Finding Stellar Masses from Binary Stars

These are formulae for determining the masses of stars in binary star systems.

Visual Binaries:

Kepler's Third Law can be written as

$$(M_1 + M_2)P^2 = a^3$$

where: masses are in $M_\odot$, P is period in years, a is semimajor axis in AU.

The semi-major axis is measured as an "angular semi-major axis" in arcsec, and converted to AU by

$$a_{AU} = a'' / \varpi$$

where: a" is the angular semi-major axis in arcsec, and ϖ is the parallax in arcsec.

Re-writing Kepler's Third Law in terms of the observables and solving for the masses gives:

$$(M_1 + M_2) = \frac{(a'')^3}{\varpi^3 P^2}$$

We can also measure the semi-major axes of the two stars relative to their center of mass, a_1 and a_2, and separate out the masses:

$$a = a_1 + a_2$$

$$\frac{M_1}{M_2} = \frac{a_2}{a_1}$$

Note that for visual binary stars, the parallax, ϖ, enters as the third power. If distance is the primary source of uncertainty, a 10% error in parallax translates into a 30% error in the derived masses.

Spectroscopic Binaries:

For double-lined spectroscopic binaries, we observe only the period, P, and the line-of-sight velocities of each of the stars:

$$K_1 = v_1 \sin i$$

$$K_2 = v_2 \sin i$$

Where sin(i) is the sine of the inclination of the orbit relative to the line of sight, defined such that i=90° is edge-on.

In the case of circular orbits,

$$v_1 = \frac{K_1}{\sin i} = \frac{2\pi a_1}{P}$$

$$v_2 = \frac{K_2}{\sin i} = \frac{2\pi a_2}{P}$$

And, as before, $a=a_1+a_2$. Since in principle the a's are unobservable because the two stars are not resolved, we can derive a form of Kepler's Third Law that uses the line-of-sight velocities instead of the semi-major axis as follows. Start with the usual expression:

$$P^2 = \frac{4\pi^2 a^3}{G(M_1+M_2)}$$

and then substitute in the form of the observed velocities, K_1 and K_2 for $a=a_1+a_2$:

$$(M_1+M_2)\sin^3 i = \frac{P\left(K_1+K_2\right)^3}{2\pi G}$$

Here we have the sum of the masses, times $\sin^3 i$, as a function of the three observables, P, K_1, and K_2. [Note: the book incorrectly has the velocities to the 2nd instead of 3rd power in Eqn 3.11]

The advantage of this mass determination is that it is *distance independent*, unlike the case of visual binaries. The disadvantage is that because we cannot a priori determine the projection of the orbit onto the sky, we are stuck with the sin(i) term unless we can find some way to figure out the projection. There are currently two basic ways to go about this:

1. <u>Eclipsing Binaries</u>: the fact it eclipses means that $i\approx 90°$, and careful observations of the eclipse timing can give more precise estimates. *Most of the best-determined stellar masses come from observations of eclipsing binaries.*
2. <u>Interferometry</u>: A number of spectroscopic binaries are beginning to be resolved into pairs using interferometry, allowing direct determination of the inclination, sin(i), and the angular semi-major axis, a". Note that we still don't need to know the distance (i.e., parallax) to determine the masses. In fact, in cases where you can determine a" and sin(i) via interferometry, you can use the version of Kepler's Third Law for visual binaries and solve for the "dynamical parallax":

$$\varpi_{dyn} = \frac{a''}{(M_1+M_2)^{1/3} P^{2/3}}$$

   Because the sum of the masses enters as the 1/3rd power, even a 30% mass determination from spectroscopy leads to a 10% estimate of the dynamical parallax.

Problem #2 on Homework 1 has you work through both cases (visual and "resolved" spectroscopic binary) for the star Capella (α Aurigae), which has a conveniently circular orbit (or close enough for us to use the circular approximation), and recent interferometric imaging from the MkIII experiment.

If the orbits are elliptical, a more general form can be derived that includes a 4th observable, the eccentricity of the orbit, ε, derived by measuring the departure from purely sinusoidal radial velocity curves expected for circular orbits. In either case, you are still left with sin(i) terms unless a way to determine the projection geometry is found.