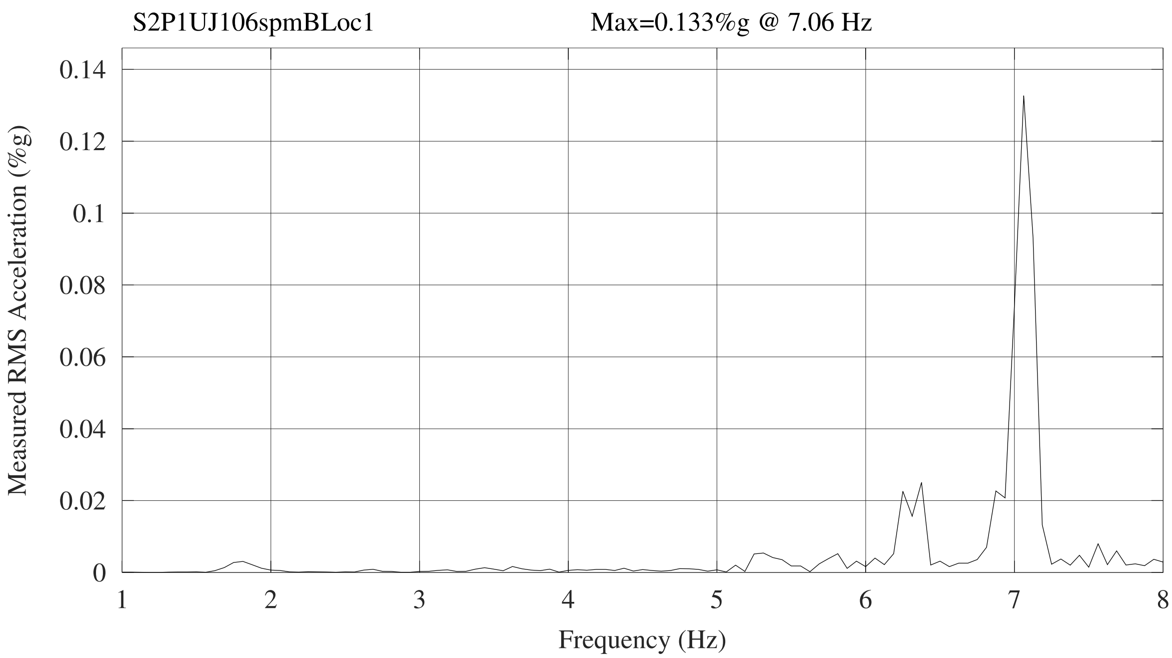

S2P1UJ106spmALoc2
ESPA=0.0927%g
Measured Acceleration (%g)
0.1
0
−0.1
0
5
10
15
Time (sec.)

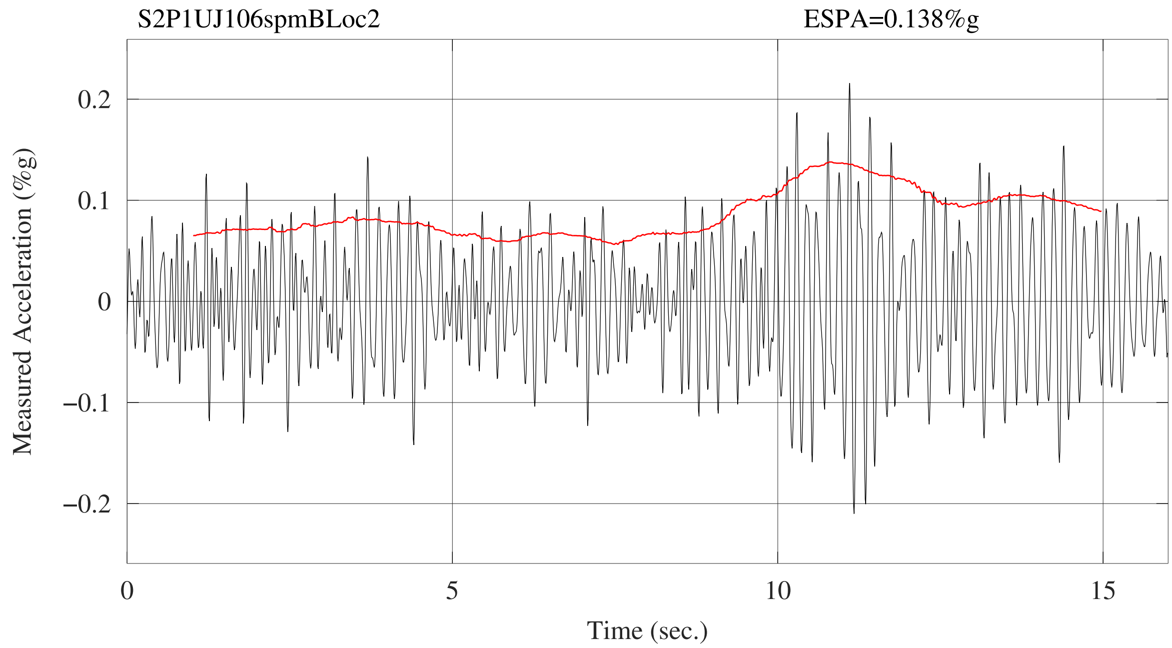

S2P1UJ106spmALoc2
Max=0.0164%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)

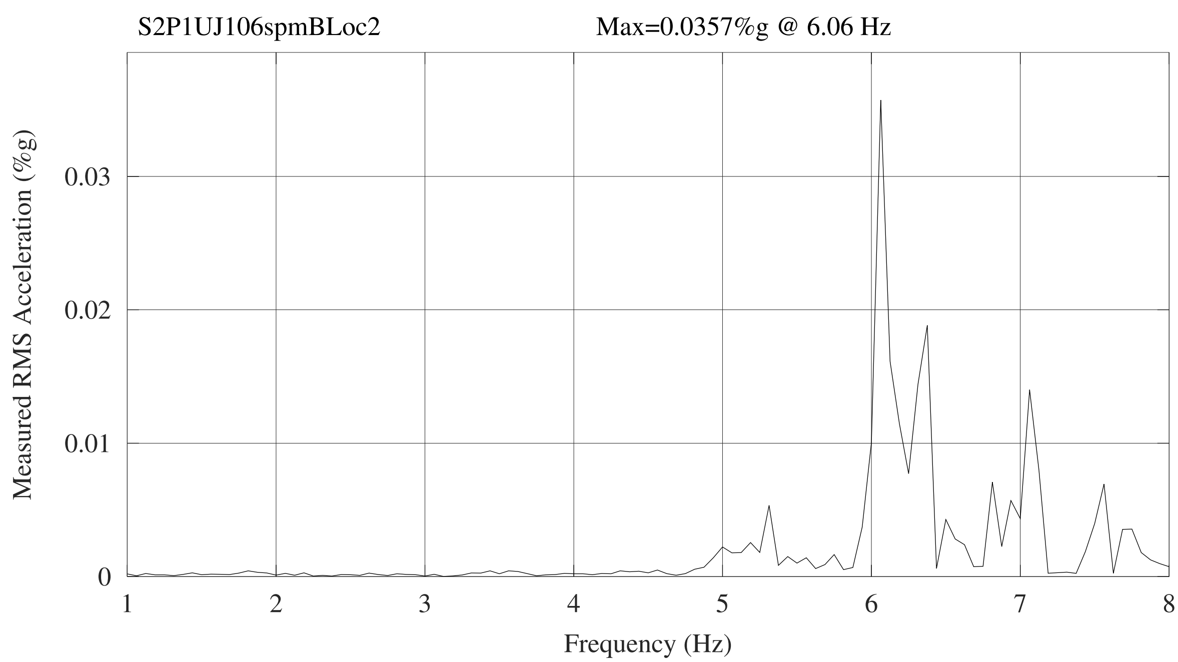

S2P1UJ106spmALoc3
ESPA=0.13%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

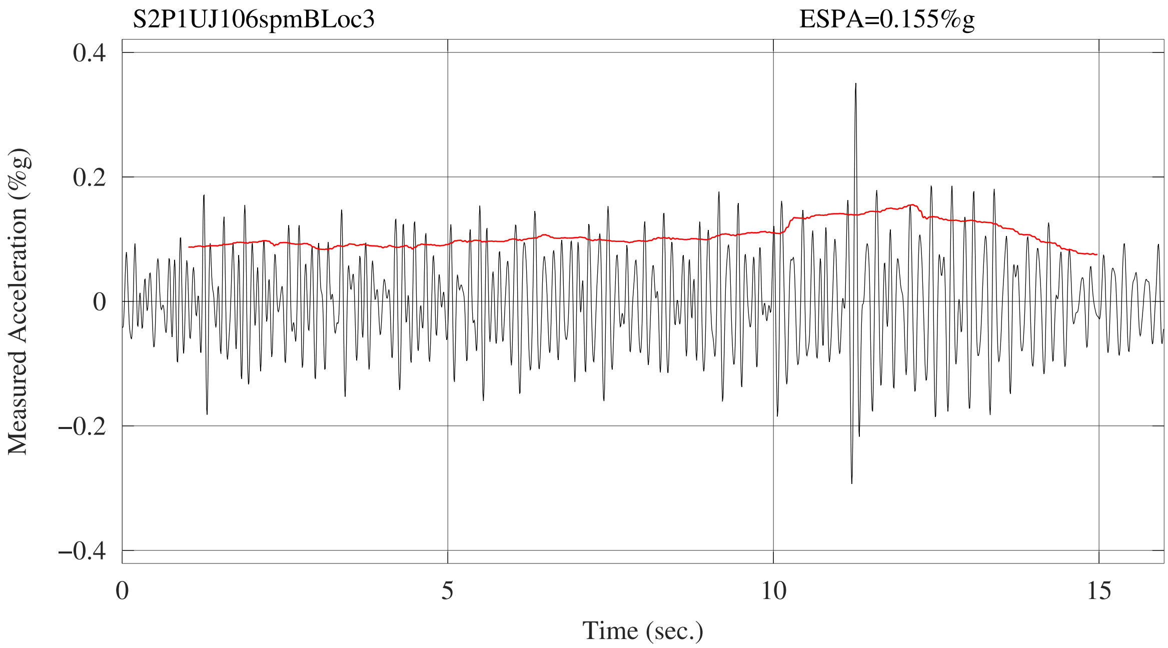

S2P1UJ106spmALoc3
Max=0.0225%g @ 7.12 Hz
Measured RMS Acceleration (%g)
0.02
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P1UJ106spmBLoc1
ESPA=0.438%g
Measured Acceleration (%g)
Time (sec.)

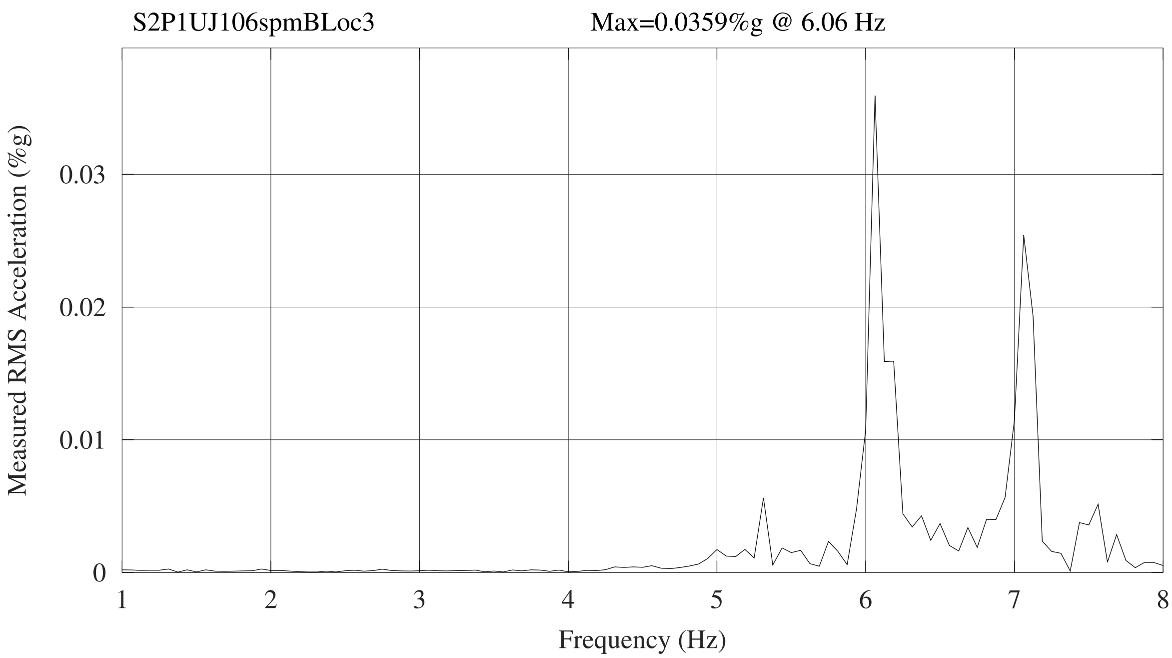

S2P1UJ106spmBLoc1
Max=0.133%g @ 7.06 Hz
Measured RMS Acceleration (%g)
Frequency (Hz)


S2P1UJ106spmBLoc2
ESPA=0.138%g
Measured Acceleration (%g)
0.2
0.1
0
−0.1
−0.2
0
5
10
15
Time (sec.)

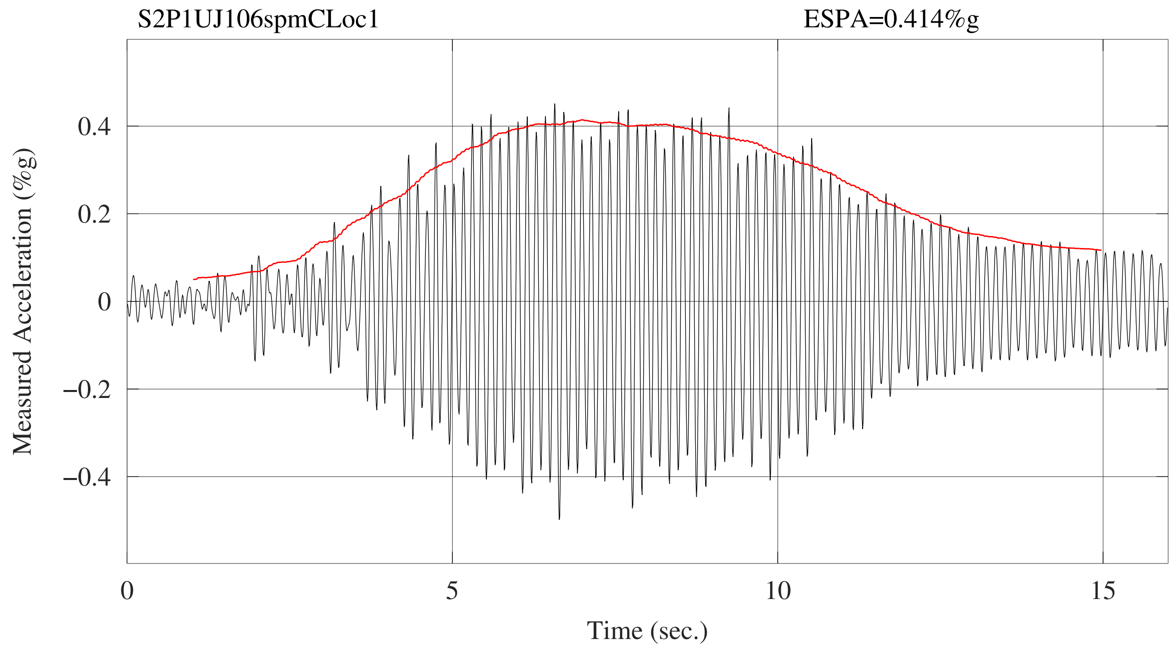

S2P1UJ106spmBLoc2
Max=0.0357%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

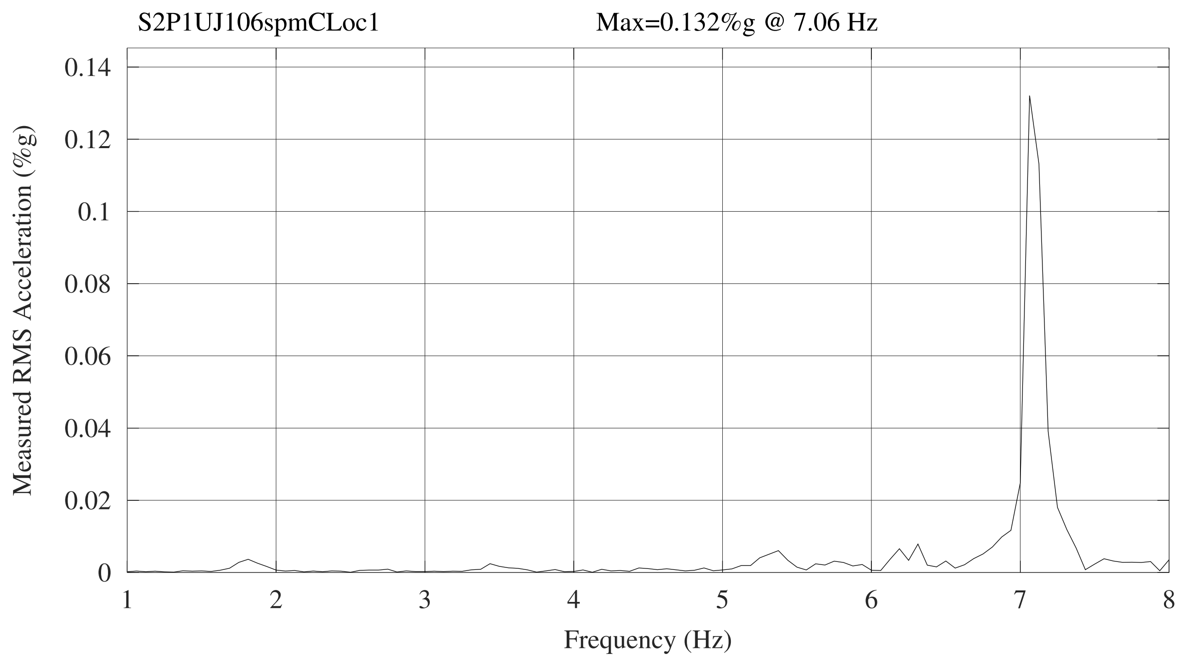


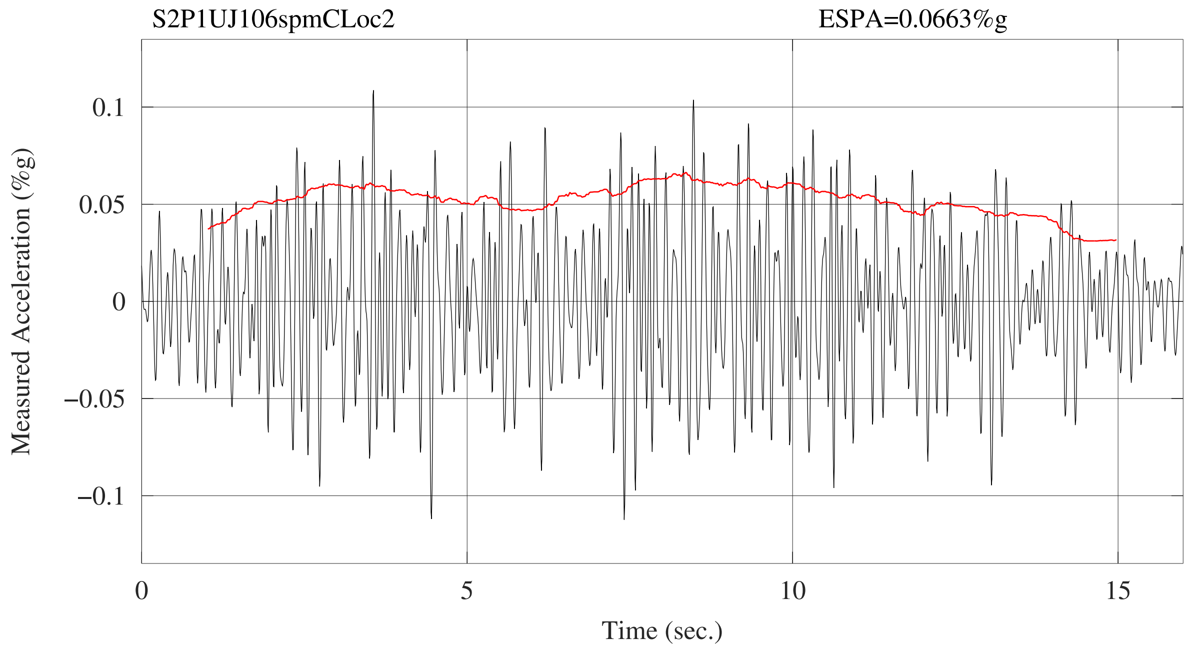
S2P1UJ106spmBLoc3
ESPA=0.155%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)



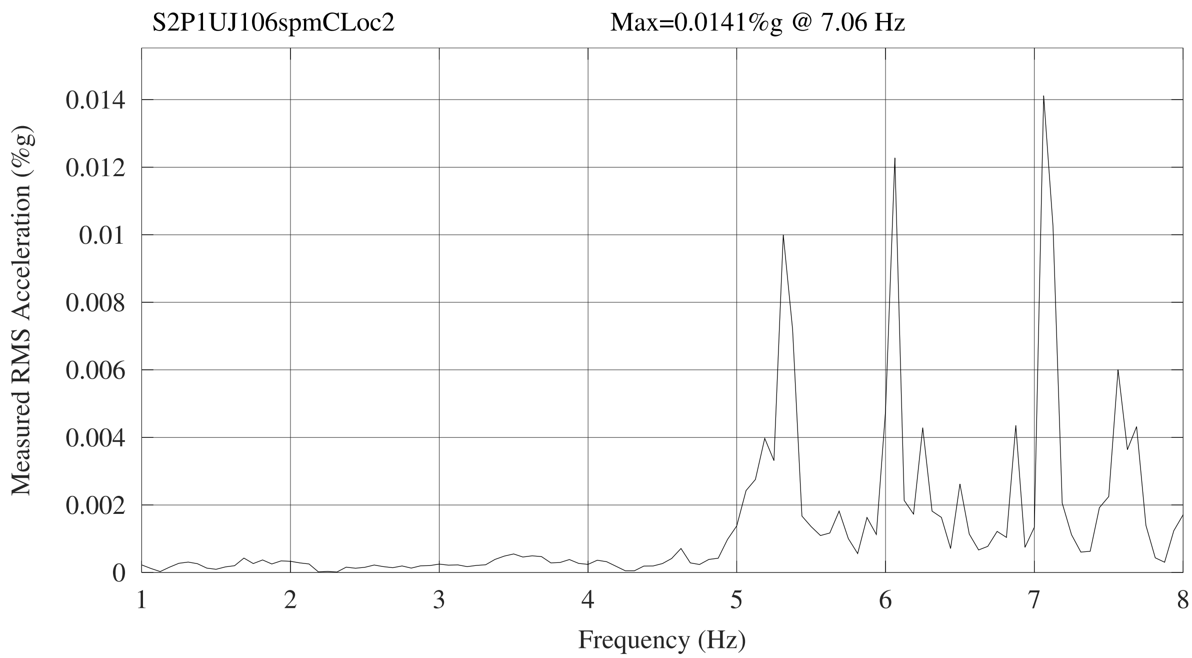
S2P1UJ106spmBLoc3
Max=0.0359%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P1UJ106spmCLoc1
ESPA=0.414%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

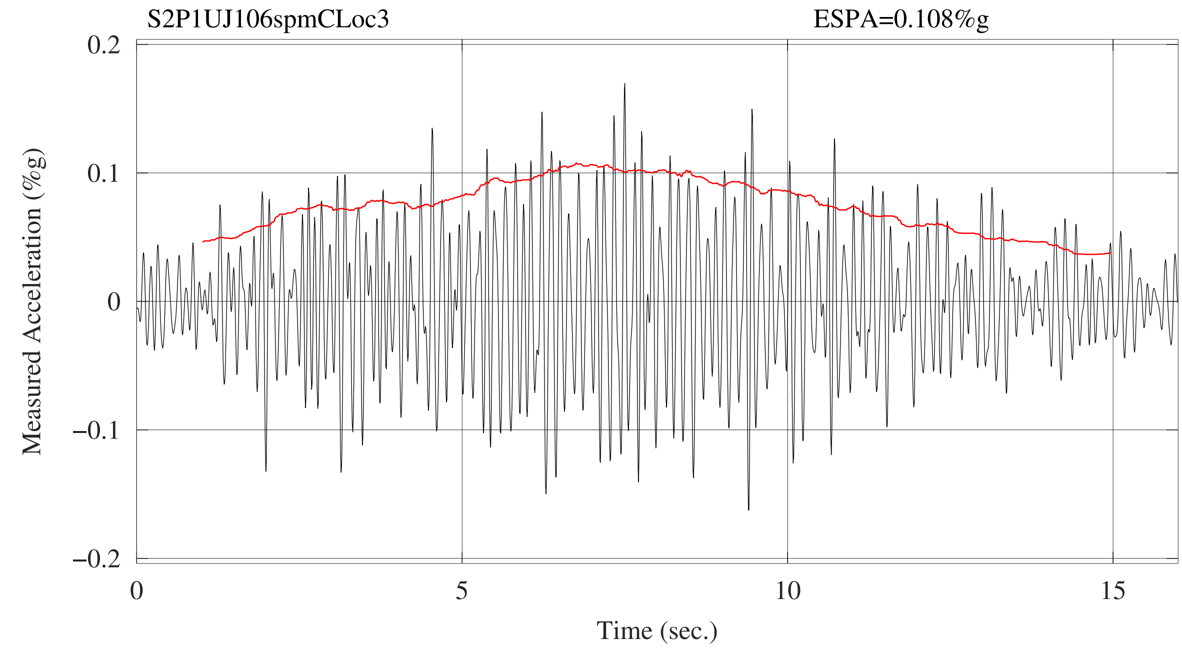

S2P1UJ106spmCLoc1
Max=0.132%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

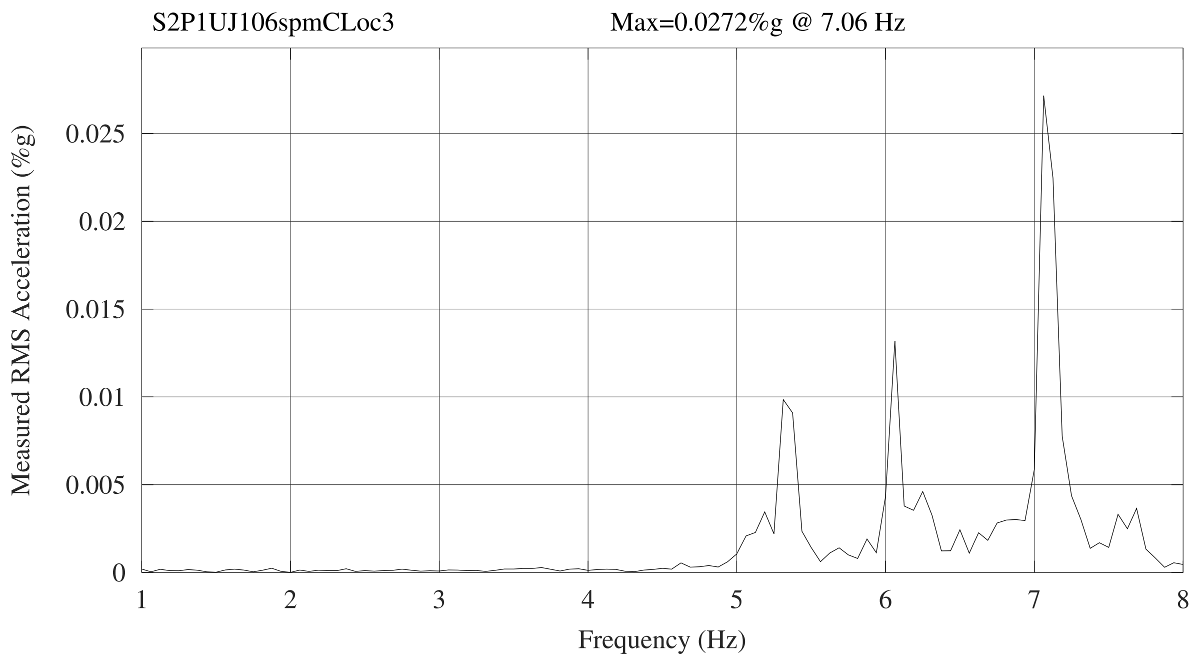


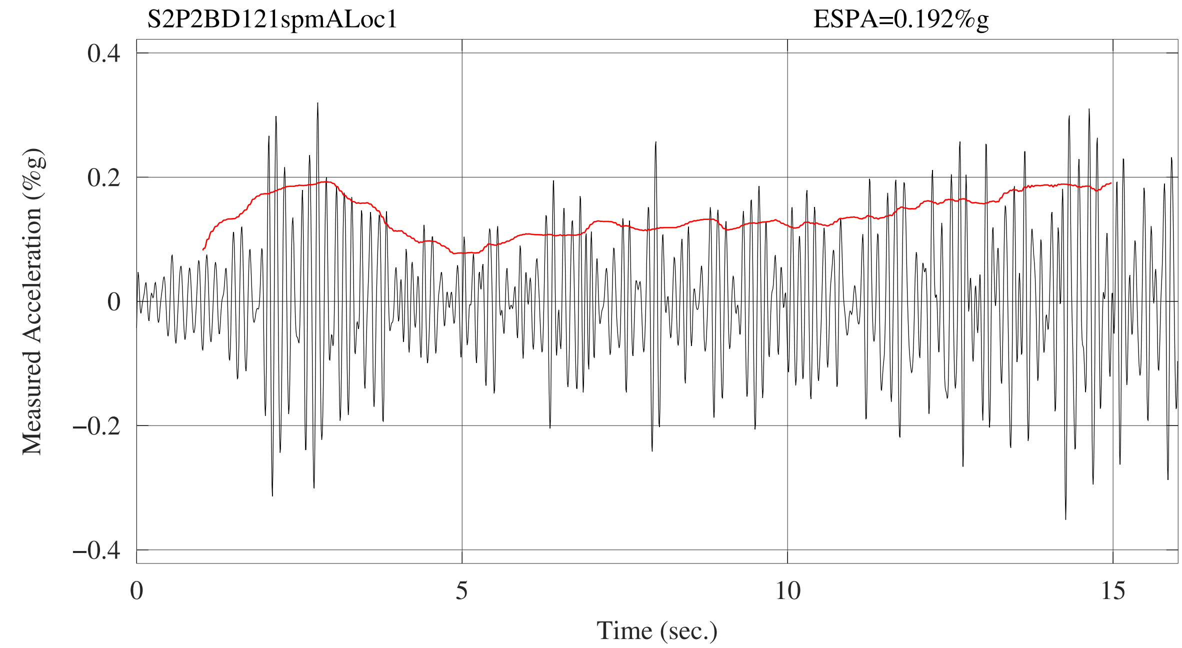
S2P1UJ106spmCLoc2
ESPA=0.0663%g
Measured Acceleration (%g)
0.1
0.05
0
−0.05
−0.1
0
5
10
15
Time (sec.)

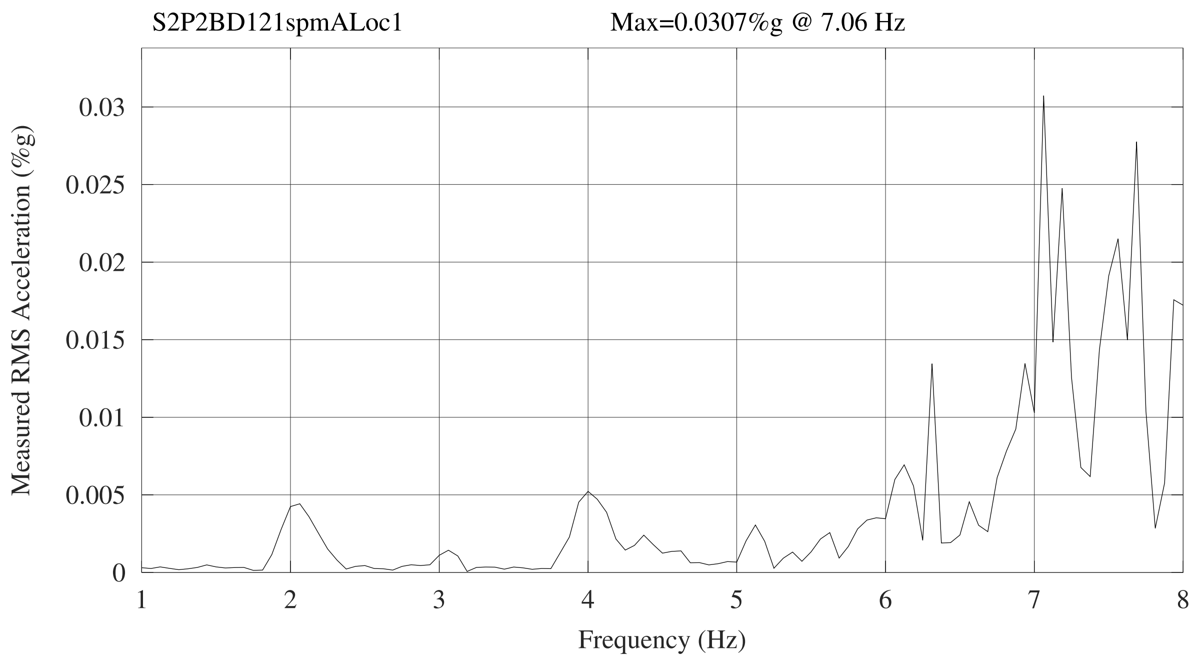


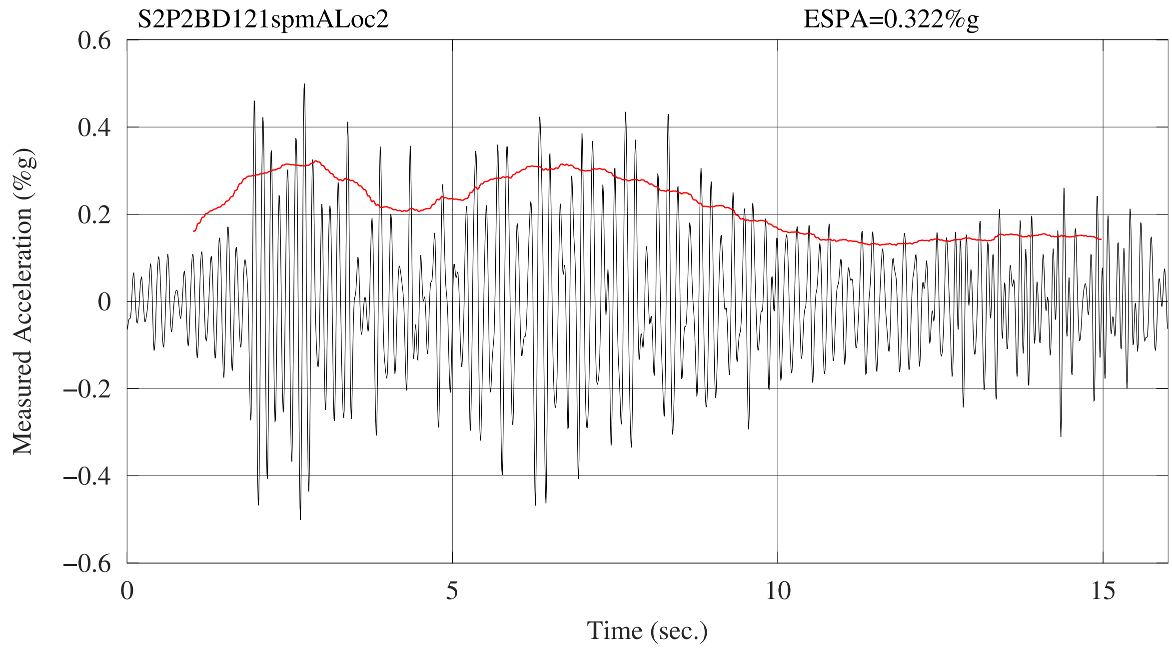
S2P1UJ106spmCLoc2
Max=0.0141%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P1UJ106spmCLoc3
ESPA=0.108%g
0.2
0.1
0
−0.1
−0.2
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

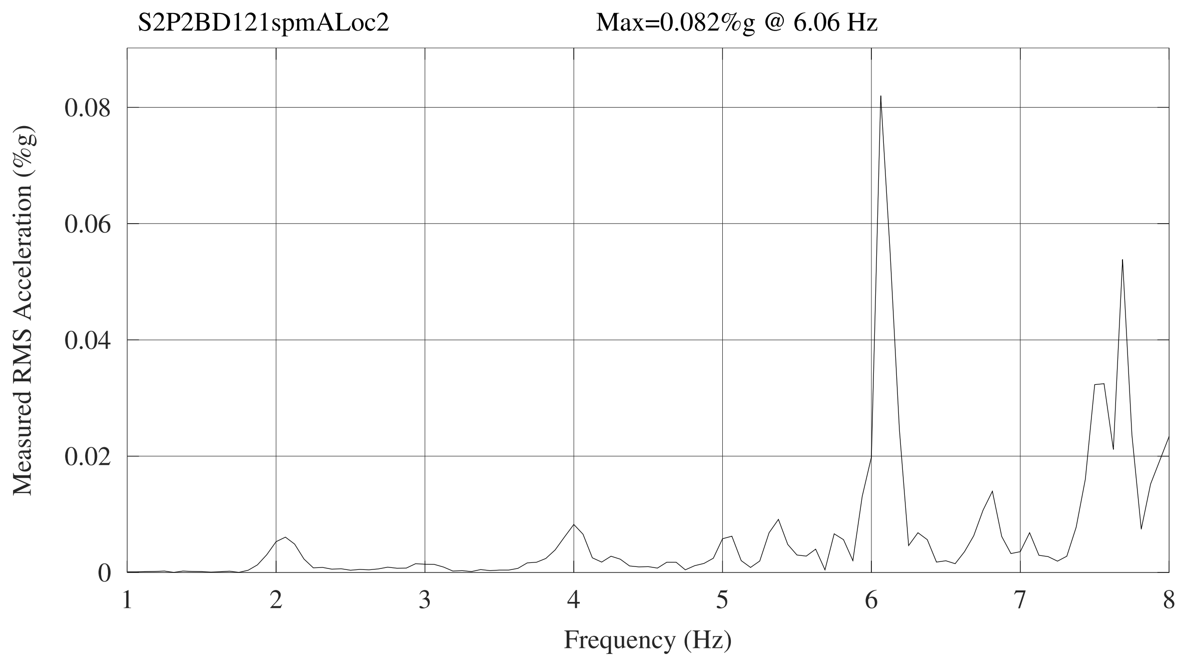

S2P1UJ106spmCLoc3
Max=0.0272%g @ 7.06 Hz
0.025
0.02
0.015
0.01
0.005
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P2BD121spmALoc1
ESPA=0.192%g
0.4
0.2
0
−0.2
−0.4
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

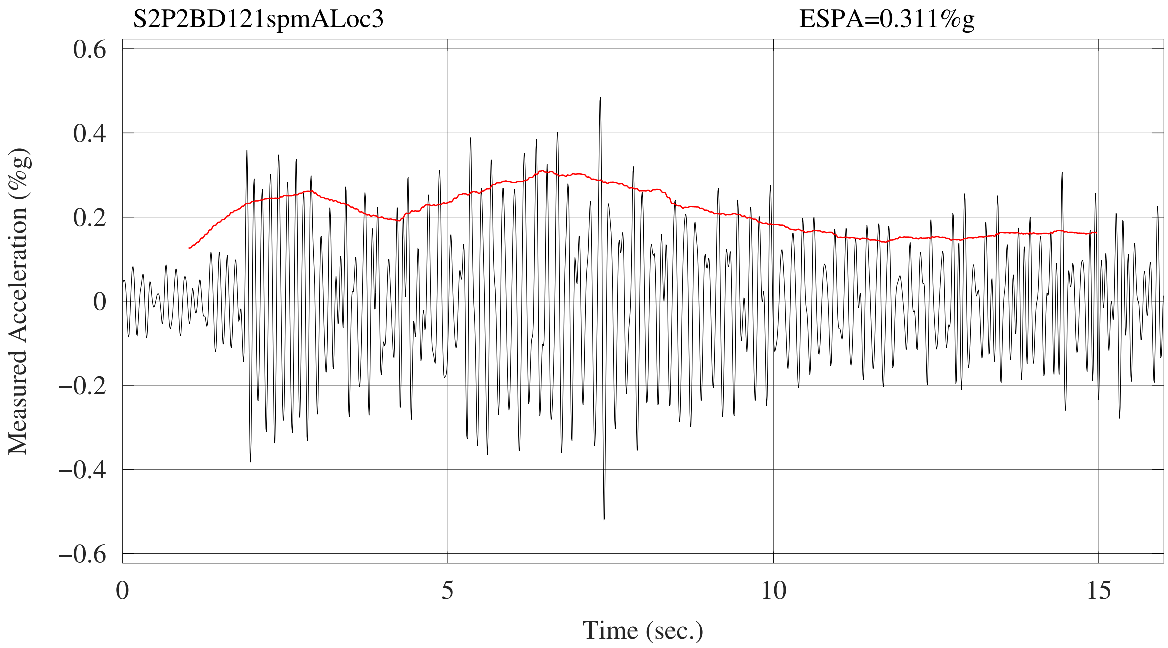

S2P2BD121spmALoc1
Max=0.0307%g @ 7.06 Hz
0.03
0.025
0.02
0.015
0.01
0.005
0
Measured RMS Acceleration (%g)
1
2
3
4
5
6
7
8
Frequency (Hz)

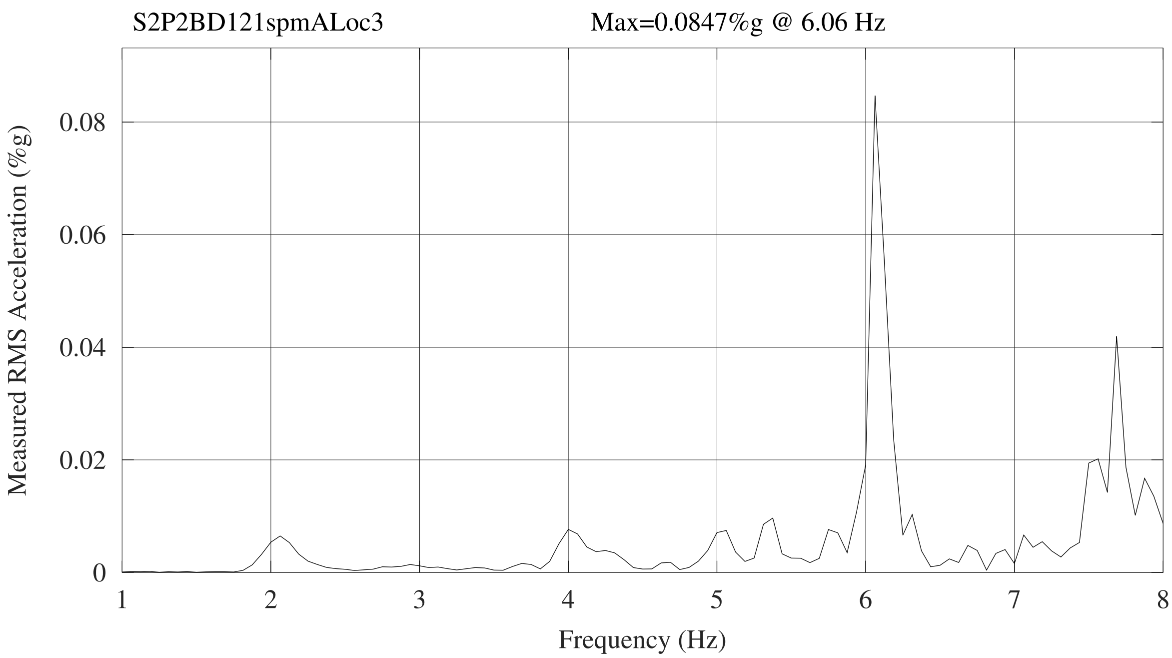


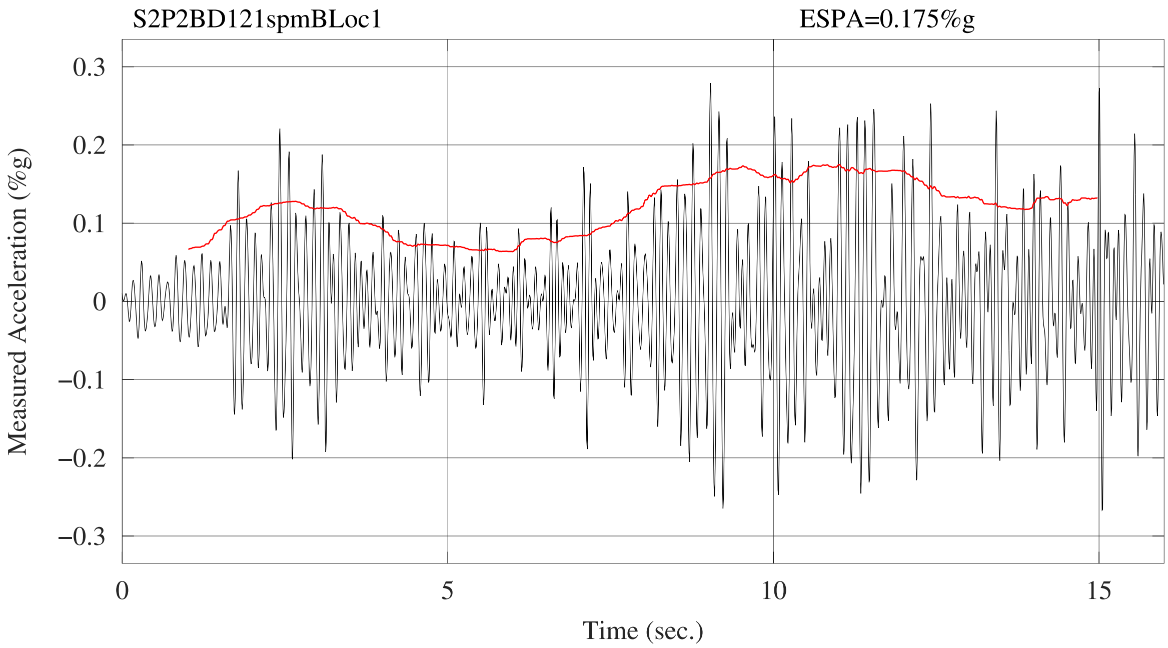
S2P2BD121spmALoc2
ESPA=0.322%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

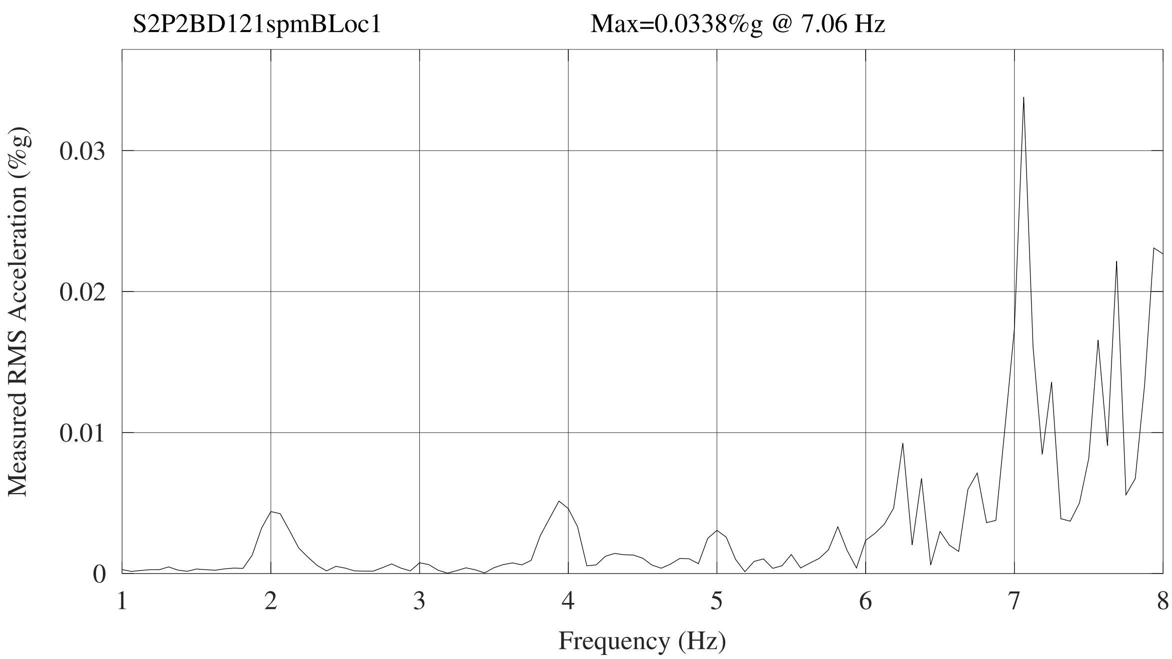


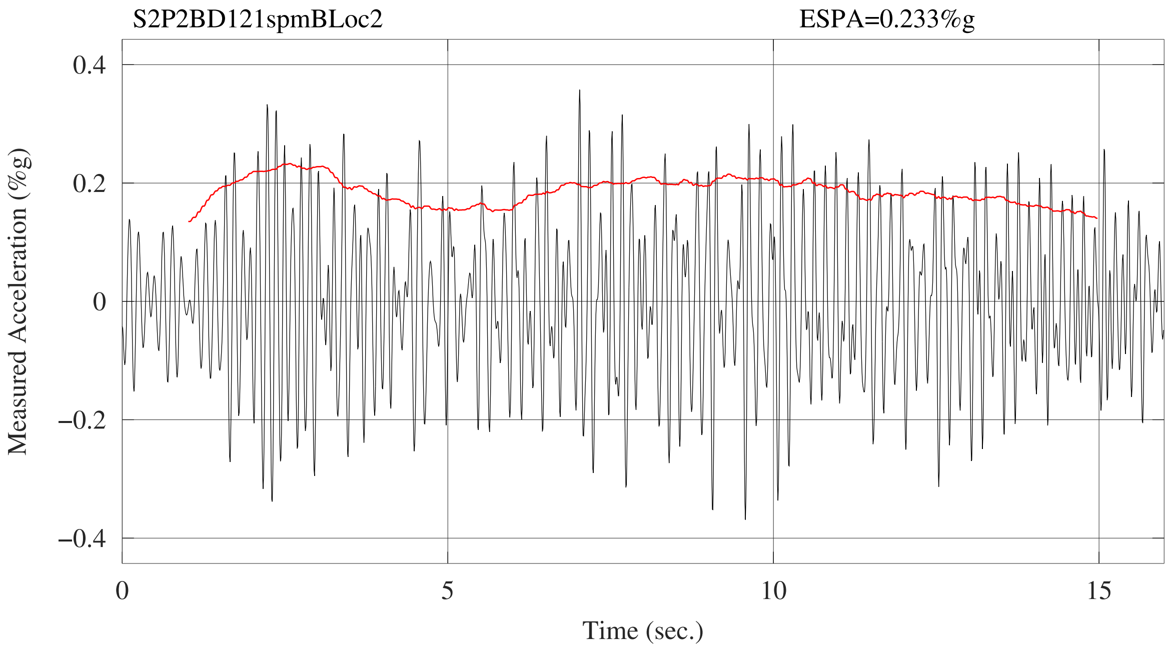
S2P2BD121spmALoc2
Max=0.082%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

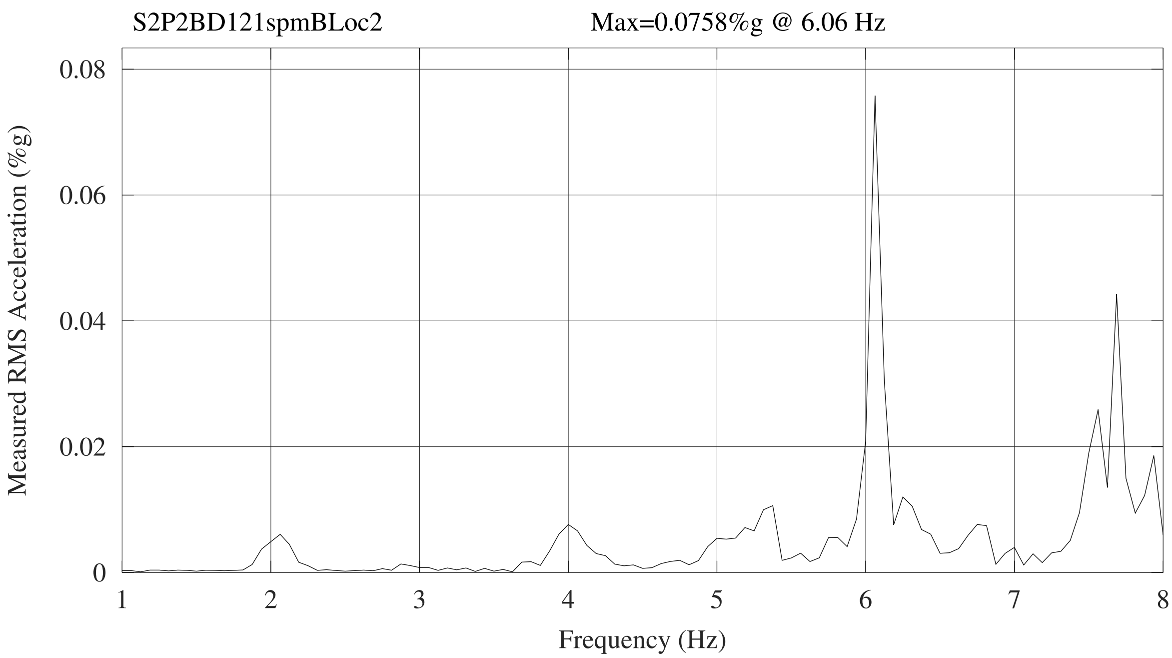

S2P2BD121spmALoc3
ESPA=0.311%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

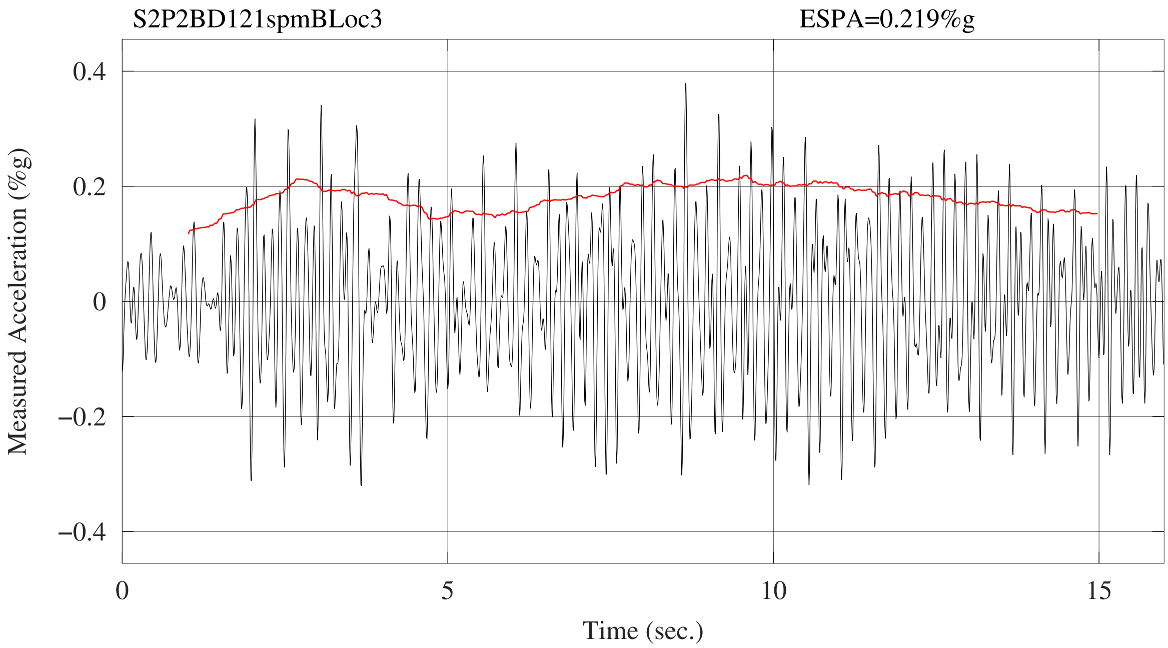

S2P2BD121spmALoc3
Max=0.0847%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P2BD121spmBLoc1
ESPA=0.175%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

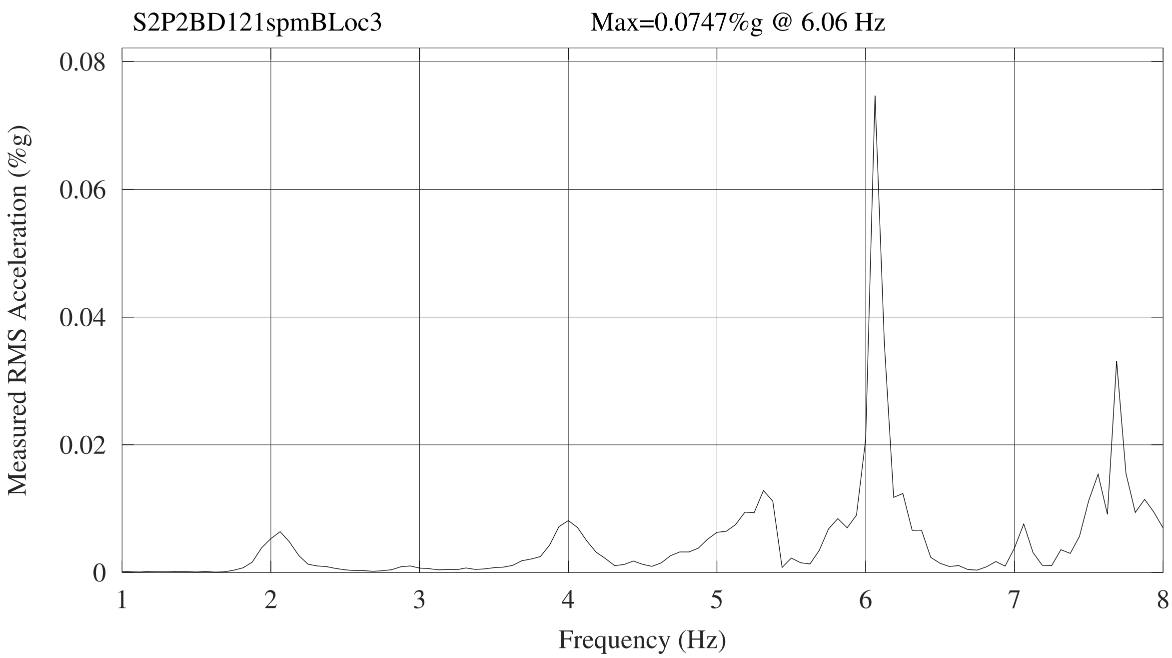

S2P2BD121spmBLoc1
Max=0.0338%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P2BD121spmBLoc2
ESPA=0.233%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

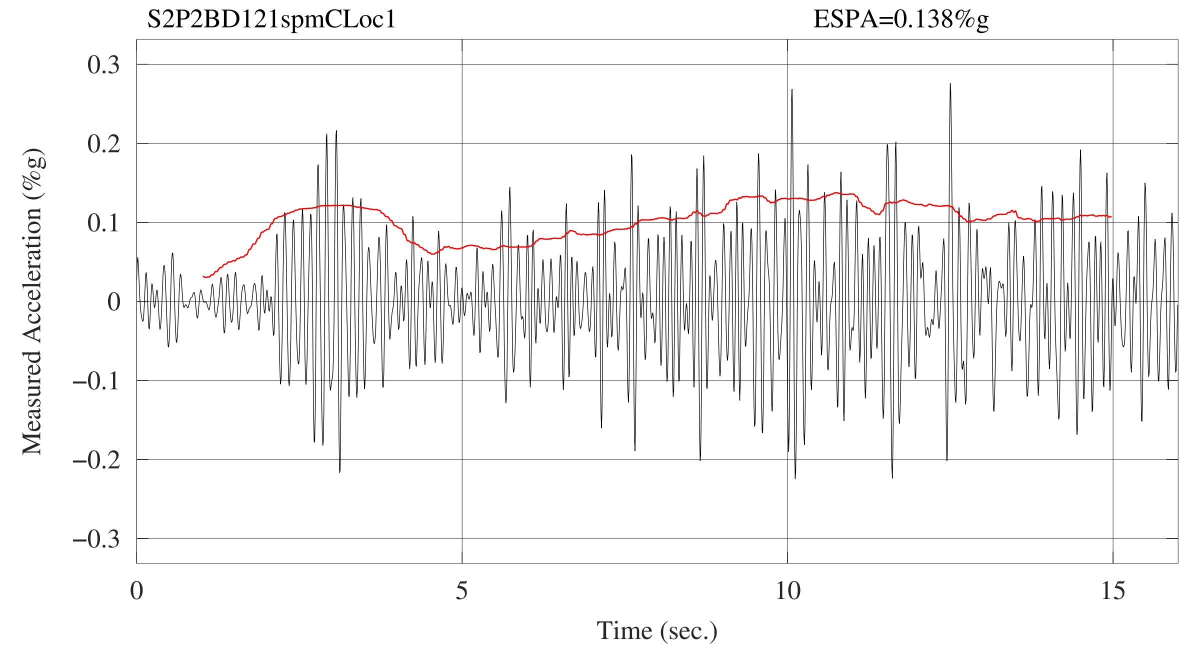

S2P2BD121spmBLoc2
Max=0.0758%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

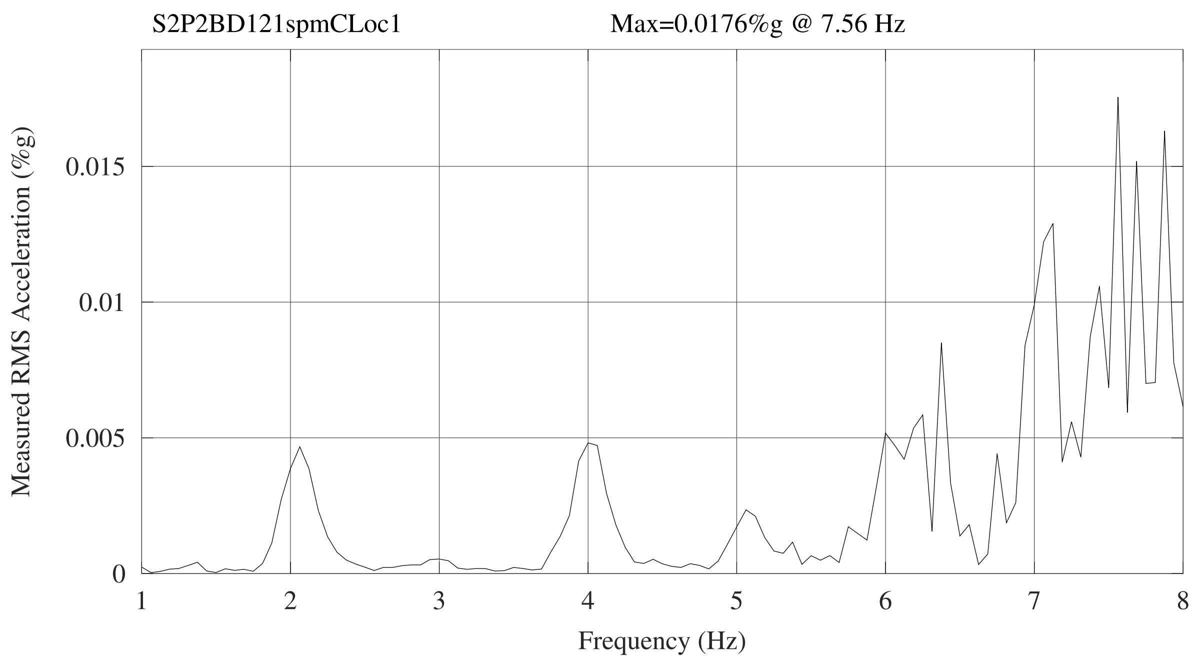


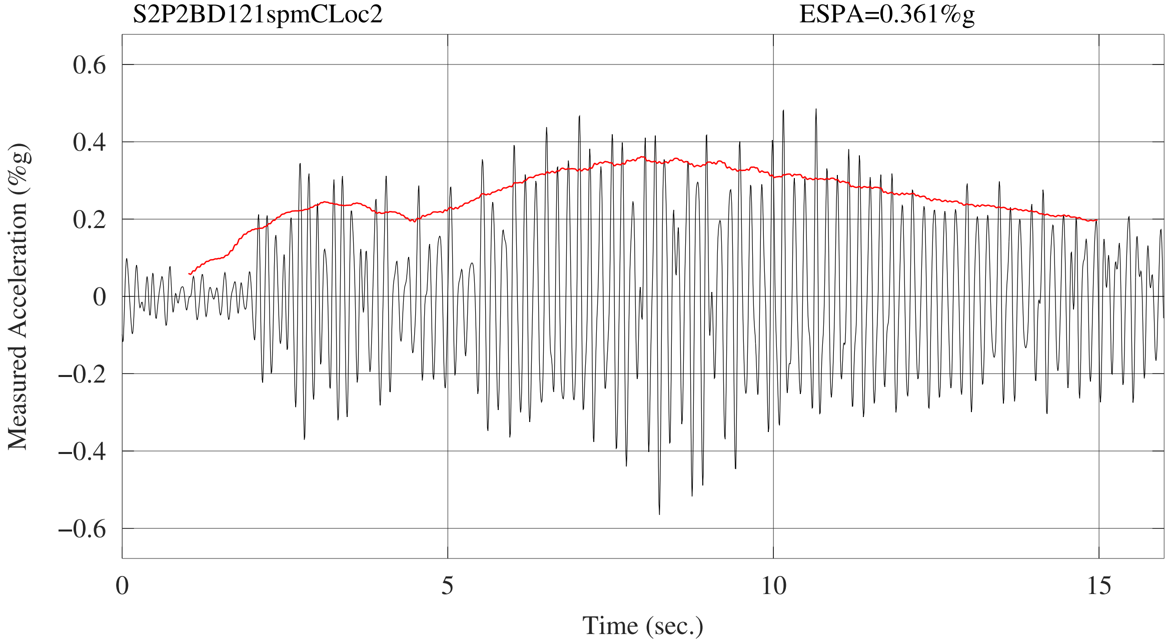
S2P2BD121spmBLoc3
ESPA=0.219%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

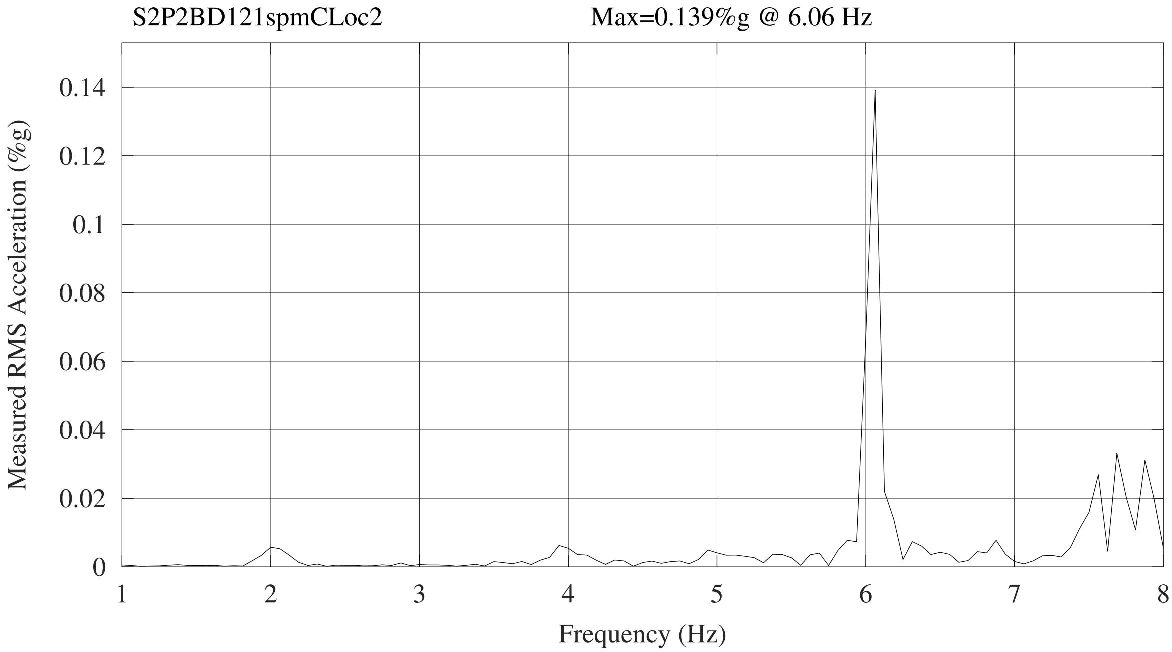


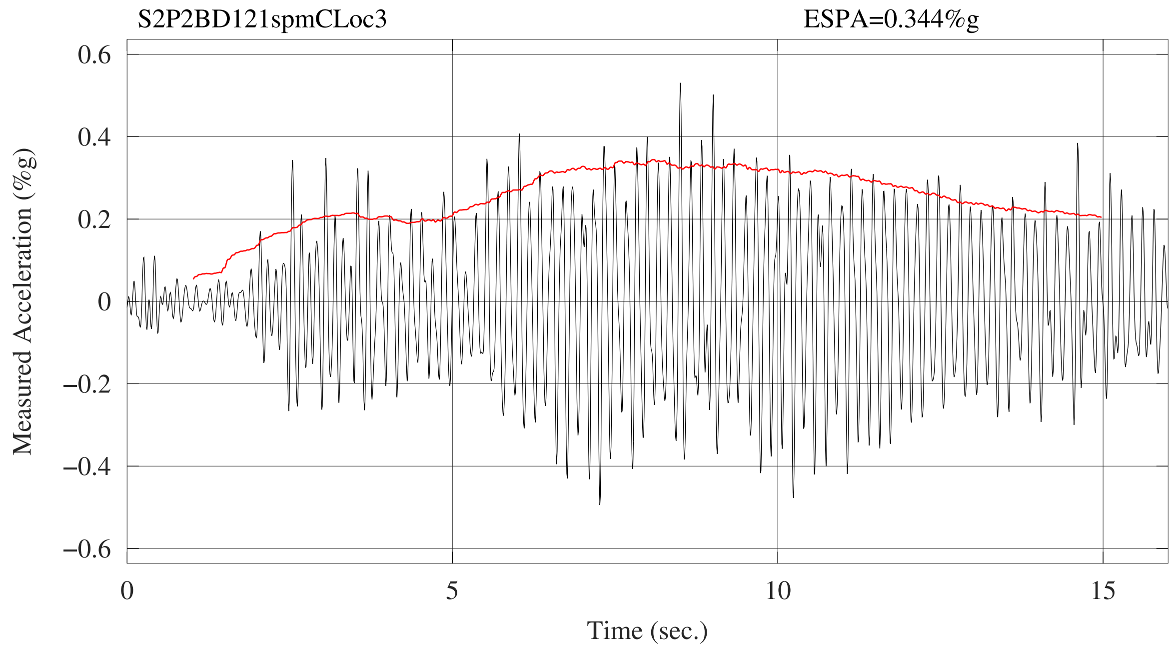
S2P2BD121spmBLoc3
Max=0.0747%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

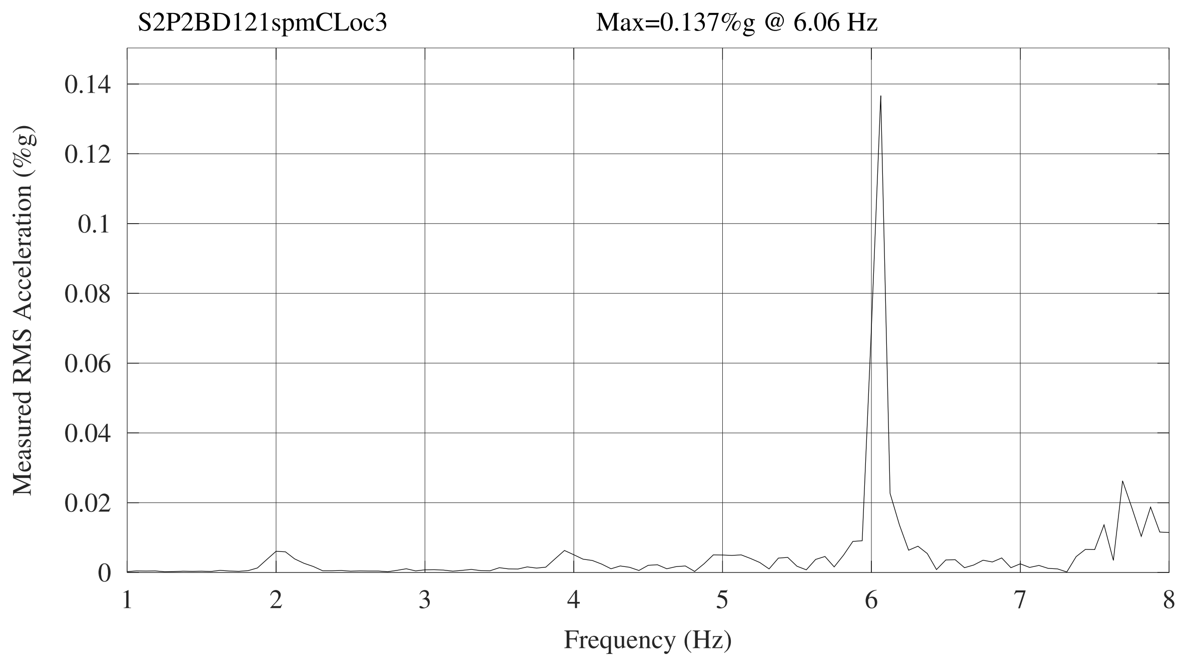

S2P2BD121spmCLoc1
ESPA=0.138%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

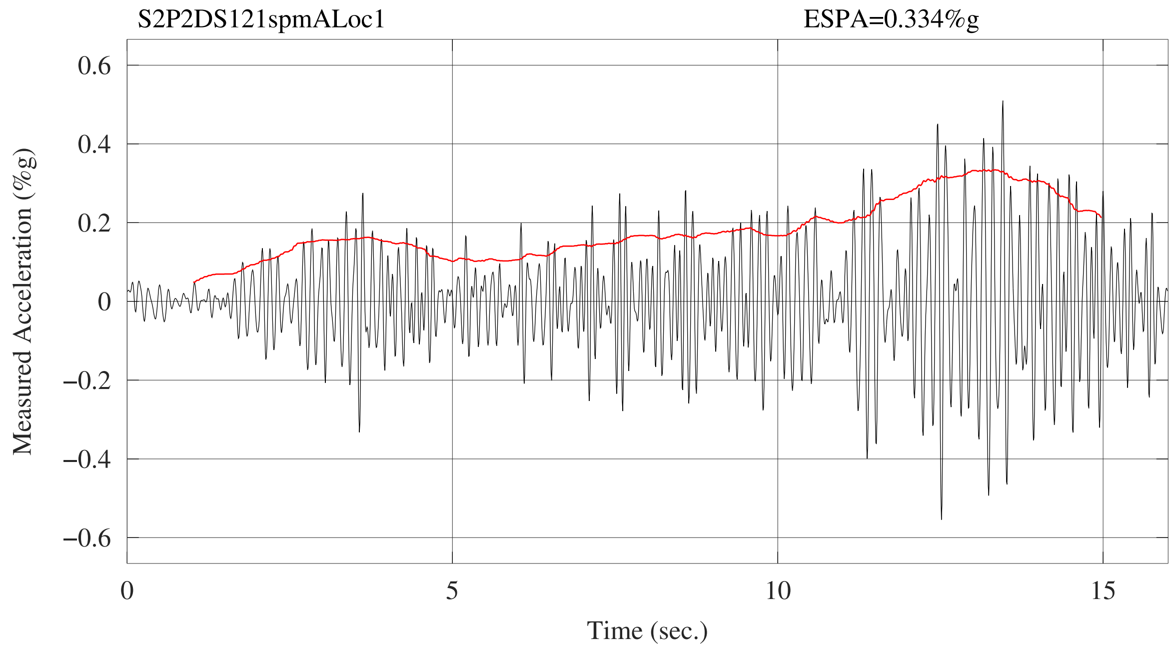

S2P2BD121spmCLoc1
Max=0.0176%g @ 7.56 Hz
Measured RMS Acceleration (%g)
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)

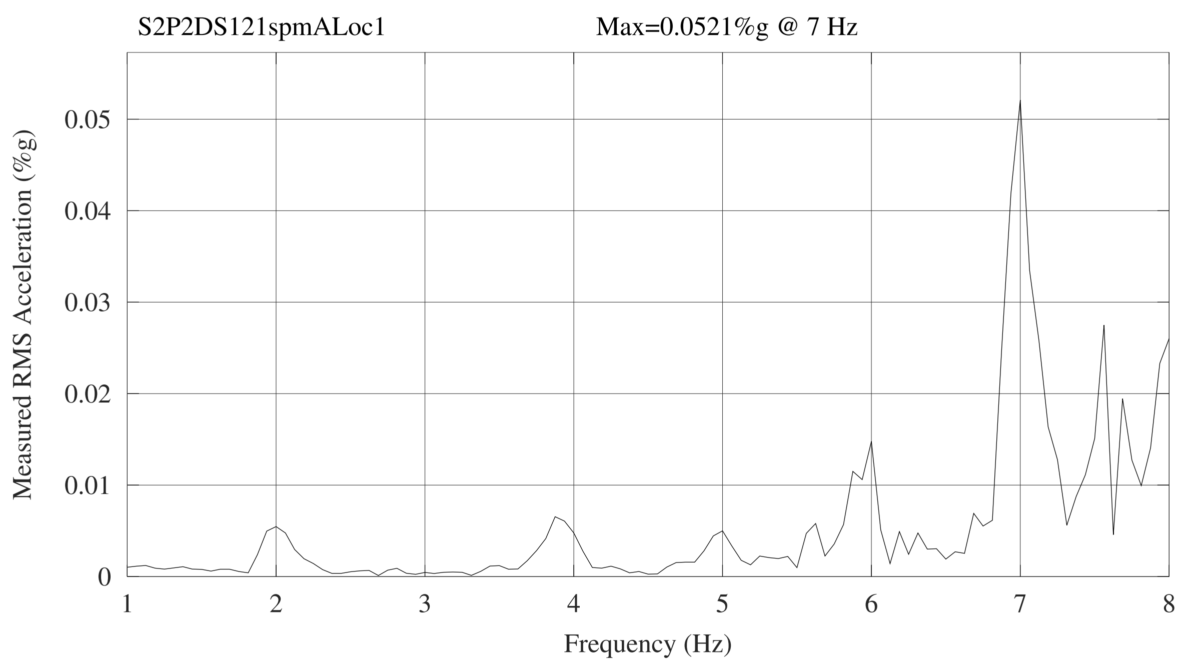

S2P2BD121spmCLoc2
ESPA=0.361%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

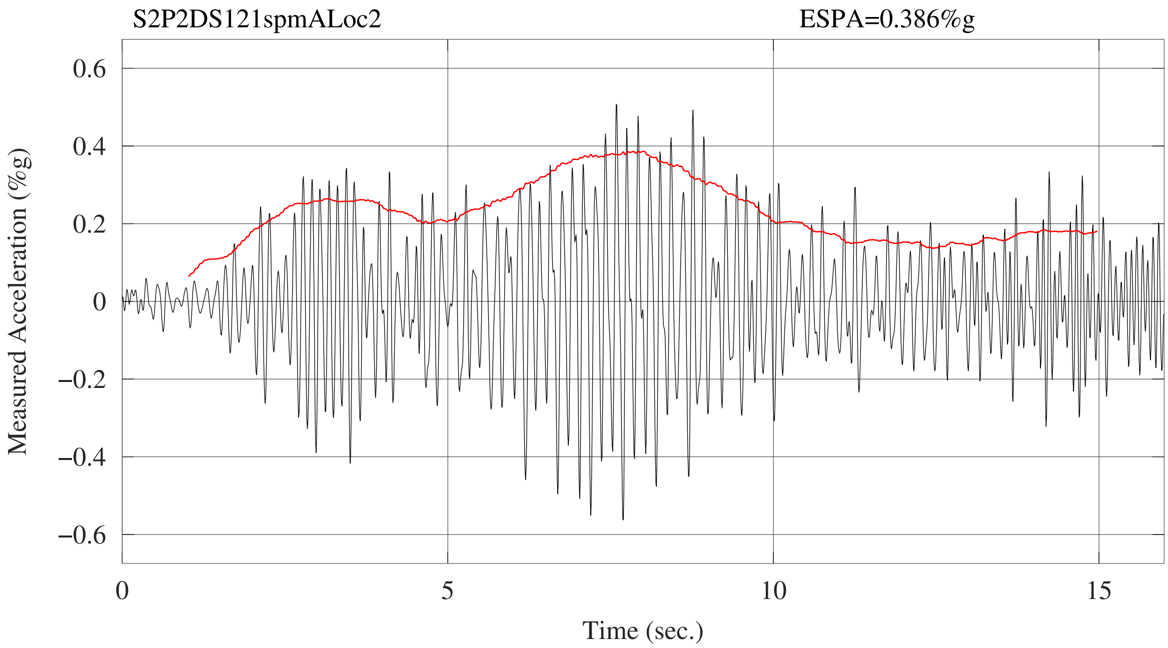

S2P2BD121spmCLoc2
Max=0.139%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

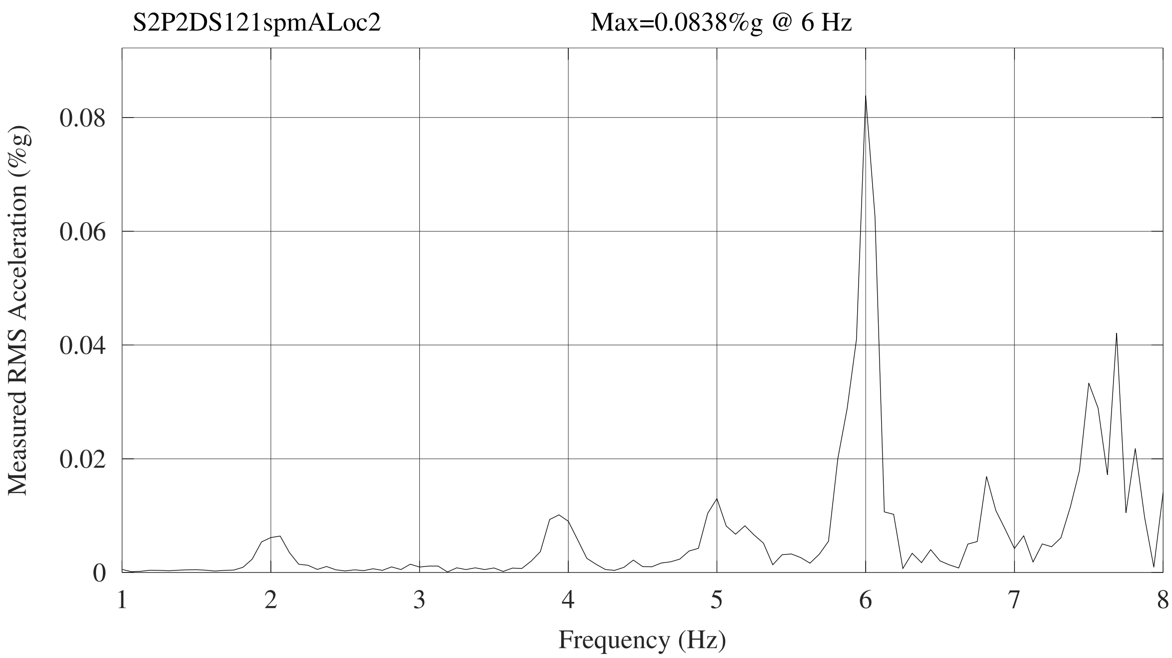

S2P2BD121spmCLoc3
ESPA=0.344%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

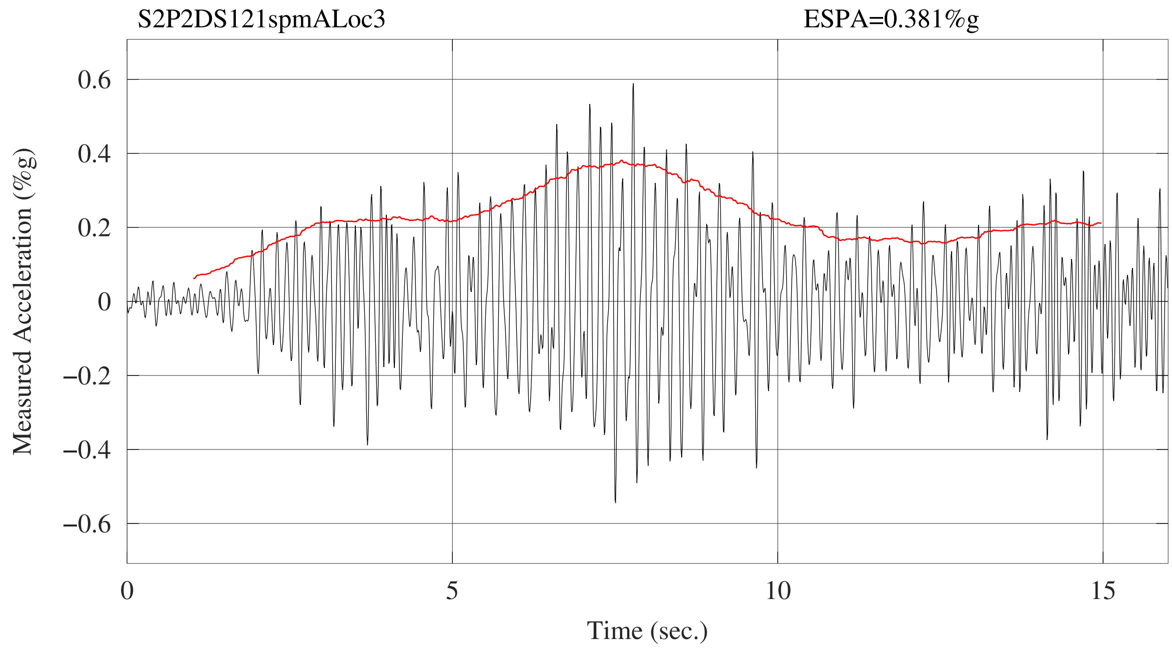

S2P2BD121spmCLoc3
Max=0.137%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P2DS121spmALoc1
ESPA=0.334%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

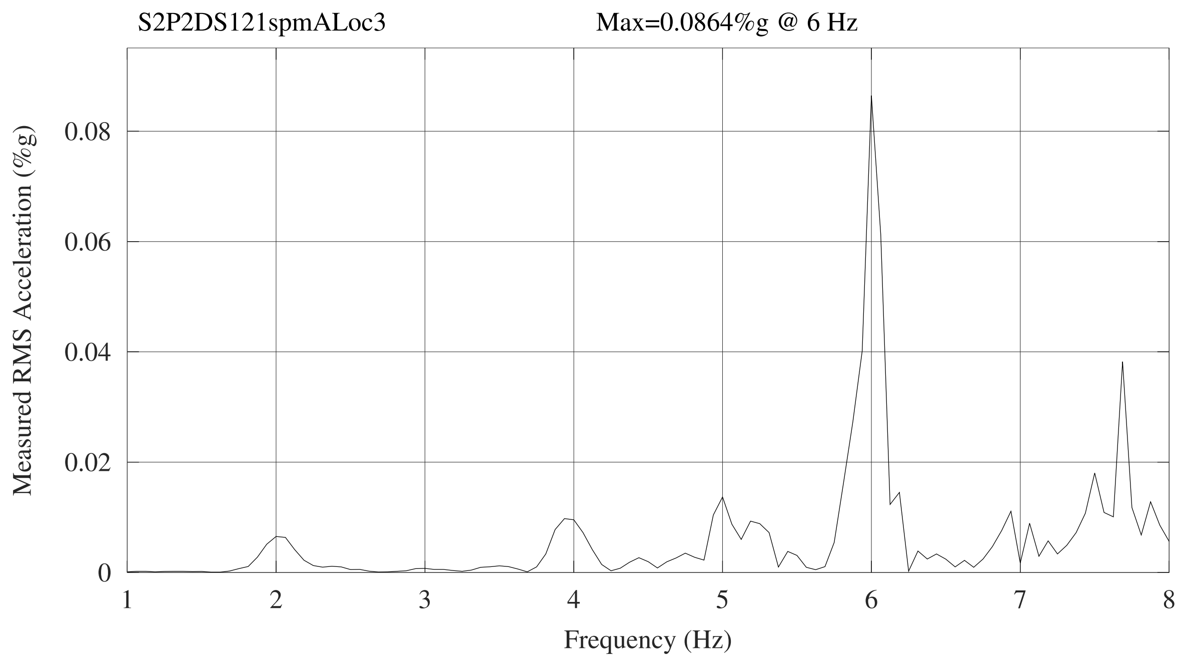

S2P2DS121spmALoc1
Max=0.0521%g @ 7 Hz
Measured RMS Acceleration (%g)
0.05
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P2DS121spmALoc2
ESPA=0.386%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

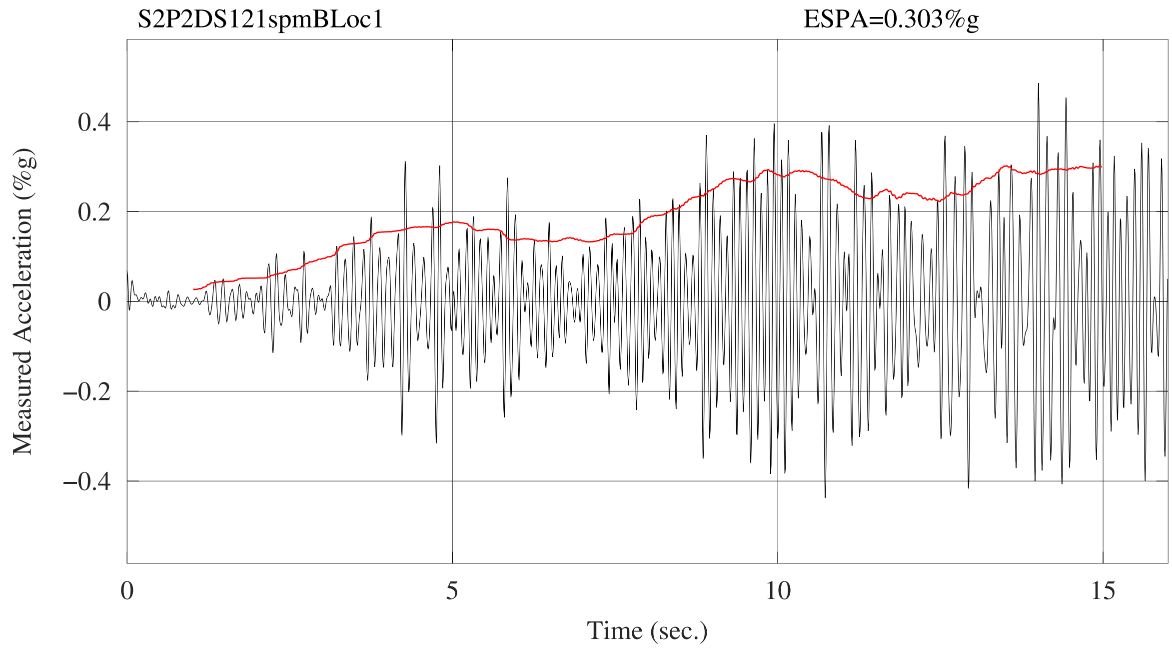

S2P2DS121spmALoc2
Max=0.0838%g @ 6 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

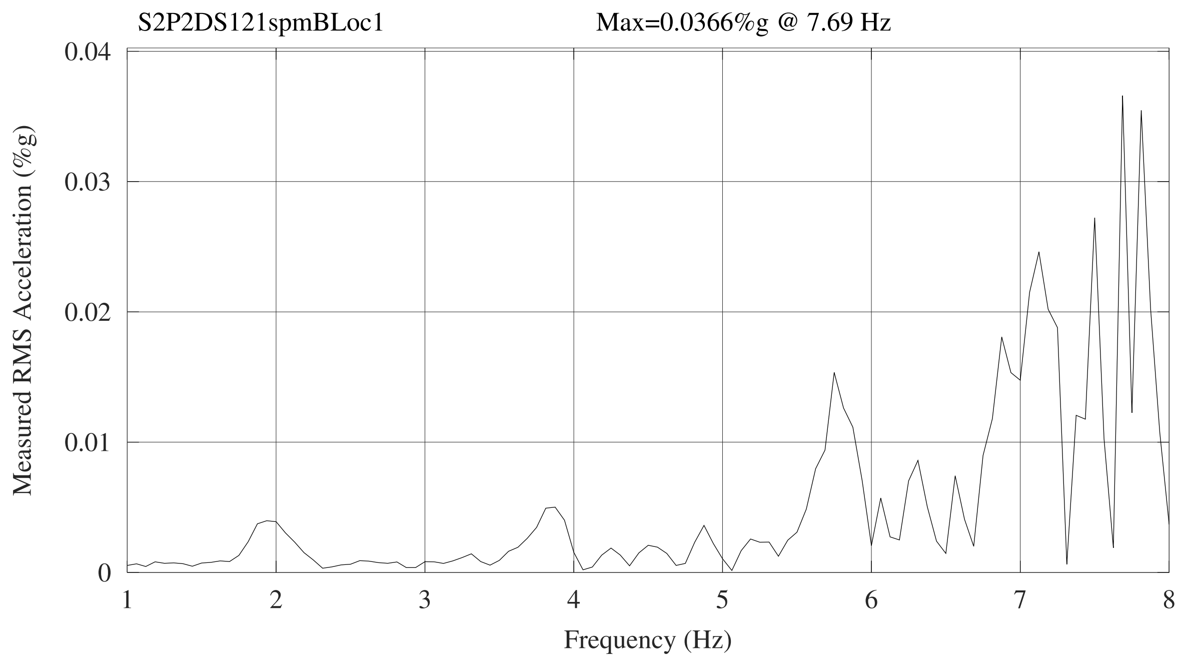


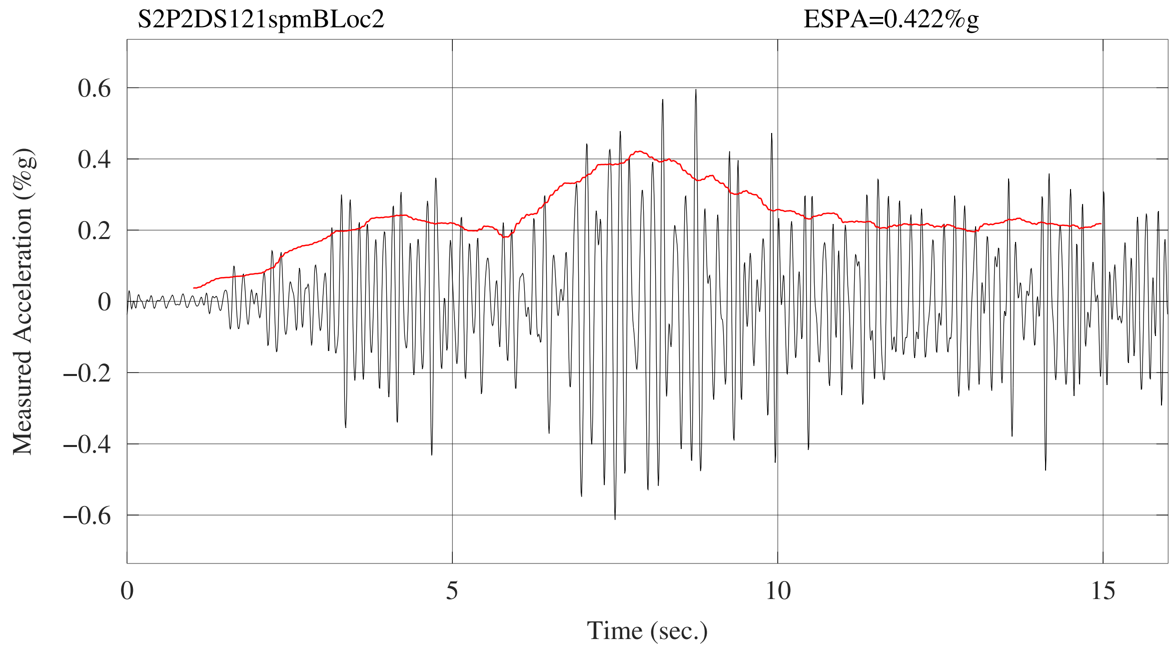
S2P2DS121spmALoc3
ESPA=0.381%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

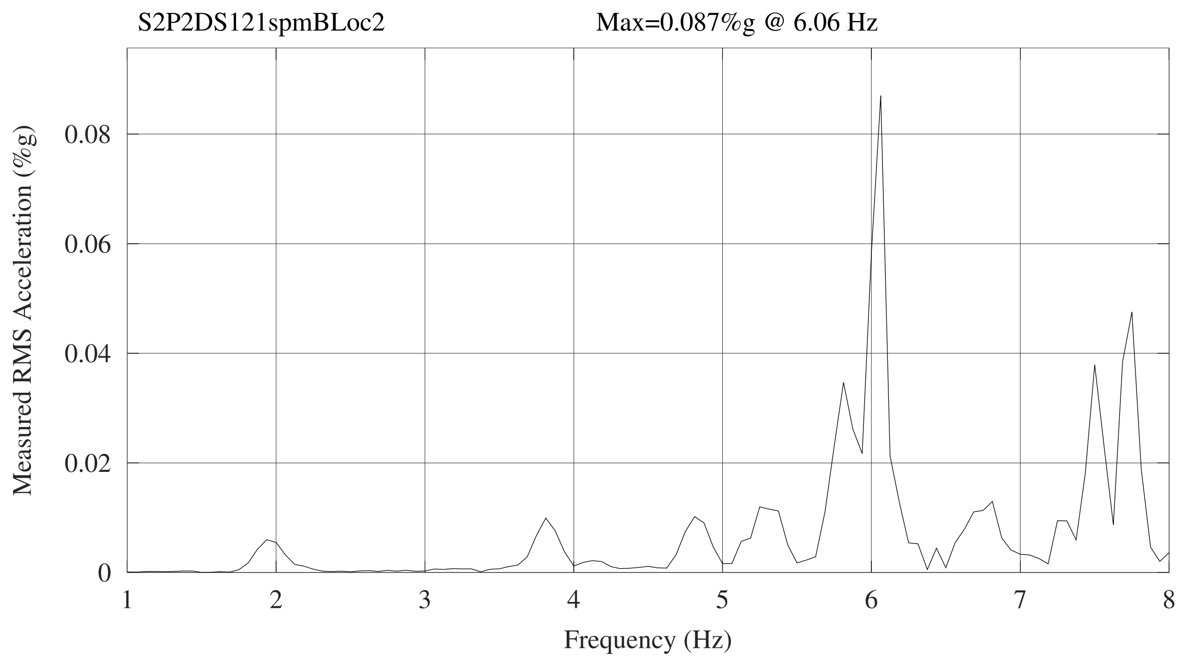


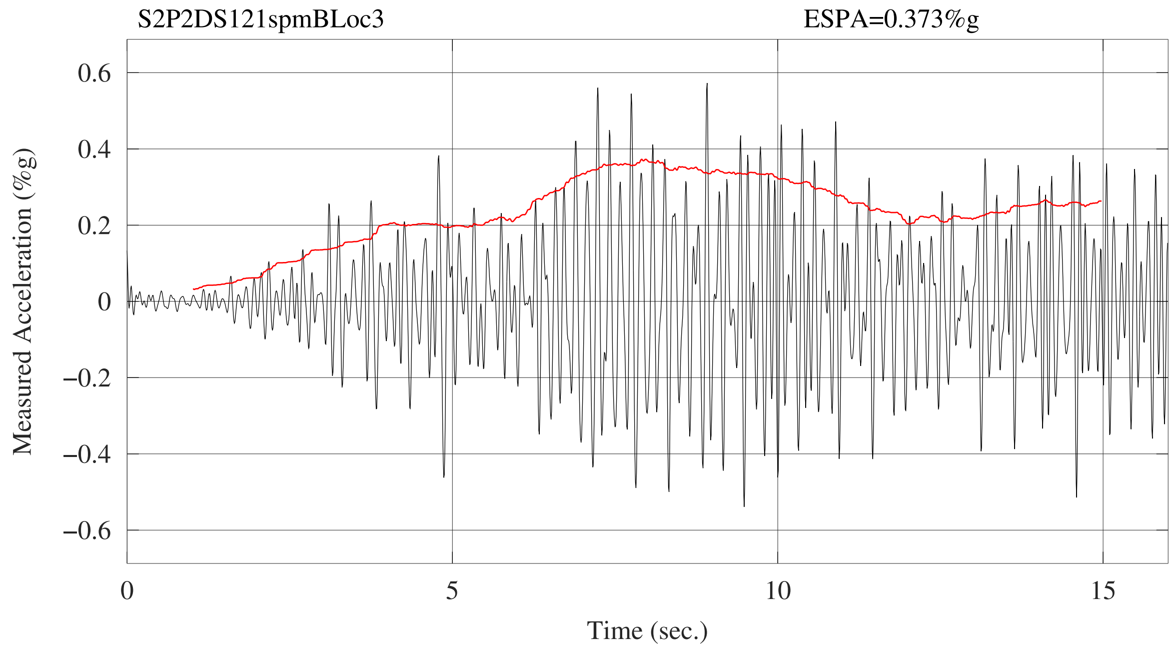
S2P2DS121spmALoc3
Max=0.0864%g @ 6 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

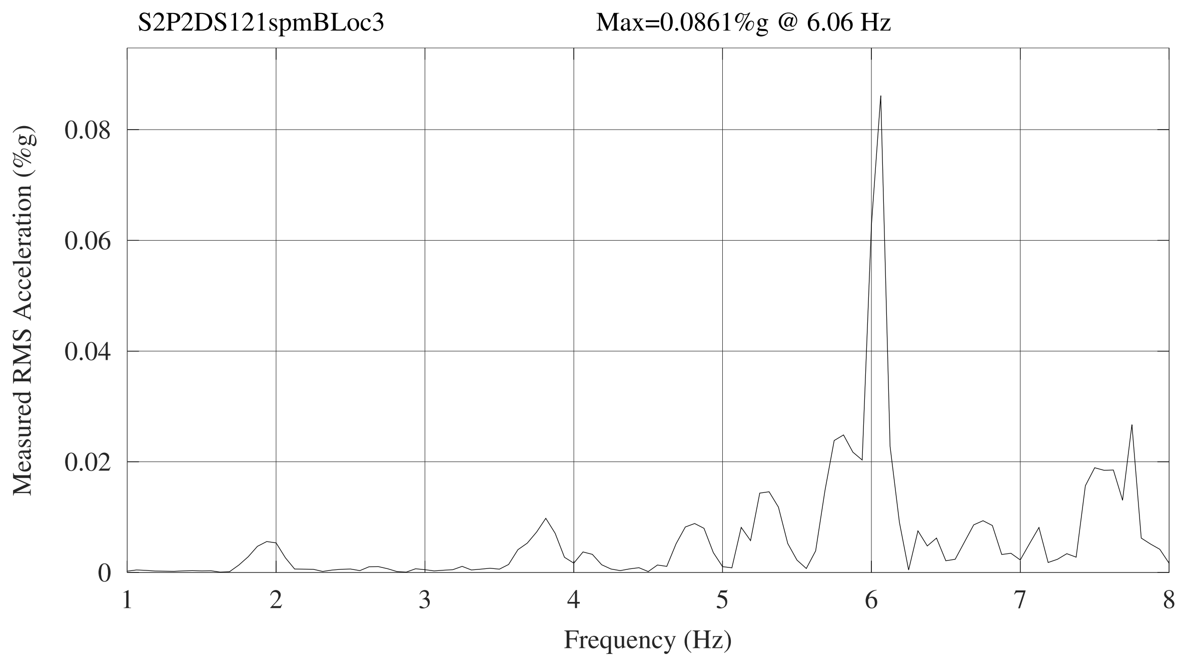

S2P2DS121spmBLoc1
ESPA=0.303%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

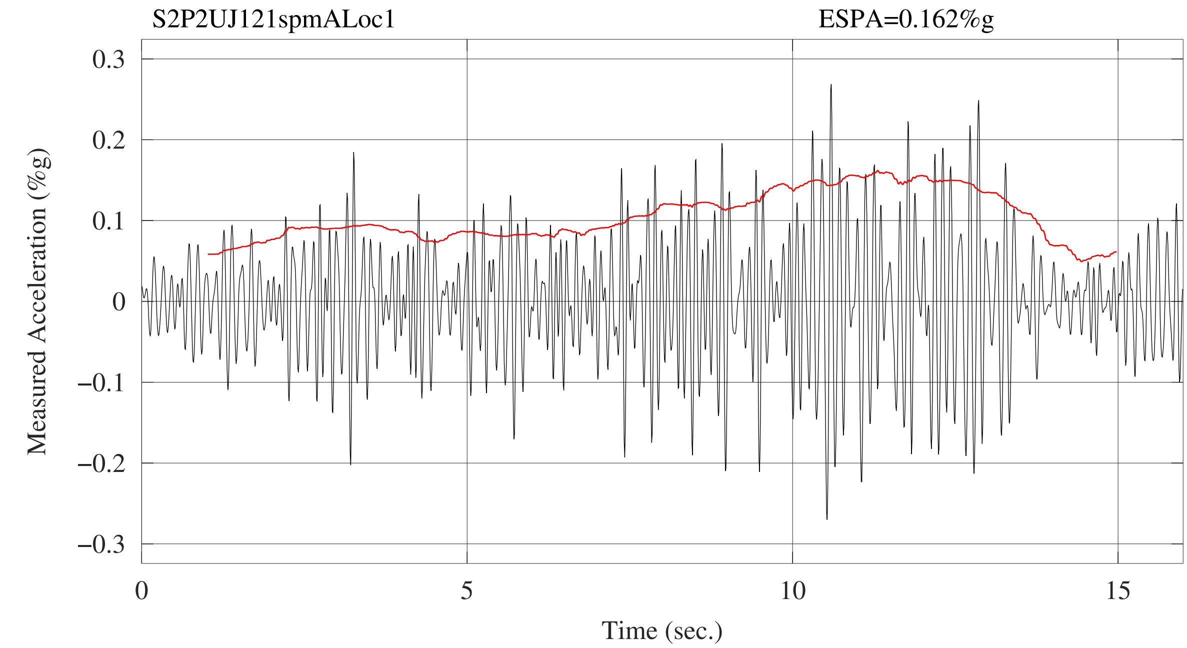

S2P2DS121spmBLoc1
Max=0.0366%g @ 7.69 Hz
Measured RMS Acceleration (%g)
0.04
0.03
0.02
0.01
0
1
2
3
4
5
6
7
8
Frequency (Hz)

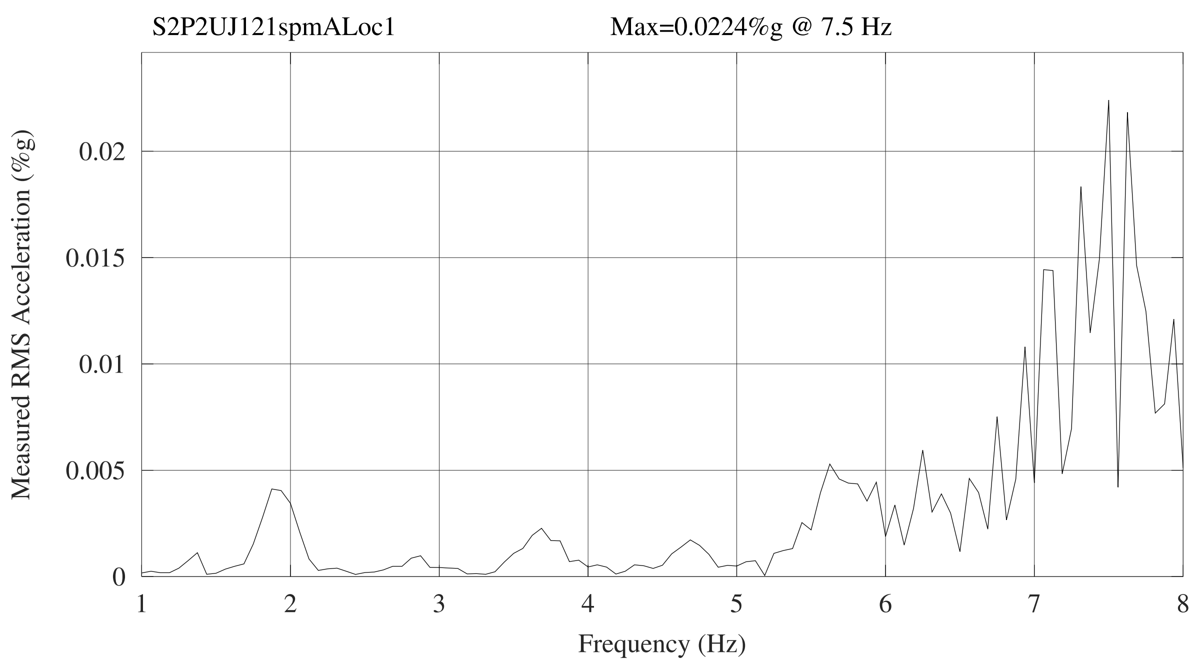

S2P2DS121spmBLoc2
ESPA=0.422%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

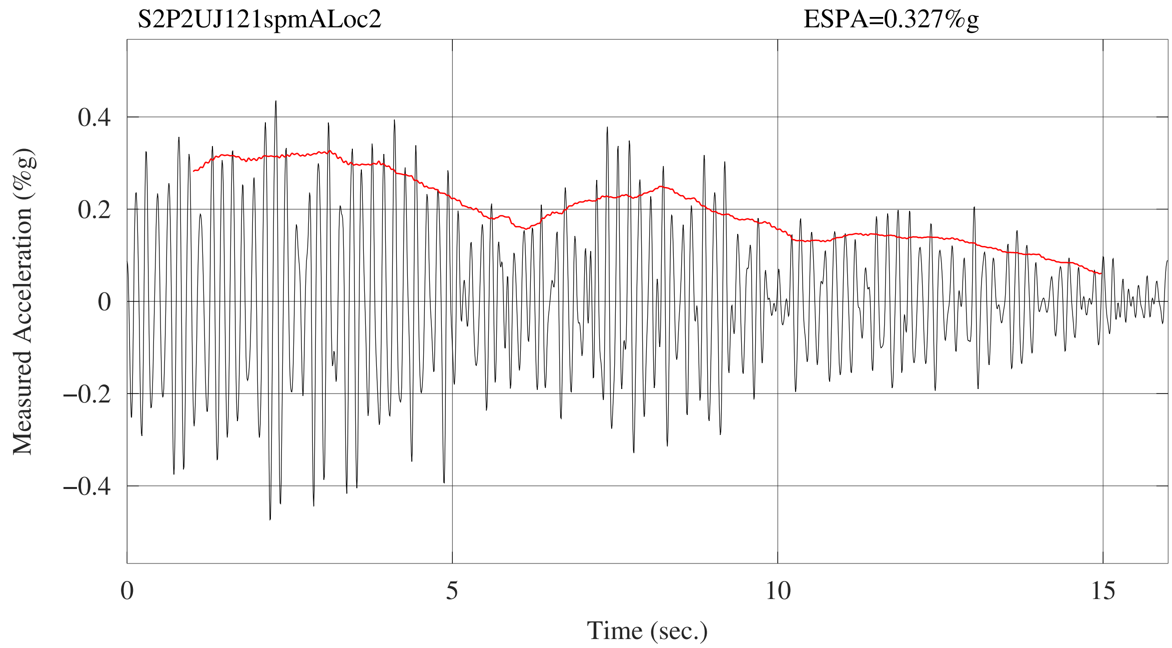

S2P2DS121spmBLoc2
Max=0.087%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

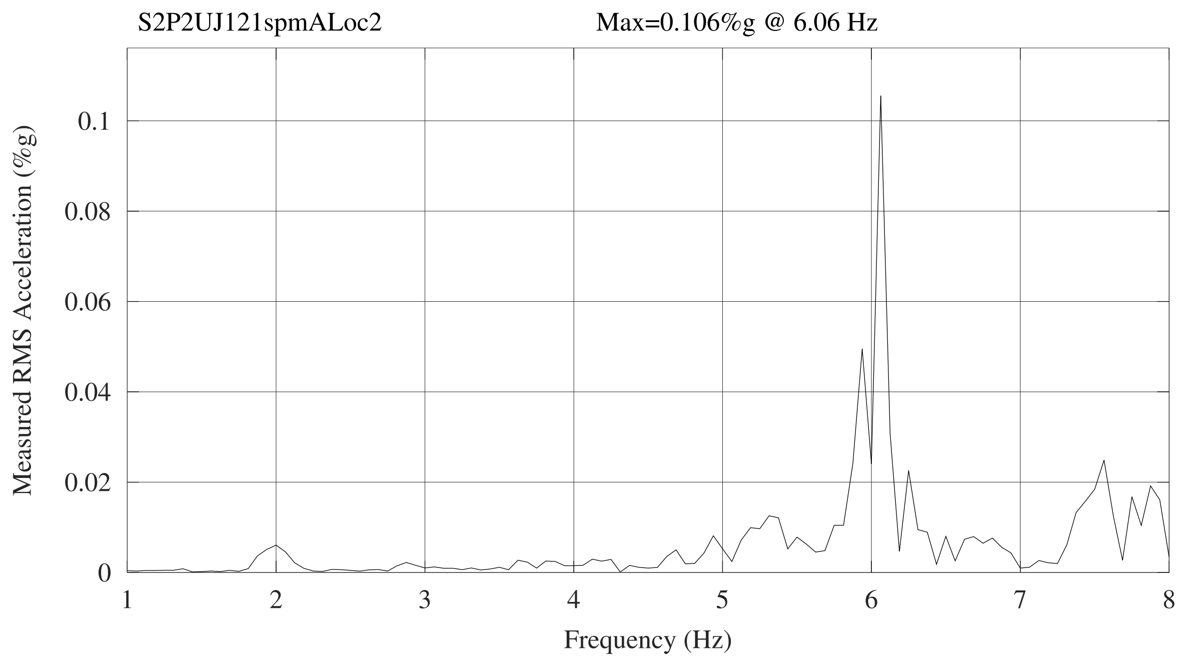

S2P2DS121spmBLoc3
ESPA=0.373%g
Measured Acceleration (%g)
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
5
10
15
Time (sec.)

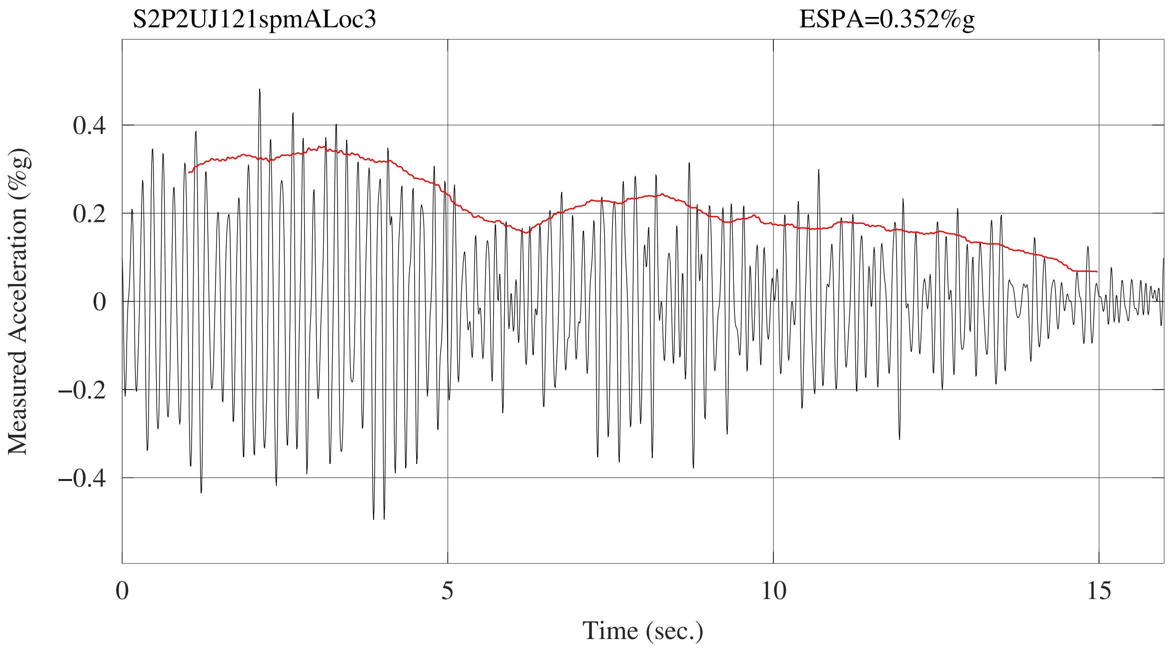

S2P2DS121spmBLoc3
Max=0.0861%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

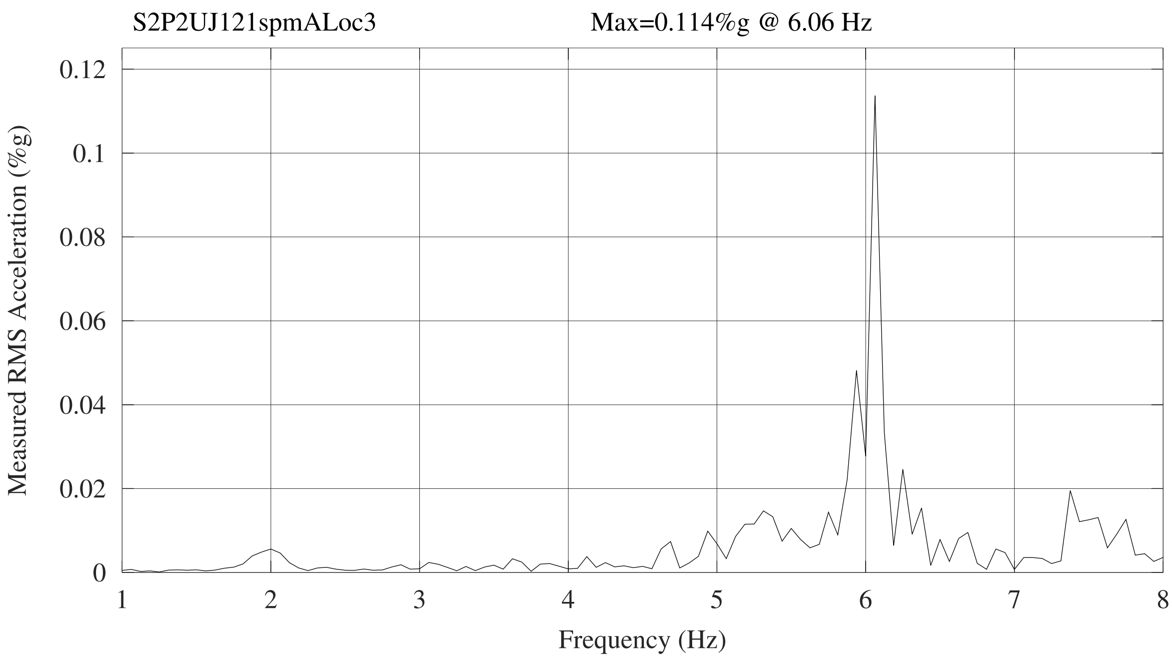

S2P2UJ121spmALoc1
ESPA=0.162%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

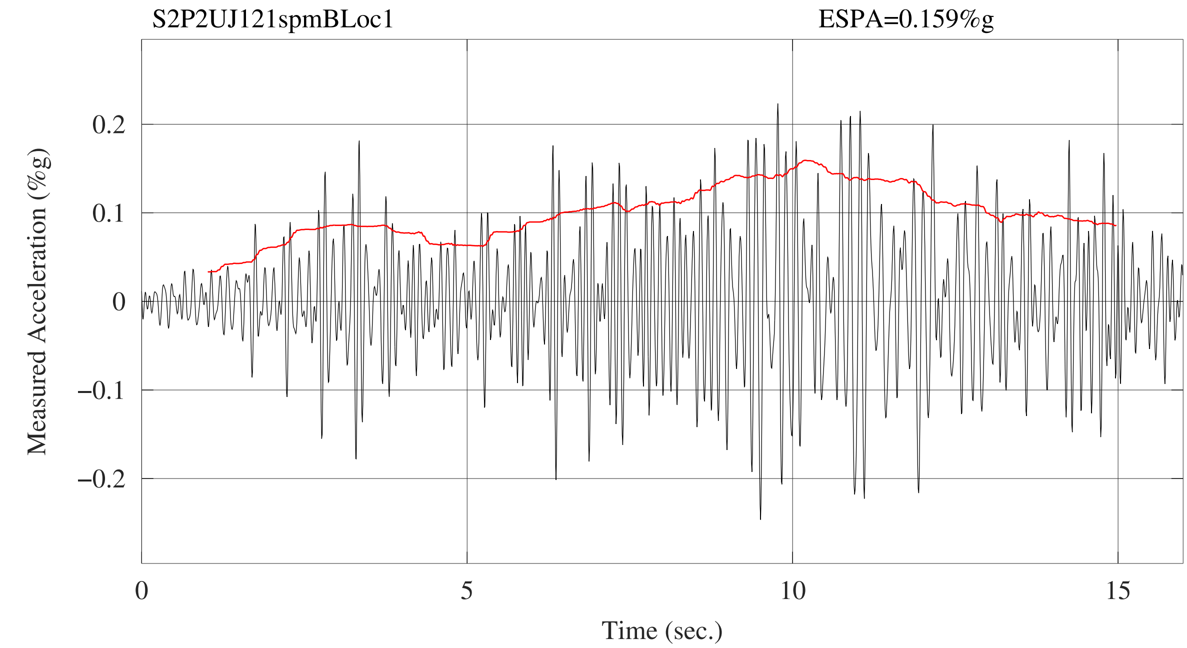

S2P2UJ121spmALoc1
Max=0.0224%g @ 7.5 Hz
Measured RMS Acceleration (%g)
0.02
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)

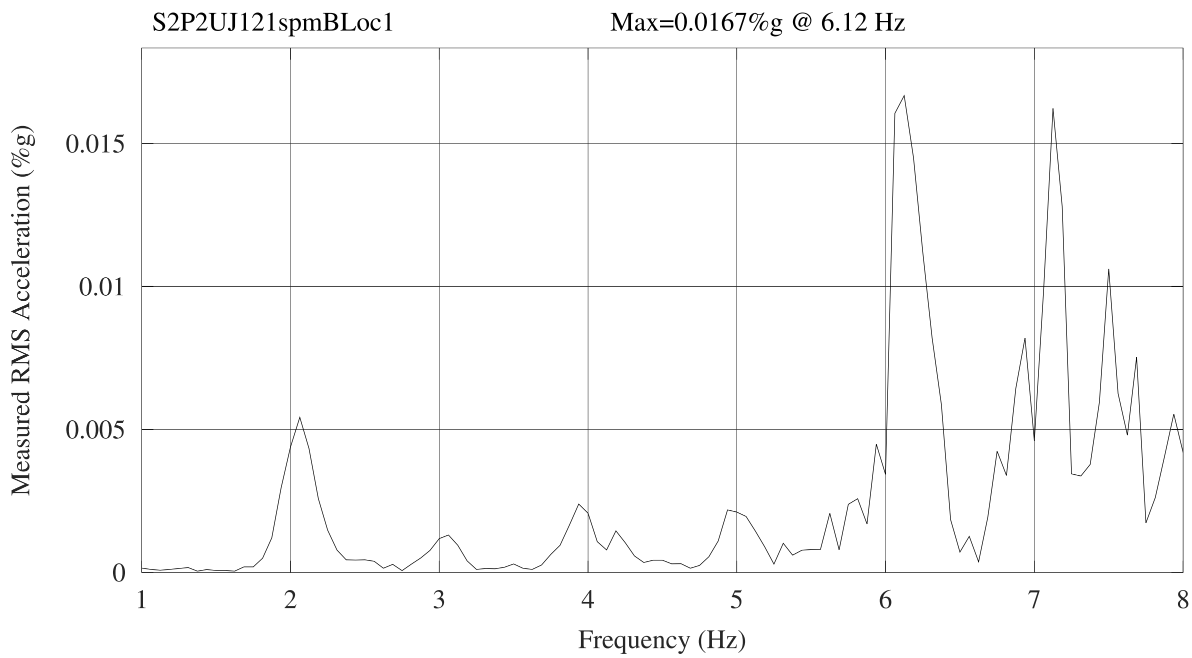

S2P2UJ121spmALoc2
ESPA=0.327%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

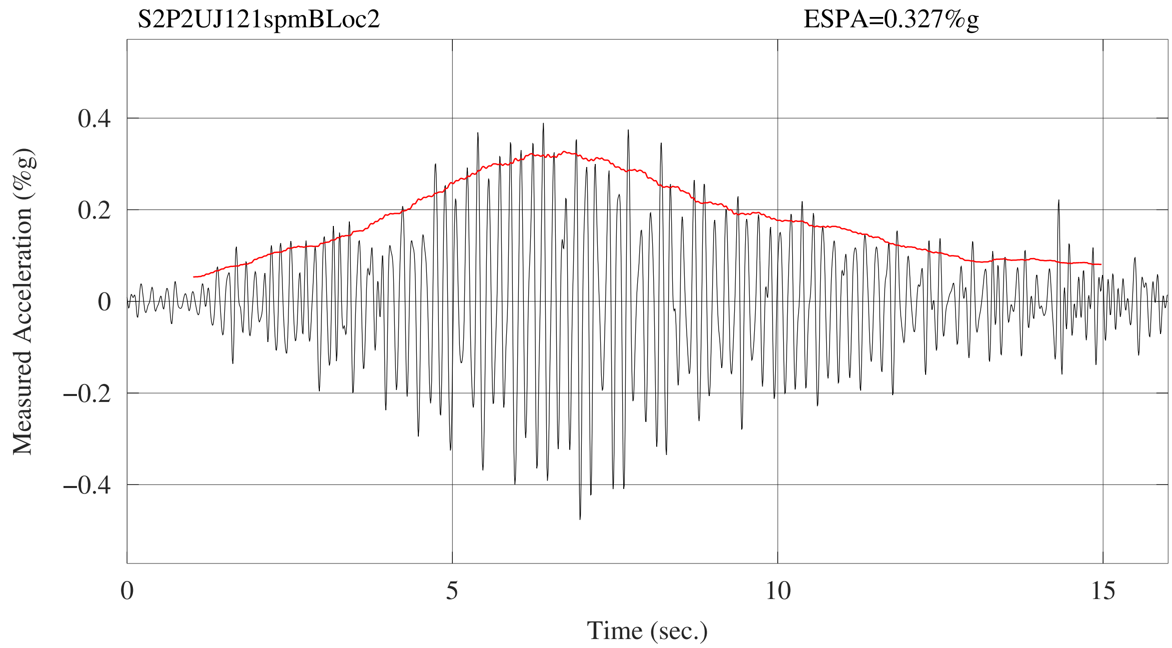

S2P2UJ121spmALoc2
Max=0.106%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

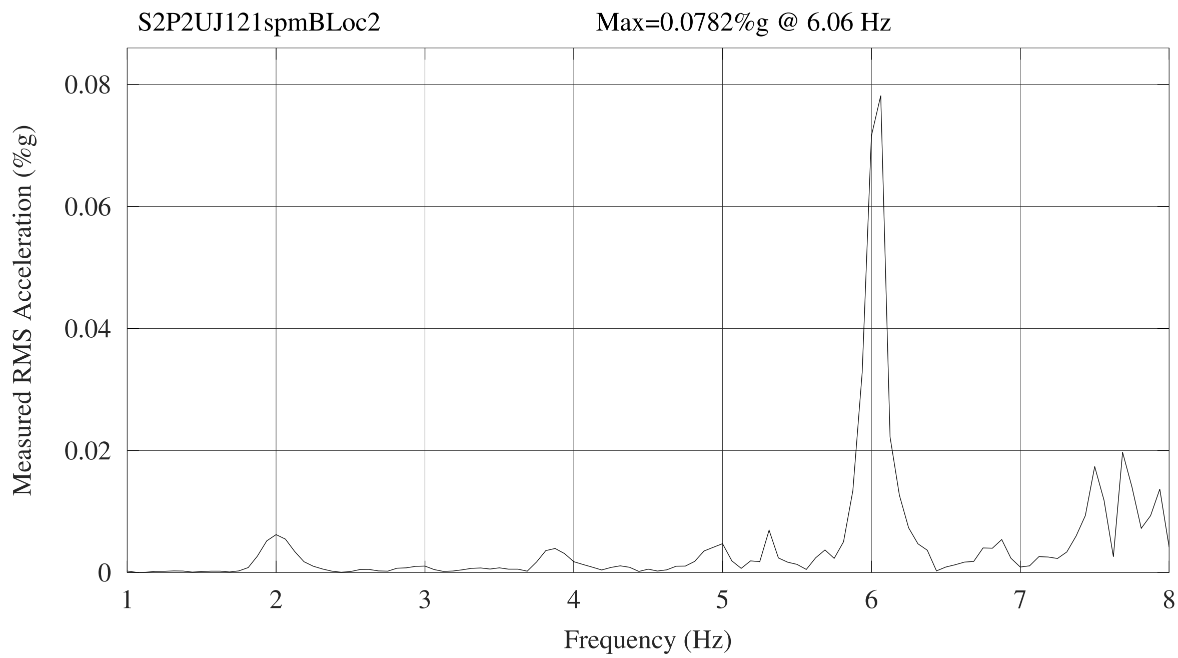

S2P2UJ121spmALoc3
ESPA=0.352%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

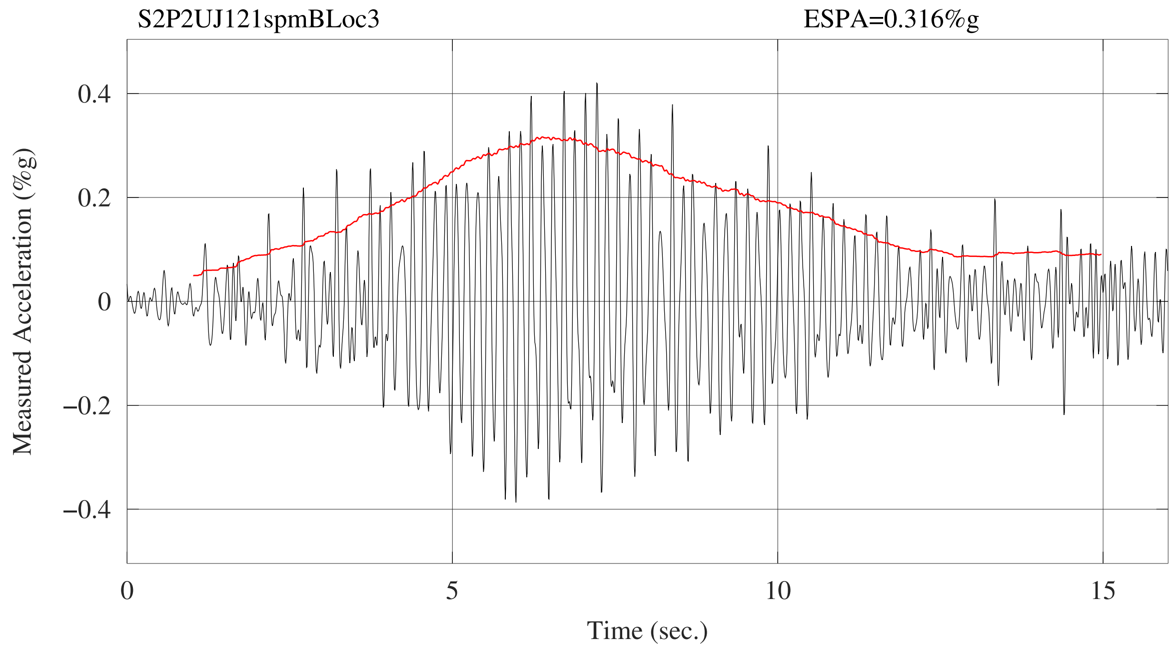

S2P2UJ121spmALoc3
Max=0.114%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

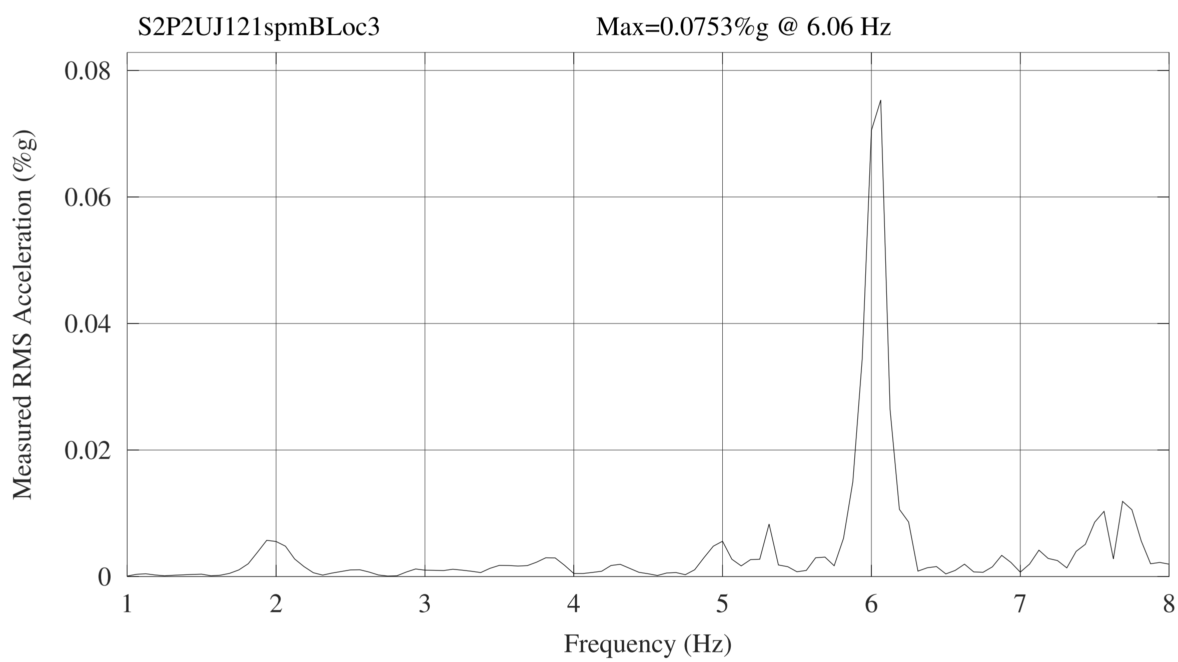

S2P2UJ121spmBLoc1
ESPA=0.159%g
Measured Acceleration (%g)
0.2
0.1
0
−0.1
−0.2
0
5
10
15
Time (sec.)

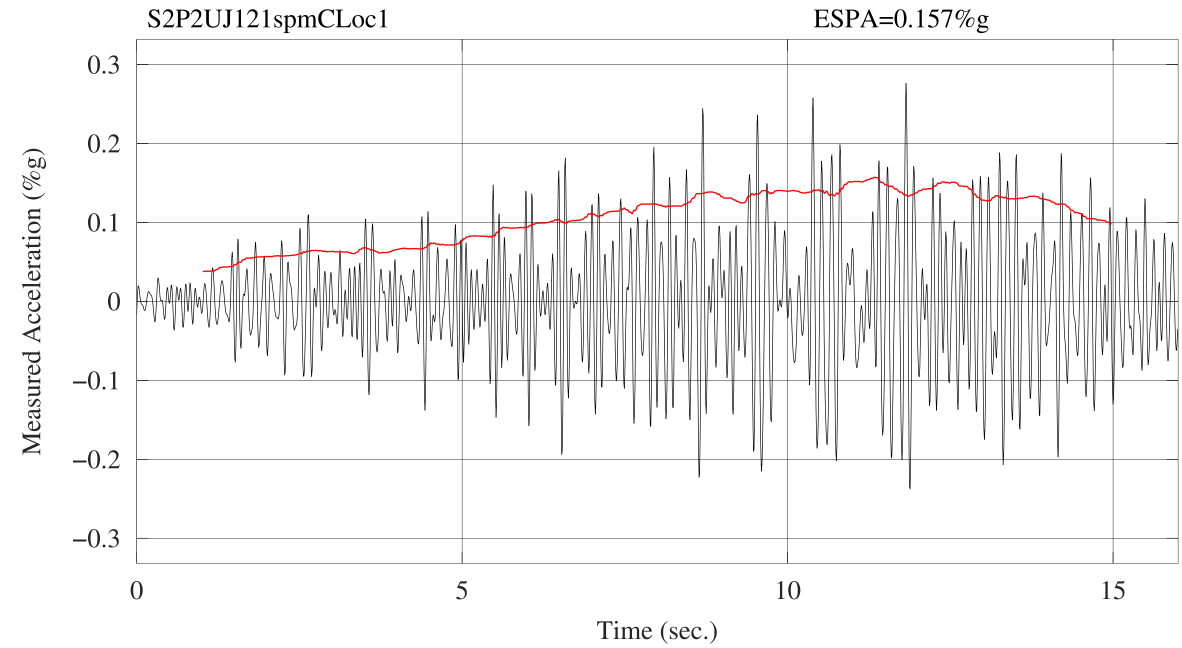

S2P2UJ121spmBLoc1
Max=0.0167%g @ 6.12 Hz
Measured RMS Acceleration (%g)
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P2UJ121spmBLoc2
ESPA=0.327%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

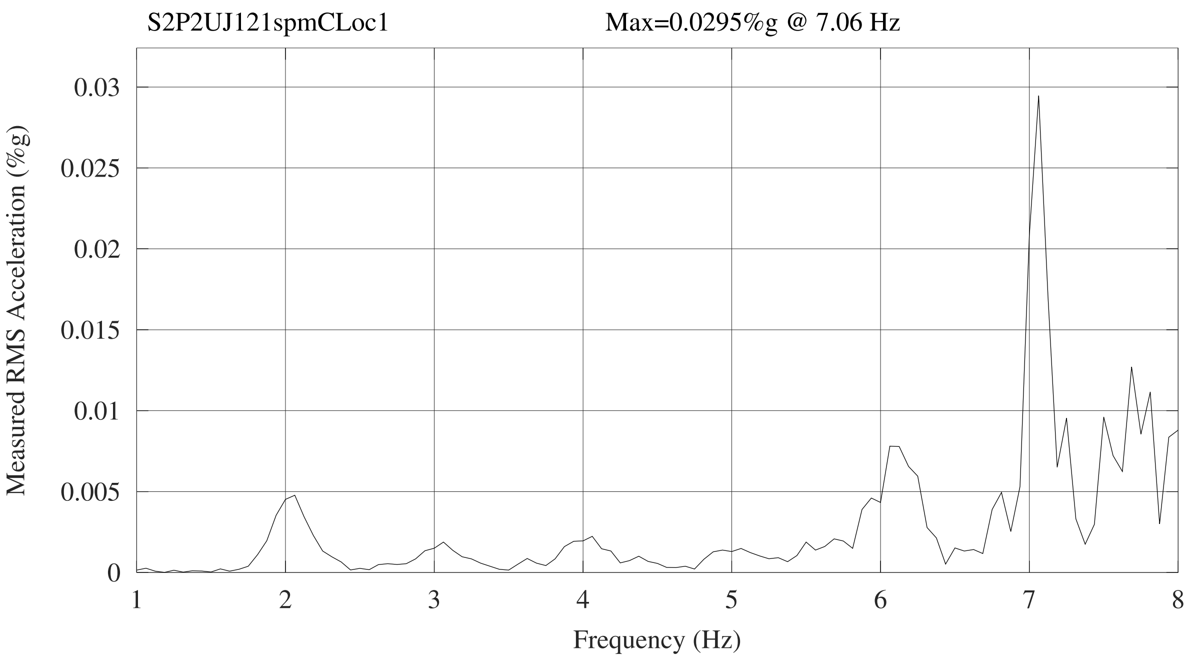

S2P2UJ121spmBLoc2
Max=0.0782%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)


S2P2UJ121spmBLoc3
ESPA=0.316%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

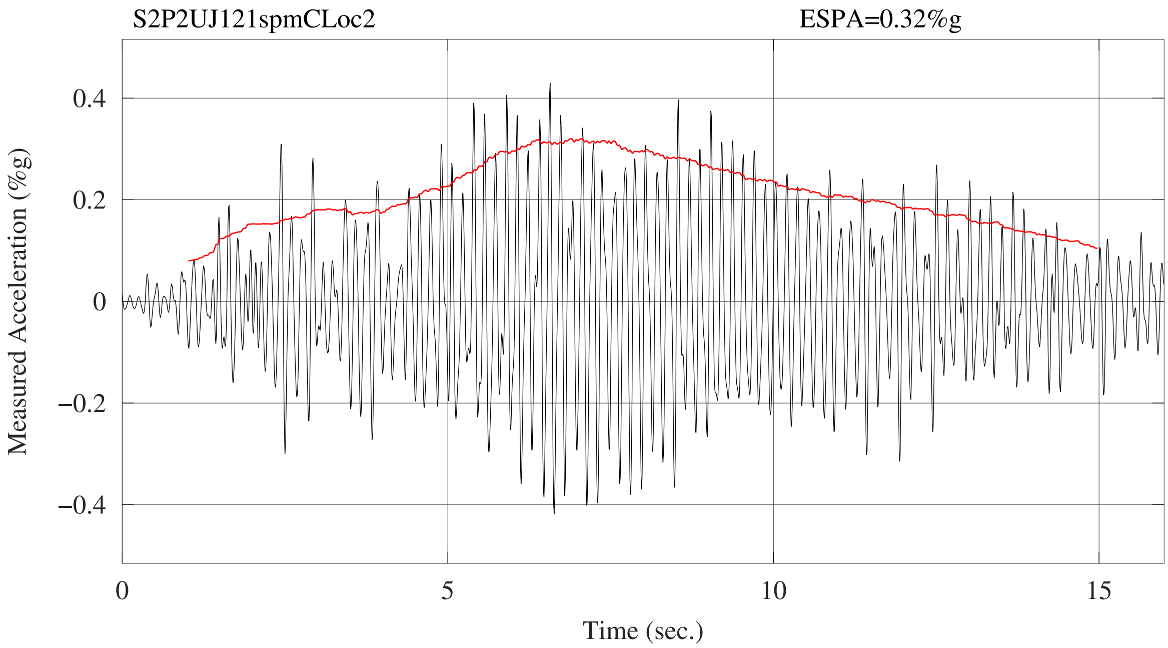

S2P2UJ121spmBLoc3
Max=0.0753%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

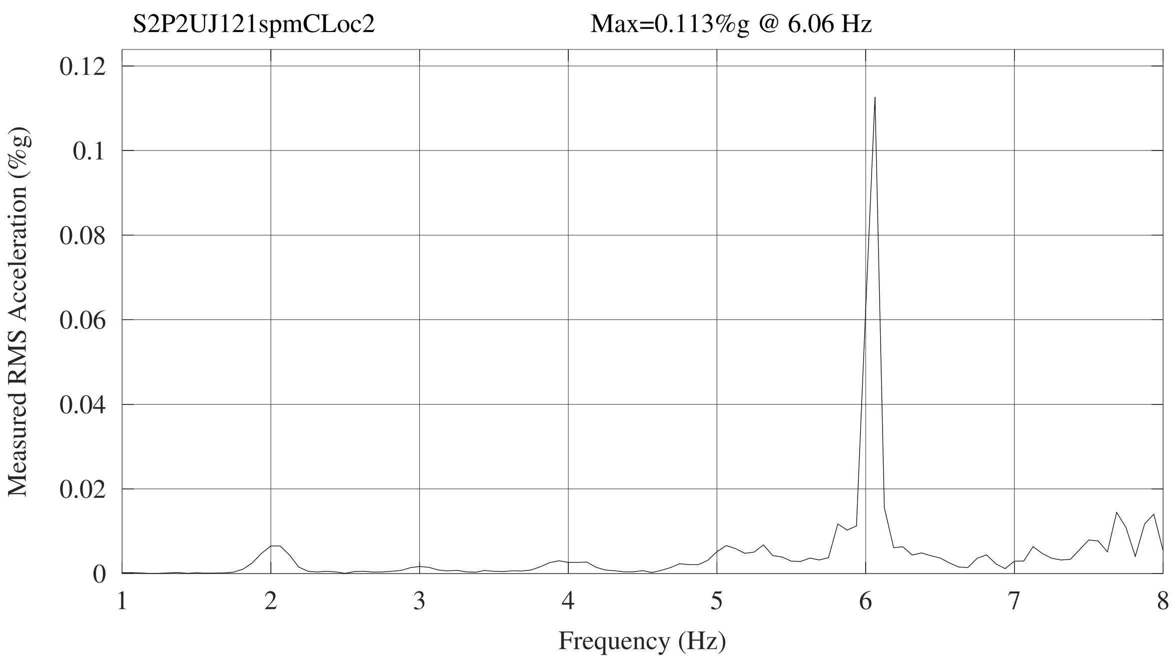


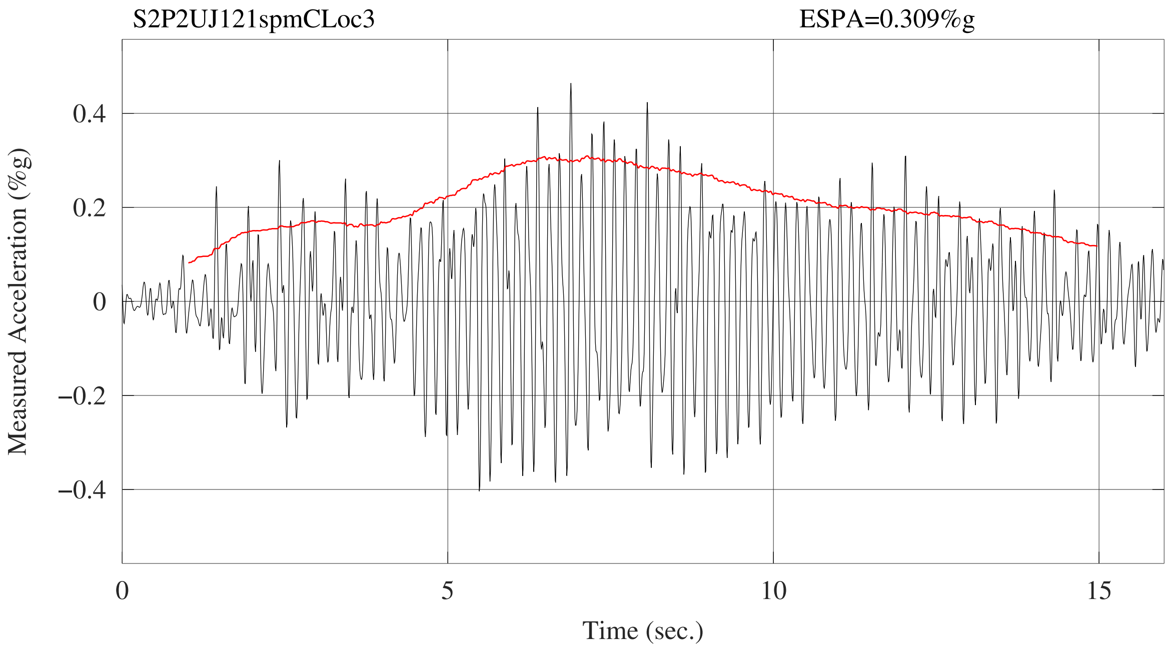
S2P2UJ121spmCLoc1
ESPA=0.157%g
Measured Acceleration (%g)
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
0
5
10
15
Time (sec.)

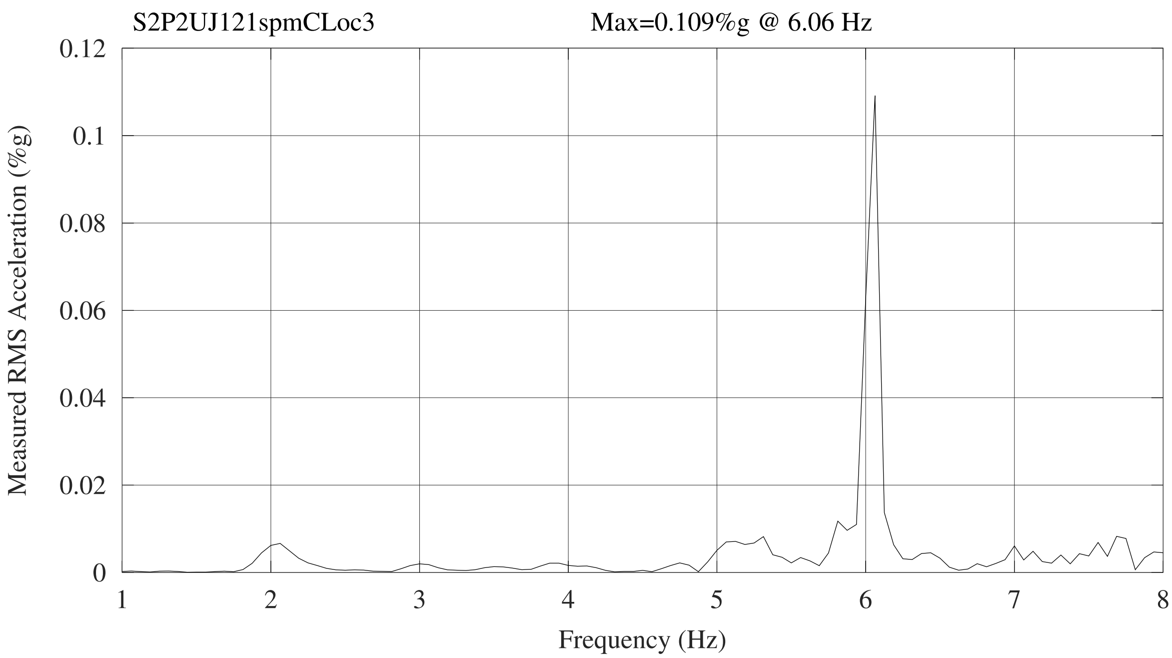


S2P2UJ121spmCLoc1
Max=0.0295%g @ 7.06 Hz
Measured RMS Acceleration (%g)
0.03
0.025
0.02
0.015
0.01
0.005
0
1
2
3
4
5
6
7
8
Frequency (Hz)

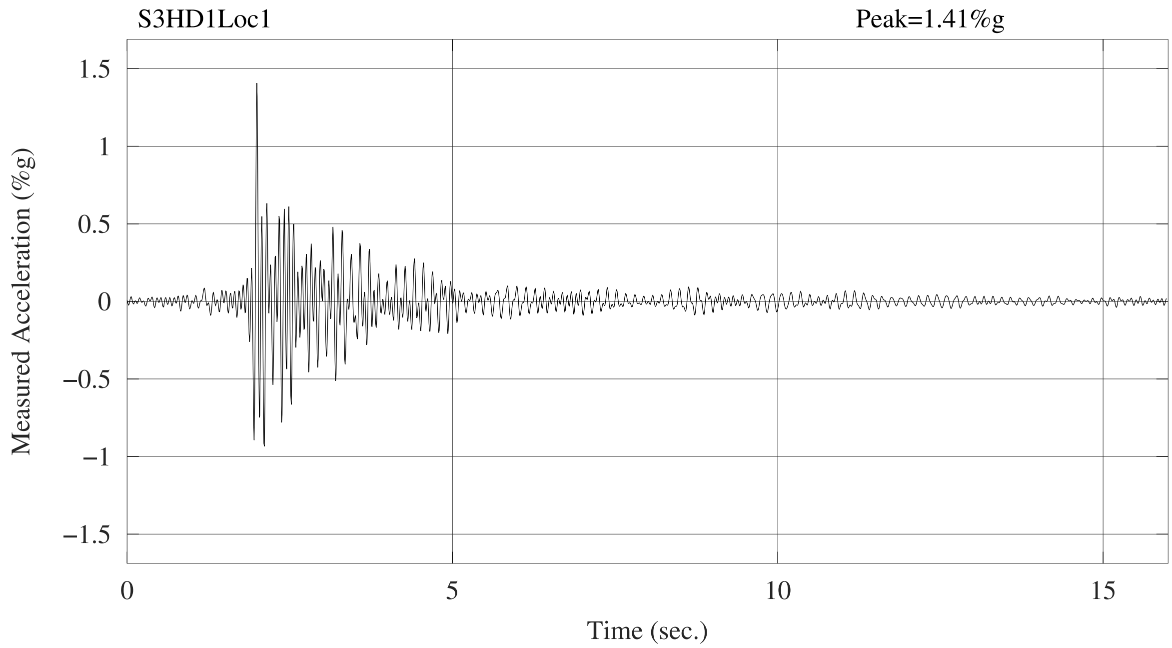

S2P2UJ121spmCLoc2
ESPA=0.32%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

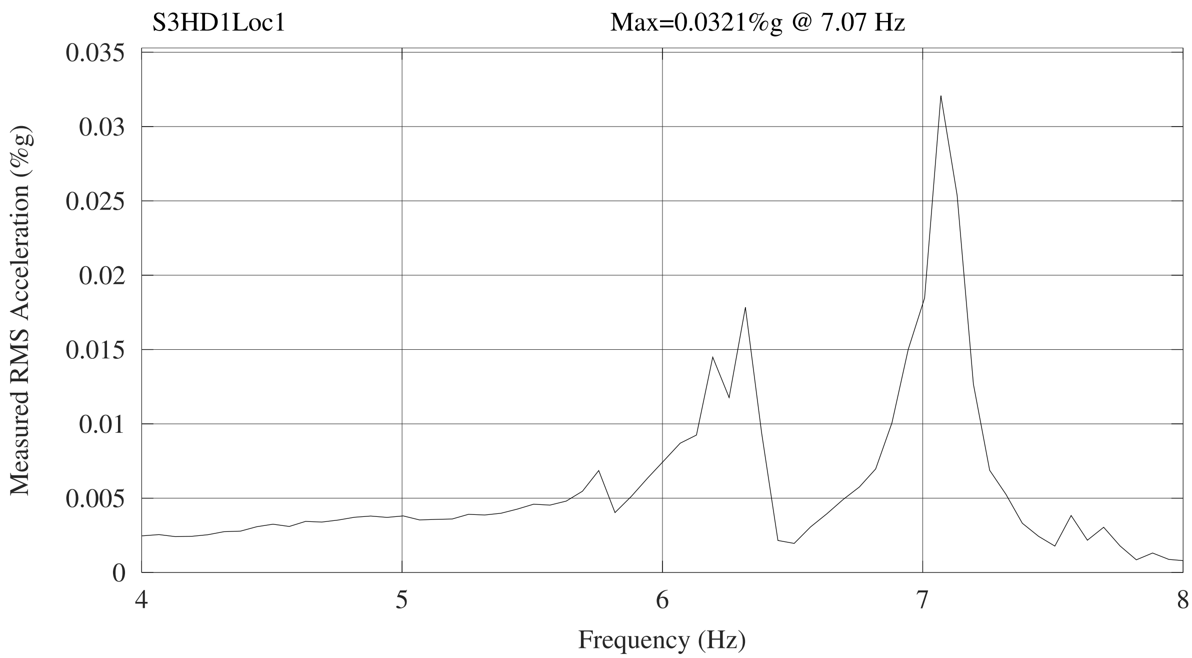

S2P2UJ121spmCLoc2
Max=0.113%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

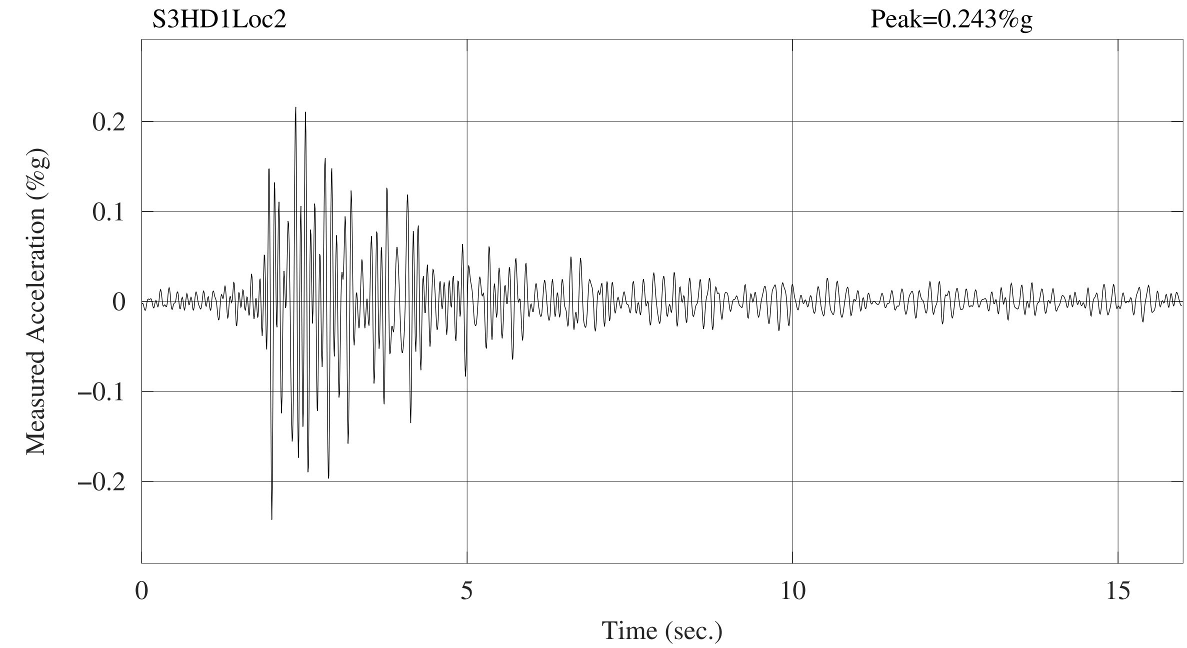

S2P2UJ121spmCLoc3
ESPA=0.309%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

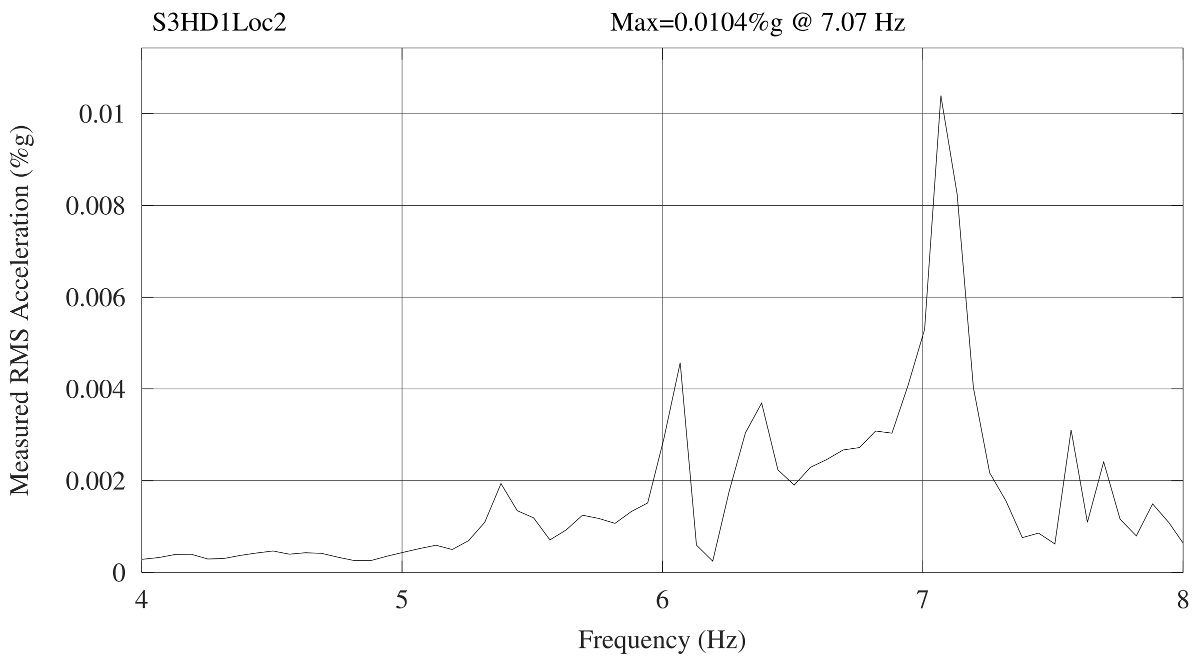

S2P2UJ121spmCLoc3
Max=0.109%g @ 6.06 Hz
Measured RMS Acceleration (%g)
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
Frequency (Hz)

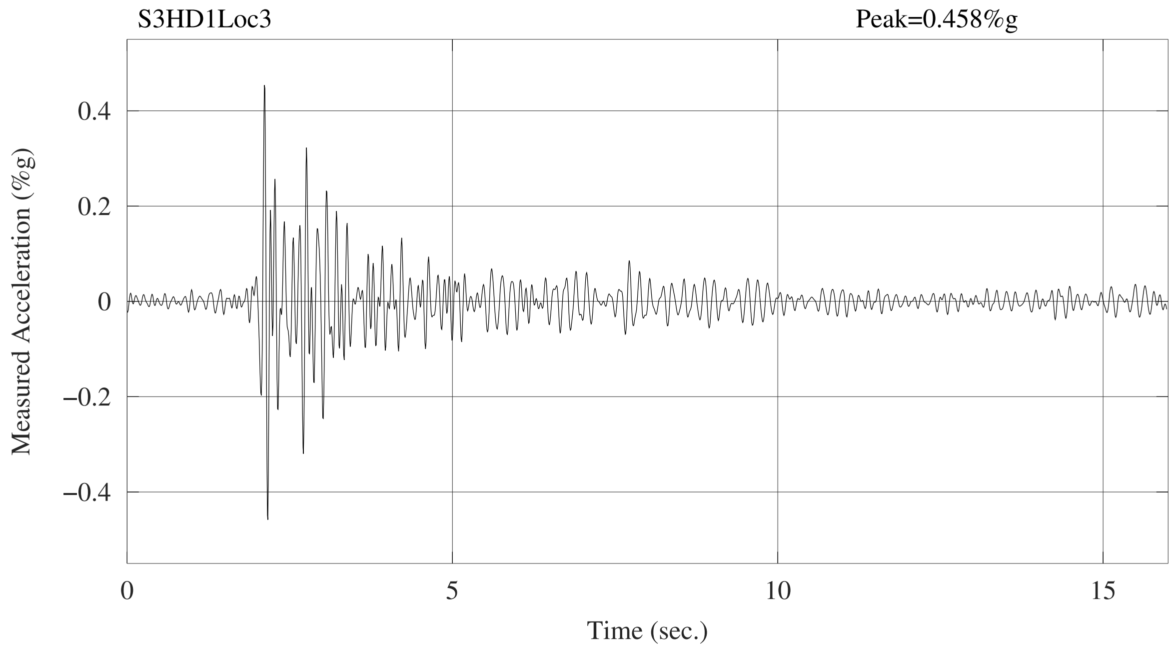
Appendix E - Heel-Drop Test Results for Setup 3

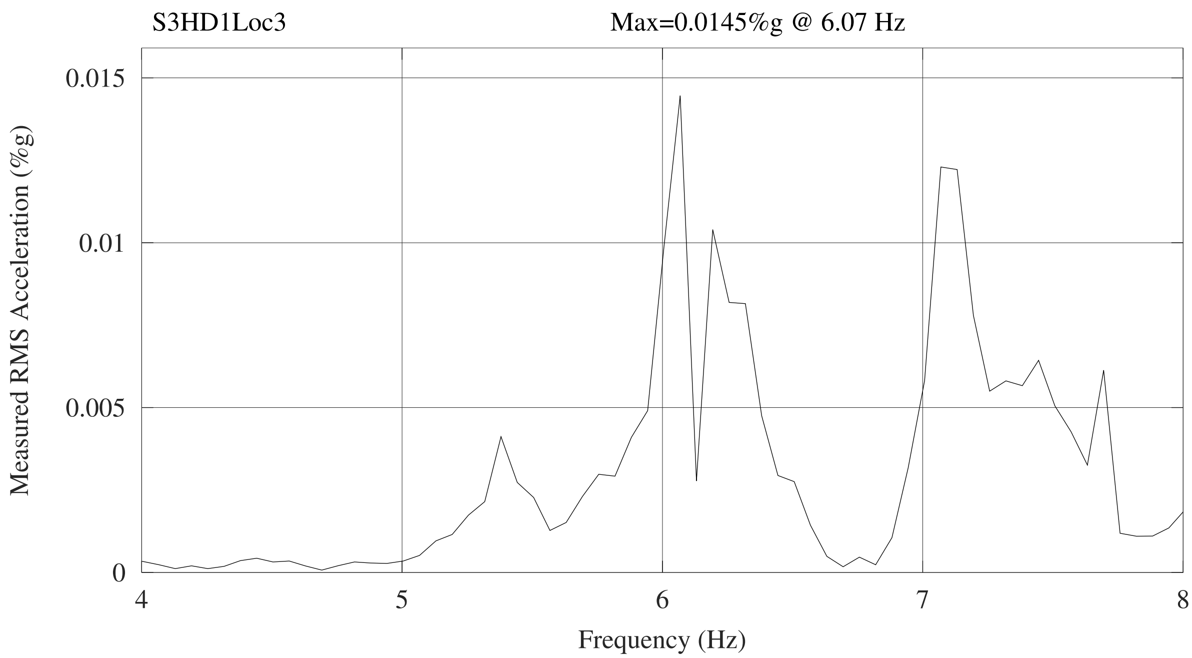


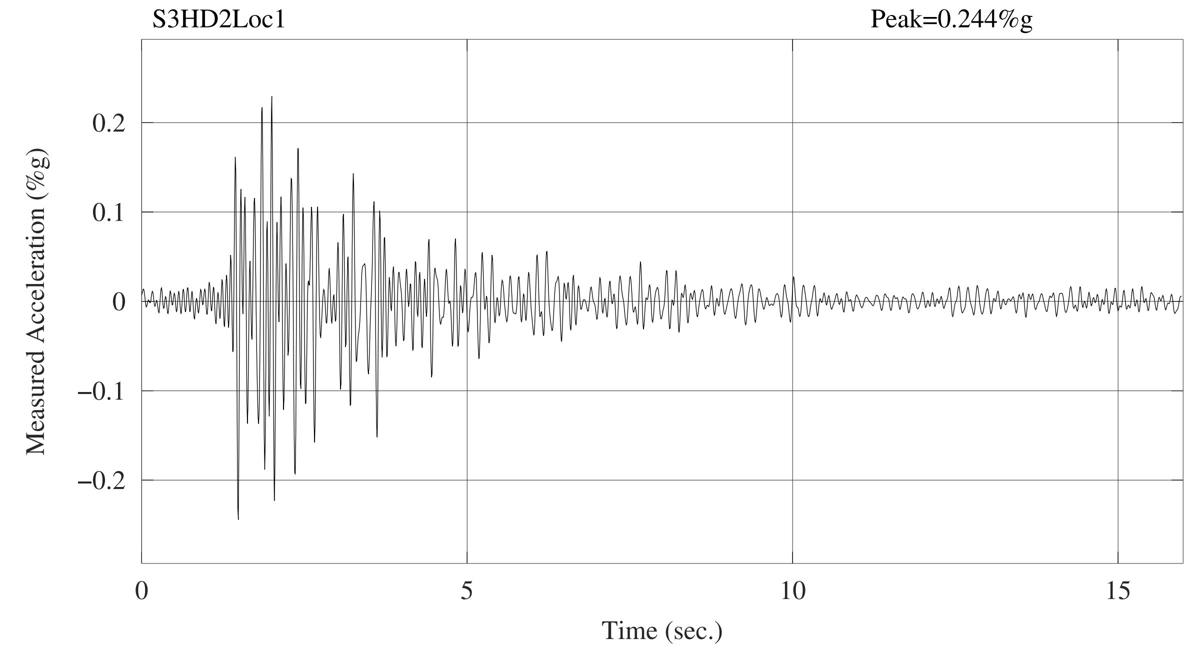


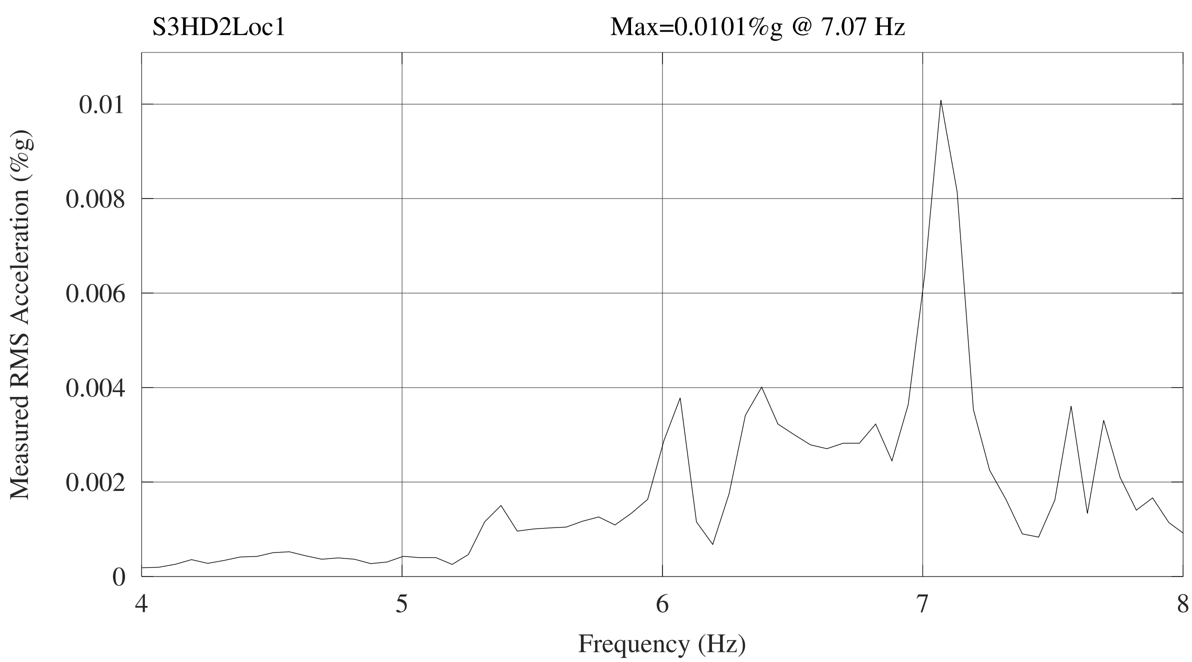

S3HD1Loc2
Peak=0.243%g
Measured Acceleration (%g)
0.2
0.1
0
−0.1
−0.2
0
5
10
15
Time (sec.)

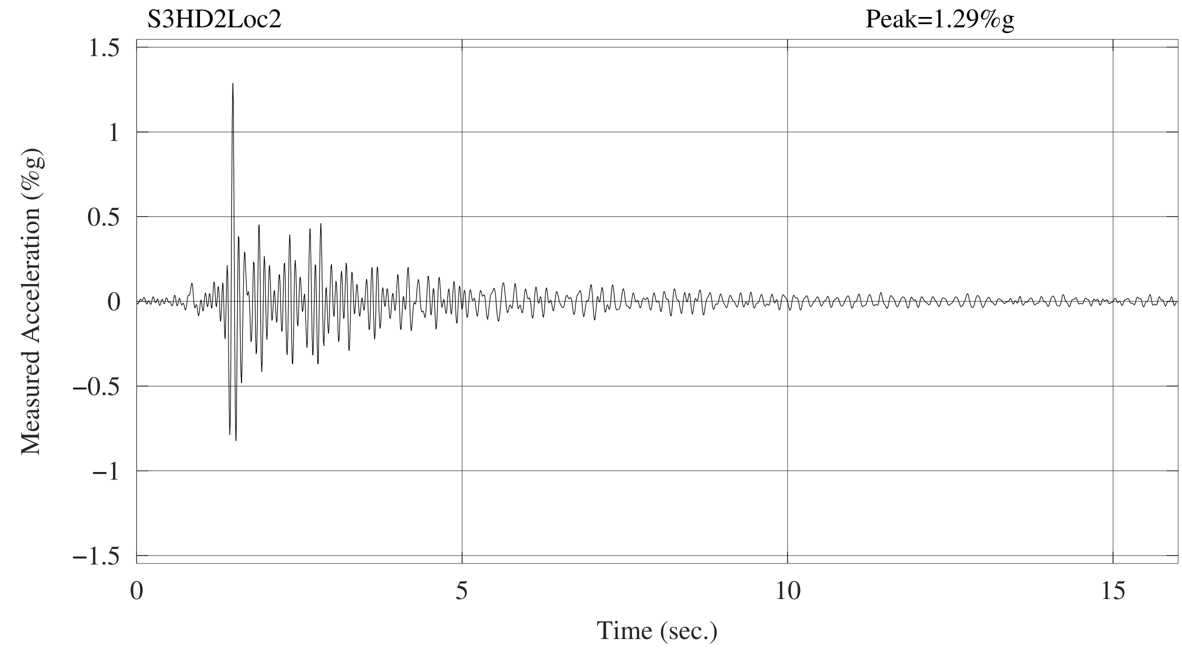

S3HD1Loc2
Max=0.0104%g @ 7.07 Hz
Measured RMS Acceleration (%g)
0.01
0.008
0.006
0.004
0.002
0
4
5
6
7
8
Frequency (Hz)

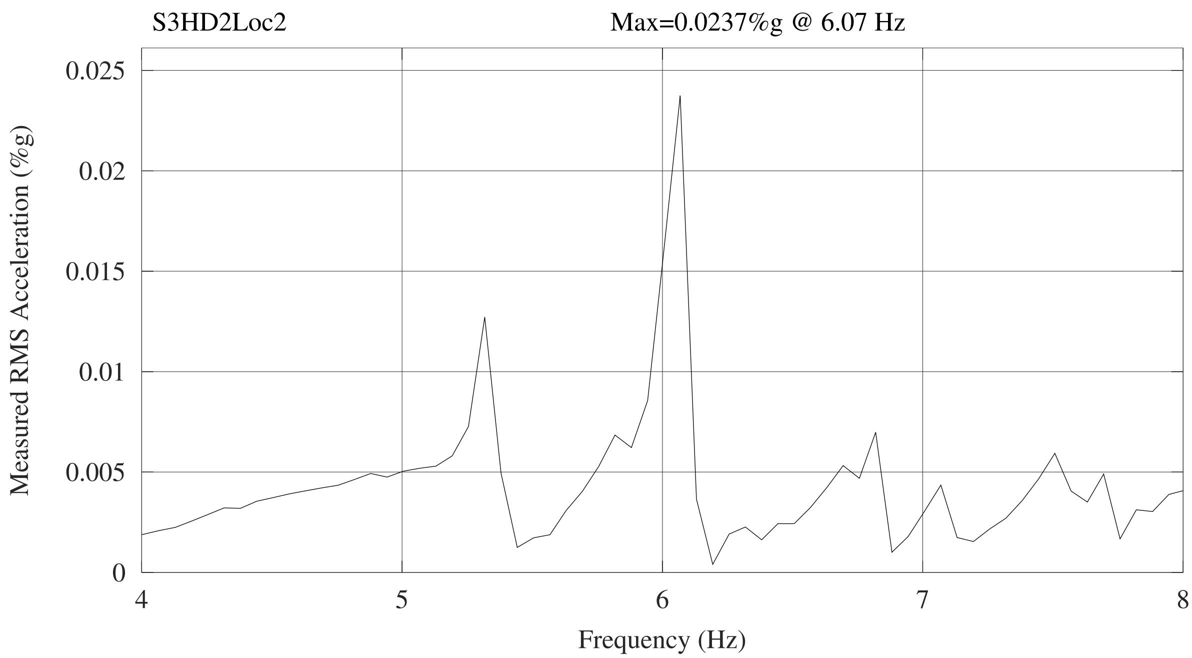

S3HD1Loc3
Peak=0.458%g
Measured Acceleration (%g)
0.4
0.2
0
−0.2
−0.4
0
5
10
15
Time (sec.)

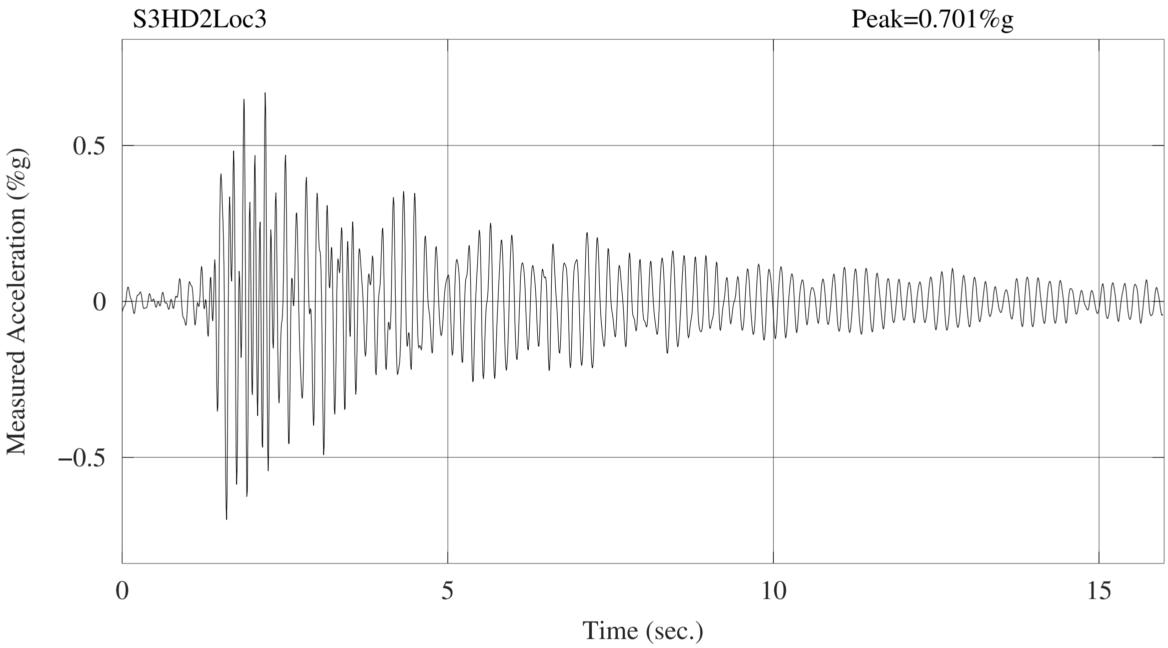

S3HD1Loc3
Max=0.0145%g @ 6.07 Hz
Measured RMS Acceleration (%g)
0.015
0.01
0.005
0
4
5
6
7
8
Frequency (Hz)

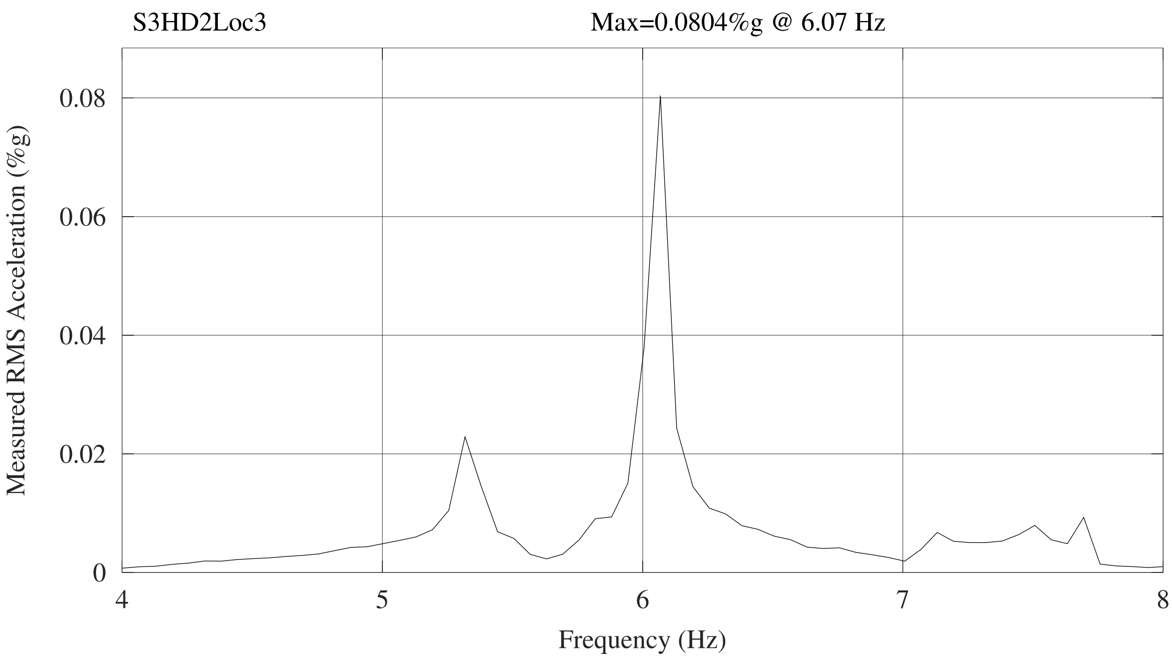

S3HD2Loc1
Peak=0.244%g
Measured Acceleration (%g)
0.2
0.1
0
−0.1
−0.2
0
5
10
15
Time (sec.)


S3HD2Loc1
Max=0.0101%g @ 7.07 Hz
Measured RMS Acceleration (%g)
0.01
0.008
0.006
0.004
0.002
0
4
5
6
7
8
Frequency (Hz)

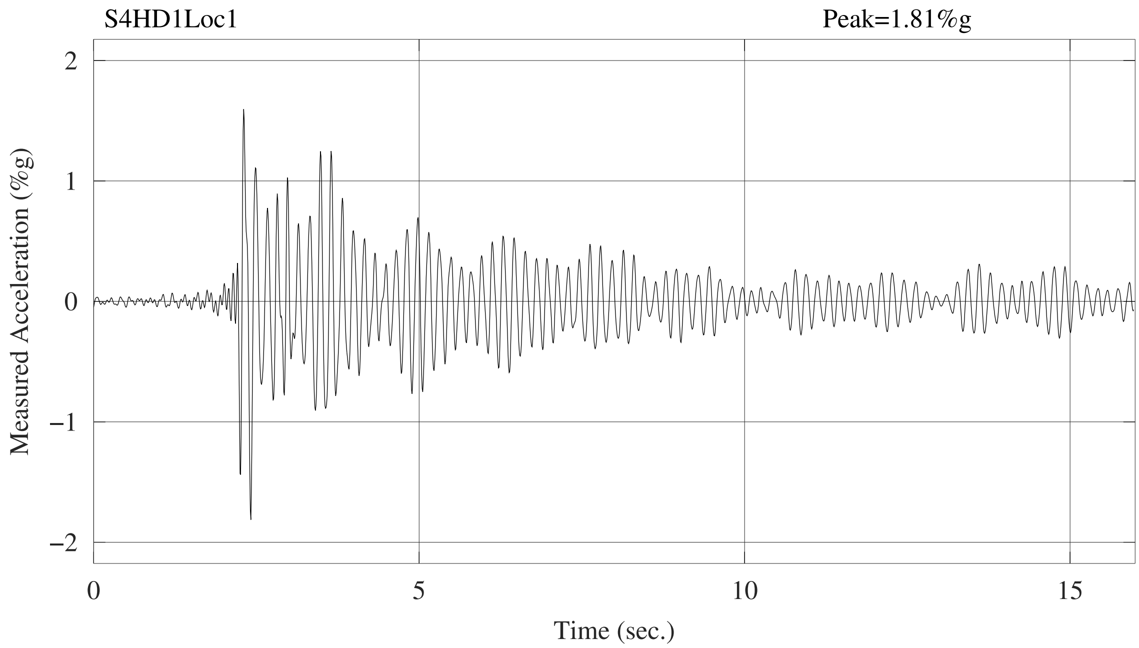

S3HD2Loc2
Peak=1.29%g
Measured Acceleration (%g)
1.5
1
0.5
0
−0.5
−1
−1.5
0
5
10
15
Time (sec.)

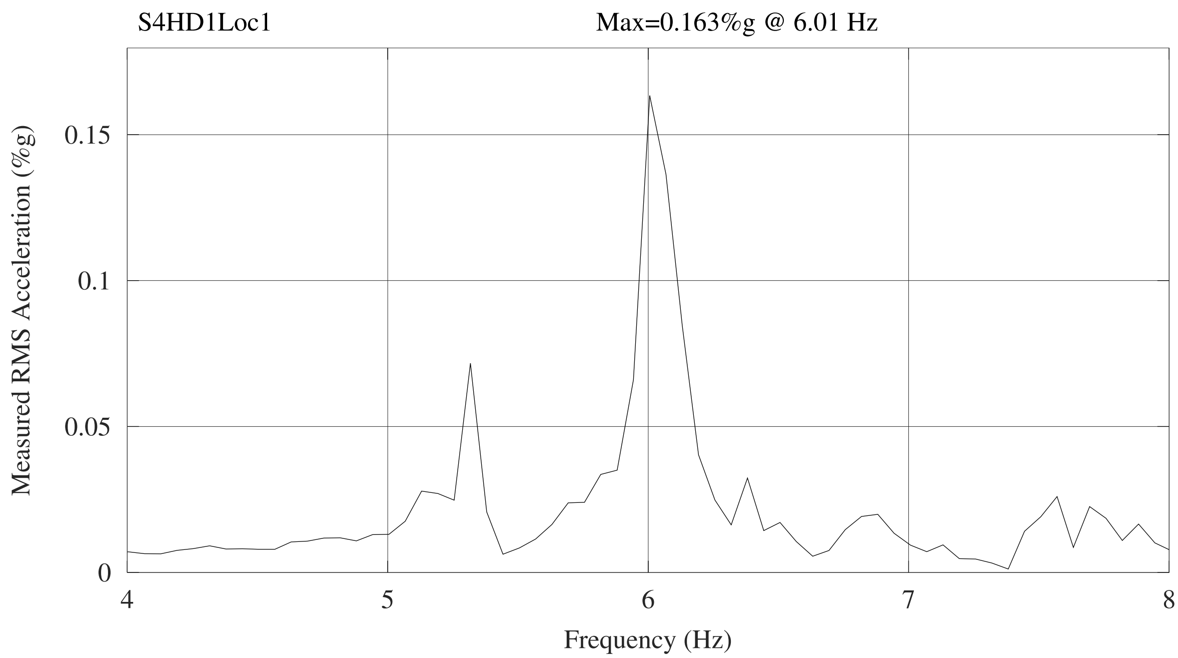

S3HD2Loc2
Max=0.0237%g @ 6.07 Hz
Measured RMS Acceleration (%g)
0.025
0.02
0.015
0.01
0.005
0
4
5
6
7
8
Frequency (Hz)

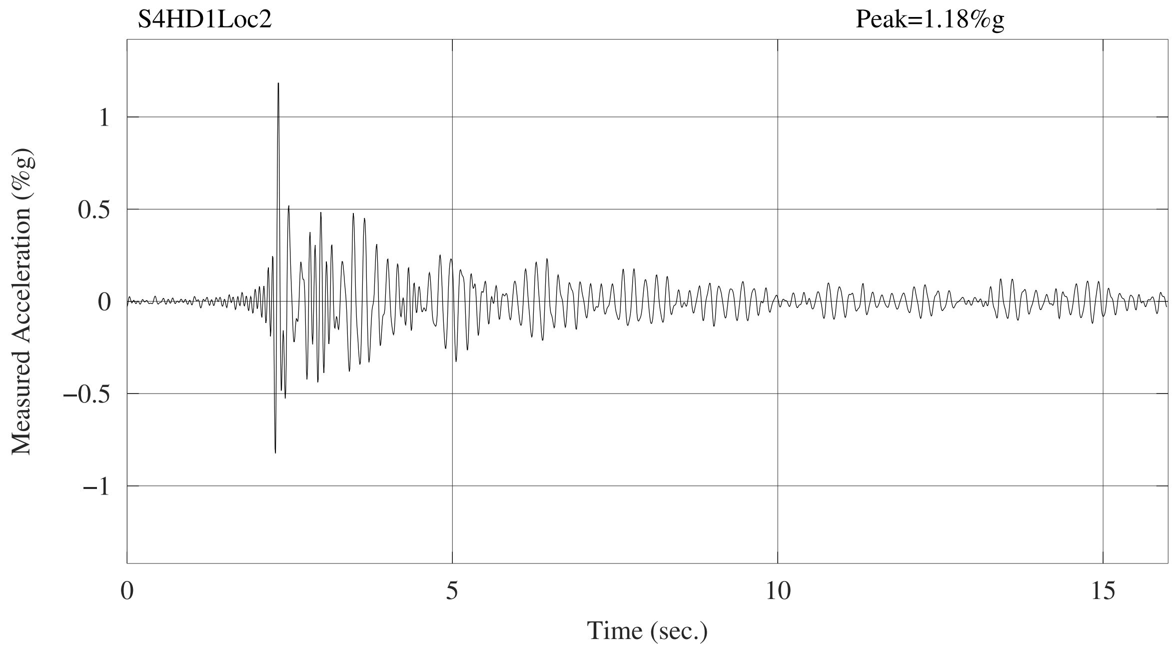

S3HD2Loc3
Peak=0.701%g
0.5
0
−0.5
Measured Acceleration (%g)
0
5
10
15
Time (sec.)

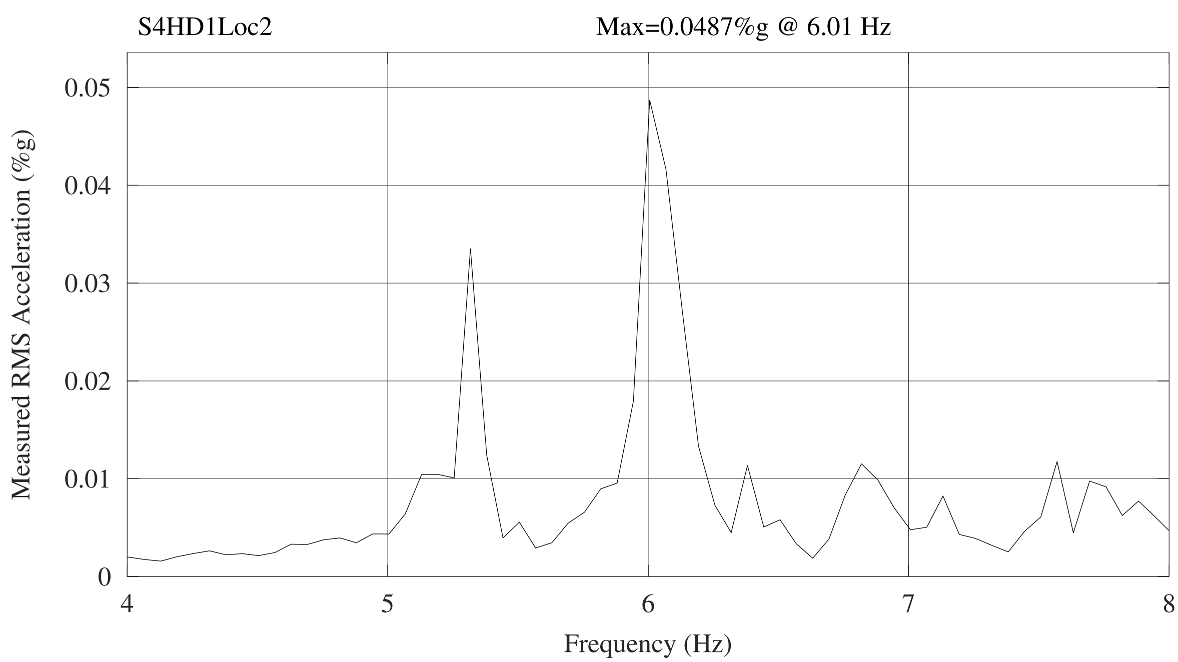

S3HD2Loc3
Max=0.0804%g @ 6.07 Hz
0.08
0.06
0.04
0.02
0
Measured RMS Acceleration (%g)
4
5
6
7
8
Frequency (Hz)

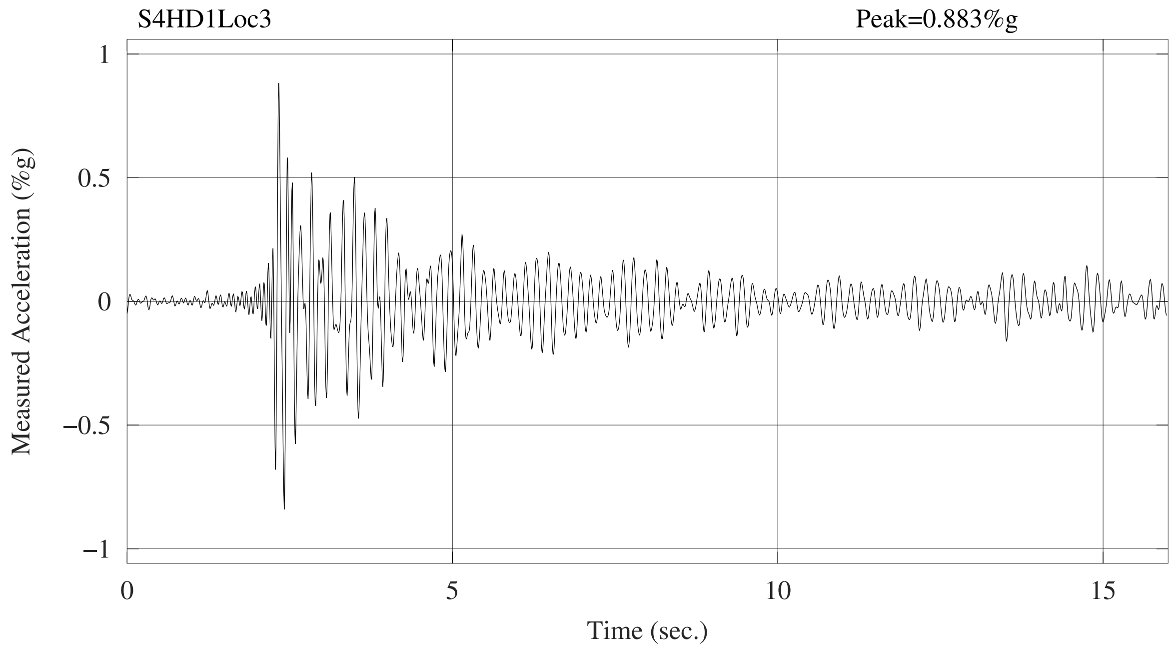
Appendix F - Heel-Drop Test Results for Setup 4

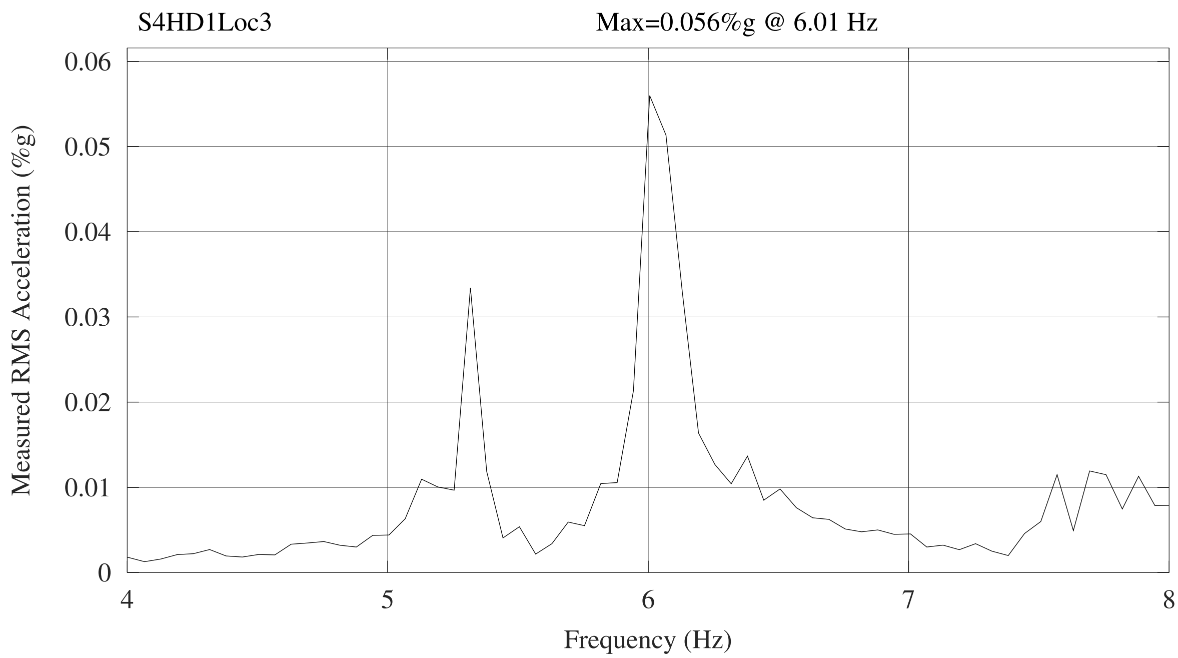





S4HD1Loc2
Peak=1.18%g
Measured Acceleration (%g)
1
0.5
0
−0.5
−1
0
5
10
15
Time (sec.)


S4HD1Loc2
Max=0.0487%g @ 6.01 Hz
Measured RMS Acceleration (%g)
0.05
0.04
0.03
0.02
0.01
0
4
5
6
7
8
Frequency (Hz)


S4HD1Loc3
Peak=0.883%g
Measured Acceleration (%g)
1
0.5
0
−0.5
−1
0
5
10
15
Time (sec.)


S4HD1Loc3
Max=0.056%g @ 6.01 Hz
Measured RMS Acceleration (%g)
0.06
0.05
0.04
0.03
0.02
0.01
0
4
5
6
7
8
Frequency (Hz)

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SHEET OF
DATE: 2/23/2021
PAGE: 1 BY: DBD

Project ID: Vulcraft Boston Tests
Project # :
Bay ID : Setup 1, Sixth Floor, Location 2

VIBRATION ANALYSIS:

Activity: Walking
Occupancy Category: User Defined
Evaluation Criterion: Walking
References: Murray, T.M., Allen, D.E., Ungar, E.E, Davis, D.B. "Vibrations of Steel-Framed Structural Systems Due To Human Activity", AISC Design Guide #11 2nd Ed, 2016
Murray, T.M., and Davis, B., "Vibration of Steel Joist-Concrete Floor Systems", SJI Technical Digest No. 5, 2014
Constant Force, P_o = 65. lb
Modal Damping Ratio, β = 0.010
Acceleration Limit, a_o/g x 100% = 0.00 %
Joist bottom chords are not extended
Joist seats are stiffened
Girders are not continuous at columns

PARAMETER SUMMARY:

	Section	w, plf	I_{eff}, in^4	f, Hz
Joist/Truss	J38 (30CJ1734)	514.8	5083.2	6.05
Left Girder	W30X90	1523.0	10336.3	11.62
Right Girder	W24X55	1230.6	4199.6	8.24
Bay (Using smaller girder frequency)				4.88

EVALUATION: Combined mode a_p/g= 1.600 % > 0.00 %
The system DOES NOT SATISFY THE CRITERION.

FRAMING:

Girder Span = 30.00 ft
Joist Spans:
Left = 10.00 ft
Center = 45.67 ft
Right = 0.00 ft

Girders/Walls:
Left -W30X90
Right-W24X55

Joist -J38 (30CJ1734)
w/ 0.00 in seats
3 spaces at 120.00 in
Floor Width = 96.00 ft
Floor Length= 45.67 ft

Concrete: dc = 6.25 in
f'c = 4.00 ksi
wt. = 115.0 pcf
Deck Height = 3.00 in

Loading: Dead = 0.00 psf
Live = 0.00 psf
Collateral = 0.00 psf

C:\Users\User\DSE LLC Dropbox\Brad Davis\Projects\2017-V6 Vulcraft Research\Boston 2019\Calcs\FV3\Sixth Floor, Setup 1, Location 2, Actual Chords.flb

LOADING DATA:

Slab + 2.0 psf Deck = 47.5 psf
Dead loads = 0.0 psf
Collateral loads = 0.0 psf
Live loads = 0.0 psf
Actual beam and girder weights

Tributary width for left girder = 10.00/2 + 45.67/2 = 27.83 ft
Tributary width for right girder = 45.67/2 = 22.83 ft

CONCRETE/SLAB DATA:

Concrete dc= 6.25 in f'c= 4.0 ksi
wt= 115. pcf Ec = 2466 ksi

Dynamic Modular ratio, n= $E_s/(1.35\ E_c)$ = 8.71

Deck height: 3.00 in
Effective concrete thickness in deck: 1.50 in

JOIST CALCULATIONS:

User Defined Joist/Truss Chords: J38 (30CJ1734) (39.6 plf) Seat Depth= 0.00 in
Top Chord: 2L 3.500x 3.500x 0.344 Bottom Chord: 2L 4.000x 4.000x 0.500
d=30.000 in A=12.079 in^2 I_{chords}= 2216. in^4 y_c= 18.27 in

Spacing: S = 120. in
Span: L_j = 45.67 ft
Uniform load: w_j = (47.5 + 0.0 + 0.0 + 0.0) x 120.00/12 + 39.6
= 514.8 plf

Effective moment of inertia:
Effective concrete width = min(0.4 L_j, S) = 120.000 in
Effective concrete depth = 3.250 in
Transformed concrete width = 13.778 in
Transformed concrete area = 44.779 in^2
Distance to neutral axis = 18.033 in (Above beam c. g.)
Transformed inertia = I_{comp} = 7243.1 in^4
$C_r=0.9\ [1-e^{(-0.28\ Lj/D)}]^{2.8}$ = 0.885 since 6 <= L_g/D= 18.27 <= 24
γ = $(1/C_r) - 1$ = 0.130
Effective moment of inertia = $1/[\gamma/I_{chords}+1/I_{comp}]$ = 5083.2 in^4

$$\delta_j = \frac{5\ w_j\ L_j^4}{384\ E_s\ I_j} = \frac{5 \times 514.8 \times 45.67^4 \times 1728}{384 \times E_s \times 5083.2} = 0.342 \text{ in}$$

$$\text{Frequency} = 0.18 \times \left[\frac{g}{\delta_j}\right]^{0.5} = 0.18 \times \left[\frac{386}{0.342}\right]^{0.5} = 6.05 \text{ Hz}$$

C_j = 2.0
Floor Width= 96.00 ft
$D_s = (12\ de^3)/(12\ n) = (12 \times 4.75^3)/(12 \times 8.71)$ = 12.31 in^4/ft
$D_j = I_j/S$ = 5083.2/ 10.00 = 508.32 in^4/ft
$B_j = \min[C_j\ (D_s/D_j)^{0.25}\ L_j$= 36.03 ft; 2/3 x 96.00 ft= 64.00 ft]
= 36.03 ft
Continuity Factor= 1.0 since joist bottom chords are not extended
W_j = 1.0 x (0.515/ 10.00) x 36.03 x 45.67 = 84.7 Kips

LEFT GIRDER CALCULATIONS:

Girder section: W30X90 d=29.500 in A= 26.30 in^2 I_x= 3610. in^4

Tributary width = 27.83 ft
Span: L_g = 30.00 ft
Equivalent uniform load: w_g = 27.83 x (514.8/ 10.00) + 90.0
= 1523.0 plf

Effective moment of inertia:
min(0.2 L_g, 10.00x12/2) + min(0.2 L_g, 45.67x12/2)
=132.000 in (15.156 in transformed)

```
+----------------------+
|----------------------| -+
|-|           |--------| -+\__  3.250 in
  +-----------+          -+\__  3.000 in
                         -+\__  0.0 in (Joist Seat)
      66.000 in (  7.578 in transformed)
  +-----------+
```

Effective concrete width =132.000 in and 66.000 in
Transformed concrete width = 15.156 in and 7.578 in
Transformed concrete area = 49.257 in^2 and 22.734 in^2
Joist seat depth = 0.0 in
Distance to neutral axis = 13.469 in (Above girder c. g.)
Transformed inertia = I_{tr} =10336.3 in^4 (Stiffened joist seats)

$$\delta_g = \frac{5\ w_g\ L_g^4}{384\ E_s\ I_g} = \frac{5 \times 1523.0 \times 30.00^4 \times 1728}{384 \times E_s \times 10336.26} = 0.093 \text{ in}$$

$$\text{Frequency} = 0.18 \times \left[\frac{g}{\delta_g}\right]^{0.5}$$

$$= 0.18 \times \left[\frac{386}{0.093}\right]^{0.5} = 11.62 \text{ Hz}$$

C_g = 1.6
Floor Length= 45.67 ft
$D_j = I_j/S$ = 5083.2/ 10.00 = 508.32 in^4/ft
$D_g = I_g$/Avg. L_j =10336.3/ 27.83 = 371.34 in^4/ft
B_g = min[C_g $(D_j/D_g)^{0.25}$ L_g= 51.92 ft; 2/3 x 45.67 ft= 30.45 ft]
= 30.45 ft
Continuity Factor= 1.0 since Not Continuous
W_g = 1.0 x (1.523/ 27.83) x 30.45 x 30.00 = 50.0 Kips

RIGHT GIRDER CALCULATIONS:

Girder section: W24X55 d=23.600 in A= 16.20 in^2 I_x= 1350. in^4

Tributary width = 22.83 ft
Span: L_g = 30.00 ft
Equivalent uniform load: w_g = 22.83 x (514.8/ 10.00) + 55.0
= 1230.6 plf

Effective moment of inertia:
min(0.2 L_g, L_b/2) = 72.000 in (8.267 in transformed)

```
+--------------------+
|--------------------| -+
|           |--------| -+\__  3.250 in
|-----------|           -+\__  3.000 in
                        -+\__  0.0 in (Joist Seat)
    36.000 in (  4.133 in transformed)
+-----------+
```

Effective concrete width = 72.000 in and 36.000 in
Transformed concrete width = 8.267 in and 4.133 in
Transformed concrete area = 26.868 in^2 and 12.400 in^2
Joist seat depth = 0.0 in
Distance to neutral axis = 10.930 in (Above girder c. g.)
Transformed inertia = I_{tr} = 4199.6 in^4 (Stiffened joist seats)

$$\delta_g = \frac{5\ w_g\ L_g^4}{384\ E_s\ I_g} = \frac{5 \times 1230.6 \times 30.00^4 \times 1728}{384 \times E_s \times 4199.55} = 0.184 \text{ in}$$

$$\text{Frequency} = 0.18 \times \left[\frac{g}{\delta_g}\right]^{0.5}$$

$$= 0.18 \times \left[\frac{386}{0.184}\right]^{0.5} = 8.24 \text{ Hz}$$

C_g = 1.6
Floor Length= 45.67 ft
D_j = I_j/S = 5083.2/ 10.00 = 508.32 in^4/ft
D_g = I_g/Avg. L_j =4199.6/ 22.83 = 183.91 in^4/ft
B_g = min[C_g (D_j/D_g)$^{0.25}$ L_g= 61.89 ft; 2/3 x 45.67 ft= 30.45 ft]
= 30.45 ft
Continuity Factor= 1.0 since Not Continuous
W_g = 1.0 x (1.231/ 22.83) x 30.45 x 30.00 = 49.2 Kips

COMBINED MODE CALCULATIONS:

Using girder with smaller frequency:
δ_j = 0.342 in δ_{rg} = 0.184 in (Right girder controls)

System frequency, $f_n = 0.18 \times \left[\frac{386}{\delta_j + \delta_g}\right]^{0.5} = 4.88$ Hz

Because the girder span, L_g= 30.00 ft, is less than B_j= 36.03 ft,
the controlling girder deflection is reduced by the factor = L_g/B_j >= 0.5.
$\max(L_g/B_j, 0.5) = \max(30.00/36.03, 0.5) = 0.833$

Therefore:
$\delta'_g = 0.184 \times 0.833 = 0.153$ in

W_j= 84.7 Kips W_g = 49.2 kips

$$W_c = \frac{\delta_j}{\delta_j + \delta'_g} W_j + \frac{\delta'_g}{\delta_j + \delta'_g} W_g$$

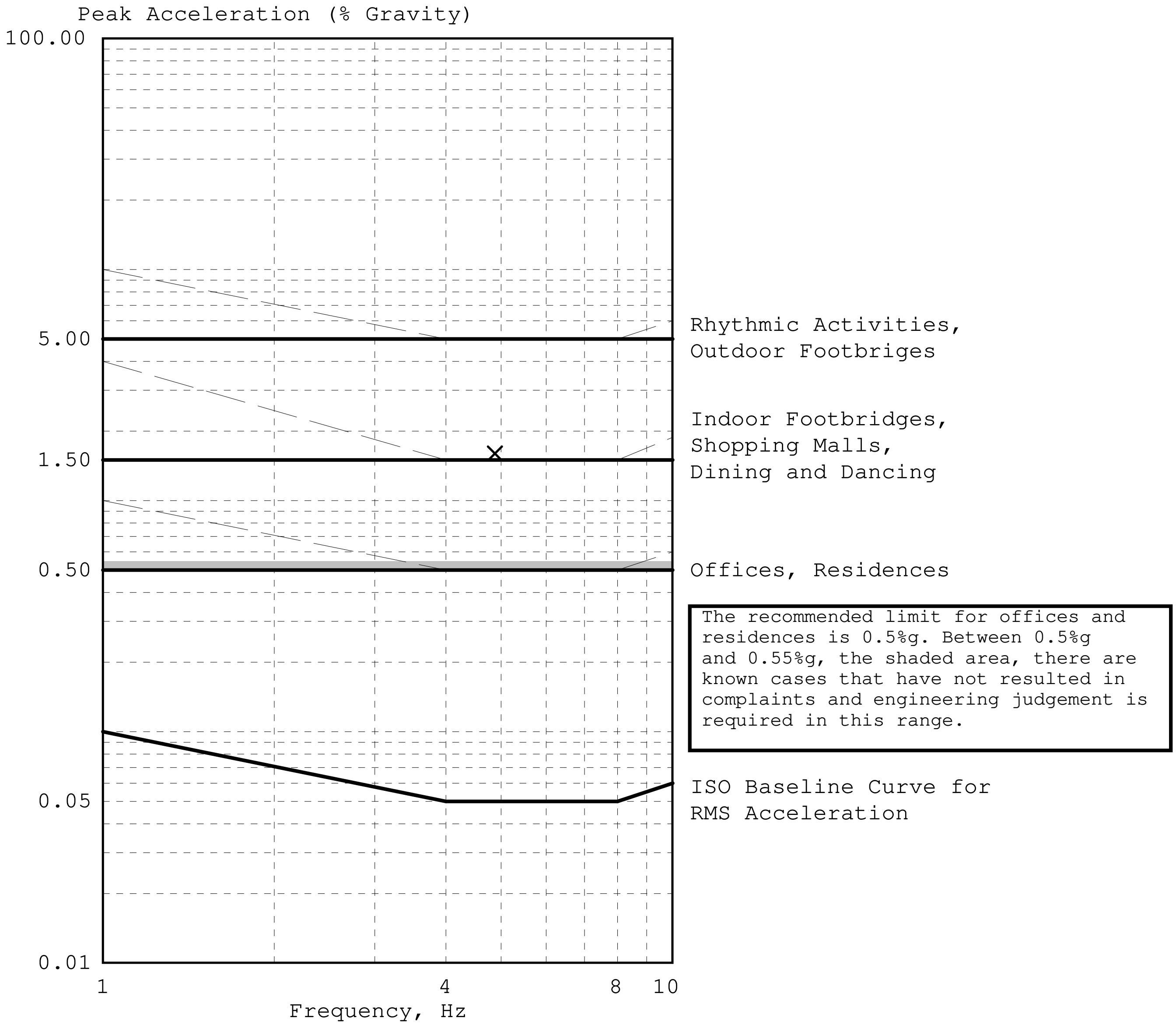
$$= \frac{0.342}{0.495} \times 84.7 + \frac{0.153}{0.495} \times 49.2 = 73.7 \text{ Kips} = 73720. \text{ lbs}$$

β = modal damping ratio = 0.010

$(a_p/g) = P_o \exp^{(-0.35\ fn)}/(\beta\ W_c)$
$=65. \exp^{(-0.35 \times 4.88)}/(0.010 \times 73720.)$
= 1.600 % > 0.00 % - DOES NOT SATISFY CRITERION

Note: The user has elected to ignore the following condition(s):

The effective girder panel width cannot exceed 2/3 times the total floor length
(floor dimension perpendicular to the girder span).
The length of the floor is normally not less than the sum of the specified adjacent
beam span(s) and the beam span. For the specified data
Floor length = 45.67 ft < 10.00 + 45.67 + 0.00 = 55.67 ft



LOW FREQUENCY FLOOR ACCEPTANCE CRITERIA

Note: Floors with a frequency below 3 Hz are not recommended

Project ID: Vulcraft Boston Tests
Project # :
Bay ID : Setup 2, Fifth Floor, Location 1

VIBRATION ANALYSIS:

Activity: Walking
Occupancy Category: User Defined
Evaluation Criterion: Walking
References: Murray, T.M., Allen, D.E., Ungar, E.E, Davis, D.B.
"Vibrations of Steel-Framed Structural Systems
Due To Human Activity", AISC Design Guide #11 2nd Ed, 2016
Murray, T.M., and Davis, B.,
"Vibration of Steel Joist-Concrete Floor Systems",
SJI Technical Digest No. 5, 2014
Constant Force, P_o = 65. lb
Modal Damping Ratio, β = 0.010
Acceleration Limit, a_o/g x 100% = 0.00 %
Joist bottom chords are not extended
Joist seats are stiffened
Girders are not continuous at columns

PARAMETER SUMMARY:	Section	w, plf	I_{eff}, in^4	f, Hz
Joist/Truss	J54 (30CJ1474)	527.1	5156.5	7.44
Left Wall				
Right Girder	W24X55	1140.6	4197.1	8.60
Bay				5.63

EVALUATION: Combined mode a_p/g= 1.511 % > 0.00 %
The system DOES NOT SATISFY THE CRITERION.

FRAMING:

Girder Span = 29.92 ft
Joist Spans:
Left = 0.00 ft
Center = 41.08 ft
Right = 0.00 ft

Girders/Walls:
Left -Wall
Right-W24X55

Joist -J54 (30CJ1474)
w/ 0.00 in seats
3 spaces at 119.68 in
Floor Width = 89.76 ft
Floor Length= 41.08 ft

Concrete: dc = 6.25 in
f'c = 4.00 ksi
wt. = 115.0 pcf
Deck Height = 3.00 in

Loading: Dead = 0.00 psf
Live = 0.00 psf
Collateral = 0.00 psf

LOADING DATA:

```
Slab + 2.0 psf Deck =  47.5 psf
Dead loads          =   0.0 psf
Collateral loads    =   0.0 psf
Live loads          =   0.0 psf
Actual beam and girder weights

Tributary width for girder  =  41.08/2 =  20.54 ft
```

CONCRETE/SLAB DATA:

```
Concrete      dc=  6.25 in              f'c=  4.0 ksi
              wt=  115. pcf             Ec = 2466 ksi
```

Dynamic Modular ratio, $n= E_s/(1.35\ E_c)$ = 8.71

Deck height: 3.00 in
Effective concrete thickness in deck: 1.50 in

JOIST CALCULATIONS:

User Defined Joist/Truss Chords: J54 (30CJ1474) (53.2 plf) Seat Depth= 0.00 in
Top Chord: 2L 3.500x 3.500x 0.313 Bottom Chord: 2L 5.000x 5.000x 0.438
d=30.000 in A=12.562 in^2 I_{chords}= 2151. in^4 y_c= 19.39 in

Spacing: S = 120. in
Span: L_j = 41.08 ft
Uniform load: w_j = (47.5 + 0.0 + 0.0 + 0.0) x 119.68/12 + 53.2
= 527.1 plf

Effective moment of inertia:
Effective concrete width = min(0.4 L_j, S) = 119.680 in
Effective concrete depth = 3.250 in
Transformed concrete width = 13.742 in
Transformed concrete area = 44.660 in^2
Distance to neutral axis = 18.744 in (Above beam c. g.)
Transformed inertia = I_{comp} = 7845.5 in^4
$C_r=0.9\ [1-e^{(-0.28\ Lj/D)}]^{2.8}$ = 0.875 since 6 <= L_g/D= 16.43 <= 24
$\gamma = (1/C_r) - 1 =$ 0.143
Effective moment of inertia = $1/[\gamma/I_{chords}+1/I_{comp}]$ = 5156.5 in^4

$$\delta_j = \frac{5\ w_j\ L_j^4}{384\ E_s\ I_j} = \frac{5 \times 527.1 \times 41.08^4 \times 1728}{384 \times E_s \times 5156.5} = 0.226 \text{ in}$$

$$\text{Frequency} = 0.18 \times \left[\frac{g}{\delta_j}\right]^{0.5} = 0.18 \times \left[\frac{386}{0.226}\right]^{0.5} = 7.44 \text{ Hz}$$

C_j = 2.0
Floor Width= 89.76 ft
D_s = (12 de^3)/(12 n) = (12 x 4.75^3)/(12 x 8.71) = 12.31 in^4/ft
D_j = I_j/S = 5156.5/ 9.97 = 517.03 in^4/ft
B_j = min[C_j $(D_s/D_j)^{0.25}$ L_j= 32.27 ft; 2/3 x 89.76 ft= 59.84 ft]
= 32.27 ft
Continuity Factor= 1.0 since joist bottom chords are not extended
W_j = 1.0 x (0.527/ 9.97) x 32.27 x 41.08 = 70.1 Kips

RIGHT GIRDER CALCULATIONS:

Girder section: W24X55 d=23.600 in A= 16.20 in^2 I_x= 1350. in^4

```
Tributary width              =  20.54 ft
Span:                   Lg =  29.92 ft
Equivalent uniform load: wg =  20.54 x ( 527.1/   9.97) +  55.0
                             = 1140.6 plf

Effective moment of inertia:
           min(0.2 Lg, Lb/2) = 71.808 in (  8.245 in transformed)
                          +---------------------+
                          |---------------------| -+
                          |           |---------| -+\__  3.250 in
                          |-----------|           -+\__  3.000 in
                                                  -+\__  0.0 in (Joist Seat)
                              35.904 in (  4.122 in transformed)
                          +-----------+
```

Effective concrete width = 71.808 in and 35.904 in
Transformed concrete width = 8.245 in and 4.122 in
Transformed concrete area = 26.796 in^2 and 12.367 in^2
Joist seat depth = 0.0 in
Distance to neutral axis = 10.921 in (Above girder c. g.)
Transformed inertia = I_{tr} = 4197.1 in^4 (Stiffened joist seats)

$$\delta_g = \frac{5\ w_g\ L_g^4}{384\ E_s\ I_g} = \frac{5 \times 1140.6 \times 29.92^4 \times 1728}{384 \times E_s \times 4197.11} = 0.169 \text{ in}$$

$$\text{Frequency} = 0.18 \times \left[\frac{g}{\delta_g}\right]^{0.5}$$

$$= 0.18 \times \left[\frac{386}{0.169}\right]^{0.5} = 8.60 \text{ Hz}$$

C_g = 1.6
Floor Length= 41.08 ft
$D_j = I_j/S$ = 5156.5/ 9.97 = 517.03 in^4/ft
$D_g = I_g$/Avg. L_j =4197.1/ 20.54 = 204.34 in^4/ft
$B_g = \min[C_g\ (D_j/D_g)^{0.25}\ L_g$= 60.38 ft; 2/3 x 41.08 ft= 27.39 ft]
= 27.39 ft
Continuity Factor= 1.0 since Not Continuous
W_g = 1.0 x (1.141/ 20.54) x 27.39 x 29.92 = 45.5 Kips

COMBINED MODE CALCULATIONS:

Using girder with smaller frequency:

δ_j = 0.226 in δ_{rg} = 0.169 in (Right girder controls)

$$\text{System frequency, } f_n = 0.18 \times \left[\frac{386}{\delta_j + \delta_g}\right]^{0.5} = 5.63 \text{ Hz}$$

Because the girder span, L_g= 29.92 ft, is less than B_j= 32.27 ft,
the controlling girder deflection is reduced by the factor = L_g/B_j >= 0.5.
max(L_g/B_j, 0.5) = max(29.92/ 32.27, 0.5) = 0.927

Therefore:

δ'_g = 0.169 x 0.927 = 0.157 in

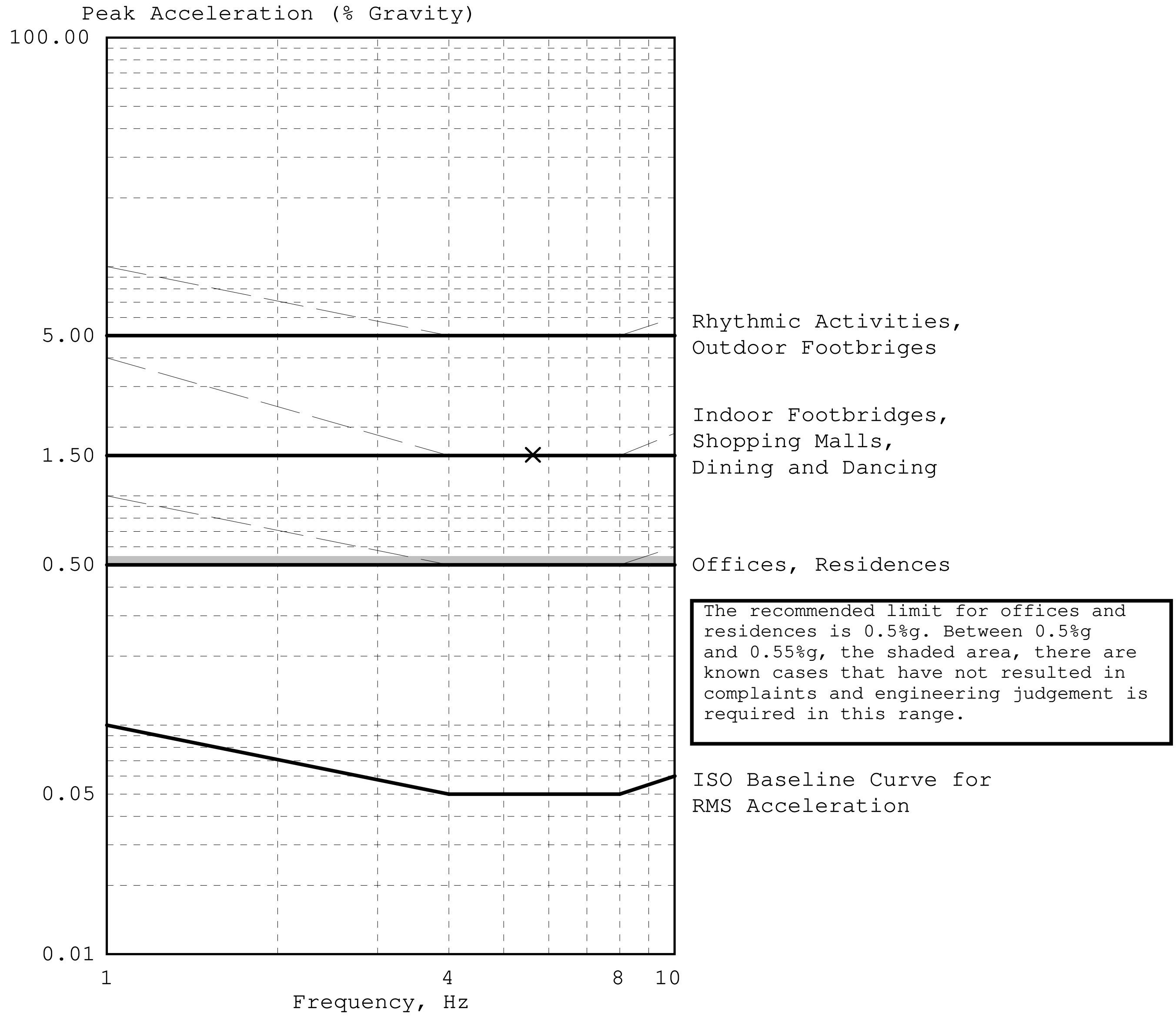
W_j= 70.1 Kips W_g = 45.5 kips

$$W_c = \frac{\delta_j}{\delta_j + \delta'_g} W_j + \frac{\delta'_g}{\delta_j + \delta'_g} W_g$$

$$= \frac{0.226}{0.383} \times 70.1 + \frac{0.157}{0.383} \times 45.5 = 60.0 \text{ Kips} = 60008. \text{ lbs}$$

β = modal damping ratio = 0.010

$(a_p/g) = P_o \exp^{(-0.35\ fn)}/(\beta\ W_c)$
$=65. \exp^{(-0.35 \times 5.63)}/(0.010 \times 60008.)$
= 1.511 % > 0.00 % - DOES NOT SATISFY CRITERION



LOW FREQUENCY FLOOR ACCEPTANCE CRITERIA

Note: Floors with a frequency below 3 Hz are not recommended

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Project ID: Vulcraft Boston Tests
Project # :
Bay ID : Setup 2, Fifth Floor, Location 2

VIBRATION ANALYSIS:

Activity: Walking
Occupancy Category: User Defined
Evaluation Criterion: Walking
References: Murray, T.M., Allen, D.E., Ungar, E.E, Davis, D.B. "Vibrations of Steel-Framed Structural Systems Due To Human Activity", AISC Design Guide #11 2nd Ed, 2016
Murray, T.M., and Davis, B., "Vibration of Steel Joist-Concrete Floor Systems", SJI Technical Digest No. 5, 2014
Constant Force, P_o = 65. lb
Modal Damping Ratio, β = 0.010
Acceleration Limit, a_o/g x 100% = 0.00 %
Joist bottom chords are not extended
Joist seats are stiffened
Girders are not continuous at columns

PARAMETER SUMMARY:

	Section	w, plf	I_{eff}, in^4	f, Hz
Joist/Truss	J50 (30CJ1734)	487.7	4975.2	6.78
Left Girder	W16X45	1388.4	2242.2	6.75
Right Girder	W24X55	1212.2	4119.2	9.79
Bay (Using smaller girder frequency)				4.78

EVALUATION: Combined mode a_p/g= 1.852 % > 0.00 %
The system DOES NOT SATISFY THE CRITERION.

FRAMING:

Girder Span = 27.50 ft
Joist Spans:
Left = 7.00 ft
Center = 43.50 ft
Right = 0.00 ft

Girders/Walls:
Left -W16X45
Right-W24X55

Joist -J50 (30CJ1734)
w/ 0.00 in seats
3 spaces at 110.00 in
Floor Width = 82.50 ft
Floor Length= 50.50 ft

Concrete: dc = 6.25 in
f'c = 4.00 ksi
wt. = 115.0 pcf
Deck Height = 3.00 in

Loading: Dead = 0.00 psf
Live = 0.00 psf
Collateral = 0.00 psf

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LOADING DATA:

```
Slab + 2.0 psf Deck =  47.5 psf
Dead loads          =   0.0 psf
Collateral loads    =   0.0 psf
Live loads          =   0.0 psf
Actual beam and girder weights

Tributary width for left girder  =   7.00/2 +  43.50/2 =  25.25 ft
Tributary width for right girder =  43.50/2 =  21.75 ft
```

CONCRETE/SLAB DATA:

```
Concrete      dc=  6.25 in              f'c=  4.0 ksi
              wt=  115. pcf             Ec = 2466 ksi
```

Dynamic Modular ratio, $n= E_s/(1.35\ E_c)$ = 8.71

Deck height: 3.00 in
Effective concrete thickness in deck: 1.50 in

JOIST CALCULATIONS:

User Defined Joist/Truss Chords: J50 (30CJ1734) (52.1 plf) Seat Depth= 0.00 in
Top Chord: 2L 3.500x 3.500x 0.344 Bottom Chord: 2L 4.000x 4.000x 0.500
d=30.000 in $A=12.079\ in^2$ $I_{chords}= 2216.\ in^4$ $y_c= 18.27$ in

Spacing: S = 110. in
Span: L_j = 43.50 ft
Uniform load: w_j = (47.5 + 0.0 + 0.0 + 0.0) x 110.00/12 + 52.1
= 487.7 plf

Effective moment of inertia:
Effective concrete width = min(0.4 L_j, S) = 110.000 in
Effective concrete depth = 3.250 in
Transformed concrete width = 12.630 in
Transformed concrete area = 41.048 in^2
Distance to neutral axis = 17.691 in (Above beam c. g.)
Transformed inertia = I_{comp} = 7145.3 in^4
$C_r=0.9\ [1-e^{(-0.28\ Lj/D)}]^{2.8}$ = 0.881 since 6 <= L_g/D= 17.40 <= 24
$\gamma = (1/C_r) - 1 =$ 0.135
Effective moment of inertia = $1/[\gamma/I_{chords}+1/I_{comp}]$ = 4975.2 in^4

$$\delta_j = \frac{5\ w_j\ L_j^4}{384\ E_s\ I_j} = \frac{5 \times 487.7 \times 43.50^4 \times 1728}{384 \times E_s \times 4975.2} = 0.272 \text{ in}$$

$$\text{Frequency} = 0.18 \times \left[\frac{g}{\delta_j}\right]^{0.5} = 0.18 \times \left[\frac{386}{0.272}\right]^{0.5} = 6.78 \text{ Hz}$$

C_j = 2.0
Floor Width= 82.50 ft
$D_s = (12\ de^3)/(12\ n) = (12 \times 4.75^3)/(12 \times 8.71) = 12.31\ in^4/ft$
$D_j = I_j/S = 4975.2/\ 9.17 = 542.75\ in^4/ft$
$B_j = \min[C_j\ (D_s/D_j)^{0.25}\ L_j$= 33.76 ft; 2/3 x 82.50 ft= 55.00 ft]
= 33.76 ft
Continuity Factor= 1.0 since joist bottom chords are not extended
W_j = 1.0 x (0.488/ 9.17) x 33.76 x 43.50 = 78.1 Kips

LEFT GIRDER CALCULATIONS:

Girder section: W16X45 d=16.100 in A= 13.30 in^2 I_x= 586. in^4

Tributary width = 25.25 ft
Span: L_g = 27.50 ft
Equivalent uniform load: w_g = 25.25 x (487.7/ 9.17) + 45.0
= 1388.4 plf

Effective moment of inertia:
min(0.2 L_g, 7.00x12/2) + min(0.2 L_g, 43.50x12/2)
=108.000 in (12.400 in transformed)

```
+-----------------------+
|-----------------------| -+
|-|           |--------| -+\__  3.250 in
  +-----------+           -+\__  3.000 in
                          -+\__  0.0 in (Joist Seat)
     54.000 in (  6.200 in transformed)
  +-----------+
```

Effective concrete width =108.000 in and 54.000 in
Transformed concrete width = 12.400 in and 6.200 in
Transformed concrete area = 40.301 in^2 and 18.601 in^2
Joist seat depth = 0.0 in
Distance to neutral axis = 9.536 in (Above girder c. g.)
Transformed inertia = I_{tr} = 2242.2 in^4 (Stiffened joist seats)

$$\delta_g = \frac{5\ w_g\ L_g^4}{384\ E_s\ I_g} = \frac{5 \times 1388.4 \times 27.50^4 \times 1728}{384 \times E_s \times 2242.22} = 0.275 \text{ in}$$

$$\text{Frequency} = 0.18 \times \left[\frac{g}{\delta_g}\right]^{0.5}$$

$$= 0.18 \times \left[\frac{386}{0.275}\right]^{0.5} = 6.75 \text{ Hz}$$

C_g = 1.6
Floor Length= 50.50 ft
$D_j = I_j/S$ = 4975.2/ 9.17 = 542.75 in^4/ft
$D_g = I_g/\text{Avg. } L_j$ =2242.2/ 25.25 = 88.80 in^4/ft
$B_g = \min[C_g\ (D_j/D_g)^{0.25}\ L_g$= 69.18 ft; 2/3 x 50.50 ft= 33.67 ft]
= 33.67 ft
Continuity Factor= 1.0 since Not Continuous
W_g = 1.0 x (1.388/ 25.25) x 33.67 x 27.50 = 50.9 Kips

RIGHT GIRDER CALCULATIONS:

Girder section: W24X55 d=23.600 in A= 16.20 in^2 I_x= 1350. in^4

Tributary width = 21.75 ft
Span: L_g = 27.50 ft
Equivalent uniform load: w_g = 21.75 x (487.7/ 9.17) + 55.0
= 1212.2 plf

Effective moment of inertia:
min(0.2 L_g, L_b/2) = 66.000 in (7.578 in transformed)

```
                        +---------------------+
                        |---------------------| -+
                        |           |---------| -+\__  3.250 in
                        |-----------|           -+\__  3.000 in
                                                -+\__  0.0 in (Joist Seat)
                            33.000 in (  3.789 in transformed)
                        +-----------+
```

Effective concrete width = 66.000 in and 33.000 in
Transformed concrete width = 7.578 in and 3.789 in
Transformed concrete area = 24.629 in^2 and 11.367 in^2
Joist seat depth = 0.0 in
Distance to neutral axis = 10.647 in (Above girder c. g.)
Transformed inertia = I_{tr} = 4119.2 in^4 (Stiffened joist seats)

$$\delta_g = \frac{5\ w_g\ L_g^4}{384\ E_s\ I_g} = \frac{5 \times 1212.2 \times 27.50^4 \times 1728}{384 \times E_s \times 4119.20} = 0.131 \text{ in}$$

$$\text{Frequency} = 0.18 \times \left[\frac{g}{\delta_g}\right]^{0.5}$$

$$= 0.18 \times \left[\frac{386}{0.131}\right]^{0.5} = 9.79 \text{ Hz}$$

C_g = 1.6
Floor Length= 50.50 ft
D_j = I_j/S = 4975.2/ 9.17 = 542.75 in^4/ft
D_g = I_g/Avg. L_j =4119.2/ 21.75 = 189.39 in^4/ft
B_g = min[C_g $(D_j/D_g)^{0.25}$ L_g= 57.25 ft; 2/3 x 50.50 ft= 33.67 ft]
= 33.67 ft
Continuity Factor= 1.0 since Not Continuous
W_g = 1.0 x (1.212/ 21.75) x 33.67 x 27.50 = 51.6 Kips

COMBINED MODE CALCULATIONS:

Using girder with smaller frequency:

δ_j = 0.272 in δ_{lg} = 0.275 in (Left girder controls)

$$\text{System frequency, } f_n = 0.18 \times \left[\frac{386}{\delta_j + \delta_g}\right]^{0.5} = 4.78 \text{ Hz}$$

Because the girder span, L_g= 27.50 ft, is less than B_j= 33.76 ft,
the controlling girder deflection is reduced by the factor = L_g/B_j >= 0.5.
max(L_g/B_j, 0.5) = max(27.50/ 33.76, 0.5) = 0.815

Therefore:

δ'_g = 0.275 x 0.815 = 0.224 in

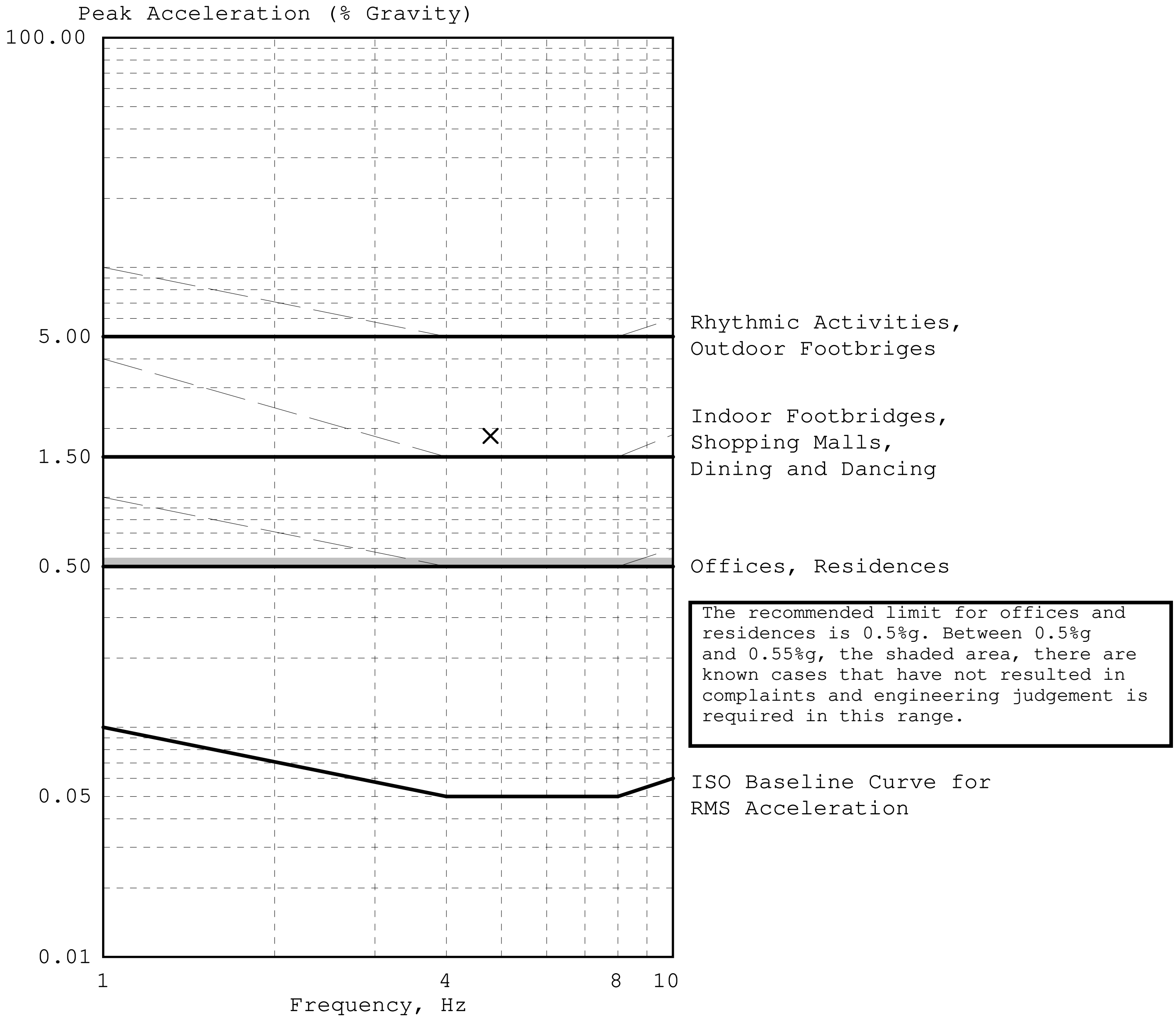
W_j= 78.1 Kips W_g = 50.9 kips

$$W_c = \frac{\delta_j}{\delta_j + \delta'_g} W_j + \frac{\delta'_g}{\delta_j + \delta'_g} W_g$$

$$= \frac{0.272}{0.496} \times 78.1 + \frac{0.224}{0.496} \times 50.9 = 65.9 \text{ Kips} = 65851. \text{ lbs}$$

β = modal damping ratio = 0.010

$(a_p/g) = P_o \exp^{(-0.35\ fn)}/(\beta\ W_c)$
$=65. \exp^{(-0.35 \times 4.78)}/(0.010 \times 65851.)$
= 1.852 % > 0.00 % - DOES NOT SATISFY CRITERION



LOW FREQUENCY FLOOR ACCEPTANCE CRITERIA

Note: Floors with a frequency below 3 Hz are not recommended