ESANN'2002 proceedings - European Symposium on Artificial Neural Networks
Bruges (Belgium), 24-26 April 2002, d-side publi., ISBN 2-930307-02-1, pp. 257-262

Novelty detection for strain-gauge degradation using maximally correlated components

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Abstract. A new method for the detection of the degradation of strain-gauges attached to airframes is developed, using novelty-detection techniques and maximally correlated components. This considerably improves upon the previous method for the detection of changes in the response-line gradient.

1 Introduction

Strain-gauges are attached to airframes to assess the damage caused by a flight to the aircraft. Each stress cycle measured by such a gauge is classified into a cell determined by the mean and alternating strain (the difference between the maximum and minimum) of the cycle. These cells make up a *Frequency Of Occurrence Matrix* (FOOM), and each cell contributes to a weighted sum evaluating the expected wear caused by the flight (see Hickinbotham and Austin [1] a detailed account). The weights are chosen to reflect the damage caused by a stress cycle of given severity, and a combination of high mean and alternating strains is considered to be sufficiently damaging to put the aircraft out of service. Thus, cells corresponding to extreme values of both strains are excluded from the FOOM, which then has a triangular shape.

Unfortunately, strain-gauges themselves may be subject to degradation. There are 4 modes of strain-gauge failure: spread noise (randomly elevated or depressed counts in random cells), spike noise (large elevations of counts in one cell), intercept shift (the linear relation $r = as$ between strain and strain-gauge response becomes $r = as + b$) and gradient shift ($r = as$ becomes $r = a's$).

The detection of noise and of intercept shifts (*Non-Gradient-Shift-Corruption Detection*, ngscd) by examining the FOOM's themselves has been successfully

Work funded by BAe Systems
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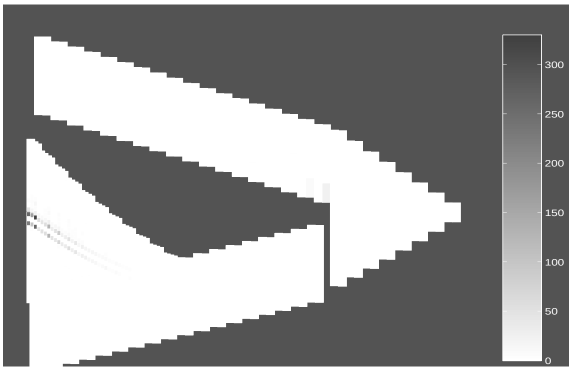

Figure 1: A FOOM — the area outside the FOOM is black, and cells with zero counts are white

carried out in [1]: this is done by extracting 2 *unlikeliness* features and 2 "eigen-FOOM" features (*Principal Components* — PC's — see, e.g., Mardia *et al.* [2][1]) and feeding them into a Gaussian Basis Function Network (GBFN — see, Bishop [4]). However, this technique fails in the detection of gradient shifts (gsd).

Further information about the flights is available in the form of auxiliary data (a vector of g-levels attained during the flight), which can be used to deal with gsd. The approach here is to extract features from this auxiliary data, and to correlate these with the FOOM's (considered as 1728-component vectors).

2 Algorithm

The unlikeliness features used in [1] are maintained, as they are important in ngscd (especially noise corruption). For an arbitrary FOOM, these are a count of unlikely cells and an unlikelihood measure. A cell is *unlikely* if the probability density of its count is low according to a Gaussian pdf whose mean, μ_i, and variance, σ_i^2, are derived from a cell-by-cell analysis of a training set of uncorrupted FOOM's. The unlikelihood measure is $\sum_i |C_i - \mu_i|/\sigma_i$, over the unlikely cells.

The PC features used in [1] are: if $\mathbf{v}$ is a PC of the training set of FOOM's (i.e., a vector in $\mathbb{R}^{1728}$) and $\mathbf{f}$ an arbitrary FOOM, then the feature value for $\mathbf{f}$ corresponding to $\mathbf{v}$ is simply $\mathbf{v}^{\mathrm{T}}\mathbf{f}$.

Here we find vectors $\mathbf{v} \in \mathbb{R}^{1728}$ in the subspace spanned by the training set of FOOM's which are *maximally correlated* with vectors of auxiliary data corresponding to the flights generating the FOOM's. These will then be used to define *combined* (flight/FOOM) features. The method differs from that of *canonical components* (see [2]), which puts the auxiliary and FOOM data on an equal footing, instead of treating the former as primary. The relation between the two methods wil be addressed in a later paper.

[1]or Turk and Pentland [5], in a pattern recognition context

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2.1 Mathematical development

Let sets of n vectors $\mathbf{x}^k \in \mathbb{R}^m$ (corresponding to a training set of FOOM's), $\mathbf{y}^k \in \mathbb{R}^p$ (corresponding to auxiliary data from the flights giving rise to the FOOM's), where $p \geq n \geq m$, be given. Suppose $\mathbf{u} \in \mathbb{R}^m$ is also given. We wish to find $\mathbf{v} \in [\mathbf{y}^1, \ldots, \mathbf{y}^n] \subset \mathbb{R}^p$ such that the $t_k = \mathbf{v}^{\mathrm{T}}\mathbf{y}^k$ are maximally correlated with the $s_k = \mathbf{u}^{\mathrm{T}}\mathbf{x}^k$, and $||\mathbf{v}|| = 1$. Set $\mathbf{v} = \mathbf{Y}\boldsymbol{\beta}$, where $\mathbf{Y} = (\mathbf{y}^1, \ldots, \mathbf{y}^n) \in \mathbb{R}^{p\times n}$.

Now

$$\bar{s} = \frac{1}{n}\sum_{k=1}^{n} s_k = \mathbf{u}^{\mathrm{T}}\left(\frac{1}{n}\sum_{k=1}^{n}\mathbf{x}^k\right) = \mathbf{u}^{\mathrm{T}}\bar{\mathbf{x}} = \mathbf{u}^{\mathrm{T}}\mathbf{X}\mathbf{e}, \tag{1}$$

where $\mathbf{X} = (\mathbf{x}^1, \ldots, \mathbf{x}^n) \in \mathbb{R}^{m\times n}$, $\mathbf{e} = \frac{1}{n}(1, \ldots, 1)^{\mathrm{T}} \in \mathbb{R}^n$, and $\bar{t} = \boldsymbol{\beta}^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\mathbf{e}$.

But

$$\sigma_s^2 = \frac{1}{n}\sum_{k=1}^{n} s_k^2 - \bar{s}^2 = \frac{1}{n}\mathbf{u}^{\mathrm{T}}\mathbf{X}[\mathrm{I}_n - n\mathbf{e}\mathbf{e}^{\mathrm{T}}]\mathbf{X}^{\mathrm{T}}\mathbf{u} = \frac{1}{n}\mathbf{u}^{\mathrm{T}}\mathbf{X}\mathbf{P}\mathbf{X}^{\mathrm{T}}\mathbf{u}, \tag{2}$$

where $\mathbf{P} = \mathrm{I}_n - n\mathbf{e}\mathbf{e}^{\mathrm{T}}$, and, $\sigma_{st} = \frac{1}{n}\mathbf{u}\mathbf{X}^{\mathrm{T}}\mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\beta$ and $\sigma_t^2 = \frac{1}{n}\beta^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\beta$.

Now, we wish to solve the problem:

$$\textit{Maximise } r_{\mathrm{corr}} = \frac{\sigma_{st}}{\sigma_s\sigma_t} \textit{ with respect to } \boldsymbol{\beta}, \textit{ subject to } ||\mathbf{Y}\boldsymbol{\beta}|| = 1. \tag{3}$$

But r_{corr} is independent of the scaling of $\boldsymbol{\beta}$, and σ_s is entirely independent of $\boldsymbol{\beta}$, so the problem (3) is equivalent to:

$$\begin{aligned} &\textit{Maximise } n\sigma_{st} = \mathbf{u}^{\mathrm{T}}\mathbf{X}\mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\boldsymbol{\gamma}, \textit{ w.r.t. } \boldsymbol{\gamma}, \\ &\quad \textit{subject to } n\sigma_t^2 = \boldsymbol{\gamma}^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\boldsymbol{\gamma} = 1, \quad \boldsymbol{\beta} = \boldsymbol{\gamma}/||\mathbf{Y}\boldsymbol{\gamma}||. \end{aligned} \tag{4}$$

As $\mathbf{P}$ is a projection, $\boldsymbol{\gamma}^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\boldsymbol{\gamma} = \boldsymbol{\gamma}^{\mathrm{T}}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\mathbf{P}^{\mathrm{T}}\mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\boldsymbol{\gamma} = ||\mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}\boldsymbol{\gamma}||^2$, and problem (4) is equivalent to:

$$\textit{Maximise } \mathbf{u}^{\mathrm{T}}\mathbf{X}\mathbf{Z}\boldsymbol{\gamma}, \textit{ w.r.t. } \mathbf{w}, \textit{ subject to } ||\mathbf{Z}\boldsymbol{\gamma}|| = 1, \boldsymbol{\beta} = \boldsymbol{\gamma}/||\mathbf{Y}\boldsymbol{\gamma}||, \tag{5}$$

where $\mathbf{Z} = \mathbf{P}\mathbf{Y}^{\mathrm{T}}\mathbf{Y}$.

Let column-orthogonal $\mathbf{U} \in \mathbb{R}^{p\times n}$, non-negative diagonal $\mathbf{S} \in \mathbb{R}^{n\times n}$ (with nonzero elements nonincreasing down the diagonal) and orthogonal $\mathbf{V} \in \mathbb{R}^{n\times n}$ be a singular value decomposition for $\mathbf{Z}$: $\mathbf{Z} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}}$.[2] Then $S^\dagger$ given by

$$S_{ij}^\dagger = 0, \text{ if } S_{ij} = 0;\ 1/S_{ij} \text{ otherwise} \tag{6}$$

is a pseudo-inverse for $\mathbf{S}$, and $\mathbf{Z}^\dagger = \mathbf{V}\mathbf{S}^\dagger\mathbf{U}^{\mathrm{T}}$ is a pseudo-inverse for $\mathbf{Z}$.

Now $\mathbf{Z}$ is not of full rank as $\mathbf{P}$ is a projection, and so at least one diagonal element of $\mathbf{S}$ is 0. But, if $\mathbf{Z} = \mathbf{P}\mathbf{Y}^{\mathrm{T}} = \mathbf{Y}^{\mathrm{T}} - n\mathbf{e}\bar{\mathbf{y}}^{\mathrm{T}} = 0$, then $\mathbf{y}^1 = \cdots = \mathbf{y}^n = \bar{\mathbf{y}}$, which we assume is not so.

Hence, if we put $\boldsymbol{\gamma} = \mathbf{Z}^\dagger\boldsymbol{\eta}$, problem (5) is solved by the solution of

$$\textit{Maximise } \mathbf{u}^{\mathrm{T}}\mathbf{X}\mathbf{Z}\mathbf{Z}^\dagger\boldsymbol{\eta}, \textit{ w.r.t. } \boldsymbol{\eta}, \quad \textit{subject to } ||\mathbf{Z}\mathbf{Z}^\dagger\boldsymbol{\eta}|| = 1. \tag{7}$$

[2] The triplet $\mathbf{U}, \mathbf{S}, \mathbf{V}$ is only unique if $\mathbf{Z}\mathbf{Z}^{\mathrm{T}}$ has no equal eigenvalues

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Now let $r = \mathrm{rank}(\mathbf{Z}) \leq n-1$. Then $\mathbf{Z}\mathbf{Z}^\dagger = \mathbf{U}\mathbf{S}\mathbf{S}^\dagger\mathbf{U}^\mathrm{T} = \mathbf{U}\mathrm{I}^{(r)}\mathbf{U}^\mathrm{T}$, where

$$\mathrm{I}^{(r)} = \begin{bmatrix} \mathrm{I}_r & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}. \tag{8}$$

Consequently, problem (7) is equivalent to

$$\textit{Maximise } \mathbf{u}^\mathrm{T}\mathbf{X}\mathbf{U}\mathrm{I}^{(r)}(\mathrm{I}^{(r)}\mathbf{U}^\mathrm{T}\boldsymbol{\eta}), \ \textit{w.r.t. } \boldsymbol{\eta}, \quad \textit{subject to } ||\mathrm{I}^{(r)}\mathbf{U}^\mathrm{T}\boldsymbol{\eta}|| = 1, \tag{9}$$

which has the solution $\mathrm{I}^{(r)}\mathbf{U}^\mathrm{T}\boldsymbol{\eta} = \mathrm{I}^{(r)}\mathbf{U}^\mathrm{T}\mathbf{X}^\mathrm{T}\mathbf{u}$, (as the projection of the condition is now isotropic in the subspace in which we are looking for a solution), so $\mathbf{V}\mathbf{S}^\dagger\mathrm{I}^{(r)}\mathbf{U}^\mathrm{T}\boldsymbol{\eta} = \mathbf{V}\mathbf{S}^\dagger\mathbf{U}^\mathrm{T}\boldsymbol{\eta} = \mathbf{Z}^\dagger\boldsymbol{\eta} = \boldsymbol{\gamma} = \mathbf{Z}^\dagger\mathbf{X}^\mathrm{T}\mathbf{u}$, and

$$\mathbf{v} = \mathbf{Y}\mathbf{Z}^\dagger\mathbf{X}^\mathrm{T}\mathbf{u}/||\mathbf{Y}\mathbf{Z}^\dagger\mathbf{X}^\mathrm{T}\mathbf{u}||. \tag{10}$$

This is the component in $[\mathbf{y}^1, \ldots, \mathbf{y}^n]$ *maximally correlated* with $\mathbf{u}$.

Letting $\mathbf{s} = (s_1, \ldots, s_n)^\mathrm{T} = \mathbf{X}^\mathrm{T}\mathbf{u}$, $\mathbf{t} = (t_1, \ldots, t_n)^\mathrm{T} = \mathbf{Y}^\mathrm{T}\mathbf{v}$, we perform a linear regression of $\mathbf{t}$ on $\mathbf{s}$: we put $\mathbf{t}_m = \mathbf{U}_r\boldsymbol{\theta}$, where $\mathbf{U}_r = [n\mathbf{e}, \mathbf{s}] \in \mathbb{R}^{n\times 2}$, $\boldsymbol{\theta} \in \mathbb{R}^2$ and minimise $||\mathbf{t} - \mathbf{t}_m||^2 = (\mathbf{U}_r\boldsymbol{\theta} - \mathbf{t})^\mathrm{T}(\mathbf{U}_r\boldsymbol{\theta} - \mathbf{t})$ with respect to $\boldsymbol{\theta}$, so

$$\boldsymbol{\theta} = \left(\mathbf{U}_r{}^\mathrm{T}\mathbf{U}_r\right)^{-1}\mathbf{U}_r{}^\mathrm{T}\mathbf{t} = \frac{1}{\mathbf{s}^\mathrm{T}\mathbf{P}\mathbf{s}}\begin{bmatrix} \mathbf{s}^\mathrm{T}(\mathbf{s}\mathbf{t}^\mathrm{T} - \mathbf{t}\mathbf{s}^\mathrm{T}) \\ \mathbf{s}^\mathrm{T}\mathbf{P}\mathbf{t} \end{bmatrix}. \tag{11}$$

For an arbitrary FOOM $\mathbf{f}$ and corresponding auxiliary data vector $\mathbf{g}$, we define the feature value as $\mathbf{u}_1{}^\mathrm{T}\mathbf{g} - \theta_1 - \theta_2\mathbf{v}_1{}^\mathrm{T}\mathbf{f}$, where $\mathbf{v}_1$ is the component maximally correlated (over a set of training FOOM's) with the component $\mathbf{u}_1$ of the corresponding auxiliary data.

2.2 Use of the combined feature

Before extracting a further combined feature from the training FOOM's and associated auxiliary data, we project the training FOOM's into the space orthogonal to $\mathbf{v}_1$, and, similarly, before extracting the $(i+1)$st combined feature, we project orthogonal to $\mathbf{v}_i$, so that components which are used to "explain" earlier features of the auxiliary data are not also used on later features.

For the first feature of this type, we chose $\mathbf{u}$ to be the mean of the auxiliary data corresponding to the training FOOM's, and, for the second, we use the first PC of the auxiliary data.

3 Simulations

For our simulations, we used data split into two sets of 50 uncorrupted FOOM's as training and validation sets. As test FOOM's we used a set of 50 uncorrupted FOOM's and sets of 50 FOOM's corrupted by (simulated) spread noise, spike noise, positive and negative intercept shifts and increased and reduced gradients.

Cell-by-cell statistics, maximally correlated components and regression coefficients were extracted from the training set alone. The corresponding features

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were extracted from this set and used to train 10 randomly initialised GBFN's. The trained GBFN's were presented with features extracted from the validation set, and the GBFN with the least maximum *novelty* (as defined in [1]) over this set was selected. The features extracted from the test data were then presented to this GBFN. The novelty was thresholded at various levels, dividing the test FOOM's into familiar (presumed uncorrupted) and unfamiliar FOOM's (presumed corrupt). The *Receiver Operator Characteristic* curve (ROC) (see Figure 2 and, e.g., Scott *et al.* [3]) plotting true positive (here, a corrupt FOOM classified as such) rates against false positive (an uncorrupted FOOM classified as corrupt) rates and AUROC's (*Area Under the ROC*) were then calculated for the test data. The AUROC measures the performance of a family of classifiers, a family with an AUROC of 1 being perfect and a family of random classifiers having an AUROC of 0.5.

This procedure was repeated 10 times for a range of GBFN parameter values (the number of GBF's: 2-8; and the minimum allowed determinant of the covariance of a GBF: 2^{-73}-2^{-64}) to obtain a sample of AUROC's for each parameter.

The parameter value with the highest value of $\mu_{\text{AUROC}} - 2\sigma_{\text{AUROC}}$ is chosen as a compromise between reliability and optimum performance.

3.1 Results

The simulation was carried out for gauges in 2 locations, one considered relatively easy to analyse aerodynamically and one considered to be hard to analyse.

Although the present method has no parameter values for which $\mu_{\text{AUROC}} - 2\sigma_{\text{AUROC}}$ is less than 0.5 for gsd, $\mu_{\text{AUROC}} - 2\sigma_{\text{AUROC}}$ for the old method was less than 0.5 for gsd at every parameter value for the "easy" location, and at 60 out of 70 for the "hard" location. For ngscd, no parameter value led to $\mu_{\text{AUROC}} - 2\sigma_{\text{AUROC}}$ being less than 0.5 for either method.

corruption	**location**	**μ, old method**	**μ, new method**	**improvement**	**σ, old method**	**σ, new method**
ngscd	**"easy"**	0.92902	0.80246	-14%	0.01492	0.00638
	"hard"	0.85674	0.87146	2%	0.01278	0.01552
gs	**"easy"**	0.45152	0.80232	78%	0.02136	0.01363
	"hard"	0.57979	0.75202	30%	0.02285	0.01018

Table 1: AUROC statistics for ngscd and gsd

Table 1 and Figure 2 show that the new method dramatically outperforms the old on gsd. Moreover, the new method is also more reliable, as shown by the reduced σ's for its AUROC's, and less dependent on a correct choice of GBFN parameters, as shown by the massive reduction in the number of parameter values for which $\mu_{\text{AUROC}} - 2\sigma_{\text{AUROC}} < 0.5$. On the problem of ngscd, the new method is not as good as the old, but this is of minor relevance, as this problem has already been solved.

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Once the best parameter value for a given stress location has been determined, the full training algorithm takes $\sim$ 100s and the classification of an unseen FOOM by a trained GBFN $\sim$ 0.6ms, on a 550MHz PC.

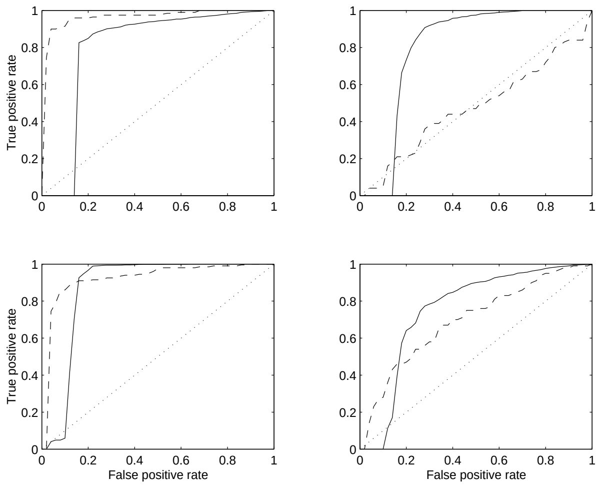

Figure 2: ROC's for the new method (solid) and that of [1] (dashed). Clockwise from top left: ngscd at "easy" location; gsd, "easy"; gsd, "hard"; ngscd, "hard"

4 Conclusion

The new method presented here has a much improved performance for gsd, enabling it to be used in combination with the previous method to detect all 4 modes of strain-gauge degradation in practical applications.

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