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Markovian blind separation of non-stationary temporally correlated sources

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Abstract. In a previous work, we developed a quasi-efficient maximum likelihood approach for blindly separating stationary, temporally correlated sources modeled by Markov processes. In this paper, we propose to extend this idea to separate mixtures of non-stationary sources. To handle non-stationarity, two methods based respectively on blocking and kernel smoothing are used to find parametric estimates of the score functions of the sources, required for implementing the maximum likelihood approach. Then, the proposed methods exploit simultaneously non-Gaussianity, non-stationarity and time correlation in a quasi-efficient manner. Experimental results using artificial and real data show clearly the better performance of the proposed methods with respect to classical source separation methods.

1 Introduction

Linear instantaneous Blind Source Separation (BSS) methods aim at recovering a set of unobserved source signals from several observations which are supposed to be linear transformations of these sources. In its simplest form, this problem can be formulated as follows. Assume that we have N samples of K linear instantaneous mixtures of K source signals. The noiseless linear mixture model is defined as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t)$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_K(t)]^T$ and $\mathbf{s}(t) = [s_1(t), \ldots, s_K(t)]^T$ are, respectively, observations and source vectors and $\mathbf{A}$ is an unknown $K \times K$ mixing matrix. The aim of blind source separation is to find an estimate of the matrix $\mathbf{A}$ up to scaling and permutation. It has been proved that this problem can be solved by exploiting non-Gaussianity, autocorrelation or non-stationarity [1].

In a previous work [2], we proposed a blind source separation method based on a maximum likelihood approach where a Markov model was used to simplify the joint Probability Density Functions (PDF) of successive samples of temporally correlated *stationary* one-dimensional independent sources. This method exploits both source non-Gaussianity and autocorrelation and has the advantage of providing an asymptotically efficient estimator. In [3], we extended this approach to images and proposed two major modifications to reduce the computational cost of the algorithm.

In this work, we extend the method of [2] to non-stationary one-dimensional sources by allowing their PDF to change with time. As a result, the proposed

approach can exploit simultaneously non-Gaussianity, non-stationarity and time correlation in a quasi-optimal manner.
Moreover, while most of the BSS methods based on non-stationarity [4, 5, 6] are second-order methods exploiting only the variations of the source variances, our method is able to also take advantage of higher-order non-stationarities.

2 Maximum Likelihood separation method

Assuming the linear instantaneous mixture model $\mathbf{x} = \mathbf{A}\mathbf{s}$, our goal is to estimate a separating matrix $\mathbf{B} = \mathbf{A}^{-1}$ up to a permutation and a scaling matrix. In a maximum likelihood approach, this can be done by maximizing, with respect to the separating matrix $\mathbf{B}$, the joint PDF of all the samples of all the components of the observation vector $\mathbf{x}$, denoted by

$$f_{\mathbf{x}}(x_1(1), \dots, x_K(1), \dots\dots, x_1(N), \dots, x_K(N)) \tag{1}$$

Supposing source signals to be mutually independent and q-th order Markov sequences [1], this joint PDF can be written as

$$\Big(\frac{1}{|\det(\mathbf{B}^{-1})|}\Big)^N \prod_{i=1}^{K} \Big[f_{s_i(t)}(\mathbf{e}_i^T\mathbf{B}\mathbf{x}(1), \dots, \mathbf{e}_i^T\mathbf{B}\mathbf{x}(q)) \prod_{t=q+1}^{N} f_{s_i(t)}(\mathbf{e}_i^T\mathbf{B}\mathbf{x}(t)|\mathbf{e}_i^T\mathbf{B}\mathbf{x}(t-1), \dots, \mathbf{e}_i^T\mathbf{B}\mathbf{x}(t-q))\Big] \tag{2}$$

where $\mathbf{e_i}$ is the i^{th} column of the identity matrix. Defining the conditional score function at time t of a source s_i with respect to a source sample $s_i(t-l)$ by

$$\psi^l_{s_i(t)}(s_i(t)|s_i(t-1), \dots, s_i(t-q)) = \frac{-\partial log f_{s_i(t)}(s_i(t)|s_i(t-1), \dots, s_i(t-q))}{\partial s_i(t-l)}, \quad \forall \quad 0 \le l \le q$$

the maximization of the logarithm of (2) yields finally the following system of $K(K-1)$ estimating equations in the same way as in [2]

$$E_{N-q}\Big[\sum_{l=0}^{q} \psi^l_{s_i(t)}(s_i(t)|s_i(t-1), \dots, s_i(t-q))s_j(t-l)\Big] = 0 \quad i \neq j = 1, \dots, K \tag{3}$$

where $E_{N-q}[.]$ is the temporal mean over $(N-q)$ samples of the source signals. Replacing $s_i(.)$ by $\mathbf{e_i}^T\mathbf{B}\mathbf{x}(.)$, this system may be solved to estimate $\mathbf{B}$ up to a diagonal and a permutation matrix as in [2]. This may be done for example using a modified equivariant version of the Newton-Raphson algorithm, which

[1] Markov sequence is used to model time correlation of the sources and to simplify the joint PDFs involved in likelihood functions. Contrary to ARMA models, it is able to model the nonlinear dependence among the samples of each source.

can be considered as a one-dimensional version of the algorithm presented in [3], for possibly non-stationary sources. In practice, the actual sources being unkown, their score functions may be estimated only via the reconstructed sources $\hat{\mathbf{s}} = \hat{\mathbf{B}}\mathbf{x}(t)$ using an iterative algorithm. In [2], we supposed the sources were stationary so that $\psi^l_{s_i(t)} = \psi^l_{s_i}$ did not vary in time. In this paper, however, we are concerned with non-stationary sources, so that $\psi_{s_i(t)}$ may vary with t and we have no prior knowledge about the way they vary, except that they should change slowly. In the following section, two alternative approaches are proposed to handle this non-stationarity.

3 Non-stationary estimation of the score functions

In order to model the non-stationarity of the score functions, we adapt the two methods, proposed in [7], initially used to handle the non-stationarity of *temporally uncorrelated* sources.

1. *Blocking method:* The temporal interval $[0, T]$ is split into L subintervals $T_j,\ j = 1, \ldots, L$. Assuming that the score functions vary slowly with time, they are considered to be constant over each subinterval and denoted as $\psi^l_{s_i(t)} = \psi^l_{s_i}(j), \quad \forall \quad t \in T_j$. In each interval T_j, the conditional score functions may be estimated by the non-parametric estimator used in [2]. However, in order to reduce the computational cost, we use a third-order polynomial parametric estimator which may be considered as a one-dimensional version of the estimator presented, for stationary sources, in [3].

2. *Kernel smoothing method:* The estimation of the score functions by third-order polynomials requires the computation of some mathematical expectations generally denoted $E\big[\phi(x_i(t), x_i(t-1) \ldots, x_i(t-q))\big]$ where $\phi(.)$ is a nonlinear function (see [3] or [7] for more details).
In the blocking method, these expectations are estimated by the temporal mean of $\phi(.)$ on each block. In the kernel smoothing method, however, the expectations are estimated using the following formula

$$\hat{E}\big[\phi(x_i(t), \ldots, x_i(t-q))\big] = \frac{\sum_{\tau=q+1}^{N} \kappa(\frac{\tau-t}{\nu})\phi(x_i(\tau), \ldots, x_i(\tau-q))}{\sum_{\tau=q+1}^{N} \kappa(\frac{\tau-t}{\nu})} \tag{4}$$

where $\kappa(.)$ is a kernel function and ν is a window width parameter. The kernel smoothing method provides a local average of the function around the time point of interest and should be better than the blocking method, especially when signal statistics change rapidly. However, it is computationally more expensive. To reduce the computational cost, the kernel smoother (4) can be approximated using sparser averaging, which yields the following estimator

$$\hat{E}\big[\phi(x_i(t), \ldots, x_i(t-q))\big] = \frac{\sum_{l=l_1}^{L} \kappa(\frac{\frac{lN}{L}-t}{\nu})\phi(x_i(\frac{lN}{L}), \ldots, x_i(\frac{lN}{L}-q))}{\sum_{l=l_1}^{L} \kappa(\frac{\frac{lN}{L}-t}{\nu})}$$

where L is chosen so that $\frac{N}{L}$ is an integer and l_1 is the first integer greater than $\frac{L(q+1)}{N}$. Moreover, the choice of L should be adapted to the smoothness of the signal.
It can be noted that the kernel smoothing approach is more flexible than the blocking approach since the window parameter ν can be adapted, at each iteration, to the reconstructed source statistics.

4 Algorithm

The BSS method which results from the above principles operates as follows.

1. initialization of the separating matrix: $\hat{\mathbf{B}} = \mathbf{I}$
2. Repeat until $\hat{\mathbf{B}}$ does not change significantly:
   (a) computation of output signals $\hat{\mathbf{s}} = \hat{\mathbf{B}}\mathbf{x}$.
   (b) normalization of $\hat{\mathbf{s}}$ to obtain unit variances.
   (c) non-stationary parametric estimation of score functions.
   (d) resolution of the system of equations (3), to obtain a new estimate $\hat{\mathbf{B}}$.

5 Experimental Results

5.1 Artificial data

In the following simulations, we compare our non-stationary Markovian methods with the standard Markovian maximum likelihood separation approach as presented in [2]. In the first experiment, two independent white and uniformly distributed signals $e_1(t)$ and $e_2(t)$ are filtered by two autoregressive (AR) filters in order to generate two 1^{st}-order Markovian sources following the model $s_i(t) = e_i(t) + \rho_i s_i(t-1)$. The filter coefficients ρ_1 and ρ_2 are set to 0.2 and 0.9, respectively. Each resulting signal is then split into P blocks and each block is multiplied by a different coefficient α_p, $p = 1, \ldots, P$ to obtain non-stationary sources. The generated sources are then artificially mixed by the mixing matrix $\mathbf{A} = \begin{pmatrix} 1 & 0.99 \\ 0.99 & 1 \end{pmatrix}$. Then, our blocking method using L blocks is used to obtain the estimated sources $\hat{s}_i(t)$. For each experiment, the output Signal to Interference Ratio (in dB) is computed by $SIR = \frac{1}{K}\sum_{i=1}^{K} 10\log_{10}\frac{E[s_i^2]}{E[(\hat{s}_i - s_i)^2]}$, after normalizing the estimated sources, $\hat{s}_i$, so that they have the same variances and signs as the source signals, s_i.
The mean of SIR over 100 Monte Carlo simulations, with $N = 1000$ and $P = 8$, is computed and shown in Fig. 1 as a function of the number of blocks L used in our algorithm.
The case of one block, which yields an average SIR of 13 dB, corresponds to the standard Markovian Likelihood Method. It can be noticed that the non-stationary version of our Markovian algorithm outperforms the standard one,

whatever the number of blocks considered in the proposed model. The best performance is reached when $L = P$, so that the mean and the standard deviation of SIR were 37 dB and 7 dB, respectively. Nevertheless, even if signals are over-blocked, we still obtain an SIR higher than 35 dB provided that L is lower than 20, so that the number of samples in each block is sufficient for score function estimation.

In the second experiment, we want to highlight the advantage of the kernel smoothing non-stationary method compared to the blocking one. Two independent white and uniformly distributed signals are generated and filtered by the same AR filters as above. The resulting 1000-sample signals are split into 200 blocks, and each block is multiplied by a different coefficient so that the variances change quite rapidly. The sources are mixed artificially by the same matrix $\mathbf{A}$ as in the first experiment and the mean of SIR over 100 Monte Carlo simulations using a Gaussian kernel is computed and shown in Fig. 2 as a function of the kernel standard deviation σ. The kernel smoothing method led to an average SIR of 41 dB for $\sigma = 100$ while the blocking method led to 35-dB at best. The standard deviation of the SIR for 100 Monte Carlo simulations is quite similar to the blocking method one, in this case.
Nevertheless, this approach is very time consuming compared to the blocking algorithm, especially for long signals. Using a mixture of two 1000-sample non-stationary source signals of P=8 blocks, the running times of blocking and kernel methods on a 1.53 GHz AMD-athlon PC were 0.2 seconds and 25 seconds respectively, for each iteration.

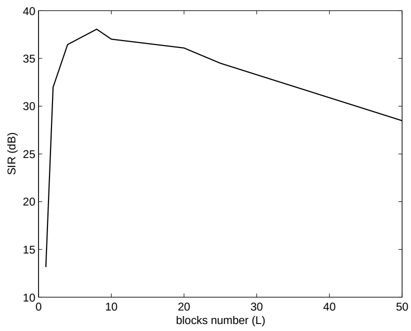

Figure 1: Mean of SIR as a function of the number of blocks.

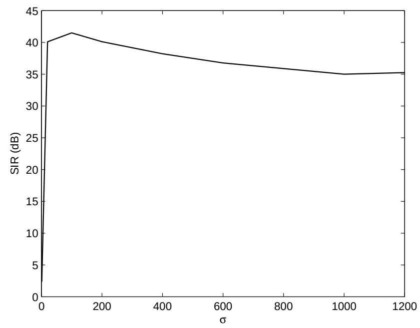

Figure 2: Mean of SIR as a function of the Gaussian kernel standard deviation.

5.2 Real data

In another experiment, we applied our blocking non-stationary Markovian method to a linear instantaneous mixture of two 100000-sample speech signals. The source signals were artificially mixed using the same matrix **A** as in the previous section. The mean of output SIRs computed over 10 couples of speech signals using our method is compared to the mean of the SIRs achieved by 19 standard algorithms available in the ICALAB Toolbox [8, 9]. Our method outperforms the 19 algorithms, with an average SIR of 91 dB while ICALAB algorithms led to 58-dB mean SIR at best (SYM-WHITE method). In the last experiment, we applied our method and the 19 ICALAB algorithms to an artificial linear instantaneous mixture of 8 real speech signals. While our algorithm led to a 66-dB SIR, the best performance achieved by ICALAB algorithms was a 36-dB SIR (SYM-WHITE algorithm).

6 Conclusion

In this paper, we proposed an extension to non-stationary sources of our Markovian blind source separation algorithm. Two approaches, based respectively on blocking and kernel smoothing have been used in order to take into account non-stationarity. Experimental results, especially with real speech signals, proved the better performance of our non-stationary method in comparison to a large number of standard algorithms available in the ICALAB toolbox. An extension of this method to bi-dimensional sources will be developed in future works.

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