SPECIAL LOW-ORDER IIR FILTER BANK DESIGN

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ABSTRACT

The model matching problem $\mathbf{min}_{R(z)}\|W(z)-R(z)E(z)\|$ involved in multirate filter banks design is addressed. A method is presented to design the synthesis filter bank $R(z)$ with the order of $W(z)$ which is the polyphase representation of the time delay of the reconstructed signal. The existence conditions of such low-order $R(z)$ are given in linear matrix inequalities (LMIs). The corresponding H_2 model matching problem is solved in the same fashion. The results are illustrated through examples. Examples demonstrate the possibility of improving signal reconstruction error and reducing the order of synthesis filters simultaneously by increasing moderately time delay.

1. INTRODUCTION

Multitate filter banks (FBs) have found a wide range of applications in signal processing and telecommunications and have been the subject of intensive research in recent years. In [4], the design problem of FBs is cast into that of optimal model matching. This approach is further extended in [12, 13] to design FBs subject to subband noises. The advantages of optimal model matching approach are that the design problems of FBs can be handled in a unified framework as in robust filtering, and that there are reliable algorithms and software for globally optimal solutions. The main limitation of this approach is that the structural features of FBs are ignored, which generally results in the FBs with very high-order. To make this approach practically applicable, optimal model matching with reduced order FBs is required. Although reduced order filtering have been extensively studied in recent years, eg [6, 8, 11, 14] and references therein, it is far from being completely solved. As feasibility of rank-constrained LMI is always associated with reduced order design problems, it is extremely hard, if not impossible, to find the globally optimal solutions to the generic design problems of reduced order filters.

In many practical applications such as in digital communications, the analysis filters of FBs, which are at the transmitter side, can be designed to accommodate not only the requirements for transmitter but also those of the synthesis filters at the receiver side. Therefore, it is reasonable to impose some necessary conditions for analysis filters which make easier the design of reduced order synthesis filters. This paper exploits this idea in the design of reduced order synthesis filters.

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The rest of this paper is organized as follows. Section 2 presents a brief analysis of model matching problems in FBs. Section 3 presents an LMI-based design method of reduced-order synthesis filters which minimizes a single induced (H or H_2) norm. Two examples are given in section 4 to illustrate the results of Section 3. The examples show that low-order filters can give rise to better performances than high-order filters with minor sacrifice of the value of time delay. Section 5 concludes the paper.

Throughout the paper, the superscript T denotes matrix transpose and superscript H denotes conjugate transpose of complex matrices.

2. PRELIMINARIES

As shown in [4], the design of synthesis filters in FBs can be cast into the following model matching problem: Given transfer function matrices $W(z)$ and $E(z)$ with compatible dimensions, find a transfer function matrix $R(z)$ such that

$$\|W(z)-R(z)E(z)\| \quad = \min.$$

In the above formula, $E(z)$, $R(z)$ and $W(z)$ are respectively the polyphase representations of analysis filters, synthesis filters and the required time delay between input and output signals. To simplify presentation, in the sequel we will consider only two channel FBs. The results of the paper carry over easily to multi-channel FBs.

In two-channel FBs, $E(z)$, $R(z)$ and $W(z)$ are 2×2 transfer function matrices. Generally, $W(z)$ is a transfer function matrix composed of time-delay operators only and takes on the following form [4]

$$W(z)=\begin{pmatrix} z^{-d} & 0 \\ 0 & z^{-d} \end{pmatrix}, \quad \text{if } m=2d+1,$$

$$W(z)=\begin{pmatrix} 0 & z^{-d+1} \\ z^{-d} & 0 \end{pmatrix}, \quad \text{if } m=2d,$$

where m is the required value of time delay.

First we see the special structure of $W(z)$. Taking a standard state-space realization of z^{-d} as follows

$$A_d=\begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & 0 & 1 \\ & & & 0 \end{pmatrix}, b_d=\begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, c_d^T=\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

it is a routine to obtain the state-space realization of $W(z)$ by combining A_d, b_d and c_d. It can be seen from this standard realization that

$$A_d^T b_d=0, \quad c_d^T c_d+A_d^T A_d=I, \quad b_d^T b_d=1.$$

So if we suppose that the state-space realization of $W(z)$ is (A_W, B_W, C_W), we can get

$$A_W^T B_W = 0, \quad C_W^T C_W + A_W^T A_W = I, \quad B_W^T B_W = I.$$

In addition, $E(z)$ in FBs also has a special structure. Because $E(z)$ is the polyphase representation of analysis filters, its structure is determined by the analysis filters $H_0(z)$ and $H_1(z) = H_0(-z)$ [10]. Let the state-space realizations of $H_0(z)$ and $H_1(z)$ be $(A_{H_0}, B_{H_0}, C_{H_0}, D_{H_0})$ and $(A_{H_1}, B_{H_1}, C_{H_1}, D_{H_1})$, respectively. According to [10], $E(z)$ is given by

$$E(z) = \begin{pmatrix} E_{11}(z) & E_{12}(z) \\ E_{21}(z) & E_{22}(z) \end{pmatrix} := \left(\begin{array}{cc|cc} A_{H_0}^2 & 0 & A_{H_0}B_{H_0} & A_{H_0}^2 B_{H_0} \\ 0 & A_{H_1}^2 & A_{H_1}B_{H_1} & A_{H_1}^2 B_{H_1} \\ \hline C_{H_0} & 0 & D_{H_0} & C_{H_0}B_{H_0} \\ 0 & C_{H_1} & D_{H_1} & C_{H_1}B_{H_1} \end{array}\right).$$

In fact, from the realizations of $H_0(z)$ and $H_1(z)$ and the relationship $H_1(z) = H_0(-z)$, we can get

$$A_{H_1} = -A_{H_0}, B_{H_1} = -B_{H_0}, C_{H_1} = C_{H_0}, D_{H_1} = D_{H_0}$$

$$E_{11} = E_{21}(z), \quad E_{12}(z) = -E_{22}(z).$$

Furthermore, we have

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} E(z) = \begin{pmatrix} 2E_{11}(z) & 0 \\ 0 & 2E_{12}(z) \end{pmatrix}.$$

From the special structures of $W(z)$ and $E(z)$, it is easy to see that the model-matching problem in FBs is actually a problem that only involves single-input and single-output (SISO) systems. With these special features we can simplify the LMI conditions for low-order $R(z)$ design in the forthcoming section.

3. SPECIAL LOW-ORDER FILTER DESIGN

In this section, we design special low-order $R(z)$ for the model-matching problem $\min_{R(z)} \|W(z) - R(z)E(z)\|$.

Suppose that the state-space realizations of $W(z)$, $E(z)$ and $R(z)$ are $W(z) := \left(\begin{array}{c|c} A_W & B_W \\ \hline C_W & 0 \end{array}\right)$, $E(z) = \left(\begin{array}{c|c} A_E & B_E \\ \hline C_E & D_E \end{array}\right)$, $R(z) := \left(\begin{array}{c|c} A_R & B_R \\ \hline C_R & D_R \end{array}\right)$, respectively. Then the state space realization of $W(z) - R(z)E(z)$ is

$$\left(\begin{array}{c|c} A_{cl} & B_{cl} \\ \hline C_{cl} & D_{cl} \end{array}\right) := \left(\begin{array}{ccc|c} A_W & 0 & 0 & B_W \\ 0 & A_E & 0 & B_E \\ 0 & B_R C_E & A_R & B_R D_E \\ \hline C_W & -D_R C_E & -C_R & -D_R D_E \end{array}\right)$$

$$:= \left(\begin{array}{cc|c} A & 0 & B \\ B_R C_{E0} & A_R & B_R D_E \\ \hline C_{W0} - D_R C_{E0} & -C_R & -D_R D_E \end{array}\right)$$

where $A = \begin{pmatrix} A_W & 0 \\ 0 & A_E \end{pmatrix}$, $B = \begin{pmatrix} B_W \\ B_E \end{pmatrix}$, $C_{E0} = \begin{pmatrix} 0 & C_E \end{pmatrix}$, $C_{W0} = \begin{pmatrix} C_W & 0 \end{pmatrix}$.

Assume $A_W \in \mathbf{R}^{n_W \times n_W}$ and $A_E \in \mathbf{R}^{n_E \times n_E}$. Then it is apparent from the above that a full order $R(z)$ will have order equal to $n_W + n_E$. By using the canonical LMI method[2, 5], we can design low-order $R(z)$ with order equal to n_W.

Theorem 1 Suppose that $E(z)$ and $W(z)$ are given as above. Then for a given $\ > 0$, there exists $R(z)$ with $A_R \in \mathbf{R}^{n_W \times n_W}$ such that

$$\|W(z) - R(z)E(z)\| \ <$$

if there are $S = S^T > 0$, $Q_1 = Q_1^T > 0, Q_1 \in \mathbf{R}^{n_W \times n_W}$ and a scalar $\ > 0$ such that

$$\begin{pmatrix} S & 0 \\ 0 & I \end{pmatrix} - \begin{pmatrix} A^T \\ B^T \end{pmatrix} S \begin{pmatrix} A & B \end{pmatrix} > 0, \tag{1}$$

and

$$\begin{pmatrix} S + Q - \ ^{-1}C_{W0}^T C_{W0} + \ C_{E0}^T C_{E0} & C_{E0}^T D_E \\ D_E^T C_{E0} & I + \ D_E^T D_E \end{pmatrix} - \begin{pmatrix} A^T \\ B^T \end{pmatrix} (S+Q) \begin{pmatrix} A & B \end{pmatrix} > 0, \tag{2}$$

where $Q = diag(Q_1, 0)$.

Remark 1 For any choice of positive semi-definite matrix Q with rank k, one can get k-th order $R(z)$. Why should we choose Q as in Theorem 1? In fact, from the discussions above, we can see that A is diagonal and (A, C_{W0}) is not observable. So we just take Q to meet the observable part of (A, C_{W0}). In this way, at least Q can eliminate the influence of $\ ^{-1}C_{W0}^T C_{W0}$ in (2). One can also search low-order $R(z)$ by the method given in [8, 11], but Theorem 1 gives a direct method to design n_W-order $R(z)$. Theorem 1 can be viewed as a special case of [9] which is based on a searching algorithm. However, in some practical applications, the fast FB design algorithms are critical and the searching is computationally too heavy to use. It is therefore necessary to exploit the properties of special problems to simplify the design procedure. From Theorem 1, we can observe some simple properties and some advantages of our choice of Q, which are summarized in the following corollaries. These special properties and advantages are not observed in [9].

Simplifying Q further and using the special feature of $W(z)$ discussed in section 2, we can get the following simplified result.

Corollary 1 Suppose that $E(z)$ and $W(z)$ are given as above. For a given $\ > 0$, if there are $P_E = P_E^T > 0, P_E \in \mathbf{R}^{n_E \times n_E}$ and a scalar $\ > 0$ such that

$$\begin{pmatrix} P_E & 0 \\ 0 & I \end{pmatrix} - \begin{pmatrix} A_E^T \\ B_E^T \end{pmatrix} P_E \begin{pmatrix} A_E & B_E \end{pmatrix} > 0 \tag{3}$$

and

$$\begin{pmatrix} P_E + \ C_E^T C_E & C_E^T D_E \\ D_E^T C_E & (\ - \ ^{-1})I + \ D_E^T D_E \end{pmatrix} - \begin{pmatrix} A_E^T \\ B_E^T \end{pmatrix} P_E \begin{pmatrix} A_E & B_E \end{pmatrix} > 0, \tag{4}$$

then there exists $R(z)$ with $A_R \in \mathbf{R}^{n_W \times n_W}$ such that $\|W(z) - R(z)E(z)\| \ < \ .$

Remark 2 The key constraint of this corollary is that two matrix inequalities (3) and (4) share a common positive matrix P_E. If we consider (3) and (4) separately, we can see that (3) is equivalent to

$$P_E^{-1} - AP_E^{-1}A^T - \ ^{-1}BB^T > 0.$$

Then (3) holds naturally under the stability of A. (4) is a simple constraint on $E(z)$. In fact by using KYP lemma [7], if we do not require $P_E > 0$, (4) is equivalent to the existence of a scalar $\ \ > 0$ such that the frequency-domain inequality

$$E_1^H(e^{jw})E_1(e^{jw}) + E_1^H(e^{jw})D_E + D_E^T E_1(e^{jw}) \\ +(\ \ -\ ^{-1})/\ \ I + D_E^T D_E > 0$$

holds, where $E_1(z) = E(z) - D_E$. Obviously, this frequency-domain inequality is related to strictly positive realness (SPR) of $E_1(z)$. If we take $D_E = I$ without losing generality and $\ \ $ large enough such that $(\ \ -\ ^{-1})/\ \ I + I > 0$, then one can see that the condition (6) is weaker than the SPR condition of $E_1(z)$. So if we require that the strictly proper part of $D_E E(z)$ is strict positive real, then (4) holds naturally. The key fact here is that we need a trade-off between (3) and (4) (a common S).

If D_E is nonsingular, we can suppose that $D_E = I$ without lost of generality. Then one can get the following result,

Corollary 2 Let $D_E = I$. Suppose that $E(z)$ is given as above. Given $\ \ > 0$, if there exists $P_E = P_E^T > 0$ such that

$$\begin{pmatrix} P_E & 0 \\ 0 & I \end{pmatrix} - \begin{pmatrix} A_E^T \\ B_E^T \end{pmatrix} P_E \begin{pmatrix} A_E & B_E \end{pmatrix} > 0 \qquad (5)$$

and

$$\begin{pmatrix} P_E & P_E(A_E - B_E C_E) \\ (A_E - B_E C_E)^T P_E & P_E + (\ \ -\ ^{-1}) C_E^T C_E \end{pmatrix} > 0, \qquad (6)$$

then there exists $R(z)$ with $A_R \in \mathbf{R}^{n_W \times n_W}$ such that $\|W(z) - R(z)E(z)\|\ \ <\ \ $.

Remark 3 In fact, noticing $D_E = I$ in Corollary 2, (6) is equivalent to the stability of $E^{-1}(z)$. The key constraint is a common P_E for (5) and (6).

The conditions in Corollaries 1 and 2 are expressed in LMIs, which is convenient for computation using the ready-made software of LMI [5], but we cannot see their obvious meanings. Returning LMIs in Corollary 2 to frequency-domain condition, we can get the following result.

Corollary 3 Let $D_E = I$. Suppose that $E(z)$ is as given above. Given $\ \ > 0$, there exists $P_E = P_E^T > 0$ such that (5) and (6) hold if, and only if, there exists a constant matrix D such that

$$\|C_E(zI - A_E)^{-1} B_E - DE(z)\|\ \ <\ \ . \qquad (7)$$

Remark 4 In fact for a given small $\ \ $, (7) means $E(z)$ is near to a constant matrix. Intuitionally, at this time of course there is a $R(z)$ with the same order of $W(z)$ such that $\|W(z) - R(z)E(z)\|\ \ $ is small. For two channel problems in multirate filter banks, given an analysis filter $H_0(z)$, $E(z)$ is determined by (see discussions in section 2)

$$E_{11}(z) = C_{H_0}(zI - A_{H_0}^2)^{-1} A_{H_0} B_{H_0} + D_{H_0},$$

$$E_{12}(z) = C_{H_0}(zI - A_{H_0}^2)^{-1} A_{H_0}^2 B_{H_0} + C_{H_0} B_{H_0}.$$

From this equality, one can see that if A_{H_0} is a near 0 matrix, then $E(z)$ is close to a constant matrix. Generally, Corollaries 1, 2 and 3 are more conservative than Theorem 1. If one of the corollaries above holds for a small $\ \ $, the actual reconstruction error would be much smaller. Because Corollaries 1, 2 and 3 are all independent of $W(z)$, when one of these three Corollaries holds one can take the smallest time delay value $m = 1$, i.e., $W(z) = I$.

Besides H norm optimization, H_2 and mixed H_2/H norm optimizations have also been used in $R(z)$ design [1, 12, 13]. Obviously, the method given above can be generalized to H_2 norm optimization.

Theorem 2 Suppose that $E(z)$ and $W(z)$ are given as above. Then for a given $\ _2 > 0$, there exists $R(z)$ with $A_R \in \mathbf{R}^{n_W \times n_W}$ such that

$$\|W(z) - R(z)E(z)\|_2 <\ _2$$

if there exist $S = S^T > 0$, $Q_1 = Q_1^T > 0, Q_1 \in \mathbf{R}^{n_W \times n_W}$ and a scalar $\ \ > 0$ such that

$$\begin{pmatrix} S & 0 \\ 0 & I \end{pmatrix} - \begin{pmatrix} A^T \\ B^T \end{pmatrix} S \begin{pmatrix} A & B \end{pmatrix} > 0, \qquad (8)$$

$$\begin{pmatrix} S + Q + \ C_{E0}^T C_{E0} & C_{E0}^T D_E \\ D_E^T C_{E0} & I + \ D_E^T D_E \end{pmatrix} - \begin{pmatrix} A^T \\ B^T \end{pmatrix} (S + Q) \begin{pmatrix} A & B \end{pmatrix} > 0, \qquad (9)$$

and

$$\begin{pmatrix} Z & C_{W0} & 0 \\ C_{W0}^T & S + Q + \ C_{E0}^T C_{E0} & C_{E0}^T D_E \\ 0 & D_E^T C_{E0} & I + \ D_E^T D_E \end{pmatrix} > 0, \qquad (10)$$
$$tr(Z) <\ _2^2,$$

where $Q = diag(Q_1, 0)$.

Similar to Corollary 2, one can get

Corollary 4 Let $D_E = I$. Suppose that $E(z)$ and $W(z)$ are given as above. For a given $\ _2 > 0$, if there are $S = S^T > 0$, $Q_1 = Q_1^T > 0, Q_1 \in \mathbf{R}^{n_W \times n_W}$ such that

$$\begin{pmatrix} S & 0 \\ 0 & I \end{pmatrix} - \begin{pmatrix} A^T \\ B^T \end{pmatrix} S \begin{pmatrix} A & B \end{pmatrix} > 0, \qquad (11)$$

$$\begin{pmatrix} S + Q & (S + Q)(A - BC_{E0}) \\ (A - BC_{E0})^T (S + Q) & S + Q + C_{E0}^T C_{E0} \end{pmatrix} > 0, \qquad (12)$$

and

$$\begin{pmatrix} Z & C_{W0} \\ C_{W0}^T & S + Q + C_{E0}^T C_{E0} \end{pmatrix} > 0, \quad tr(Z) <\ _2^2, \qquad (13)$$

where $Q = diag(Q_1, 0)$, then there exists $R(z)$ with $A_R \in \mathbf{R}^{n_W \times n_W}$ such that small

$$\|W(z) - R(z)E(z)\|_2 <\ _2.$$

Remark 5 From the discussions in section 2, one can see that FB problems can be dealt with as SISO systems. It is known from [3] that for SISO discrete systems, H_2 norm is bounded by H norm. Then by Corollary 3, when $E(z)$ is close to a constant matrix, $\min_{R(z)} \|W(z) - R(z)E(z)\|_2$ is small.

Remark 6 When the inequalities in Theorems 1 and 2 are feasible, one can parameterize all solutions of $R(z)$ by LMI method.

4. EXAMPLES

Example 1 This is Example 2 of [1]. The analysis filters are third-order IIR filters given by

$$H_0(z) = \frac{0.1412z^3 + 0.3805z^2 + 0.3805z + 0.1412}{z^3 - 0.3011z^2 + 0.3694z - 0.0250},$$

$$H_1(z) = H_0(-z).$$

Example 2 This is Example 1 of [4]. The analysis filters are third-order IIR filters given by

$$H_0(z) = \frac{z^3 + 3z^2 + 3z + 1}{6z^3 + 2z}, \; H_1(z) = H_0(-z).$$

Setting time delay $m = 7$ and $m = 9$, respectively, three different types of synthesis filters are designed for these two examples using the methods of Theorems 1 and 2 and the conventional full order design. The performances of these designs are compared in Tables 1 and 2. In the tables, Th means Theorem, FO means full-Order, OSF means order of synthesis filter.

Table 1 Comparison of H performance

	Th $1, m = 7$	FO$, m = 7$	Th $1, m = 9$
Ex 1	$= 0.067$	$= 0.024$	$= 0.022$
Ex 2	$= 0.075$	$= 0.038$	$= 0.025$
OSF	13	25	17

Table 2 Comparison of H_2 performance

	Th $2, m = 7$	FO$, m = 7$	Th $2, m = 9$
Ex 1	$_2 = 0.041$	$_2 = 0.023$	$_2 = 0.014$
Ex 2	$_2 = 0.051$	$_2 = 0.035$	$_2 = 0.017$
OSF	13	25	17

Observations: The order of synthesis filters is $n_S = 2n_R + 1$, where n_R is the order of $R(z)$, see [10] for details. For full-order $R(z)$, $n_S = 2n_E + 2m - 1$ [4], while for low-order $R(z)$ designed with Theorems 1 and 2, $n_S = 2m - 1$. Hence, the order of synthesis filters is reduced by $2n_E$. As can be seen from table 1, for these two examples the low-order filters attain better performances than higher-order filters with a moderate increase of time-delay. Therefore, time-delay can be used as a parameter to design reduced-order filters using the method given in Theorems 1 and 2. It should be pointed out that the performance bounds of low order designs presented in the tables are computed directly using the conditions in Theorems 1 and 2, the actual bounds may be much smaller. Apparently, the low order design methods are especially effective for high-order analysis filters.

5. CONCLUSIONS AND FUTURE WORKS

Since the generic problems of reduced order filter design are very hard to solve, it is an interesting topic to consider design methods of special low order filters which use the structural features of given problems to overcome the difficulties in reduced order design.

This paper presents a special low-order filter design method for FBs. The order of synthesis filter is determined by the order of time-delay operator. In telecommunication applications, analysis filters are at the transmitter side and synthesis filters are at the receiver side. Due to hardware constraints, often receiver should be simpler and transmitter can be more complicated. The presented method is suitable for such problems. Designing analysis filters which satisfy simultaneously the signal reconstruction requirements of FBS and the reduced-order conditions in Theorems 1 and 2 is a further research topic.

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