

New Methods for Evaluating Students' Answerscripts Using Vague Values

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Abstract

In this paper, we present two new methods for evaluating students' answerscripts using vague values, where the evaluating marks awarded to the questions in the students' answerscripts are represented by vague values. The vague mark awarded to each question of a student's answerscript can be regarded as a vague set, where each element in the universe of discourse belonging to the vague set is represented by a vague value. The proposed methods can evaluate students' answerscripts in a more flexible and more intelligent manner.

Keywords: Vague Sets, Vague Grade Sheets, Generalized Vague Grade Sheets, Index of Optimism, Fuzzy Sets.

1. Introduction

In recent years, some methods [1], [2], [7], [9]-[12], [14]-[17] for students' evaluation have been presented. In [1] and [7], the fuzzy marks awarded to answers in the students' answerscripts are represented by fuzzy sets [18]. However, if we can allow the marks awarded to the questions of the students' answerscripts to be represented by vague values [13], then there is room for more flexibility.

In this paper, we present two new methods for evaluating students' answerscripts using vague values, where the evaluating marks awarded to the questions in the students' answerscripts are represented by vague values. The vague mark awarded to each question of a student's answerscript can be regarded as a vague set, where each element in the universe of discourse belonging to a vague set is represented by a vague value. The proposed methods can evaluate students' answerscripts in a more flexible and more intelligent manner.

2. Vague Sets and Vague Values

A vague set \tilde{A} [13] in the universe of discourse U is characterized by a truth-membership function $t_{\tilde{A}}$ and a false-membership function $f_{\tilde{A}}$, where $t_{\tilde{A}}: U \rightarrow [0, 1]$, $f_{\tilde{A}}: U \rightarrow [0, 1]$, $t_{\tilde{A}}(u_i)$ is a lower bound of the grade of membership of u_i derived from the evidence for u_i , $f_{\tilde{A}}(u_i)$ is a lower bound of the negation of u_i derived from the evidence against u_i , $t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$, and $u_i \in U$. The grade of membership of u_i in the vague set \tilde{A} is bounded by a subinterval $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ of $[0, 1]$. The vague value $[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i)]$ indicates that the exact grade of membership $\mu_{\tilde{A}}(u_i)$ of u_i is bounded by $t_{\tilde{A}}(u_i) \leq \mu_{\tilde{A}}(u_i) \leq 1 - f_{\tilde{A}}(u_i)$, where $t_{\tilde{A}}(u_i) + f_{\tilde{A}}(u_i) \leq 1$. If the universe of discourse U is a finite set, then a vague set \tilde{A} of the universe of discourse U can be represented as follows:

$$\tilde{A} = \sum_{i=1}^n \left[t_{\tilde{A}}(u_i), 1 - f_{\tilde{A}}(u_i) \right] / u_i. \quad (1)$$

In this paper, we present the concept of the expected truth value $E(X)$ of a vague value X . Let X be a vague value, where $X = [t_x, 1 - f_x]$, t_x denotes the degree of truth, f_x denotes the degree of false, $t_x \in [0, 1]$, $f_x \in [0, 1]$, and $t_x + f_x \leq 1$. Let $f_x^* = 1 - f_x$, i.e., the vague value $X = [t_x, 1 - f_x] = [t_x, f_x^*]$. Then, the unknown part $N(X)$ of the vague value X is defined as follows:

$$N(X) = f_x^* - t_x = (1 - f_x) - t_x. \quad (2)$$

Assume that the index of optimism determined by the evaluator is λ , where $\lambda \in [0, 1]$. If $0 \leq \lambda < 0.5$, then the evaluator is a pessimistic evaluator. If $\lambda = 0.5$, then the evaluator is a normal evaluator. If $0.5 < \lambda \leq 1.0$, then the evaluator is an optimistic evaluator. The larger the value of λ , the more optimistic the evaluator. The less the value of λ , the more pessimistic the evaluator. Then, the expected truth value $E(X)$ of the vague value X based on the index of optimism λ determined by the evaluator is calculated as follows:

$$\begin{aligned} E(X) &= (1 - \lambda) \times t_x + \lambda \times (t_x + N(X)), \\ &= (1 - \lambda) \times t_x + \lambda \times (t_x + f_x^* - t_x), \\ &= (1 - \lambda) \times t_x + \lambda \times (t_x + (1 - f_x) - t_x), \\ &= (1 - \lambda) \times t_x + \lambda \times (1 - f_x), \end{aligned}$$

$$= (1 - \lambda) \times t_x + \lambda \times f_x^* \quad (3)$$

where $0 \leq t_x \leq E(X) \leq f_x^* \leq 1$ and $f_x^* = 1 - f_x$.

3. A New Method for Students' Answerscripts Evaluation Using Vague Values

In this section, we present a new method for students' answerscripts evaluation using vague values, where the vague marks awarded to the questions of the students' answerscripts are represented by vague values in $[0, 1]$. An index of optimism λ determined by the evaluator is used to indicate the degree of optimism of the evaluator for evaluating students' answerscripts, where $\lambda \in [0, 1]$. If $0 \leq \lambda < 0.5$, then the evaluator is a pessimistic evaluator. If $\lambda = 0.5$, then the evaluator is a normal evaluator. If $0.5 < \lambda \leq 1.0$, then the evaluator is an optimistic evaluator. The larger the value of λ , the more optimistic the evaluator. The less the value of λ , the more pessimistic the evaluator. In this paper, eleven satisfaction levels shown in Table 1 are used to evaluate the students' answerscripts regarding a question of a test/examination, where the corresponding vague truth values of these eleven satisfaction levels are also shown in Table 1. Let T be a mapping function for mapping a set of satisfaction levels into their corresponding vague truth values. From Table 1, we can see that

$$\begin{aligned} T(EG) &= [1, 1], \\ T(VVG) &= [0.90, 0.99], \\ T(VG) &= [0.80, 0.89], \\ T(G) &= [0.70, 0.79], \\ T(MG) &= [0.60, 0.69], \\ T(F) &= [0.50, 0.59], \\ T(MB) &= [0.40, 0.49], \\ T(B) &= [0.25, 0.39], \\ T(VB) &= [0.10, 0.24], \\ T(VVB) &= [0.01, 0.09], \\ T(EB) &= [0, 0]. \end{aligned} \quad (4)$$

Assume that an evaluator evaluates the students' answerscripts by using a vague grade sheet as shown in Table 2, where X_i denotes a vague truth value defined in $[0, 1]$ and $1 \leq i \leq 11$. In any row of Table 2, the columns from the second to the twelfth indicate the vague mark awarded to the answer to the corresponding question shown in the first column, where the vague mark is represented as a vague set. The last (i.e., the thirteenth) column of the vague grade sheet shown in Table 2 indicates the degree of satisfaction evaluated by the proposed method awarded to each question. The box at the bottom of the vague grade sheet shown in Table 2 indicates the total mark awarded to the student.

Table 1. Satisfaction Levels and Their Corresponding Vague Truth Values

| Satisfaction Levels | Vague Satisfaction Values |
|---------------------------------|---------------------------|
| extremely good (<i>EG</i>) | [1, 1] |
| very very good (<i>VVG</i>) | [0.90, 0.99] |
| very good (<i>VG</i>) | [0.80, 0.89] |
| good (<i>G</i>) | [0.70, 0.79] |
| more or less good (<i>MG</i>) | [0.60, 0.69] |
| fair (<i>F</i>) | [0.50, 0.59] |
| more or less bad (<i>MB</i>) | [0.40, 0.49] |
| bad (<i>B</i>) | [0.25, 0.39] |
| very bad (<i>VB</i>) | [0.10, 0.24] |
| very very bad (<i>VVB</i>) | [0.01, 0.09] |
| extremely bad (<i>EB</i>) | [0, 0] |

Table 2. Vague Mark Represented by Vague Values of the Question $Q.i$ in A Vague Grade Sheet

| Question No. | Satisfaction Levels | | | | | | | | | | | Degree of Satisfaction |
|--------------|---------------------|------------|-----------|----------|-----------|----------|-----------|----------|-----------|------------|-----------|------------------------|
| | <i>EG</i> | <i>VVG</i> | <i>VG</i> | <i>G</i> | <i>MG</i> | <i>F</i> | <i>MB</i> | <i>B</i> | <i>VB</i> | <i>VVB</i> | <i>EB</i> | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $Q.i$ | X_1 | X_2 | X_3 | X_4 | X_5 | X_6 | X_7 | X_8 | X_9 | X_{10} | X_{11} | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | | | | | | | | | | | | Total Mark = |

Assume that the vague mark of the question $Q.i$ of a student's answerscript evaluated by an evaluator is as shown in Table 2, where X_i is a vague value in the universe of discourse $[0, 1]$ and $1 \leq i \leq 11$. Assume that the degree of optimism of the evaluator determined by the evaluator for evaluating students' answerscript is λ , where $\lambda \in [0, 1]$. The proposed method for students' answerscripts evaluation using vague values is now presented as follows:

Step 1: Based on formula (3) and the index of optimism λ determined by the evaluator, where $\lambda \in [0, 1]$, calculate the expected satisfaction value $E(X_i)$ of each vague satisfaction value X_i in the vague grade sheet shown in Table 2, where $E(X_i) \in [0, 1]$ and $1 \leq i \leq 11$, as shown in Table 3.

Table 3. Expected Vague Values of the Vague Mark of the Question $Q.i$ of Table 2

| Question No. | Satisfaction Levels | | | | | | | | | | | Degree of Satisfaction |
|--------------|---------------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|--------------|--------------|------------------------|
| | <i>EG</i> | <i>VVG</i> | <i>VG</i> | <i>G</i> | <i>MG</i> | <i>F</i> | <i>MB</i> | <i>B</i> | <i>VB</i> | <i>VVB</i> | <i>EB</i> | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| $Q.i$ | $E(X_{i1})$ | $E(X_{i2})$ | $E(X_{i3})$ | $E(X_{i4})$ | $E(X_{i5})$ | $E(X_{i6})$ | $E(X_{i7})$ | $E(X_{i8})$ | $E(X_{i9})$ | $E(X_{i10})$ | $E(X_{i11})$ | |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | | | | | | | | | | | | Total Mark = |

Step 2: From Table 1, we can see that the corresponding vague satisfaction truth values of the satisfaction levels $EG, VVG, VG, G, MG, F, MB, B, VB, VVB$ and EB are as follows: $T(EG) = [1, 1]$, $T(VVG) = [0.90, 0.99]$, $T(VG) = [0.80, 0.89]$, $T(G) = [0.70, 0.79]$, $T(MG) = [0.60, 0.69]$, $T(F) = [0.50, 0.59]$, $T(MB) = [0.40, 0.49]$, $T(B) = [0.25, 0.39]$, $T(VB) = [0.10, 0.24]$, $T(VVB) = [0.01, 0.09]$ and $T(EB) = [0, 0]$, where $[1, 1]$, $[0.90, 0.99]$, $[0.80, 0.89]$, $[0.70, 0.79]$, $[0.60, 0.69]$, $[0.50, 0.59]$, $[0.40, 0.49]$, $[0.25, 0.39]$, $[0.10, 0.24]$, $[0.01, 0.09]$ and $[0, 0]$ are vague truth values. Based on formula (3) and the index of optimism λ determined by the evaluator, where $\lambda \in [0, 1]$, calculate the corresponding expected truth value $E(Y)$ of each satisfaction level Y in the vague grade sheet shown in Table 2, where $Y \in \{EG, VVG, VG, G, MG, F, MB, B, VB, VVB, EB\}$ and $E(Y) \in [0, 1]$. For example, from Table 6, we can see that $T(VG) = [0.80, 0.89]$, where $t_{VG} = 0.80$ and $1 - f_{VG} = 0.89$. Assume that the index of optimism λ determined by the evaluator is 0.60 (i.e., $\lambda = 0.60$), then based on formula (3), we can see that the expected truth value $E(VG)$ of the satisfaction level VG is calculated as follows:

$$\begin{aligned} E(VG) &= (1 - \lambda) \times t_{VG} + \lambda \times (1 - f_{VG}) \\ &= (1 - 0.60) \times 0.80 + 0.60 \times 0.89 \\ &= 0.32 + 0.534 \\ &= 0.854. \end{aligned}$$

It indicates that the expected truth value of the satisfaction level VG is 0.854 when the index of optimism λ determined by the evaluator is 0.60 (i.e., $\lambda = 0.60$). The degree of satisfaction $D(Q.i)$ of the question $Q.i$ of the student's answerscript can be evaluated by the function D ,

$$D(Q.i) = \frac{E(X_{i1}) \times E(EG) + E(X_{i2}) \times E(VVG) + \dots + E(X_{i11}) \times E(EB)}{E(X_{i1}) + E(X_{i2}) + \dots + E(X_{i11})}, \quad (5)$$

where $E(X_i)$ denotes the expected satisfaction value of the vague satisfaction value X_i , $1 \leq i \leq 11$, and $0 \leq D(Q.i) \leq 1$. The larger the value of $D(Q.i)$, the higher the degree of satisfaction that the question $Q.i$ of the student's answerscript satisfies the evaluator's opinion.

Step 3: Consider the situation that the total mark of a student's answerscript to an examination is 100 marks. Assume that there are n questions to be answered, i.e.,

$$\begin{aligned} \text{TOTAL MARKS} &= 100, \\ Q.1 &\text{ carries } s_1 \text{ marks,} \\ Q.2 &\text{ carries } s_2 \text{ marks,} \\ &\vdots \\ Q.n &\text{ carries } s_n \text{ marks,} \end{aligned}$$

where $\sum_{i=1}^n s_i = 100$, $0 \leq s_i \leq 100$, and $1 \leq i \leq n$. Assume

that the evaluated degrees of satisfaction of the questions $Q.1, Q.2, \dots$, and $Q.n$ are $D(Q.1),$

$D(Q.2), \dots$, and $D(Q.n)$, respectively, then the total mark of the student is evaluated as follows:

$$s_1 \times D(Q.1) + s_2 \times D(Q.2) + \dots + s_n \times D(Q.n). \quad (6)$$

Put this total mark in the appropriate box at the bottom of the extended fuzzy grade sheet.

4. A Generalized Method for Students' Answerscripts Evaluation Using Vague Values

In this section, we present a generalized students' answerscripts evaluation method using vague values. Consider a student's answerscript to an examination of 100 marks. Assume that there are n questions to be answered:

$$\begin{aligned} \text{TOTAL MARKS} &= 100, \\ Q.1 &\text{ carries } s_1 \text{ marks,} \\ Q.2 &\text{ carries } s_2 \text{ marks,} \\ &\vdots \\ Q.n &\text{ carries } s_n \text{ marks,} \end{aligned}$$

where $\sum_{i=1}^n s_i = 100$, $0 \leq s_i \leq 100$, and $1 \leq i \leq n$. Assume

that the degree of optimism of the evaluator is λ , where $\lambda \in [0, 1]$. The larger the value of λ , the more optimistic the evaluator. The less the value of λ , the more pessimistic the evaluator. Assume that an evaluator evaluates the answers of students' answerscripts using the following four criteria [1]:

$$\begin{aligned} C_1: &\text{ Accuracy of information,} \\ C_2: &\text{ Adequate coverage,} \\ C_3: &\text{ Conciseness,} \\ C_4: &\text{ Clear expression,} \end{aligned}$$

and assume that the weights of the criteria C_1, C_2, C_3 and C_4 are w_1, w_2, w_3 and w_4 , respectively, where $0 \leq w_i \leq 1$ and $1 \leq i \leq 4$. Furthermore, assume that the evaluator can evaluate each question of students' answerscripts using the above four criteria based on the method described in Section 3. In this case, an evaluator can evaluate students' answerscripts using vague values and the degrees of satisfaction of the question $Q.i$ of a student's answerscript regarding to the criteria C_1, C_2, C_3 and C_4 evaluated by the proposed method presented in Section 3 are $D(C_{i1}), D(C_{i2}), D(C_{i3}),$ and $D(C_{i4})$, respectively, where $0 \leq D(C_{i1}) \leq 1, 0 \leq D(C_{i2}) \leq 1, 0 \leq D(C_{i3}) \leq 1, 0 \leq D(C_{i4}) \leq 1$, and $1 \leq i \leq n$. The degree of satisfaction $P(Q.i)$ of the question $Q.i$ of the student's answerscript can be evaluated as follows:

$$P(Q.i) = \frac{w_1 \times D(C_{i1}) + w_2 \times D(C_{i2}) + w_3 \times D(C_{i3}) + w_4 \times D(C_{i4})}{w_1 + w_2 + w_3 + w_4}, \quad (7)$$

where $0 \leq P(Q,i) \leq 1$ and $1 \leq i \leq n$. The total mark of the student can be evaluated and is equal to

$$s_1 \times P(Q,1) + s_2 \times P(Q,2) + \dots + s_n \times P(Q,n). \quad (8)$$

5. Conclusions

In this paper, we have presented two new methods for evaluating students' answerscripts using vague values, where the evaluating marks awarded to the questions in the students' answerscripts are represented by vague values. The vague mark awarded to each question of a student's answerscript can be regarded as a vague set, where each element in the universe of discourse belonging to a vague set is represented by a vague value in $[0, 1]$. An index of optimism λ determined by the evaluator is used to indicate the degree of optimism of the evaluator. The proposed methods can evaluate students' answerscripts in a more flexible and more intelligent manner.

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