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Abstract

Many markets are characterized by a high level of inter-firm collaboration in R&D activity. This paper develops a simple model of strategic networks which captures two distinctive features of such collaboration activity: bilateral agreements and non-exclusive relationships. We study the effects of collaborations on individual R&D effort, cost reduction, and market performance. We then examine the incentives of firms to form collaborative links and the architecture of strategically stable networks.

Our analysis highlights the interaction between market competition and R&D network structure. We find that if firms are Cournot competitors then individual R&D effort is declining in the level of collaborative activity. However, cost reduction and social welfare are maximized under an intermediate level of collaboration. In some cases, firms can gain market power, and even induce exit of rival firms, by forming suitable collaboration agreements. Moreover, under certain circumstances, such asymmetric collaboration networks are also strategically stable. By contrast, if firms operate in independent markets then individual R&D effort is increasing in the level of collaborative activity. Cost reduction and social welfare are maximized under the complete network, which is also strategically stable.

Keywords: strategic alliances, networks, research and development
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1 Introduction

Many markets are characterized by a high level of inter-firm collaboration in R&D activity. There is empirical evidence to suggest that such collaboration has increased in recent years.[1] Moreover, a significant proportion of such collaboration takes place between firms that are horizontally related, i.e., where firms exhibit some degree of market rivalry.[2] Collaboration agreements and alliances vary greatly in their form. They may range from loose and informal agreements (such as a memorandum of understanding), all the way to the formation of common legal organizations with strong equity ties (such as a joint venture).[3] Similarly, the nature of collaborative activity varies widely. In some cases, the collaboration involves sharing of information (concerning new technologies and market conditions), while in others the focus is on sharing facilities (such as distribution channels and complementary assets) or on setting market standards.

In this paper, we study the nature of collaboration between horizontally related firms. We suppose that the form of the agreement is relatively loose and that the benefits arise from sharing of knowledge about a cost-reducing technology. When firms collaborate, their individual R&D efforts lower costs of their partners too. The impact of these spillovers is intimately related to the nature of market rivalry between the firms. Thus market competition has a bearing on the incentives for collaboration. However, by forming collaborations, firms alter the competitive position of different firms and in turn influence market structure and performance. This two-way flow of influence is central to our analysis. In particular, we address the following questions:

- What are the effects of collaborative activity on individual R&D and industry performance?

[1] Hagedoorn and Schakenraad (1990) report that, during the 1980's, on the average there were an additional 100 collaborative agreements every year in biotechnology, and over 200 every year in information technologies. Burgers, Hill and Kim (1993) find that the 23 largest firms in the world automobile market had been involved in 58 bilateral linkages by 1988. The rapid growth in the number of technological alliances between firms in the 1980's has also been documented by Ghemawat, Porter, Rawlinson (1986), Hagedoorn and Narula (1996), Harrigan (1988) and Morris and Hergert (1987). Delapierre and Mytelka (1998) present evidence for a similar trend in strategic alliance formation in the 1990's, for electronics and biotechnology firms.

[2] For empirical work on this, see Harrigan (1988). Collaboration between firms also arise between partners who are *vertically* related, as in manufacturer-supplier relationships. See Artz and Brush (2000) for an empirical study of vertical manufacturer-supplier relationships.

[3] WISe and IVANS have recently signed a "letter of understanding" as the basis for a strategic alliance (see National Underwriter, Chicago, April 24, 2000). Another example is the memorandum of understanding recently signed by Presstek and Xerox (see Printing Impressions, Philadelphia, March, 2000). Hagedoorn and Schakenraad (1990) and Delapierre and Mytelka (1998) present evidence suggesting that non-RJV forms of collaboration have become more important in recent years.

- What are the incentives of firms to collaborate and what is the architecture of "incentive-compatible" networks?
- Are individual incentives to collaborate adequate from a social welfare point of view?

We also study how the answers to these questions depend on the nature of market competition.

A distinctive feature of collaboration agreements is that they are often bilateral and are embedded in a broader network of collaborations (Delapierre and Mytelka, 1998; Mody, 1993). This translates into situations where firms i and j, and j and k have a collaborative relationship, respectively, while firms i and k do not have any collaborative arrangement, i.e., collaborative relationships are non-exclusive (Milgrom and Roberts, 1992, p. 576).[4] These structural features lead us to formulate a strategic model of R&D networks.

The structure of the model is as follows. We consider an oligopoly with (ex-ante) identical firms. Prior to market interaction, each firm has an opportunity to form *pair-wise* collaborative links with other firms. The purpose of these ties is sharing R&D knowledge about a cost-reducing technology. The collection of pair-wise links between the firms in the industry defines a *network* of collaboration. Our model allows for *non-exclusive* ties, and generates a rich class of possible collaborative structures.[5] Given a collaboration network, firms choose a (costly) level of effort in R&D *unilaterally*, aimed at reducing production costs.[6] The level of effort of different firms and the network of collaboration define the effective costs of the different firms in the market. Given these costs firms operate in the market by setting quantities. We consider two types of market competition: in the first case, firms operate in a homogeneous product market, while in the second case, they operate in independent markets.

We start with a consideration of *symmetric* networks, i.e., networks in which all firms maintain the same number of collaborative ties. For symmetric networks, the level of collaborative activity is naturally measured in terms of the number of ties

[4]This pattern can be discerned in collaborative ties between pharmaceutical firms. For example, in the late 1980's and early 1990's, Merck and Ciba-Giegy had a series of collaborative ties. Similarly Bristol-Mayers and Bayer had such bilateral collaborations. At the same time, these firms also had collaboration ties with non-overlapping sets of firms. For example, Bayer had collaboration arrangements with Hoechst, while Bristol-Mayers had no such collaboration. Delapierre and Mytelka (1998) present information on a variety of such collaborative ties.

[5]For example, with n firms there are $2^{n(n-1)/2}$ possible networks of collaboration.

[6]The unilateral choice of R&D effort is consistent with the model of competitive RJV, proposed in Kamien, Muller and Zang (1992). See Anbarci, Lemke and Roy (2000) for a detailed discussion of the incentive problems involved in cooperative R&D; these arguments suggest that it is more plausible to model R&D effort as being unilaterally (non-cooperatively) determined.

of a typical firm. Our first result pertains to the relationship between the level of collaboration and individual R&D. We show that *if firms compete in a homogeneous product market, individual firm R&D effort is declining in the level of collaborative activity. In contrast, when firms operate in independent markets, individual R&D effort increases monotonically in the level of collaborative activity.* Thus, in the former case individual R&D attains its minimum while in the latter case it attains its maximum under the complete network.[7] This simple relationship highlights the dominating influence of the competition effect. The intuition is that collaboration is associated with reduced appropriability of R&D results, which, in the case of intense market rivalry brings about substantial negative business-stealing effects.

We examine next the level of cost reduction under different levels of collaboration. For any given level of R&D effort, adding a collaboration link leads to lower costs for firms. However, as argued above, in the standardized product market setting additional collaborative ties bring about detrimental business-stealing effects which lower firms' research efforts. Thus we have to compare the relative magnitude of these two effects. We find that *in a homogeneous product setting, the level of cost reduction is initially increasing and then decreasing in the level of collaborative activity,* i.e., it is non-monotonic with respect to the number of collaborations. By contrast, *when firms operate in independent markets, individual R&D effort and hence, by implication, the cost reduction, is maximal under the complete network.*

Our next result pertains to the incentives of firms to form collaborative alliances. We show that irrespective of the degree of market competition firms have an incentive to form collaborative relations; in other words, the empty network is never incentive compatible. Further, we show that the incentives to form collaborations are quite large in both settings: *the complete network is a strategically stable network, irrespective of the market setting.*[8]

We then examine aggregate industry performance in relation to collaboration between firms. We show that *if firms compete in a homogeneous product market, industry performance both in terms of aggregate profits and social welfare is highest when firms have an intermediate level of collaboration with other firms.*[9] In other words, both the empty and the complete network are dominated by intermediate levels of collaboration. Thus, in the case of intense market rivalry, the incentives of firms to form collaborative ties may be excessive both from an industry profit maximizing perspective as well as from a social welfare point of view. By contrast,

[7] A network is said to be complete if every pair of firms has a collaborative agreement.

[8] In the general setting, we have been unable to obtain a complete characterization of strategically stable networks. In section 4, we characterize such networks in the context of an example of three firms.

[9] More precisely, the industry profit maximizing network is typically denser than the social welfare maximizing network. This is consistent with earlier work in the coalition formation literature (see Yi, 1998).

if firms interact in independent markets the aggregate industry profits as well as the social welfare are highest under the complete network.

Our results on welfare and profit properties of collaboration may provide some explanation as to why a large number of strategic alliances are unstable or are terminated early and also why some alliances work well.[10] In highly competitive environments, firms would 'collectively' prefer not to form many collaborative ties, since in this way they could attain higher profits. However, a pair of individual firms would gain competitive advantage over the rivals by forming a collaboration and thus increase their profits. This implies that firms may have incentives to form too many links and consequently achieve a poor overall performance.[11] In the case of independent markets, this dilemma does not appear because firms' R&D effort is not declining with the number of links. Seen from another perspective, the results suggest that firms should be more successful in sustaining collaboration in independent markets.[12] Similar considerations should apply in differentiated product markets.

In the first part of the paper we restrict attention to symmetric network structures. In such structures, every firm is ex-post in a similar situation. One of the primary motivations for firms to form collaboration alliances is to gain competitive advantage vis-a-vis their rivals (Hagedoorn and Schakenraad, 1990). We next examine the role of collaborations in generating such competitive advantages and their influence on market structure and industry performance. This motivates an examination of *asymmetric networks*. A general analysis of asymmetric networks however turns out to be very complicated. To gain some insight we work with an example of three firms and completely characterize the solution to this model.

We show that asymmetric networks such as the *star* or the *unconnected* network perform quite well from the social as well as from the private point of view.[13] Indeed, the star network always dominates the complete network from both perspectives and moreover, for some parameter values, the star is both profit and welfare maximizing.[14]

[10] For empirical work on these issues see, for instance, Harrigan (1988b), Kogut (1988, 1989), Levine and Byrne (1986), Mooney (2000), Parkhe (1993) and Porter (1987).

[11] This is in line with comments in Axelrod, Mitchell, Thomas, Bennet and Bruderer (1995) concerning standard-setting alliances.

[12] These results are consistent with the empirical findings of Roller, Tombak and Siebert (1998) on the relative success of research joint ventures between non-competing firms.

[13] The star is a network in which there is a central firm which is directly linked to every other firm, while none of the other firms have a direct link with each other. In our 3 firm example, the unconnected network refers to a configuration in which two firms have a link while the third firm is isolated.

[14] These results may help explain the empirically observed pattern of dominant firms at the center of stars, with the other firms at the spokes (see e.g., Delapierre and Mytelka, 1998, Figures 4.3 and 4.4. Marín, Siotis and Hernán, 2000, also find large firms being more active in EU sponsored R&D collaborative programs).

We also find that asymmetric forms of collaboration may alter the market structure by causing large disparities between firms, or even leading to the exit of firms and, that this is not necessarily detrimental from a social stand-point. Indeed, under certain circumstances, the unconnected network is strategically stable, and both industry-profit and social welfare maximizing.

Our paper should be seen as a contribution to the study of group formation and cooperation in oligopolies. Our approach to strategic collaborations is inspired by recent work on strategic models of network formation, see e.g. Aumann and Myerson (1989), Bala and Goyal (1999), Dutta, van den Nouweland and Tijs (1995), Goyal (1993), Goyal and Joshi (1999), Jackson and Wolinsky (1996) and Kranton and Minehart (1999).

The work of Kranton and Minehart (1999) deals with networks between vertically related firms. In contrast, our paper studies collaborative ties between horizontally related firms, i.e., firms which compete in the market subsequently. This leads us to incorporate an explicit market competition element in our collaboration model. Our paper should be seen as complementary to their work. The results of our analysis suggest that the nature of market competition has major implications for the nature of collaboration networks as well as the overall welfare effects of collaboration.

We next discuss the relationship with Goyal and Joshi (1999). Their paper also studies networks of collaboration between oligopolistic firms. The principal difference concerns the way we model R&D effort. In their paper it is assumed that a collaboration link between two firms induces an exogenously specified cost reduction for both the firms. By contrast, in the present paper we study the impact of collaboration relations on the incentives of firms to expend effort and lower their own costs. Thus the issues examined in the two papers are quite different. We would like to draw out one particular implication of endogenizing the effort level. In the context of a linear Cournot model, they find that the complete network is strategically stable as well as efficient. By contrast, we find that the complete network is stable but not efficient. This difference in results highlights the importance of endogenizing the level of R&D effort.

Issues relating to group formation and cooperation have long been a central concern of economic theory, and game theory in particular. The traditional approach to these issues is in terms of coalitions. In recent years, there has been considerable work on coalition formation in games; see e.g., Bloch (1995), Yi (1998a,1998b) and Yi and Shin (2000). For a survey of this work, refer to Bloch (1997). One application of this theory is to the formation of groups in oligopolies. In this literature, group formation is modeled in terms of a *coalition structure* which is a partition of the set of firms. Each firm therefore can belong to one and only one element of the partition, referred to as a *coalition.*

In our paper, we consider two-player relationships. In this sense, our model is somewhat restrictive as compared to the work referred to above, which allows for groups of arbitrary size. However, the principal distinction concerns the nature of collaboration structures we examine. Our approach accommodates collaborative relations that are *non-exclusive*. Both from a conceptual as well as a descriptive point of view, this distinction is substantive. In particular, we allow for relationships across coalitions and this leads us to a class of cooperative structures - which includes stars and arbitrary symmetric networks – which is significantly different from those studied in the coalition formation literature. Moreover, the network approach allows a simple measure – the number of links of a firm in a symmetric network – of the level of collaboration activity. We exploit this measure to study the effects of collaboration on R&D.

Our model is also related to the literature on R&D cooperation in oligopoly.[15] In the terminology of Kamien *et al.* (1992) our model is a Research Joint Venture competition type of model, where firms forming a collaboration commit to completely share the R&D results arising from research efforts decided unilaterally. This literature typically compares the properties of the grand coalition RJV with the autarchy situation, both in terms of social efficiency as well as from the point of view of industry profits. Perhaps the main contribution of our paper to this literature is the finding that social welfare as well as industry profits are maximized at intermediate levels of collaboration. This finding also brings out the methodological contribution of our paper: we obtain this result in a strategic model of networks, which allows consideration of a very general class of collaborative relationships.

Our result potentially has implications for policy: When market rivalry is substantial, there is a large set of networks of collaboration which dominate both the empty network and the complete network in terms of social efficiency as well as on the basis of industry profits. We feel that these networks may be implementable with an appropriate set of transfers. The work of Kamien *et al.* (1992) suggests that RJV competition type of collaborative agreements are likely to be welfare reducing and thus should be dealt with by competition authorities with caution. Our results suggest that this prescription may need to be reconsidered.

The rest of the paper is organized as follows. The model is presented in section 2. In section 3 we present the results on symmetric networks. Section 4 explores the role of asymmetric networks and knowledge spillovers. Section 5 concludes.

2 The model

We consider a three stage game. In the first stage, firms form pair-wise collaboration links. In the second stage, each firm chooses a level of effort in R&D which lowers the cost of production of the firm. This effort also has positive spillovers and helps reduce

[15] See the seminal contributions of d'Aspremont and Jacquemin (1988), Kamien *et al.* (1992), Katz (1986) and Suzumura (1992).

the costs of other firms. If two firms have a collaboration link, then the spillovers are assumed to be perfect. The R&D efforts define the costs of the firms. In stage three, the firms operate in the market, taking as given the costs of production.

We are interested in the networks of collaboration that emerge in two different settings. We will first study collaborations in a market with strong firms rivalry. As a case of strong competition, we will consider a homogeneous product oligopoly market with quantity-setting firms. We shall then study collaborations in a market with weak rivalry. As an extreme case of reduced competition, we will consider the case of independent producers.[16] We will also examine the effects of collaboration links on the nature of market outcomes. Finally, we shall compare stable collaboration networks with efficient networks. We now develop the notation and define our notions of stability and efficiency.

2.1 Networks

For any pair of firms $i, j \in N$, the pair-wise relationship between the two firms is represented by a binary variable $g_{ij} \in \{0, 1\}$. When $g_{ij} = 1$, this means that the two firms are linked, while $g_{ij} = 0$ refers to the case of no link. A network g is then a collection of links, i.e. $g = \{g_{ij}\}_{i,j \in N}$. Let $g - g_{ij}$ denote the network obtained by severing an existing link between firms i and j from network g, while $g + g_{ij}$ is the network obtained by adding a new link between firms i and j in network g.

2.2 Effort levels and spillovers

Given a network g, every firm chooses a R&D effort level. This effort helps lower its own cost of production. Individual efforts also have spillovers on the costs of other firms. We assume that these spillovers are positive. Moreover, we assume that if two firms have a collaboration link then this spillover is perfect, while if they do not have a collaboration link then this spillover is imperfect. Let $\beta \in [0, 1)$ be a parameter which reflects the level of spillovers among firms with no collaboration links. Note that $\beta = 1$ for linked firms. Also, let $N_i(g)$ be the set of firms which are linked to firm i in network g. In the first part of the paper we shall study the case where $\beta = 0$, i.e., there will be no knowledge spillovers between firms that do not collaborate. In section 4 we shall examine the role of spillovers.

We assume that firms are initially symmetric, with zero fixed costs and identical constant marginal costs $\bar{c}$. R&D efforts help lower these marginal costs. Given a network g and the collection of effort levels $\{e_i(g)\}_{i \in N}$ the effective cost of firm i is

[16] We also considered a slightly more general model with a product differentiated parameter. The analysis of that model yielded similar insights. The additional parameter made the computations cumbursome and so we decided to restrict attention to these two limiting cases.

given as follows:

$$c_i(\{e_i(g)\}_{i\in N}) = \bar{c} - \sum_{k\in N_i(g)} e_k - \beta \sum_{l\notin N_i(g)} e_l$$

The total cost reduction for firm i stems from its own research e_i, and the research knowledge of other firms, which is either fully absorbed, in the case of existing collaborative links, or partially absorbed, in the case of spillovers. We refer to this total cost reduction as *effective* R&D level. We shall assume that R&D effort is costly. Formally, this is modelled via a cost of effort function. Given a level e_i of effort, the cost of effort is $Z(e_i) = \gamma e_i^2$, $\gamma > 0$. Under this specification, the cost of R&D effort is an increasing function and exhibits decreasing returns. The parameter γ measures the curvature of the cost of effort function. Throughout, we shall assume γ to be sufficiently large so that the firms' decision problems have interior solutions.

2.3 Payoffs

A network of collaboration g leads to a vector of R&D efforts $\{e_i(g)\}_{i\in N}$, which in turn defines the firms' production costs $\{c_i(g)\}_{i\in N}$. Given these marginal costs, firms operate in the market by choosing quantities $\{q_i(g)\}_{i\in N}$. The demand is assumed to be linear and given by $Q = a - p$, $a > \bar{c}$.[17] In the homogeneous good market with quantity-setting firms, $Q = \sum_{i=1}^{N} q_i$. Thus, the profits of firm i in collaboration network g are given by

$$\pi_i(g) = \left[a - q_i(g) - \sum_{j\neq i} q_j(g) - c_i(g)\right] q_i(g) - \gamma e_i^2(g). \tag{1}$$

In the independent market case each firm faces demand $q_i = a - p$ and the profits of firm i in collaboration network g are $\pi_i(g) = [a - q_i - c_i(g)]\, q_i - \gamma e_i^2(g)$.

2.4 Stability and efficiency

We shall say that a network g is *stable* if and only if for all $i, j \in N$:

(i) If $g_{ij} = 1$, then $\pi_i(g) \geq \pi_i(g - g_{ij})$ and $\pi_j(g) \geq \pi_j(g - g_{ij})$

(ii) If $g_{ij} = 0$ and $\pi_i(g + g_{ij}) > \pi_i(g)$ then $\pi_j(g + g_{ij}) < \pi_j(g)$.

This definition of stability, which is taken from Jackson and Wolinsky (1996), is quite weak and should be seen as a necessary condition for strategic stability.

[17] This is admittedly a special setting. We find that even in this simple case, a complete analysis of the interaction between markets and R&D networks is quite complicated and several issues arise that cannot be completely resolved. We intend to take up a more general market specification in future work.

For any network g social welfare is defined as the sum of consumer surplus and producers' profits. We shall say that a network g is *efficient* if and only if $W(g) \geq W(g')$ for all g'.

For future reference, let $Q(g) = \sum_{i \in N} q_i(g)$ be the aggregate output in the homogeneous product oligopoly with quantity setting firms. Let $W(g)$ denote aggregate welfare in network g. In this case, it is easily seen that

$$W(g) = \frac{Q(g)^2}{2} + \sum_{i=1}^{N} \pi_i(g). \tag{2}$$

When firms operate in independent markets social welfare is

$$W(g) = \sum_{i=1}^{N} (q_i^2(g)/2 + \pi_i(g)).$$

3 Symmetric networks

We focus on *symmetric* networks here; these are networks where every firm has the same number of collaboration links.[18] Formally, let $N_i(g)$ be the set of firms with which firm i has a collaboration link in network g. Let $\eta_i(g)$ be the cardinality of set $N_i(g)$. We will focus on symmetric networks, i.e. networks where $\eta_i(g) = \eta_j(g) = \eta(g)$ for any two firms i and j. The number η_i will be referred to as the *degree* of network g, or equivalently, as the *level of collaborative activity.*[19]

3.1 Homogeneous product oligopoly

Our first set of results pertains to the effects of collaboration on R&D effort of individual firms. We proceed backwards, in the game defined above.
Market Outcome: In the market competition stage, we note that given a cost configuration of firms, $\{c_i(g)\}_{i \in N}$, the equilibrium quantity of firm i in a homogeneous product oligopoly is given by

$$q_i(g) = \frac{a - nc_i(g) + \sum_{j \neq i} c_j(g)}{n+1}, \tag{3}$$

and the profits of the Cournot competitors are given by

$$\pi_i(g) = \left(\frac{a - nc_i(g) + \sum_{j \neq i} c_j(g)}{n+1} \right)^2 - \gamma e_i^2(g). \tag{4}$$

[18] In section 4 below, we shall analyze some asymmetric networks, such as stars and unconnected networks.

[19] If the number of firms is even, then there is always a set of links l such that the resulting network is symmetric of degree k, where $k = 0, 1, ..., n-1$.

With this profit expression in hand, we can derive the equilibrium level of R&D effort by firms in a symmetric network of degree k, i.e. $\eta_i = k$ for all i. To compute the payoff to the representative firm i we note that there are three types of firms in a network g of degree k (g^k from now on), namely, (i) the firm i, (ii) k firms linked to firm i, which we represent with subscript l, and (iii) $n-k-1$ firms not linked to firm $i$which we represent with subscript m.

R&D Efforts: We only consider symmetric solutions. Let e_l denote the R&D effort undertaken by a firm linked to firm i and e_m be the corresponding effort of firms not linked to firm i. Then the cost structure of the different Cournot competitors is as follows:

$$\begin{aligned} c_i(g^k) &= \bar{c} - e_i - ke_l \\ c_l(g^k) &= \bar{c} - e_l - \sum_{j \in N_l(g)} e_j \\ c_m(g^k) &= \bar{c} - e_m - \sum_{j \in N_m(g)} e_j \end{aligned} \tag{5}$$

Plug these costs into profits function in (4), take the first order condition and invoke symmetry of effort levels. After some computations, this yields the following equilibrium level of effort:[20]

$$e(g^k) = \frac{(a-\bar{c})\,(n-k)}{\gamma(n+1)^2 - (n-k)(k+1)}. \tag{6}$$

Our first observation is that this equilibrium level of R&D effort is declining in the degree k of the network of collaboration g^k. We establish this by computing

$$e(g^k) - e(g^{k+1}) = \frac{(a-\bar{c})\,[\gamma(n+1)^2 - (n-k)(n-k-1)]}{[(n+1)^2 - (n-k)(k+1)]\,[(n+1)^2 - (n-k-1)(k+2)]\,.} \tag{7}$$

We note that the numerator is positive (from the second order condition), while the denominator is positive since $k \leq n-1$. This leads us to state the following result:

Proposition 3.1 *Suppose firms are competitors in a homogeneous product market. Then R&D effort of a firm is decreasing in the level of collaborative activity.*

We observe that the marginal cost to an increase in R&D effort is an increasing function of e_i with slope $\gamma(n+1)^2$. Observe further that this function is independent of k. The marginal revenue to an increase in R&D effort is also an increasing function of e_i, but with the smaller slope $(n-k)^2$ (see second order condition).

[20]Maximization requires the second order condition of this problem to be satisfied, i.e. $(n-k)^2 - \gamma(n+1)^2 < 0$, for all $k = 0, ..., n-1$. The condition $\gamma > n^2/(n+1)^2$ ensures maximization for all k.

Thus, Proposition 3.1 implies that the marginal return to an increase in research and development effort is, at least 'locally', declining in k. The intuition behind this result is that even though a firm's research effort helps reduce its own production cost, it also lowers rivals' costs, which makes them tougher competitors. The imperfect appropriability of R&D effort allows a firm's collaborator to gain a competitive advantage. This detrimental effect is traded off against the incentives to conduct research to reduce own marginal costs in a way such that R&D effort declines with the number of collaborators.

We now examine the nature of cost reduction under different levels of firm collaboration. We find that the level of effort and the nature of spillovers under collaboration interact in an interesting manner and generate a non-monotonic relationship between cost reduction and the degree of the network. By substituting (6) into the cost structure (5), we obtain

$$c(g^k) = \frac{\bar{c}\gamma(n+1)^2 - a(n-k)(k+1)}{\gamma(n+1)^2 - (n-k)(k+1)}.$$

From the difference,

$$c(g^k) - c(g^{k+1}) = \frac{(a-\bar{c})\gamma(n+1)^2(n-2k-2)}{[\gamma(n+1)^2 - (n-k)(k+1)]\,[\gamma(n+1)^2 - (n-k-1)(k+2)]},$$

we can see that an increase in the level of collaborations reduces the cost of the firms if and only if $k < n/2 - 1$.

Proposition 3.2 *Suppose firms are competitors in a homogeneous product market. Then the relationship between cost reduction and the level of collaborative activity is non-monotonic. Moreover, cost reduction is maximum when each firm is linked with roughly half of the other firms.*

This result shows that in relatively less dense networks the cost-reducing benefits of an extra collaboration are substantial as compared to the detrimental effects arising from the induced decrease in the R&D activity of the firms (Proposition 3.1). When the network is relatively dense, the latter effect dominates and an increase in the level of collaborative activity results in an increase in the firms' operating costs. In other words, 'effective' R&D exhibits a non-monotonic relationship with respect to the density of the network.

Strategic Stability: We now examine the nature of stable networks. The profits attained by the representative firm in a symmetric network of degree k can be obtained by substituting the equilibrium level of effort into (4):

$$\pi(g^k) = \frac{(a-\bar{c})^2\,[\gamma^2(n+1)^2 - (n-k)^2]}{[\gamma(n+1)^2 - (n-k)(k+1)]^2}. \tag{8}$$

An equilibrium requires first-stage profits to be non-negative. This imposes the restriction $\gamma > (n-k)/(n+1)$ for all $k = 0, ..., n-1$.[21]

Our first observation is that firms always have an incentive to form pair-wise collaborative R&D agreements. We prove this by showing that the empty network is not stable.

Proposition 3.3 *Suppose firms are competitors in a homogeneous product market and $\gamma = 1$. Then the empty network g^e is not stable.*

The proof is given in Appendix A. In Propositions 3.3 - 3.6 we assume that $\gamma = 1$. This allows us to explicitly compare the relevant expressions. However, we firmly believe that the results are true for general values of γ; in the general case, the expressions are complicated and clear comparisons are difficult, so we use a series of plots on the behaviour of profits and welfare. These are given in Appendix A also.

Our second result about stability of symmetric networks is that the incentives to form collaborations are quite large. We show this by proving that the complete network is stable.

Proposition 3.4 *Suppose firms are competitors in a homogeneous product market and $\gamma = 1$. Then the complete network is stable.*

The proof is given in Appendix A.

Aggregate Performance: We now examine the relationship between aggregate performance and the level of collaboration among firms in an industry. We show that aggregate profits under the complete network are relatively low. To establish this we compare the profits a firm obtains under the complete network (the network of degree $k = n-1$) with the benefits the same firm would obtain in a network of one degree less, i.e., $k = n-2$. The profits under the complete network, g^{n-1}, are

$$\pi(g^{n-1}) = \frac{(a-\overline{c})^2 \left[\gamma^2(n+1)^2 - 1\right]}{\left[\gamma(n+1)^2 - n\right]^2},$$

while the benefits under a network of degree $n-2$ are

$$\pi(g^{n-2}) = \frac{(a-\overline{c})^2 \left[\gamma^2(n+1)^2 - 4\right]}{\left[\gamma(n+1)^2 - 2(n-1)\right]^2}.$$

In Appendix A we show that

$$\pi(g^{n-2}) - \pi(g^{n-1}) > 0,$$

[21] The condition $\gamma > n/(n+1)$ thus suffices.

which implies that the complete network yields lower profits than a symmetric network of one degree less.

More generally, we note that profits do not behave monotonically with respect to the level of collaborative activity. Given the above observation, it is sufficient to show that firms obtain higher benefits under the complete network as compared to the empty network ($k = 0$):

$$\pi(g^{n-1}) - \pi(g^0) = \frac{(a-\overline{c})^2(n^2-1)}{\left[\gamma(n+1)^2-n\right]^2} > 0$$

Proposition 3.5 *Suppose firms are competitors in a homogeneous product market and $\gamma = 1$. Then there exists an intermediate level of collaborative activity $\overline{k}$ with $0 < \overline{k} < n-1$ for which firms profits are maximized.*

The intuition behind this result may be seen with the help of an example. Figure 1 depicts firms profits in an industry with 16 firms for symmetric networks with different degrees of collaboration k (parameters are $a - \overline{c} = 1$, $\gamma = 1$). At the left-end of the horizontal axis we have the case of no collaborations, i.e. empty network. At its right-end we have the case where each firm collaborates with every other firm, i.e. the complete network. It can be seen that profits are lowest under the empty network in this case. As the number of links rises, the performance of firms improves up to a point after which collaborative agreements lead to lower firms profits. Thus, the graph shows how the result about collaboration and costs in Proposition 3.2 interplays with the fact that a higher level of collaboration leads to lower dupplication of R&D and thus lower research costs, and how this translates into firms' profits.

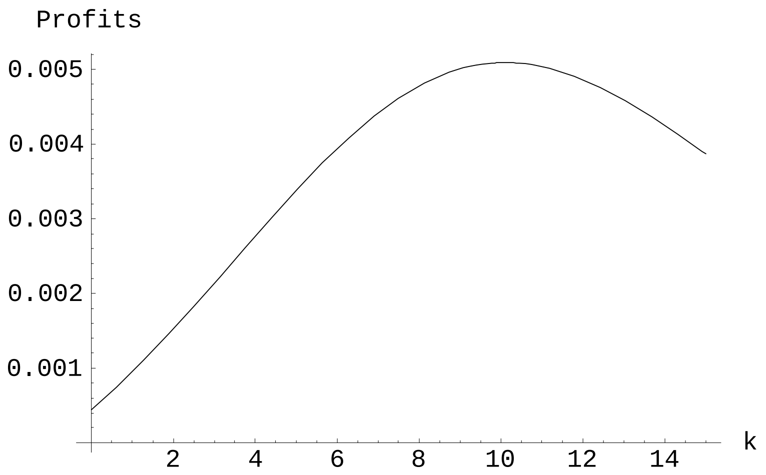

Figure 1: Firm profits in a symmetric network of degree k (n=16).

For further illustration, we can use (8) to see that in this example (for $\gamma = 1$), for any given number of firms n, the number of links k that maximizes aggregate profits satisfies the equation:

$$k^3 - 3k^2n - 3k(1 + 2n) + (n^3 + 3n^2 - 1) = 0 \tag{9}$$

The solution to this equation exhibits the property that $dk/dn > 0$, i.e. the number of collaborations maximizing aggregate profits is greater in larger markets. Table 1 reports the solution to (9) for different market sizes. It shows that, in the class of symmetric networks, profits are maximum when every firm maintains approximately $k = (2n - 1)/3$ links.

$n =$	4	6	8	10	12	14	16	18	20
$k \simeq$	2.26	3.57	4.87	6.18	7.48	8.79	10.10	11.40	12.71

Table 1: Profit maximizing symmetric networks (Example $\gamma = 1$).

We now consider the social welfare aspects of collaboration networks. The welfare attained under a network of collaboration of degree k is given by[22]

$$W(g^k) = \frac{(a - \bar{c})^2 n \left[\gamma^2 (n + 2)(n + 1)^2 - 2(n - k)^2\right]}{2\left[\gamma(n + 1)^2 - (n - k)(k + 1)\right]^2}. \tag{10}$$

Our first finding is that the empty network is not efficient. We prove this by showing that welfare under the complete collaboration agreement is higher than under absence of collaborations. Secondly, since neither industry profits nor cost reductions are the highest under the complete collaboration arrangement, this raises questions about the efficiency of the complete network. Indeed, we find that the complete network generally leads to too little effort and therefore is not socially efficient.

Proposition 3.6 *Suppose firms are competitors in a homogeneous product market and $\gamma = 1$. Then there exists some intermediate level of collaborative activity $\widetilde{k}$ with $0 < \widetilde{k} < n - 1$ for which social welfare is maximized.*

What is the structure of efficient symmetric networks? For any given number of firms n, an efficient symmetric network of collaboration has k links such that $W(g^k)$ is maximized. Solving that problem is in general difficult because the polynomials involved in the first order condition are somewhat intractable. However, we would like to illustrate that efficient networks resemble cost minimizing networks rather than stable networks by means of an example. Suppose that $\gamma = 1$. Using the

[22] Refer to the proof of Proposition 3.6 in the Appendix for details.

expression for welfare in (10), after some algebra one can show that for any given number of firms n, the efficient network has k links where k satisfies

$$2k^3 - 6k^2 n - 2kA + B = 0, \tag{11}$$

with $A = n^3 + 2n^2 + 7n + 3$ and $B = n^4 + 3n^3 + 5n^2 - n - 2$. A careful analysis of this implicit equation shows that $dk/dn > 0$, i.e. symmetric efficient networks are greater the larger the market. Table 2 reports the solution to equation (11), which suggests that for any even number of firms n, the symmetric efficient network has roughly $k = n/2$ links, which corresponds quite closely to the symmetric cost minimizing networks.

$n =$	4	6	8	10	12	14	16	18	20
$k \simeq$	1.80	2.82	3.83	4.84	5.84	6.84	7.85	8.85	9.85

Table 2: Efficient symmetric networks (Example $\gamma = 1$).

A further consequence of the previous observations on market performance is that industry-profit maximizing symmetric networks need not be social welfare maximizing. This remark is in line with Yi (1998a) and Yi and Shin (2000).

3.2 Independent markets

In the analysis so far we have assumed that firms compete in the market by choosing quantities and that the product is homogeneous. In this section we will examine the impact of collaborative links on individual R&D effort and subsequently the effect of market competition on firms' incentives to form links. With this aim in view, we will now assume that each firm is a local monopolist, in its own market. This implies that individual R&D effort has no implications for the level of market competitiveness of potential collaborators. This setting allows us to isolate the pure effects of collaboration, since in this setting firms do not have to worry about the competitive effects of collaborative work. Here we find that collaboration between firms increases the level of effort by individual firms. Thus individual R&D efforts are strategic complements. Moreover, every pair of firms has an incentive to form links and the complete network is the unique stable and efficient network. Seen together with the earlier results, this suggests that we should see more collaboration among firms that share similar production technologies but operate in highly differentiated product markets or in distinct markets.

Market Outcome: Given a network g, and the R&D efforts levels $\{e_i(g)\}_{i=1}^n$, firms choose quantities to maximize their monopoly profits. Standard derivations show that equilibrium quantities are

$$q_i(g) = \frac{a - c_i(g)}{2}, \tag{12}$$

and that firms obtain profits

$$\pi_i(g) = \left[\frac{a - c_i(g)}{2}\right]^2 - \gamma e_i^2(g). \tag{13}$$

R&D efforts: In the second stage of the game, firms choose their R&D effort levels to maximize the reduced-form profits (13). The costs $c_i(g)$ depend on the effort levels undertaken by the firms, which in turn are a function of the existing network. We again consider symmetric networks of degree k.

A firm i that forms k collaborations has a cost $c_i(g^k) = \overline{c} - e_i - ke_l$, where e_l denotes the R&D effort level of every firm linked to firm i. Plugging this cost into firm profits function in (13) we obtain

$$\pi_i(g^k) = \frac{[a - \overline{c} + e_i + ke_l]^2}{4} - \gamma e_i^2.$$

The first order condition is[23]

$$\frac{a - \overline{c} + e_i + ke_l}{4} - \gamma e_i = 0.$$

We note that efforts of linked firms are strategic complements. Invoking symmetry and solving we obtain[24]

$$e(g^k) = \frac{(a - \overline{c})}{4\gamma - k - 1}. \tag{14}$$

This yields us the following result.

Proposition 3.7 *Suppose firms operate in independent markets. Then individual R&D effort is increasing in the level of collaborative activity.*

Note also that effort is in this case independent of n, which reflects the fact that firms do not compete for the same consumers. Given the effort (14), we now examine the nature of cost reduction under different levels of firm collaboration. We find that the level of firm costs are monotonically decreasing with respect to the degree of the network. In fact,

$$c(g^k) = \overline{c} - \frac{(a - \overline{c})(k + 1)}{4\gamma - k - 1},$$

which clearly declines with k.

[23] The second order condition for maximization requires $\gamma > 1/4$.

[24] Since effort levels must be non-negative, it must be the case that $\gamma > (k + 1)/4$ for all $k = 0, ..., n - 1$. Thus, the condition $\gamma > n/4$ suffices for that.

Proposition 3.8 *Suppose firms operate in independent markets. Then costs are monotonically decreasing with respect to the level of collaboration among firms.*

The profits attained by the representative firm in a symmetric network of degree k can be obtained by plugging (14) into (13):

$$\pi(g^k) = \frac{(a-\bar{c})^2\gamma(4\gamma-1)}{(4\gamma-k-1)^2}. \tag{15}$$

Clearly profits are increasing in the level of collaborative activity.

Strategic Stability: We next examine the nature of stable networks. It is trivial to show that, in the class of symmetric networks, the only stable network is complete. This can be seen by noting that for all $k = 0, 1, ..., n-2$, any pair of firms can increase their profits by linking to each other. This statement can indeed be made general by noting that for any network g, whether symmetric or asymmetric, the R&D efforts of the (linked) firms are strategic complement variables. Thus, no firm would reduce its effort should two other firms form a link with each other, which suggests that the unique stable network when firms operate in different markets is complete. Substituting $k = n-1$ into the expressions above we get efforts, quantities and profits of the firms under the complete collaborative arrangement:

$$e(g^{n-1}) = \frac{a-\bar{c}}{4\gamma-n},\ q(g^{n-1}) = \frac{2\,(a-\bar{c})\,\gamma}{4\gamma-n},\ \pi(g^{n-1}) = \frac{(a-\bar{c})^2\,\gamma(4\gamma-1)}{(4\gamma-n)^2} \tag{16}$$

Proposition 3.9 *Suppose firms operate in independent markets. The complete network is strategically stable.*

The computations underlying this result are straightforward and are thus omitted.

Our final result looks at the efficiency aspects of collaboration networks in this extreme case of reduced rivalry between the firms. Computing welfare in a symmetric network of degree k is straight forward:

$$W(g^k) = \frac{n(a-\bar{c})^2\gamma(6\gamma-1)}{(4\gamma-k-1)^2}.$$

Welfare is clearly increasing in the number of collaborations k. The previous observation that efforts are strategic complement variables allows us to state:

Proposition 3.10 *Suppose firms operate in independent markets. The complete network is uniquely efficient in the class of symmetric networks.*

Seen together, the different results obtained in the distinct contexts of Cournot oligopoly and independent markets yield a set of observations. First, we note that firms generally have an incentive to collaborate, irrespective of the product market setting, so the empty network is never incentive-compatible. Second, while in the independent market case the firm incentives to undertake R&D effort increase with the number of collaborations each firm maintains, in the homogeneous product case these incentives decline. This is due to the fact that in the latter context, collaborations bring about detrimental business-stealing effects which dampen the firm incentives to invest in cost-reducing activities. This different firm behavior in the distinct contexts has important consequences from the point of view of social welfare. It turns out that the private incentives to collaborate are aligned with the social ones if firms operate in independent markets. In contrast, when firms are competitors, the private incentives may lead to an excessive degree of collaboration, as shown by the fact that the complete network is stable but never efficient. In the latter case, the complete network is not efficient because firms anticipate a substantial business-stealing effect and undertake very little effort in R&D.

4 Asymmetric networks and knowledge spillovers

In the analysis so far, we have restricted attention to symmetric R&D networks for the most part and also assumed that knowledge spillovers across unlinked firms are absent. In this section we model knowledge spillovers and also allow for asymmetric networks. The aim of this section is threefold. First, we show that asymmetric networks play an important role in the nature of collaboration because they may be industry-profit as well as welfare maximizing. Second, we see that asymmetric forms of collaboration may substantially alter the market structure by causing significant disparities between firms and, in extreme cases, even the exit of firms. We shall also find, somewhat surprisingly, that this is not necessarily detrimental from a social perspective. Finally, we investigate how substantial knowledge spillovers across firms interact with firms' incentives to form collaborative agreements and their consequences from a social stand-point. Undertaking this analysis in general is quite complicated. So we have chosen to restrict ourselves to the simplest possible case which allows us illustrate our points.

We consider a three firm market for a homogeneous good similar to the one presented in Section 3 with the following additional features: On the one hand, we shall assume that there is a positive spillover also between firms that have no collaboration link. We measure the extent of general spillovers via the parameter $\beta \in [0, 1]$. On the other hand, we assume $\gamma = 1$, which suffices to ensure non-negativity of all variables. There are four possible network architectures in this case: (i) the complete network, g^{n-1}, in which every pair of firms is linked, (ii) the star network, g^s, in which there is one firm which is linked to the other two firms, (iii) the unconnected network, g^u, in which two firms have a link and the third firm is isolated, and (iv) the empty network, g^e, in which there are no collaboration links. We present these networks in Figure 2 below.

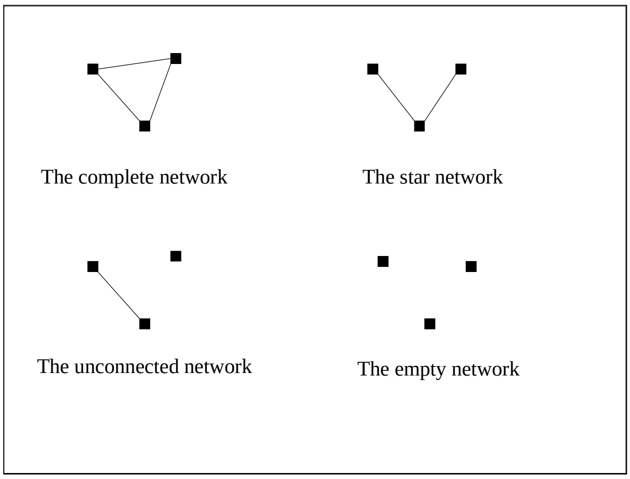

Figure 2

We first work out the market equilibrium for a given distribution of costs across firms. Given a network of collaboration, we then compute the equilibrium in R&D efforts. This allows us to state the level of R&D effort, the market equilibrium and the profits for every network. We then use these values to derive the incentives of firms to form collaborative relationships. This tells us which networks are stable from a strategic point of view. Finally, we examine the nature of efficient networks, from the perspective of firms as well as from the point of view of society at large.

Market Competition: Given the network g and the R&D efforts levels $\{e_1(g), e_2(g), e_3(g)\}$, firms choose quantities to maximize their profits in (1). Standard derivations show that

$$q_i = \left[a - 3c_i(g) + \sum_{j \neq i} c_j(g)\right] /4, \quad i, j = 1, 2, 3. \tag{17}$$

R&D efforts: In the second stage of the game firms choose their R&D effort levels to maximize the reduced-form profits

$$\pi_i(g) = \left[a - 3c_i(g) + \sum_{j \neq i} c_j(g)\right]^2 /16 - e_i^2(g). \tag{18}$$

As before, the costs $c_i(g)$, $i = 1, 2, 3$ depend on the effort levels undertaken by the firms, which in turn are a function of the existing network. We consider the different networks now and derive the equilibrium levels of effort, cost reduction and firm profits. The details of these computations are given in Appendix B.

We use these computations to study the stability of the different networks of collaboration. The following results will illustrate, first, that the complete network is generally stable. Also, that asymmetric networks may play an important role in markets with little knowledge spillovers. In particular, we shall see that the complete network is not the unique stable network when spillovers are not substantial: the unconnected network is also stable. Define $\widehat{\beta}$ as the solution to equation $\pi_l(g^u) = \pi_h(g^s)$.[25] Applying our notion of stable network presented above, the following result obtains:

Proposition 4.1 *(i) The complete network, g^{n-1}, is stable for all $\beta \in [0,1]$; (ii) The unconnected network, g^u, is stable for all $\beta \in [0,\widehat{\beta}]$; (iii) The star network, g^s, and the empty network, g^e, are never stable.*

Figure 3 plots the profit levels in the different networks. In this figure, $\pi_h(g^s)$ refers to the profits of the hub firm, while $\pi_s(g^s)$ refers to the profits of the spoke firms, respectively, in the star network. Similarly, $\pi_l(g^u)$ refers to the profits of the linked firms, while $\pi_i(g^u)$ refers to the profits of the isolated firm in the unconnected network. The other profit terms are self-explanatory. The details of the proof are given in Appendix B.

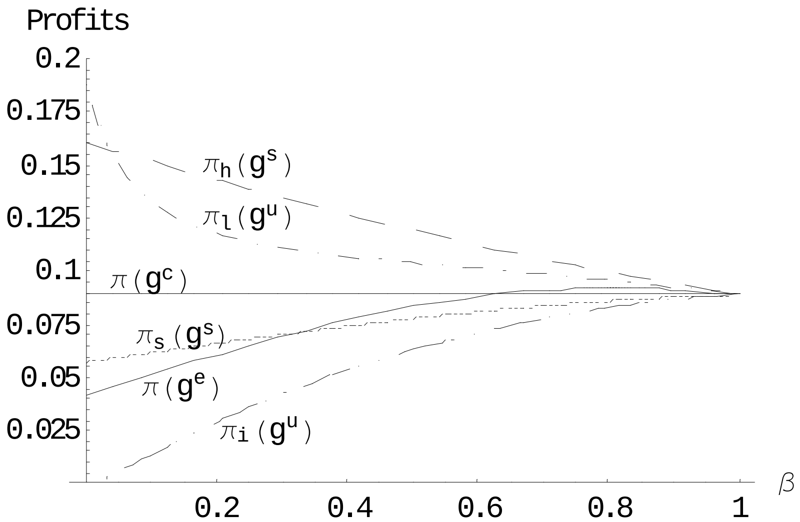

Figure 3: Profits of the different firms in the distinct networks.

The result that an asymmetric network such as the unconnected network is stable for some parameters indicates that stable collaboration agreements may alter the market structure substantially, either by driving isolated firms into a very large cost disadvantage or, in extreme cases, driving firms out of the market. Indeed, in the absence of spillovers, the unconnected network is stable and the isolated firm abandons the market by producing nothing. This result is in line with the empirically documented observation that firms often seek collaborative relationships in an attempt to alter market structure in their own benefit (Hagedoorn and Schakenraad, 1990). Seen from another perspective, the fact that firms without collaboration ties can easily be forced out of the market may be an important reason behind the recent

[25] Figure 3 below shows that $\widehat{\beta}$ exists and is unique.

observed proliferation of strategic alliances and interfirm collaboration agreements: *businesses seek partners as a survival strategy* (Delapierre and Mytelka, 1998).

Our final interest is related to the performance of general collaboration networks. We have already seen that in the presence of market rivalry, firms' private incentives to form collaborative links are not generally aligned with the social incentives. From a social perspective, collaborations have two effects working in opposite directions: on the one hand, denser networks lead to greater inter-firm spillovers and consequently minimum duplication of R&D occurs. On the other hand, well-connected firms undertake very little R&D effort in an attempt to minimize detrimental business-stealing effects from other firms.

We have seen that individual incentive considerations will lead firms either to form collaboration agreements with all other firms in the market, or to form the unconnected network. Using the profits expressions of the different firms under the distinct networks, one can easily graph aggregate profits under the different networks (Figure 4). In the graph, industry profits under the complete, empty, star and unconnected networks are represented by the flat solid line, the curved solid line, the dashed curve and the dotted curve, respectively. Interestingly, the complete network is dominated in terms of industry profits for all β. In particular, industry-profit maximizing networks are the unconnected, the star and the empty network for low, intermediate and high knowledge spillovers, respectively.

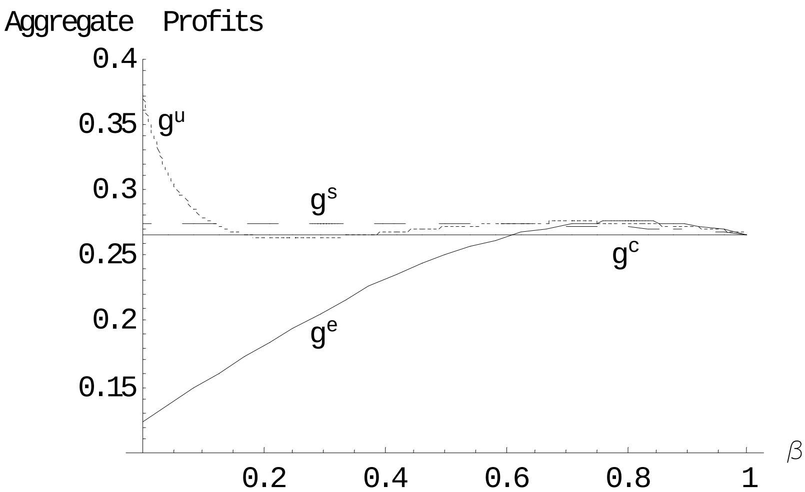

Figure 4: Industrial profits under different networks

Next we examine social welfare for every network g. As before, we shall say that network g is efficient if and only if there is no other network g' such that $W(g') > W(g)$.

To compute social welfare under the complete R&D network $W(g^{n-1})$ we plug equilibrium quantities and profits, into social welfare in (2). It obtains

$$W(g^{n-1}) = \frac{9(a-\overline{c})^2}{13}. \tag{19}$$

Analogously, by substituting equilibrium quantities and profits of the spoke firms, and the quantity and profit of the hub firm into (2) we obtain social welfare under the star network g^s:

$$W(g^s) = \frac{(a-\overline{c})^2(23\beta^4 - 138\beta^3 + 579\beta^2 - 1084\beta + 2492)}{(58 + 7\beta^2 - 13\beta)^2}. \tag{20}$$

Similarly we obtain social welfare under the unconnected network

$$W(g^u) = \frac{(a-\overline{c})^2(28 + 188\beta\beta + 449\beta^2 - 98\beta^3 - 239\beta^4 + 108\beta^5 - 12\beta^6)}{2(4 + 33\beta - 16\beta^2 + 7\beta^3 - 2\beta^4)^2}, \tag{21}$$

and finally under the empty network

$$W(g^e) = \frac{3(a-\overline{c})^2(31 + 12\beta - 4\beta^2)}{(13 - 4\beta + 4\beta^2)^2}. \tag{22}$$

Define $\underline{\beta}$ as the solution to equation $W(g^u) = W(g^s)$. Let $\overline{\beta}$ be the solution to equation $W(g^s) = W(g^e)$.[26] Now we are ready to state that:

Proposition 4.2 *For any fixed β there is a unique efficient network: (i) If $\beta \in [0, \underline{\beta}]$ the unconnected network g^u is efficient. (ii) If $\beta \in [\underline{\beta}, \overline{\beta}]$ the star network, g^s is efficient. (iii) If $\beta \in [\overline{\beta}, 1]$ the empty network g^e is efficient. Moreover, the star network g^s always dominates the complete network g^{n-1} in terms of efficiency.*

The result follows from comparing welfare under the different networks of collaboration (expressions (19), (20), (21) and (22)). We compare them by means of Figure 5. In the graph, welfare under g^{n-1}, g^s, g^u and g^e is represented by the flat line, the decreasing solid curve, the dashed curve and the dotted curve, respectively.

[26] Figure 5 below shows that $\underline{\beta}$ and $\overline{\beta}$ exist and are unique.

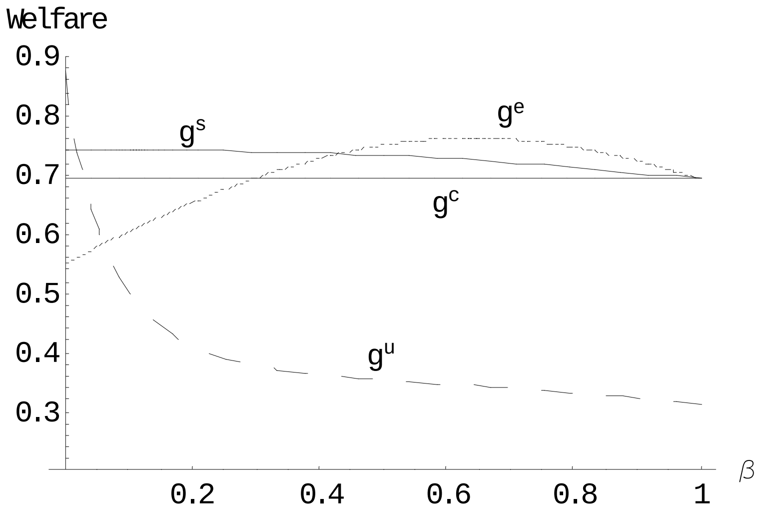

Figure 5: Welfare under the different collaborative networks.

It is clearly seen that the unconnected network dominates all other networks for low spillover parameters. When spillovers are moderate, it is the star network the arrangement which dominates all others. Finally, for large spillover parameters, social welfare is maximum under the empty network. The graph additionally shows that social welfare attained under the star network is always greater than under the complete network.

Proposition 4.2 shows that the private and the social incentives to form collaborations are in general misaligned. These results should be seen together with the previous remarks on stable networks and industry-profit maximizing networks. For low involuntary knowledge spillovers, the unconnected network is stable, socially efficient and profit-maximizing. For intermediate and high values of the spillover parameter, the star and the empty network respectively are efficient and profit maximizing, but not stable. Finally, the complete network, which is always stable, is neither profit-maximizing nor efficient.

5 Conclusion

There is a large body of work in business strategy and international business that describes the growing use of inter-firm collaboration in R&D activity. This literature has emphasized two distinctive features of collaborative relationships: they are often bilateral and non-exclusive.

This paper develops a strategic model of the formation of collaboration networks which incorporates these two structural features. The model allows us to define the level of collaborative activity naturally in terms of the number of links that a firm has in a collaboration network. We examine the impact of collaboration on cost reduction, industry profits and market structure. We then explore the incentives of firms to form collaborative agreements and the architecture of incentive compatible collaboration networks.

Our analysis highlights the interaction between market competition and R&D network structure. We find that if firms are Cournot competitors then individual R&D effort is declining in the level of collaborative activity. However, due to less duplication under collaboration, the level of collaboration has a non-monotonic relationship with cost reduction and related performance indicators such as industry profits and welfare. This suggests that an intermediate level of collaboration is desirable. We also identify strategic reasons for firms to form collaboration agreements. Such agreements improve the competitive position of collaborating firms and that this can generate very asymmetric market outcomes, with the non-collaborating firms exiting from the market. Moreover, under certain circumstances, such asymmetric collaboration relations are also strategically stable.

By contrast, if firms operate in independent markets then individual R&D effort is increasing in the level of collaborative activity. Cost reduction and social welfare are maximized under the complete network, which is also strategically stable.

Our analysis has been carried out in the simple setting of a linear demand model. We believe that the insights obtained are quite general, and hope to study the relationship between networks and markets in a more general oligopoly model in the future.

6 Appendix A

Proof of Proposition 3.3: We can derive the nature of efforts, quantities and profits in the empty network by setting $k = 0$ in the above expressions. For later reference, profits under the empty network are

$$\pi(g^e) = \frac{(a-\overline{c})^2 \left[\gamma^2(n+1)^2 - n^2\right]}{\left[\gamma(n+1)^2 - n\right]^2}. \tag{23}$$

Notice that the stability condition (i) is trivially satisfied because the empty network has no links formed at all. Thus, we only need to check that condition (ii) is satisfied. Consider that some firm i forms a link with some firm j. The resulting network is $g^e + g_{ij}$. We next show that firms i and j find such a deviation profitable. In $g^e + g_{ij}$ there are two types of firms: (i) firms i (and j), which maintain 1 link; and (ii) the rest $n-2$ firms, which have no link. We will look for symmetric solutions. Let us denote the research effort of the latter $n-2$ isolated firms as e_m, and the representative firm in this group as firm l. The costs of the different types of firms in network $g^e + g_{ij}$ are

$$\begin{aligned} c_i &= \overline{c} - e_i - e_j \\ c_l &= \overline{c} - e_l \end{aligned}$$

Using these cost expressions we can obtain firm i profits:

$$\pi_i(g^e + g_{ij}) = \frac{[a - \bar{c} + (n-1)(e_i + e_j) - e_l - (n-3)e_m]^2}{(n+1)^2} - \gamma e_i^2 \tag{24}$$

The first order condition is[27]

$$(n-1)[a - \bar{c} + (n-1)(e_i + e_j) - e_l - (n-3)e_m] - \gamma (n+1)^2 e_i = 0$$

Note that the R&D efforts of firms which form a collaboration become strategic complements.

The representative firm l in the group of firms with no links maximizes

$$\pi_l(g^e + g_{ij}) = \frac{[a - \bar{c} + ne_l - 2(e_i + e_j) - (n-3)e_m]^2}{(n+1)^2} - \gamma e_l^2.$$

The first order condition is[28]

$$n\,[a - \bar{c} + ne_l - 2(e_i + e_j) - (n-3)e_m] - \gamma (n+1)^2 e_l = 0$$

Invoking symmetry, i.e. $e_i = e_j$ and $e_l = e_m$, we obtain the R&D efforts of the different types of firms in the deviation network:

$$\begin{aligned} e_i(g^e + g_{ij}) &= \frac{(a - \bar{c})\,(n-1)(\gamma(n+1) - n)}{\gamma(\gamma-2)(n^3+1) + \gamma n(n+1)(3\gamma - 1) + 2n(n-1)} \\ e_m(g^e + g_{ij}) &= \frac{(a - \bar{c})\,n\,[\gamma(n+1) - 2n + 2]}{\gamma(\gamma-2)(n^3+1) + \gamma n(n+1)(3\gamma - 1) + 2n(n-1)} \end{aligned}$$

We can substitute these expressions into (24) to obtain deviating profits:

$$\pi_i(g^e + g_{ij}) = \frac{(a - \bar{c})^2\,\gamma\,[\gamma(n+1) - n]^2\,[(n^2+1)(\gamma - 1) + 2n(\gamma + 1)]}{[\gamma(\gamma-2)(n^3+1) + \gamma n(n+1)(3\gamma - 1) + 2n(n-1)]^2} \tag{25}$$

Now we set $\gamma = 1$ in (25) and (23). The result follows.[29]

$\triangle$

Proof of Proposition 3.4: We can derive the nature of efforts, quantities and profits in the complete network by setting $k = n - 1$ in the expressions above. For

[27] The second order condition requires that $\gamma > (n-1)^2/(n+1)^2$. The condition in footnote 20 ensures this.

[28] The second order condition in this case requires $\gamma > n^2/(n+1)^2$, which is also ensured by previous conditions.

[29] A discussion about the comparison between (25) and (23) for general γ is presented below.

later utilization, note that profits under the complete collaborative agreement are:

$$\pi(g^{n-1}) = \frac{(a-\overline{c})^2\left[\gamma^2(n+1)^2-1\right]}{\left[\gamma(n+1)^2-n\right]^2} \tag{26}$$

We only need to check that the stability condition (i) is satisfied because there is no additional link to be formed and so condition (ii) is trivially satisfied. Consider that some firm i deletes its link with some firm j. The resulting network is $g^{n-1}-g_{ij}$. We next show that firm i finds such a deviation unprofitable. In $g^{n-1}-g_{ij}$ there are two types of firms: on the one hand, firms i (and j), which have $n-2$ links; and on the other hand, the rest of the firms which have $n-1$ links. We will look for symmetric solutions. Let us denote the effort of the latter $n-2$ firms which maintain $n-1$ collaborations as e_m, and the representative firm in this group as firm l. The costs of the different types of firms are

$$\begin{aligned} c_i &= \overline{c}-e_i-(n-2)e_m \\ c_l &= \overline{c}-e_l-e_i-e_j-(n-3)e_m \end{aligned}$$

Using these cost expressions we can write down firm i profits as

$$\pi_i(g^{n-1}-g_{ij}) = \frac{\left[a-\overline{c}+2e_i-(n-1)e_j+(n-2)e_m\right]^2}{(n+1)^2}-\gamma e_i^2$$

The first order condition is[30]

$$2[a-\overline{c}+2e_i-(n-1)e_j+(n-2)e_m]-\gamma\,(n+1)^2\,e_i=0$$

Note that the R&D efforts of firms breaking a collaboration link become strategic substitutes.

The representative firm l in the group of firms with $n-1$ links maximizes

$$\pi_l(g^{n-1}-g_{ij}) = \frac{\left[a-\overline{c}+2e_i+2e_j+e_l+(n-3)e_m\right]^2}{(n+1)^2}-\gamma e_l^2.$$

The first order condition is[31]

$$a-\overline{c}+2e_i+2e_j+e_l+(n-3)e_m-\gamma\,(n+1)^2\,e_l=0$$

Invoking symmetry, i.e. $e_i=e_j$ and $e_l=e_m$, we obtain the R&D efforts of the

[30] The second order condition requires that $\gamma > 4/(n+1)^2$, which is ensured by the second order condition mentioned in footnote 20.

[31] The second order condition in this case requires $2/(n+1)^2-2<0$, which always holds.

different types of firms in the deviation network

$$
\begin{aligned}
e_i(g^{n-1} - g_{ij}) &= \frac{2\gamma\,(a-\overline{c})\,(n+1)}{\gamma^2(n^3+3n^2+3n+1)+\gamma(n^2-3n-4)-2n+4} \\
e_m(g^{n-1} - g_{ij}) &= \frac{(a-\overline{c})\,(\gamma(n+1)+2)}{\gamma^2(n^3+3n^2+3n+1)+\gamma(n^2-3n-4)-2n+4}
\end{aligned}
$$

We can substitute these expressions into (3) and (4) to obtain deviating profits

$$
\pi_i(g^{n-1} - g_{ij}) = \frac{\gamma^2\,(a-\overline{c})^2\,(n+1)^2(\gamma^2(n+1)^2-4)}{\left[\gamma^2(n^3+3n^2+3n+1)+\gamma(n^2-3n-4)-2n+4\right]^2} \tag{27}
$$

Set $\gamma = 1$ in (26) and (27). The result follows.[32]

△

Proof that $\pi(g^{n-2}) - \pi(g^{n-1}) > 0$, for all $n \geq 4$, and $\gamma > n/(n+1)$:

Recall that

$$
\begin{aligned}
\pi(g^{n-1}) &= \frac{(a-\overline{c})^2\left[\gamma^2(n+1)^2-1\right]}{\left[\gamma(n+1)^2-n\right]^2} \\
\pi(g^{n-2}) &= \frac{(a-\overline{c})^2\left[\gamma^2(n+1)^2-4\right]}{\left[\gamma(n+1)^2-2(n-1)\right]^2},
\end{aligned}
$$

and notice that this statement can only be proved for an even number of firms. The proof is based on the following observations:

(i) $\frac{d\pi(g^{n-2})}{d\gamma} = -\frac{(a-\overline{c})^2 4(\gamma(n-1)-2)}{\left[\gamma(n+1)^2-2(n-1)\right]^3} < 0$

(ii) $\frac{d\pi(g^{n-1})}{d\gamma} = -\frac{(a-\overline{c})^2 2(n+1)^2(\gamma n-1)}{\left[\gamma(n+1)^2-n\right]^2} < 0$

(iii) $\lim_{\gamma\to\frac{n}{n+1}} \pi(g^{n-2}) = \frac{n^2-4}{(n^2-n+2)^2} > \lim_{\gamma\to\frac{n}{n+1}} \pi(g^{n-1}) = \frac{n^2-1}{n^4}$

(iv) $\lim_{\gamma\to\infty} \pi(g^{n-1}) = \lim_{\gamma\to\infty} \pi(g^{n-2}) = 1/(n+1)^2$.

These remarks show that profits under the network of degree $n-2$ are above profits under the complete network for the minimum feasible γ, i.e. $\gamma = n/(n+1)$. As γ approaches infinity, both profits functions converge to the same point. Thus, to establish the result we need to prove that $\pi(g^{n-1})$ declines at a higher rate than $\pi(g^{n-2})$. In other words, using (i) and (ii), we need to show that

$$
\frac{(\gamma n-1)}{\left[\gamma(n+1)^2-n\right]^2} < \frac{2(\gamma(n-1)-2)}{\left[\gamma(n+1)^2-2(n-1)\right]^3}. \tag{28}
$$

Note first that the denominator of the left hand side of this equation is larger than the denominator of its right hand side. Observe now that the numerator

[32] A discussion about the comparison between (26) and (27) for general γ is presented below.

of the left hand side is smaller than the numerator of the right hand side whenever $2(\gamma(n-1)-2)-(\gamma n-1)>0$. In other words, as long as $\gamma(n-2)-3>0$. This expression is decreasing in γ. So, for the lowest possible $\gamma = n/(n+1)$, we have that it must be the case that $n^2-5n-3>0$. This inequality holds for all $n \geq 6$ and so expression (28) is satisfied too when $n \geq 6$.
To complete the proof we need to show that the result is also true for $n = 4$. Substituting $n = 4$ into the above expressions, we can establish the comparison

$$\frac{\pi(g^{n-2})}{\pi(g^{n-1})} = \frac{[25\gamma^2-4]\,[25\gamma-4]^2}{[25\gamma-6]^2\,[25\gamma^2-1]}.$$

With the help of software Mathematica 3.0, one easily sees that $\pi(g^{n-2}) > \pi(g^{n-1})$ for all $\gamma > 4/5$. This completes the proof.

△

Proof of Proposition 3.6: We first compute social welfare for a symmetric network of degree k. Then, we establish a comparison between the welfare attained under the empty network, i.e. when $k = 0$, and the welfare resulting from the complete network, i.e. when $k = n-1$. We then easily see that $W(g^e) < W(g^{n-1})$. Later we compare the welfare under the complete network with the welfare resulting from a network where all firms have a link less, i.e. when $k = n-2$. We finally see that $W(g^{n-1}) < W(g^{n-2})$.
Social welfare in a symmetric network of degree k when firms are competitors in a homogeneous product market is given by

$$W(g^k) = \frac{n(n+2)}{2} q(g^k)^2 - ne^2(g^k).$$

Using the optimal effort $e(g^k)$ in (6) and the cost structure (5) one easily obtains $q(g^k)$. Some more algebra leads to the expression for social welfare under a network of degree k

$$W(g^k) = \frac{(a-\overline{c})^2 n\left[\gamma^2(n+2)(n+1)^2 - 2(n-k)^2\right]}{2\left[\gamma(n+1)^2-(n-k)(k+1)\right]^2} \tag{29}$$

Substituting $k = 0$ into this expression we obtain welfare under the empty network

$$W(g^e) = \frac{(a-\overline{c})^2 n\left[\gamma^2(n+2)(n+1)^2 - 2n^2\right]}{2\left[\gamma(n+1)^2-n\right]^2}.$$

Substituting $k = n-1$ into this expression we get welfare under the complete network

$$W(g^{n-1}) = \frac{(a-\overline{c})^2 n\left[\gamma^2(n+2)(n+1)^2 - 2\right]}{2\left[\gamma(n+1)^2-n\right]^2}.$$

Substituting $k = n - 2$, instead, we obtain

$$W(g^{n-2}) = \frac{(a-\bar{c})^2 n \left[\gamma^2 (n+2)(n+1)^2 - 8\right]}{2\left[\gamma(n+1)^2 - 2n + 2\right]^2}.$$

Deducting $W(g^e)$ from $W(g^{n-1})$, we find that

$$W(g^{n-1}) - W(g^e) = \frac{(a-\bar{c})^2 n(n^2-1)}{2\left[\gamma(n+1)^2 - n\right]^2} > 0$$

We need to prove that social welfare under the complete network

$$W(g^{n-1}) = \frac{(a-\bar{c})^2 n \left[\gamma^2 (n+2)(n+1)^2 - 2\right]}{2\left[\gamma(n+1)^2 - n\right]^2} \tag{30}$$

is lower than social welfare under the symmetric network of one degree less

$$W(g^{n-2}) = \frac{(a-\bar{c})^2 n \left[\gamma^2 (n+2)(n+1)^2 - 8\right]}{2\left[\gamma(n+1)^2 - 2n + 2\right]^2}. \tag{31}$$

We substitute $\gamma = 1$ in (30) and (31). The result follows.

△

Discussion of the general γ case.
Proposition 3.3: We show that $\pi(g^e) - \pi_i(g^e + g_{ij}) < 0$ for $n \geq 4$ and $\gamma > n/(n+1)$: with the help of a plot.

Recall that

$$\begin{aligned}
\pi(g^e) &= \frac{(a-\bar{c})^2\left[\gamma^2(n+1)^2 - n^2\right]}{\left[\gamma(n+1)^2 - n\right]^2} \\
\pi_i(g^e + g_{ij}) &= \frac{(a-\bar{c})^2\gamma\left[\gamma(n+1) - n\right]^2\left[(n^2+1)(\gamma-1) + 2n(\gamma+1)\right]}{\left[\gamma(\gamma-2)(n^3+1) + \gamma n(n+1)(3\gamma-1) + 2n(n-1)\right]^2}.
\end{aligned}$$

The following observations are useful in this case:

(i) $lim_{\gamma->\frac{n}{n+1}}\pi(g^e) = lim_{\gamma->\frac{n}{n+1}}\pi(g^e + g_{ij}) = 0$ for all n.

(ii) $lim_{\gamma->\infty}\pi(g^e) = lim_{\gamma->\infty}\pi(g^e + g_{ij}) = 0$ for all n.

These remarks show that firms get no lower profits by forming a link for the extreme values of γ. To establish the result, we need to compare $\pi(g^e)$ and $\pi_i(g^e+g_{ij})$ for the rest of the parameters. This comparison is made by means of the following Figure, which has been drawn by software Mathematica 3.0. In the vertical axis we have the quantity $\pi(g^e) - \pi_i(g^e + g_{ij})$, while in the horizontal axis we have n varying from 4 to 30, and γ ranging from $n/(n+1)$ to 100. The graph shows that firms i and j find it beneficial to deviate and form a collaboration.

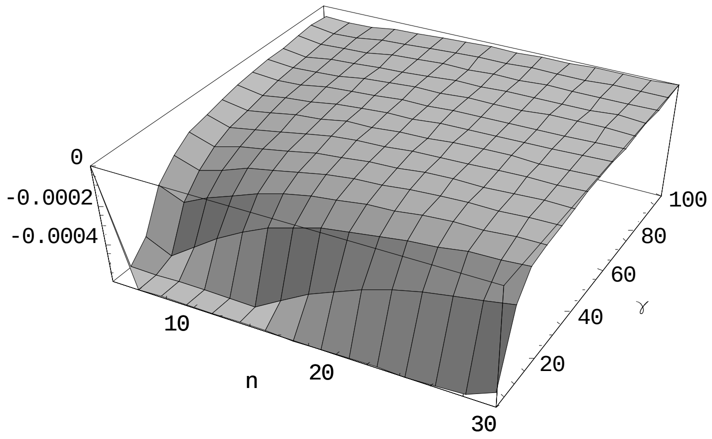

Figure 6: $\pi(g^e) - \pi_i(g^e + g_{ij})$

Proposition 3.4: We now illustrate that the above behaviour of profits obtains for all $n \geq 4$ and $\gamma > n/(n+1)$: We need to prove that equilibrium profits under the complete network

$$\pi(g^{n-1}) = \frac{(a-\overline{c})^2\left[\gamma^2(n+1)^2-1\right]}{\left[\gamma(n+1)^2-n\right]^2}$$

exceed deviating profits

$$\pi_i(g^{n-1} - g_{ij}) = \frac{\gamma^2\,(a-\overline{c})^2\,(n+1)^2(\gamma^2(n+1)^2-4)}{\left[\gamma^2(n^3+3n^2+3n+1)+\gamma(n^2-3n-4)-2n+4\right]^2}.$$

The following observations are useful:

(i) $\lim_{\gamma->\frac{n}{n+1}} \pi(g^{n-1}) = \frac{n^2-1}{n^4} > \lim_{\gamma->\frac{n}{n+1}} \pi(g^{n-1} - g_{ij}) = \frac{n^2(n+2)(n-2)}{(n^3+2n^2-6n+4)^2}$.

(ii) $\lim_{\gamma->\infty} \pi(g^{n-1}) = \lim_{\gamma->\infty} \pi(g^{n-1}) = \frac{1}{(n+1)^2}$.

The rest of the comparison is made by means of the following Figure, which has also been drawn by the software Mathematica 3.0. In the vertical axis we have the quantity $\pi_i(g^{n-1} - g_{ij}) - \pi(g^{n-1})$, while in the horizontal axis we have n varying from 4 to 30, and γ ranging from $n/(n+1)$ to 100. The graph shows that firm i does not find it beneficial to severe its collaboration with firm j.

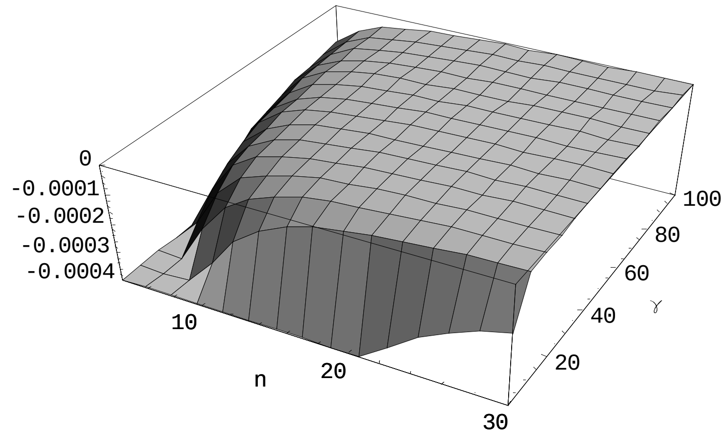

Figure 7: $\pi(g^{n-1} - g_{ij}) - \pi(g^{n-1})$

Proposition 3.6: We conjecture that $W(g^{n-1}) - W(g^{n-2}) < 0$, for all $n \geq 4$, $\gamma > n/(n+1)$. The following observations are useful:

(i) $\lim_{\gamma -> \frac{n}{n+1}} W(g^{n-1}) = \frac{1}{2} + \frac{1}{n}(\frac{n^2-1}{n^2}) < \lim_{\gamma -> \frac{n}{n+1}} W(g^{n-2}) = \frac{n(n^3+2n^2-8)}{2(n^2-n+2)^2}$.

(ii) $\lim_{\gamma -> \infty} W(g^{n-1}) = \lim_{\gamma -> \infty} W(g^{n-2}) = \frac{n(n+2)}{2(n+1)^2}$.

The rest of the comparison is made by means of the following Figure drawn by Mathematica 3.0. In the vertical axis we have the quantity $W(g^{n-1}) - W(g^{n-2})$, while in the horizontal axis we have n varying from 4 to 30, and γ ranging from $n/(n+1)$ to 100. The graph shows that welfare under the complete arrangement is always below welfare under a one-degree less dense network.

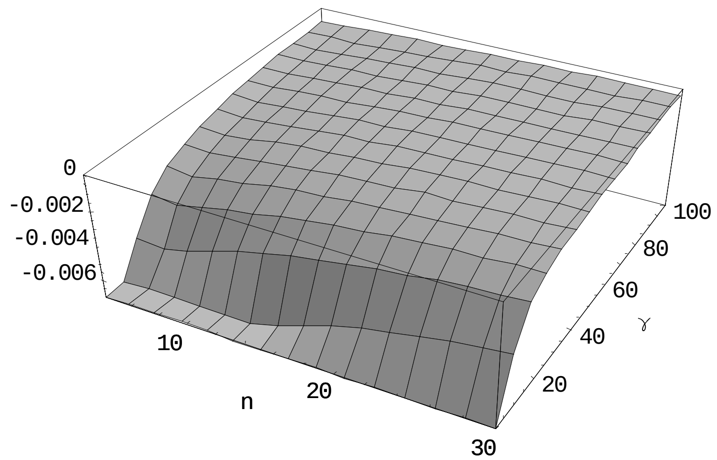

Figure 8: $W(g^{n-1}) - W(g^{n-2})$

7 Appendix B

Equilibrium computations in 3 firm example:

The complete network (g^{n-1})**:** When every pair of firms has collaboration agreements, firm i's cost is given by $c_i = \overline{c} - \sum_{j \in N} e_j$. Standard computations show that the equilibrium effort level is

$$e(g^{n-1}) = \frac{a-\overline{c}}{13}. \tag{32}$$

The equilibrium quantities and profits are then given as follows:

$$q(g^{n-1}) = \frac{4(a-\overline{c})}{13} \tag{33}$$

$$\pi(g^{n-1}) = \frac{15(a-\overline{c})^2}{169} \tag{34}$$

The star network (g^s)**:** In the star network firms are no longer in a symmetric position. Indeed, two firms have one collaboration agreement each, while the remaining firm has two collaboration agreements. Suppose that firms 1 and 2 are the *spokes* and firm 3 is the *hub* firm, without loss of generality. The cost structure under g^s is then

$$\begin{aligned} c_1 &= \overline{c} - e_1 - \beta e_2 - e_3 \\ c_2 &= \overline{c} - \beta e_1 - e_2 - e_3 \\ c_3 &= \overline{c} - \sum_{i \in N} e_i. \end{aligned} \tag{35}$$

Plugging this cost structure into (18), maximizing profits and invoking symmetry for the firms at the spokes, i.e. $e_1 = e_2 = e$, we obtain the effort levels of the different firms in the star network:

$$e_s(g^s) = \frac{4(a-\overline{c})(2-\beta)}{7\beta^2 - 13\beta + 58} \tag{36}$$

$$e_h(g^s) = \frac{(a-\overline{c})(\beta^2 - 3\beta + 6)}{7\beta^2 - 13\beta + 58} \tag{37}$$

The subscript s applies to the firms at the spokes, and the subscript h refers to the firm at the center of the star network.

Introducing (37) and (36) into (17) and (18) we obtain the equilibrium quantities and profits. For the firms at the spokes we have

$$q_s(g^s) = \frac{16(a-\overline{c})}{7\beta^2 - 13\beta + 58} \tag{38}$$

$$\pi_s(g^s) = \frac{16(a-\overline{c})^2\left(12+4\beta-\beta^2\right)}{\left(7\beta^2 - 13\beta + 58\right)^2}. \tag{39}$$

For the firm at the hub we obtain

$$q_h(g^s) = \frac{4(a-\overline{c})(\beta^2 - 3\beta + 6)}{7\beta^2 - 13\beta + 58} \tag{40}$$

$$\pi_h(g^s) = \frac{15(a-\overline{c})^2(\beta^2 - 3\beta + 6)^2}{\left(7\beta^2 - 13\beta + 58\right)^2}. \tag{41}$$

The unconnected network (g^u)**:** In the unconnected network, g^u, two firms are linked while the other firm stays isolated. Suppose without loss of generality that firms 1 and 2 are linked in g^u. The cost structure under g^u is then as follows:

$$\begin{aligned} c_1 &= c_2 = \overline{c} - e_1 - e_2 - \beta e_3 \\ c_3 &= \overline{c} - e_3 - \beta(e_1 + e_2) \end{aligned} \tag{42}$$

Proceeding as before and now invoking symmetry for the firms linked to each other, i.e. $e_1 = e_2 = e$, the subgame can be solved to obtain the research efforts of the different firms in the unconnected network g^u

$$e_l(g^u) = \frac{(a-\overline{c})(2+9\beta-9\beta^2+2\beta^3)}{2(4+33\beta-16\beta^2+7\beta^3-2\beta^4)} \tag{43}$$

$$e_u(g^u) = \frac{\beta(a-\overline{c})(9-9\beta+2\beta^2)}{(4+33\beta-16\beta^2+7\beta^3-2\beta^4)}, \tag{44}$$

where the subscript l applies to the firms connected in the graph g^u and u refers to the isolated firm in g^u.

We now obtain the equilibrium quantities and profits of the different firms in g^u by substituting (43) and (44) into (17) and (18). For the firms with collaborative agreements we have

$$q_l(g^u) = \frac{2(a-\overline{c})(1+5\beta-2\beta^2)}{4+33\beta-16\beta^2+7\beta^3-2\beta^4} \tag{45}$$

$$\pi_l(g^u) = \frac{(a-\overline{c})^2(1+5\beta-2\beta^2)^2(12+4\beta-\beta^2)}{4\left(4+33\beta-16\beta^2+7\beta^3-2\beta^4\right)^2}. \tag{46}$$

For the firm without research partners we have

$$q_u(g^u) = \frac{4(a-\overline{c})\beta(3-\beta)}{4+33\beta-16\beta^2+7\beta^3-2\beta^4} \tag{47}$$

$$\pi_u(g^u) = \frac{(a-\overline{c})^2\beta^2(3-\beta)^2\left(7+12\beta-4\beta^2\right)}{\left(4+33\beta-16\beta^2+7\beta^3-2\beta^4\right)^2} \tag{48}$$

The empty network (g^e)**:** The empty network, g^e, is the graph representing the case where no firm forms collaborations. In such a situation the firms cost structure is $c_i = \overline{c} - e_i - \beta \sum e_j$, $i, j = 1, 2, 3$, $j \neq i$. Taking advantage of the symmetry to solve the firms maximization problems for e we obtain the R&D effort under the empty network

$$e(g^e) = \frac{(a-\overline{c})(3-2\beta)}{13-4\beta+4\beta^2}. \tag{49}$$

Using (49) to obtain equilibrium quantities and profits we get

$$q(g^e) = \frac{4(a-\overline{c})}{4\beta^2-4\beta+13} \tag{50}$$

$$\pi(g^e) = \frac{(a-\overline{c})^2(-4\beta^2+12\beta+7)}{\left(4\beta^2-4\beta+13\right)^2}. \tag{51}$$

Proof of Proposition 4.1: Consider first the complete network g^{n-1}. The stability condition (i) is trivially satisfied because there are not more links to be formed. Thus, the complete collaborative agreement is stable if and only if no firm has a unilateral interest in breaking one of its ties (stability condition (ii)). To prove this, consider a firm in g^{n-1} that deviates by severing a link. Note that the resulting collaboration network is the star network g^s, and also that the deviating firm becomes a spoke firm in g^s. Then, the profits such firm would attain from this deviation would be given by $\pi_s(g^s)$ in (39). We need to establish a comparison between equilibrium profits $\pi(g^{n-1})$ in (34), and deviating benefits $\pi_s(g^s)$. Since the mathematical formulae for this comparison are lengthy, we have chosen to compare them by means of Figure 3 (given in the text). In this graph we have represented the different profits obtained by the distinct firms in the different networks of collaboration (we have normalized $a-\overline{c} = 1$, without loss of generality). The profits of a spoke firm in the star network are depicted by the dotted curve, while the benefits of a completely linked firm are represented by the flat solid curve. It can be seen that the curve $\pi_s(g^s)$ lies below $\pi(g^{n-1})$ for all spillover parameters β, which shows that no firm in the complete network would like to severe a link. Observe that this also proves that the star network is not stable.

Consider now the unconnected network g^u. To see that the stability condition (i) is satisfied, let us check that no linked firm in g^u can gain by breaking its tie. Notice that if a linked firm severs its link, the resulting network is the empty network g^e. The deviator would obtain profits $\pi(g^e)$ in (51). We need to compare the profits $\pi_c(g^u)$ in (46) with $\pi(g^e)$. Let us again resort to Figure 3 for this profits comparison. In the graph, the thinner solid curve represents the profits of a firm in the empty network, while the single-dotted dashed curve shows the profits of a linked firm in g^u. It is clearly seen that $\pi(g^e)$ lies below $\pi_c(g^u)$ for all β. Therefore, no firm in g^u has an unilateral incentive to break its collaborative link. Notice that this argument also proves that the empty network g^e is not stable because any pair of firms would deviate by forming a collaborative arrangement.

Let us now examine when stability condition (ii) is satisfied too. Notice first that the isolated firm always wishes to form a collaboration link with one of the connected firms in g^u. Indeed, if such a link is formed, then the isolated firm becomes a spoke firm in g^s, with profits $\pi_s(g^s)$ in (39). In g^u the isolated firm obtains equilibrium benefits $\pi_u(g^u)$. To compare these two profits, we use Figure 3 again. In this graph, deviating profits $\pi_s(g^s)$ are depicted by the dotted curve, while equilibrium benefits $\pi_u(g^u)$ are represented by the double-dotted dashed curve. The graph shows that the curve $\pi_u(g^u)$ lies below $\pi_s(g^s)$ for all β. This shows that the isolated firm in g^u always wishes to form a collaboration.

However, since links are pair-wise in our model of networks, we have to check whether or not a linked firm in g^u desires to form a collaborative arrangement with the isolated firm. Notice that if the linked is formed, the resulting network is the star network g^s, and that the deviating firm would be at the hub of the network. Thus, we have to compare equilibrium profits $\pi_c(g^u)$ in (46) with deviating profits $\pi_h(g^s)$ in (41). In Figure 3, the profits of a linked firm in the unconnected network are depicted by the single-dotted dashed curve, while the benefits of the hub firm in the star network are represented by the dashed curve. The graph shows that these two curves intersect approximately at the point $\widehat{\beta} = 0.03649165$, and that deviating profits curve $\pi_h(g^s)$ lies thereafter below the equilibrium profits curve $\pi_c(g^u)$. This completes the proof.

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