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Gain-Scheduling Control of Port-Fuel-Injection Processes

Andrew White, Jongeun Choi, Ryozo Nagamune, and Guoming Zhu

Abstract— **In this paper, we first obtain an event-based sampled discrete-time linear system to represent a port-fuel-injection process based on wall-wetting dynamics, and formulate it as a linear parameter varying (LPV) system. The system parameters used in the engine fuel system model are engine speed, temperature, and load. These system parameters can be measured in real-time through physical or virtual sensors. A gain-scheduling controller for the obtained LPV system is then designed based on the numerically efficient convex optimization or linear matrix inequality (LMI) technique. The simulation results show the effectiveness of the proposed scheme.**

I. INTRODUCTION

Increasing concerns about global climate change and ever-increasing demands on fossil fuel capacity call for reduced emissions and improved fuel economy. Vehicles equipped with a port-fuel-injection fuel system have been widely used today; and vehicles equipped with a direct-injection (DI) fuel system have been introduced to markets globally. In order to improve DI engine full load performance at high speed, Toyota introduced an engine with a stoichiometric direct injection system with two fuel injectors for each cylinder (see [1]). One is a DI injector generating a dual-fan-shaped spray with wide dispersion, while the other is an intake port injector. The dual-fuel system introduces one additional degree of freedom for engine optimization to reduce emissions with improved fuel economy. The use of gasoline port-fuel-injection and ethanol DI dual-fuel system to substantially increase gasoline engine efficiency is described in [2]. The main idea is to use a highly boosted small turbocharged engine to match the performance of a much larger engine. Direct injection of ethanol is used to suppress engine knock at high engine load due to its substantial air charge cooling resulting from its high heat of vaporization. This shows that with the introduction of DI fuel systems for the internal combustion engine, port-fuel-injection fuel systems will be part of the engine fuel system for improved engine performance, which is the main motivation for us to revisit the air-to-fuel ratio control problem for a port-fuel-injection fuel system.

The control of air-to-fuel (A/F) ratio is an increasingly important control problem due to the federal and state emission regulations. Spark-ignited internal combustion engines are operated at a desired air-to-fuel ratio since the highest conversion efficiency of a three-way catalyst occurs around stoichiometric air-to-fuel ratio. Due to the introduction of internal combustion engines with dual fuel systems (port-fuel-injection and DI), control of both A/F ratio and fuel ratio (ratio of port-fuel-injection fueling vs. total fueling) becomes a part of the combustion optimization problem [3].

There have been several fuel control strategies developed for internal combustion engines to improve the efficiency and exhaust emissions. A key development in the evolution was the introduction of a closed-loop fuel injection control algorithm [4], followed by the linear quadratic control method [5], and an optimal control and Kalman filtering design [6]. Specific applications of A/F ratio control based on observer measurements in the intake manifold were developed in 1991 [7]. Another approach was based on measurements of exhaust gases A/F ratio measured by the oxygen sensor and on the throttle position [8]. Hedrick also developed a nonlinear sliding mode control of A/F ratio based upon the oxygen sensor feedback [9]. Continuing research efforts of A/F ratio control include adaptive approaches [10], [11], observer-based controllers [12], H_∞ controllers [13], model predictive controllers [14], sliding mode controllers [15], and linear parameter-varying controllers [16], [17], [18]. The conventional A/F ratio control for automobiles uses both closed-loop feedback and feedforward control to have good steady state and transient responses.

For a spark-ignited engine equipped with a port-fuel-injection system, the wall-wetting dynamics is commonly used to model the fuel injection process; and the wall-wetting effects are compensated on the basis of simple linear models that are tuned and calibrated through engine tests. These models are quite effective for an engine operated at steady state or slow transition conditions but they are difficult to be used at fast transient and other special operational conditions, for instance, during engine cold start. One of the approaches to model the wall-wetting dynamics during engine cold start is to describe it using a family of linear models to approximate the system dynamics at a given engine coolant temperature, speed and load conditions, that is, to translate the fuel system model into a linear parameter varying (LPV) system.

As stated earlier, the use of LPV modeling to control the A/F ratio of a port-fuel- injection system has been reported by [16], [17], [18]. In [18], a continuous-time, LPV model is developed considering only engine speed as a time-varying parameter. Due to the simplicity of the model used, the issue of engine cold start is not addressed. Furthermore, the control synthesis method used in [18] relies on gridding the parameter space at a finite number of grid points. In [17], a large variable time delay is present in the air-fuel

Andrew White, Jongeun Choi and Guoming Zhu are with the Department of Mechanical Engineering, Michigan State University {whitea23, jchoi, zhug}@egr.msu.edu

Ryozo Nagamune is with the Department of Mechanical Engineering, Department of Mechanical Engineering, University of British Columbia nagamune@mech.ubc.ca

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ratio control loop for a lean burn spark ignition engine. LPV control methods are used to compensate for the variable time delay. In [16], a discrete-time, LPV model is developed with manifold absolute pressure, exhaust value closing, and inlet value opening as the time-varying parameters. However, only manifold absolute pressure is used as a scheduling parameter in the gain-scheduling control that is synthesized. Also, [16] does not address the issue of engine cold start. Additionally, all LPV control synthesis methods used by [16] are based in continuous time, relying on Tustin's (bilinear) transformation to convert the discrete-time system to a continuous-time system, thus fixing the engine speed and sampling rate of the discrete-time system. In contrast to each of these efforts, in this paper an event-based, gain-scheduling controller for an event-based, discrete-time LPV system with wall-wetting parameters and engine speed as time-varying parameters is designed. To cope with practical situations, the discrete-time LPV control synthesis method given by [19] is used to develop the event-based, gain-scheduling controller.

The control structure used in this study is a proportional-integral (PI) controller. PI controllers are widely used in industry since they are well understood by field control engineers. The PI gains are often calibrated in the test field for the best performance as functions of system operational conditions. However, the system stability and performance are not guaranteed for all time-varying parameters. Therefore, we propose to apply LPV techniques to design gain-scheduling PI controllers for guaranteed stability and performance for all time-varying parameters for the proposed LPV system, which will be well received by industrial control engineers.

In this paper, we first obtain an event-based sampled discrete-time linear system to represent a port-fuel-injection process based on wall-wetting dynamics, and formulate it as an LPV system. The system parameters used in the engine fuel system model are engine speed, temperature, and load. These system parameters can be obtained in real-time through physical or virtual sensors. We then obtain a gain-scheduling controller for the derived LPV system based on the numerically efficient convex optimization (or LMI) techniques. The simulation results show the effectiveness of the proposed scheme.

Standard notation is used throughout the paper. Let $\mathbb{R}$ and $\mathbb{Z}_{\geq 0}$ denote the set of real and non-negative integer numbers. The positive definiteness of a matrix A is denoted by $A \succ 0$. Other notation will be explained in due course.

II. Plant dynamics

In this section, the dynamics of the plant (Fig. 1) be will carefully explained and modeled.

A. Dynamics of the port-fuel-injection process

The discrete-time linear system is obtained by event-based sampling of the port-fuel-injection process; hence the sampling time of this discrete-time system is the period of an engine cycle (see general engine modeling techniques in [20]). The wall-wetting dynamics can be described as

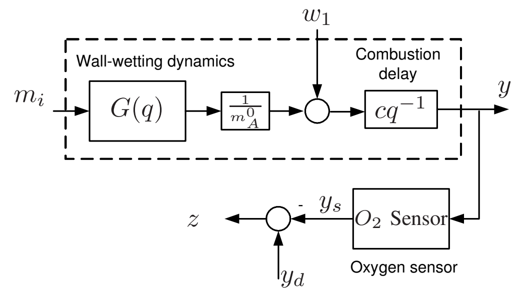

Fig. 1. The block diagram of the port-fuel-injection process and sensor dynamics.

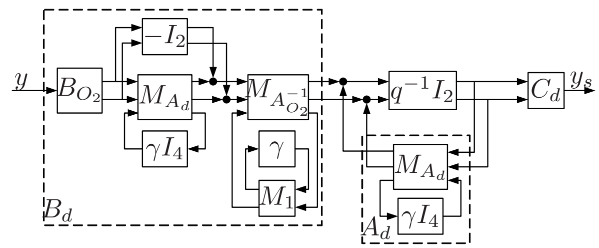

Fig. 2. Block diagram of the combined dynamics of the exhaust gas and sensor delays.

follows:

$$
\begin{aligned}
m_w(k) &= (1-\beta_k)m_i(k) + (1-\alpha_k)m_w(k-1), \\
m_c(k) &= \beta_k m_i(k) + \alpha_k m_w(k-1),
\end{aligned} \tag{1}
$$

where $k \in \mathbb{Z}_{\geq 0}$, and m_i, m_w, and m_c denote the amount of fuel, injected, on the wall, and in the cylinder respectively. The coefficients $\alpha \in [0,\, 1]$, and $\beta \in [0,\, 1]$, are the ratios of the fuel delivered from the wall to the cylinder, and of the fuel entering the cylinder from injection, respectively. These values can be estimated online through an available set of engine sensors, which allows us to apply gain-scheduling control to the plant. Using the discrete-time dynamics in (1), we obtain the transfer function $G(q)$ from m_i to m_c

$$
G(q) := \frac{m_c(k)}{m_i(k)} = \frac{\beta_k + (\alpha_k - \beta_k)q^{-1}}{1-(1-\alpha_k)q^{-1}}, \tag{2}
$$

where q is the *forward shift operator* that satisfies $qu(k) = u(k+1)$. The dotted box in the block diagram in Fig. 1 illustrates the fuel-injection process. The output of $G(q)$ is the input to the gain block of $\frac{1}{m_A^0}$, which is the nominal value of the inverse of the air amount m_A. The signal w_1 represents the deviation $\left(\frac{m_c}{m_A} - \frac{m_c}{m_A^0}\right)$, which will be treated as a disturbance in this paper. Another constant gain factor $c = 14.6$ in Fig. 1 is the value for the air-to-fuel-ratio at stoichiometric. After the combustion delay block the equivalence ratio y is generated. The diagram of the transfer function from the amount of fuel injected m_i and the disturbance w_1 to the equivalence ratio y (inverse of normalized air-to-fuel ratio) is shown in the dotted box in Fig. 1.

B. Oxygen sensor

To measure y, we use an oxygen sensor whose dynamics are modeled as a transport delay of the exhaust gas mixture as a function of engine speed, $T_D = \frac{80}{v}$, where v denotes the speed of the engine in revolutions per a minute (rpm). The combined transfer function in the continuous time domain is

$$y_s(s) = \frac{\exp(-T_D s)}{T_{O_2}s+1} y(s), \tag{3}$$

where y_s is the equivalence ratio measured by the sensor and T_{O_2} is the time constant of the oxygen sensor. Equation (3) can be approximated by the second-order system

$$y_s(s) = \frac{1}{T_D s+1}\frac{1}{T_{O_2}s+1} y(s),$$

which has the state-space representation

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \underbrace{\begin{bmatrix} -\frac{1}{T_D} & \frac{1}{T_D} \\ 0 & -\frac{1}{T_{O_2}} \end{bmatrix}}_{=:A_{0_2}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{T_{O_2}} \end{bmatrix}}_{=:B_{O_2}} y, \\ y_s &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{=:C_{O_2}} x. \end{aligned} \tag{4}$$

The event based controller updates the control every combustion event such that the sample rate is given by $t_s = \frac{120}{v}$. Using t_s as the sampling rate, the corresponding discrete system of Eq. (4) is

$$\begin{aligned} x(k+1) &= A_d x(k) + B_d y(k), \\ y_s(k) &= C_d x(k), \end{aligned}$$

where $A_d = \exp(A_{O_2} t_s)$, $B_d = A_{O_2}^{-1}(A_d - I)B_{O_2}$, and $C_d = C_{O_2}$. Since both T_D and t_s are functions of engine speed, v, naturally A_d and B_d are as well. A fourth-order Taylor series approximation is used to capture the parameter variation of A_d. To ensure that the coeffiecents of the Taylor series approximation of A_d are numerically stable the engine speed, v, must be normalized. Furthermore, due to the way that v appears in $A_{O_2}^{-1}$, it is necessary to isolate $\frac{1}{v}$ instead of v. For this reason, we normalize $\frac{1}{v} \in [\frac{1}{\bar{v}}, \frac{1}{\underline{v}}]$ to γ in the following way:

$$\gamma = \frac{\frac{1}{v} - \frac{1}{v_0}}{\frac{1}{v} + \frac{1}{v_0}}, \quad \text{where} \quad \frac{1}{v_0} = \frac{\frac{1}{\bar{v}} + \frac{1}{\underline{v}}}{2}. \tag{5}$$

The polynomial LFTs M_{A_d} and $M_{A_{O_2}^{-1}}$ are used to isolate the varying parameter [21]. The diagram of the transfer function from the equivalence ratio y to the measured equivalence ratio y_s is displayed in Fig. 2.

III. LPV GAIN SCHEDULING CONTROLLER DESIGN

The objective of the control system is to regulate the equivalence ratio y to a reference input y_d using feedback control against the disturbance signal w_1. To achieve our objective, we propose to design a gain scheduling controller $K(\Theta)$ to adapt to the time-varying parameters Θ, and add

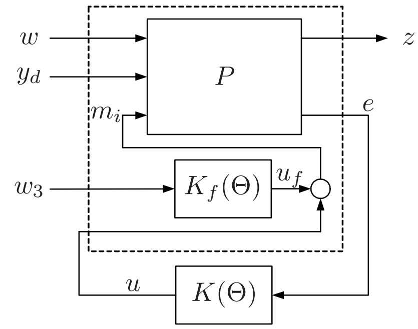

Fig. 3. The proposed control strategy for the fuel injection process. The LPV control strategy is applied to the systems inside the dotted box. Here $w_3 = m_A\ y_d$.

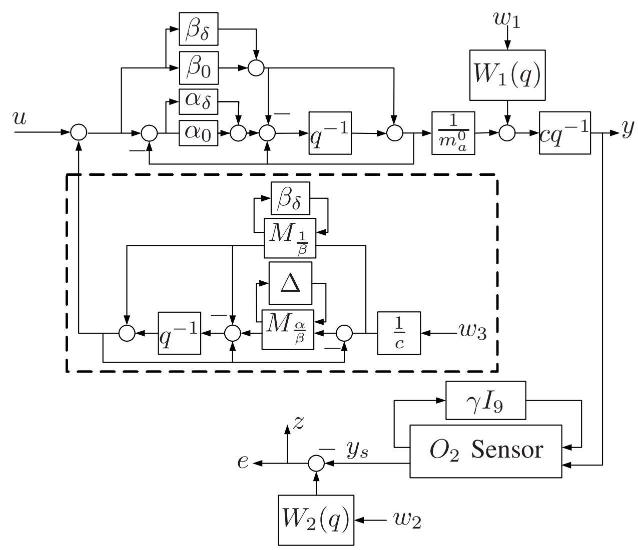

Fig. 4. The generalized plant with varying parameters and feedforward control included. Here $w_2 = y_d$.

a standard feedforward controller designed using the inverse of $cG(q)$

$$K_f(\Theta) = \frac{G^{-1}(q)}{c} = \frac{1}{c}\left(\frac{1-(1-\alpha_k)q^{-1}}{\beta_k + (\alpha_k - \beta_k)q^{-1}}\right).$$

The proposed control system is depicted in Fig. 3. Notice in Fig. 3, the generalized plant P and the feedforward control $K_f(\theta)$ are grouped together inside a dotted box. We formulate the systems inside the dotted box as a discrete-time LPV system using linear fractional transformation (LFT), and then design the gain-scheduling controller $K(\Theta)$ based on the technique in [19]. Components of the generalized plant P and feedforward controller $K_f(\Theta)$ in Fig. 3 are illustrated in Fig. 4. The feedforward controller components are depicted inside the dotted box of Fig. 4.

To use $\mathcal{L}_2$ gain for our performance criterion, we selected weighting functions $W_1(q) = \frac{0.1411}{q-0.9986}$ and $W_2(q) = \frac{0.0003982q+0.0003979}{q^2-1.997q+0.9972}$, where W_2 was chosen as a 2nd order low-pass filter with a high DC gain to provide more weight on the low frequency signals since w_2 is the step input y_d.

In our formulation, we consider α and β to be equivalent to a constant nominal value plus a time-varying fluctuation.

For instance, the parameter variation of $\alpha_k \in [\underline{\alpha}, \overline{\alpha}]$ with $\alpha_0 = \frac{\underline{\alpha}+\overline{\alpha}}{2}$ would be represented by $\alpha_\delta(k) = \alpha_k - \alpha_0 \in [\underline{\alpha}-\alpha_0, \overline{\alpha}-\alpha_0]$, so that the parameter range of α_δ is centered around zero. Hence, we replace α_k by $\alpha_0 + \alpha_\delta(k)$. The same is done for $\beta_k \in [\underline{\beta}, \overline{\beta}]$ as well. The parameter variation of v is represented by γ as shown in Eq. (5). The upper LFTs inside the dotted box in Fig. 4, $M_{\frac{1}{\beta}}$ and $M_{\frac{\alpha}{\beta}}$ are used to isolate the varying parameters β_δ and $\Delta = \mathrm{diag}(\beta_\delta, \alpha_\delta)$ [21].

With the parameter variation represented in this way, we write our system as a discrete-time LPV system with LFT parameter dependency,

$$\begin{bmatrix} l(k) \\ x(k+1) \\ z(k) \\ e(k) \end{bmatrix} = \underbrace{\left[\begin{array}{c|ccc} D_{00} & C_0 & D_{01} & D_{02} \\ \hline B_0 & A & B_1 & B_2 \\ D_{10} & C_1 & D_{11} & D_{12} \\ D_{20} & C_2 & D_{21} & D_{22} \end{array}\right]}_{=:M} \begin{bmatrix} p(k) \\ x(k) \\ w(k) \\ u(k) \end{bmatrix}, \tag{6}$$

$$p(k) = \Theta(k) l(k).$$

where $x(k) \in \mathbb{R}^n$ is the state at time k, $w(k) \in \mathbb{R}^r$ is the disturbance, $z(k) \in \mathbb{R}^p$ is the error output, $p(k), l(k) \in \mathbb{R}^{n_p}$ are the pseudo-input and output connected by $\Theta(k)$, $u(k) \in \mathbb{R}^m$ is the control input and $e(k) \in \mathbb{R}^q$ is the measurement for control. We found that the time-varying parameter Θ in Eq. (6) follows the structure

$$\begin{aligned} \Theta \in \boldsymbol{\Theta} =& \{\mathrm{diag}(\beta_\delta I_3, \alpha_\delta I_2, \gamma I_9) : \\ & |\alpha_\delta| \le \delta_1, |\beta_\delta| \le \delta_2, |\gamma| \le 1\}, \end{aligned} \tag{7}$$

where $\delta_1 = \frac{\overline{\alpha}-\underline{\alpha}}{2}$ and $\delta_2 = \frac{\overline{\beta}-\underline{\beta}}{2}$.

The $\mathcal{L}_2$ gain of the LPV system in Eq. (6) with a gain-scheduling feedback controller is defined as

$$\max_{\Theta \in \boldsymbol{\Theta}, \|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2}. \tag{8}$$

Problem : Our goal is to design a static gain-scheduling control $u(k) = K(\Theta)e(k)$ that minimizes the $\mathcal{L}_2$ gain of the closed-loop LPV system in Eq. (8).

By inspection of the LPV system in Eq. (6), we found that D_{00} is not equal to a zero block. Hence, the system matrices are not affine functions of parameters. It is noted at this juncture that LPV control techniques exist which do handle rational parameter variation, namely [22]. However, for discrete-time systems, no controller formula covering all parameter variation is given in [22]. Instead, for each set of parameters a controller must be solved for using the method given in [23]. Since a different controller is needed for each set of parameters, gridding over the parameter space [24] is necessary, which increases the complexity of implementing the controller. In contrast, [19] does not require any gridding over the parameter space. Also, as shown in Eq. (7) each of the parameters are less than 1 at all times. Therefore, neglecting the higher-order parameter variation is a justifiable approximation. Hence, to utilize the control synthesis technique in [19], we calculate the first-order Taylor series approximation of the system matrices to obtain affine functions in Θ. Notice that Eq. (6) is an upper LFT, i.e.,

$$H(\Theta) := \mathcal{F}_u(M, \Theta). \tag{9}$$

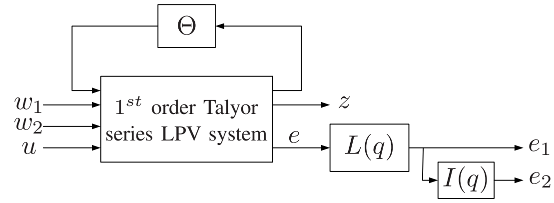

Fig. 5. Augmented 1^{st} order Taylor series LPV system

Using the Taylor series expansion at $\Theta = 0$, the system can be approximated as

$$\hat{H}(\Theta) = H(0) + \beta_\delta \left[\nabla H(0)\right]_1 + \alpha_\delta \left[\nabla H(0)\right]_2 + \gamma \left[\nabla H(0)\right]_3,$$

where $\left[\nabla H(0)\right]_i$ is the partial derivative of the LFT system $H(\Theta)$ in Eq. (9) with respect to the i-th parameter, which can be calculated as shown in [25].

The control synthesis technique in [19] also requires that the ouput matrix, $\hat{C}_2$, be independent of the time-varying parameters and the output e must not be corrupted be the disturbance input, $w(k)$ ($D_{21} = 0$ in (10)). To accomplish this, we filter the error output e with the lowpass filter (Fig. 5) $L(q) = \frac{0.9999}{q-0.0001405}$. The filter $L(q)$ was selected to have a high enough cut off frequency so that it would remain relatively unchanged for all engine speeds considered, since the sampling rate depends on engine speed. Integral action was introduced to eliminate steady-state error for the step input y_d: $I(q) := \frac{e_2(k)}{e_1(k)} = \frac{1}{q-1}$.

The LPV system was augemented with $L(q)$ and $I(q)$ as displayed in Fig. 5 to recover the discrete-time polytopic linear time-varying system

$$\begin{bmatrix} x(k+1) \\ z(k) \\ \hat{e}(k) \end{bmatrix} = \begin{bmatrix} \hat{A}_{[\lambda(k)]} & \hat{B}_{1[\lambda(k)]} & \hat{B}_{2[\lambda(k)]} \\ \hat{C}_{1[\lambda(k)]} & \hat{D}_{11[\lambda(k)]} & \hat{D}_{12[\lambda(k)]} \\ \hat{C}_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ w(k) \\ u(k) \end{bmatrix}, \tag{10}$$

where, for all $k \in \mathbb{Z}_{\ge 0}$, $\lambda(k)$ is the vector of time-varying parameter weights that belongs to the unit simplex

$$\Lambda_N = \left\{ \zeta \in \mathbb{R}^N : \sum_{i=1}^{N} \zeta_i = 1, \zeta_i \ge 0, i = 1, \cdots, N \right\}.$$

A way to compute the weight vector $\lambda(k)$ for a given $\alpha_\delta(k)$, $\beta_\delta(k)$, and $\gamma(k)$ is provided in [26]. For all $k \in \mathbb{Z}_{\ge 0}$, the rate of variation of the weights $\Delta\lambda_i(k) = \lambda_i(k+1) - \lambda_i(k)$, for $i = 1, \cdots, N$ is limited by the calculated bound $b \in [0, 1]$ such that

$$-b\lambda_i(k) \le \Delta\lambda_i(k) \le b(1 - \lambda_i(k)), \quad i = 1, \cdots, N. \tag{11}$$

The system matrices $\hat{A}_{[\lambda(k)]} \in \mathbb{R}^{n\times n}$, $\hat{B}_{1[\lambda(k)]} \in \mathbb{R}^{n\times r}$, $\hat{B}_{2[\lambda(k)]} \in \mathbb{R}^{n\times m}$, $\hat{C}_{1[\lambda(k)]} \in \mathbb{R}^{p\times n}$, $\hat{D}_{11[\lambda(k)]} \in \mathbb{R}^{p\times r}$, $\hat{D}_{12[\lambda(k)]} \in \mathbb{R}^{p\times m}$ belong to the polytope

$$\begin{aligned} \mathcal{D} = \{ & (\hat{A}, \hat{B}_1, \hat{B}_2, \hat{C}_1, \hat{D}_{11}, \hat{D}_{12})(\lambda(k)) : \\ & (\hat{A}, \hat{B}_1, \hat{B}_2, \hat{C}_1, \hat{D}_{11}, \hat{D}_{12})(\lambda(k)) \\ & = \sum_{i=1}^{N} \lambda_i(k) (\hat{A}, \hat{B}_1, \hat{B}_2, \hat{C}_1, \hat{D}_{11}, \hat{D}_{12})_i, \lambda(k) \in \Lambda_N \}. \end{aligned}$$

A finite set of LMIs in [19] can be used to design the gain-scheduling controller. This control is proved to stabilize affine parameter-dependent systems such as (10) with a guaranteed $\mathcal{H}_\infty$ performance bounded by η for all $\lambda \in \Lambda_N$ and $\Delta\lambda$ that satisfies (11).

IV. Design of LTI Feedback Controller

To demonstrate the necessity of a gain-scheduled controller over a linear time invarient (LTI) controller, we designed a fixed gain H_∞ controller based on the nominal parameters. Using the nominal parameters, the closed-loop state-space representation is

$$\begin{aligned} x(k+1) &= A_{CL}(K)x(k) + B_1 w(k), \\ z(k) &= C_{CL}(K)x(k) + D_{11}w(k), \end{aligned} \tag{12}$$

where $A_{CL}(K) = A + B_2KC_2$ and $C_{CL}(K) = C_1 + D_{12}KC_2$. Denoting the transfer function from w to z by H_{wz}, the inequality $\|H_{wz}\|_\infty^2 < \mu$ holds if, and only if, there exists a symmetric matrix P such that

$$\begin{bmatrix} P & A_{CL}(K)P & B_1 & 0 \\ PA_{CL}^T(K) & P & 0 & PC_{CL}^T(K) \\ B_1^T & 0 & I & D_{11}^T \\ 0 & C_{CL}(K)P & D_{11} & \mu I \end{bmatrix} \succ 0 \tag{13}$$

is feasible [27]. The optimal feedback controller K for the closed-loop system (12) is formulated as the optimization of the bilinear matrix inequality (BMI)

$$\min_{\mu,P,K} \mu \quad \text{subject to (13)} \tag{14}$$

where $P = P^T \in \mathbb{R}^{n\times n}$ and $K \in \mathbb{R}^{1\times 2}$. The BMI (14) was solved using PENBMI [28] in conjunction with YALMIP [29] to find the fixed H_∞ controller $K = [1.8260 \quad 0.3205]$.

V. Simulation Results

To demonstrate the effectiveness of the proposed gain-scheduling controller, we now show simulations using the original plant as shown in Eq. (6) for the following cases: engine cold start and load change. We have corrupted the time varying parameters by filtered white noise of up to 10% their nominal values to represent the slowly drifting offset that might occur in practical situations. To see transient responses, the initial conditions for Case 1 were chosen such that a little extra fuel is injected at first, giving a slightly higher equivalence ratio than 1. The initial conditions in Case 2 was set such that each controller would start with an equivalence ratio of 1. For the following simulation cases, we have used the extracted profiles of time varying parameters from engine dynamometer tests.

A. Case 1: Engine Cold Start

We simulate an engine which has been started in freezing temperatures and heats up to an operating temperature of approximately $100°C$ in just under 2 minutes at an engine speed of 1500 rpm. Look-up tables from experimental data of the wall-wetting parameters α and β were used to find the values in Fig. 6E. In Fig. 6A, notice that the gain-scheduling controller remains stable over the time-varying parameters, while the closed-loop system with the fixed H_∞ controller becomes unstable after the engine has warmed up for about 90 seconds.

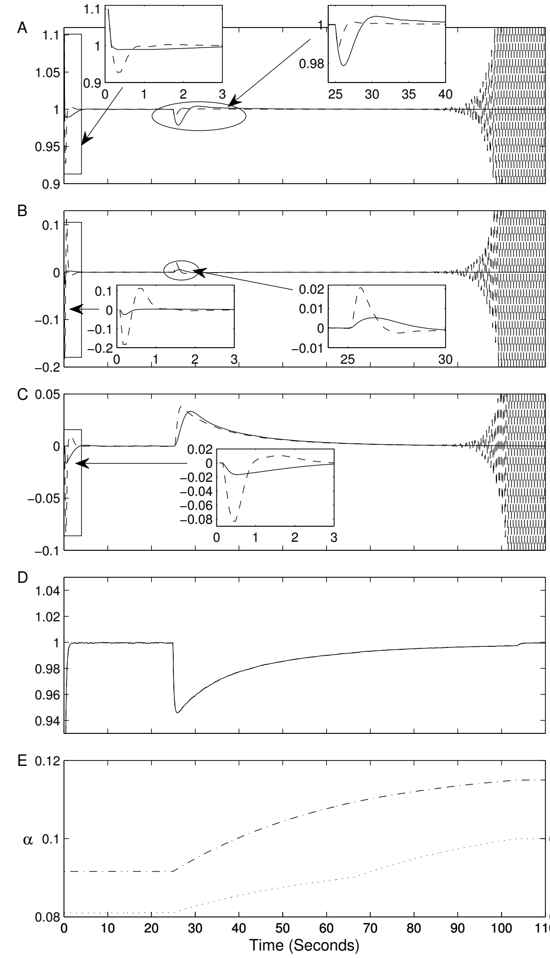

Fig. 6. Case 1 Engine Cold Start: In plots A, B, C, and D the equivalence ratio $y(k)$, proportional control $u_p(k)$, integral control $u_i(k)$, and the feedforward control are compared for the gain-scheduling feedback controller (solid line) and the fixed H_∞ controller (dashed line). The time varying parameters α (dotted line, left axis) and β (dash-dot line, right axis) are displayed in plot E.

B. Case 2: Load Change

We simulate an engine dynamometer test with the engine at an operating temperature of $80°C$ and an engine speed of 1500 rpm. A load increase is induced by increasing the throttle position from 32% to 46%. The increase in throttle position produces a slight change in the wall-wetting parameter β as shown in Fig. 7E. In Fig. 7, the throttle increase at 30 seconds results in a momentary spike in the equivalance ratio; which is quickly regulated back into safe limits by the gain-scheduled controller, but the fixed H_∞ controller takes much longer and uses more control effort.

VI. Conclusion

In this paper, we obtained an event-based sampled discrete-time linear system to represent a port-fuel-injection

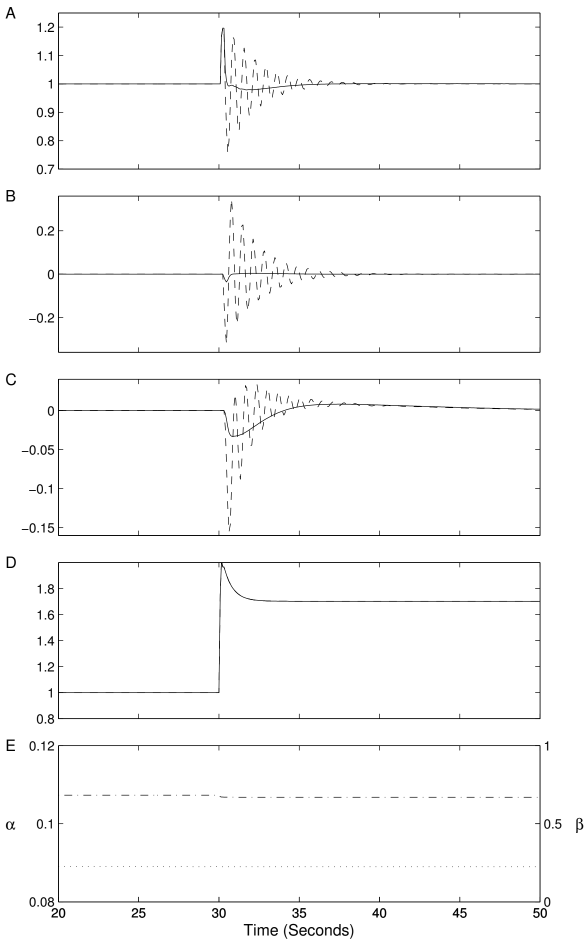

Fig. 7. Case 2 Load Change: In plots A, B, C, and D the equivalence ratio $y(k)$, proportional control $u_p(k)$, integral control $u_i(k)$, and the feedforward control are compared for the gain-scheduling feedback controller (solid line) and the fixed H_∞ controller (dashed line). The time varying parameters α (dotted line, left axis) and β (dash-dot line, right axis) are displayed in plot E.

process based on wall-wetting dynamics and we formulated it as an LPV system. The system parameters used in the engine fuel system model are engine speed, temperature, and load. A gain-scheduling controller for the obtained LPV system was then designed based on the numerically efficient convex optimization (or LMI) technique. The simulation results demonstrate the effectiveness of the proposed scheme.

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