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Stability of Neural Network Control for Uncertain Sampled-Data Systems

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Abstract. This paper derives robust stability conditions for neural network control of sampled-data systems whose parameters are uncertain. The controllers are nonlinear, full state regulators implemented as single hidden layer, feedforward neural networks. The controlled systems must be locally controllable and full-state accessible. The robust stability is confirmed by the existence of a Lyapunov function of the closed loop systems. A modified backpropagation algorithm with a model reference technique is employed to determine the weights of the controllers. Simulation results on the classical motor-driven inverted pendulum model are presented to demonstrate the applications of these conditions.

1 Introduction

Neural networks (NNs) have been proposed for use in a broad range of control applications [1]-[2]. Nowadays, there are many approaches used to design a neural network controller (NNC) [3]-[6]. Regardless of the design approach, the stability of the control system needs to be systematically verified. Moreover, the problem becomes more complex when any parameter of the system is uncertain. The control system that remains stable in the presence of the uncertainty is said to be robustly stable.

A robustifying control methodology using high-order NNs was proposed by Rovithakis [7]. In his approach, a nominal controller was first designed to guarantee a desired control performance for the nominal system. The NN was then trained to approximate the nonlinear terms that were not included in the nominal model. The parameters of the trained NN were employed to form the augmented adaptive control signal, which actually robustified the nominal system. Kuntanapreeda and Fullmer [9], [10] presented stability sufficient conditions for a class of NN control systems. The controller was a single hidden layer feedforward NN, with linear output functions at the output neurons. The controlled nonlinear system was restricted to be locally hermitian, which was later removed in [11]. The stability conditions were for nonadaptive applications. A modified backpropagation training algorithm for adjusting the weights of NNCs was also proposed in [9]. This modified algorithm imposed the stability conditions as training constraints, so the stability of the NN control systems is guaranteed.

In this paper, we extend the works in [9]-[11] by deriving robust stability conditions for NN control of sampled-data nonlinear systems, whose parameters are

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uncertain. The modified backpropagation algorithm proposed in [9] is also used here for training NNCs to satisfy with the new conditions.

2 Neural Network Control Systems

Consider NN control systems comprising a nonlinear system with parameter uncertainties and a feedforward NN closing the feedback loop as shown in Fig. 1

2.1 Controlled Systems with Parameter Uncertainties

We consider nonlinear uncertain systems of order n with the state-space model

$$\begin{aligned} \partial x(\tau) &= G(x(\tau),u(\tau),\delta) \\ 0 &= G(0,0,\delta) \end{aligned} \tag{1a}$$

where ∂ denotes the differentiation operator for continuous-time systems, or the shift operator for sampled-data systems, τ is the time-step k for sampled-data systems, $x \in \Re^n$ is the state vector with initial condition x_0, $u \in \Re^m$ is the input vector, and $\delta \in \Re^p$ is an uncertain parameter vector. It is assumed that the systems can be modeled as linear uncertain systems

$$\partial x(\tau) = [A_0 + \alpha_1 A_1]x(\tau) + B_0 u(\tau). \tag{1b}$$

Here $A_0 \in \Re^{n\times n}$ and $B_0 \in \Re^{n\times m}$ are a nominal constant system and input matrices, respectively. The system's uncertainties are represented by $A_1 \in \Re^{n\times n}$, and $\alpha_1 \in \Re$ where $|\alpha_1| \le \mu \in \Re^+$. The pair (A_0, B_0) is assumed to be controllable.

Remark 1: In (1b), A_0 and B_0, respectively, can be found by computing the Jacobian matrices of $G(\bullet)$ with respect to $x(\tau)$ and $u(\tau)$ evaluated at the equilibrium point. Also, A_1 and α_1, as well as μ, can directly be estimated from

$$\alpha_1 A_1 x(k) = G(x(k),u(k),\delta) - [A_0 x(k) + B_0 u(k)]. \tag{2}$$

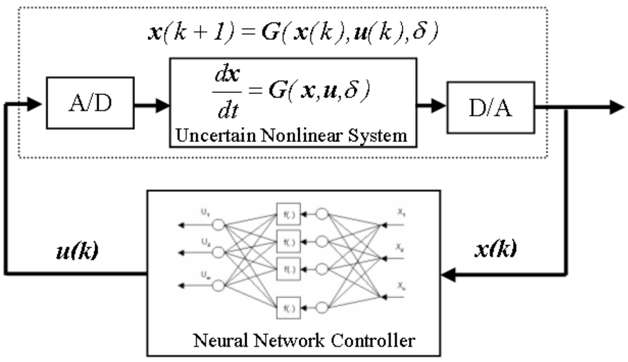

Fig. 1. Neural network control of Sampled-data systems.

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2.2 Neural Network Controllers

We consider nonlinear uncertain systems of order n with the state-space model.

The controllers are full state regulators implemented as single hidden layer feedforward NNs with a linear output layer. The hidden layer consists of p nonlinear neurons whose activation functions are hyperbolic tangent. Let $W_1 \in \Re^{p\times n}$ and $W_2 \in \Re^{m\times p}$ be the weight matrices in the hidden layer and the output layer, respectively. The control law can then be written in the form

$$u(\tau) = W_2 F(W_1 x(\tau)) = W_2 F(h) \tag{3}$$

where $F(h)$ is a p-vector function whose ith component is $f_i(h_i) = tanh(h_i)$.

3 Robust Stability Conditions

The proofs of robust stability are achieve d by showing the existence of Lyapunov functions of the closed loop control systems [11], [12].

Proposition : The sampled-data control system, as shown in Fig. 1, consisting of the uncertain system (1) with the neural network control law (3) is equilibrium stable in the presence of the uncertain parameter δ if there exists a positive symmetric definite matrix $P \in \Re^{n\times n}$ and a matrix $\Gamma \in \Re^{p\times p}$, a matrix $q \in \Re^{n\times p}$ such that

$$(1+\mu)[A_0 + aB_0W_2W_1]^T P[A_0 + aB_0W_2W_1] - P = -Q - qq^T - \mu(1+\mu)A_1^T PA_1 \tag{4a}$$

$$[A_0 + aB_0W_2W_1]^T PB_0W_2 = -W_1^T - q\Gamma^T - \mu A_1^T PB_0W_2 \tag{4b}$$

$$W_2^T B_0^T PB_0W_2 = 2I - \Gamma\Gamma^T \tag{4c}$$

where $a \in \Re^+$ is less than one, $Q \in \Re^{n\times n}$ is a positive symmetric definite matrix, and I is the identity matrix of dimension p.

Lemma 1: For any $\widetilde{A}, A_1 \in \Re^{nxn}$ and any positive symmetric definite matrix $P \in \Re^{nxn}$ the following matrix inequality holds:

$$\widetilde{A}^T PA_1 + A_1^T P\widetilde{A} \le \widetilde{A}^T P\widetilde{A} + A_1^T PA_1 . \tag{5}$$

Proof: For all $k > k_0$, let the uncertain sampled-data system be given as (1) with the NNC (3) and assumed P , q , and Γ satisfy (4). Consider the following Lyapunov function candidate:

$$V = x(k)^T Px(k) .$$

Using (2) and (3), the time difference of V along the state trajectory of the control system is

$$\begin{aligned} dV &= V(k+1) - V(k) \\ &= x(k+1)Px(k+1) - x(k)Px(k) \\ &= [(A_0 + \alpha_1 A_1)x(k) + B_0W_2F(W_1x(k))]^T P[(A_0 + \alpha_1 A_1)x(k) + B_0W_2F(W_1x(k))] \\ &\quad - x(k)Px(k). \end{aligned}$$

For convenience, we will drop the time step k of $x(k)$.

where $h = W_1 x$ and $\widetilde{F}(h) = F(h) - ah$. It is convenient to define $\widetilde{A} = A_0 + aB_0 W_2 W_1$.

$$\begin{aligned} dV &= \left[(\widetilde{A} + \alpha_1 A_1)x + B_0 W_2 \widetilde{F}(h)\right]^T P\left[(\widetilde{A} + \alpha_1 A_1)x + B_0 W_2 \widetilde{F}(h)\right] - x^T P x \\ &= x^T\left[\widetilde{A}^T P \widetilde{A} - P\right]x + \alpha_1 x^T\left[\widetilde{A}^T P A_1 + A_1^T P \widetilde{A}\right]x + \alpha_1^2 x^T A_1^T P A_1 x \\ &\quad + 2x^T \widetilde{A}^T P B_0 W_2 \widetilde{F}(h) + 2\alpha_1 x^T A_1^T P B_0 W_2 \widetilde{F}(h) + \widetilde{F}^T(h) W_2^T B_0^T P B_0 W_2 \widetilde{F}(h). \end{aligned}$$

By Lemma 1 and $|\alpha_1| \le \mu$, we obtain

$$\begin{aligned} dV &\le x^T\left[\widetilde{A}^T P \widetilde{A} - P\right]x + \alpha_1 x^T\left[\widetilde{A}^T P \widetilde{A} + A_1^T P A_1\right]x + \alpha_1^2 x^T A_1^T P A_1 x + \\ &\qquad 2x^T \widetilde{A}^T P B_0 W_2 \widetilde{F}(h) + 2\alpha_1 x^T A_1^T P B_0 W_2 \widetilde{F}(h) + \widetilde{F}^T(h) W_2^T B_0^T P B_0 W_2 \widetilde{F}(h) \\ &\le x^T\left[\widetilde{A}^T P \widetilde{A} - P\right]x + \mu x^T\left[\widetilde{A}^T P \widetilde{A} + A_1^T P A_1\right]x + \mu^2 x^T A_1^T P A_1 x + \\ &\qquad 2x^T \widetilde{A}^T P B_0 W_2 \widetilde{F}(h) + 2\mu x^T A_1^T P B_0 W_2 \widetilde{F}(h) + \widetilde{F}^T(h) W_2^T B_0^T P B_0 W_2 \widetilde{F}(h) \\ &= x^T\left[(1+\mu)\widetilde{A}^T P \widetilde{A} - P\right]x + \mu(1+\mu) x^T A_1^T P A_1 x + \\ &\qquad 2x^T \widetilde{A}^T P B_0 W_2 \widetilde{F}(h) + 2\mu x^T A_1^T P B_0 W_2 \widetilde{F}(h) + \widetilde{F}^T(h) W_2^T B_0^T P B_0 W_2 \widetilde{F}(h) \end{aligned}$$

Using (4) yields

$$\begin{aligned} dV &\le -x^T Q x - x^T q q^T x + 2x^T \widetilde{A}^T P B_0 W_2 \widetilde{F}(h) + 2\mu x^T A_1^T P B_0 W_2 \widetilde{F}(h) \\ &\quad + \widetilde{F}^T(h) W_2^T B_0^T P B_0 W_2 \widetilde{F}(h) \\ &= -x^T Q x - x^T q q^T x - 2x^T W_1 \widetilde{F}(h) - 2x^T q \Gamma^T \widetilde{F}(h) + 2\widetilde{F}^T(h)\widetilde{F}(h) - \widetilde{F}^T(h)\Gamma\Gamma^T \widetilde{F}(h) \\ &= -x^T Q x - \left\|x^T q + \widetilde{F}^T(h)\Gamma\right\|^2 - 2\left[h^T \widetilde{F}(h) - \widetilde{F}^T(h)\widetilde{F}(h)\right] \\ &= -x^T Q x - \left\|x^T q + \widetilde{F}^T(h)\Gamma\right\|^2 - 2\sum_{i=1}^{p} \widetilde{f}_i(h_i)\left(h_i - \widetilde{f}_i(h_i)\right). \end{aligned}$$

Thus, following the same argument as in Proposition 1 [13], we have $dV \le 0$. Therefore, V is a Lyapunov function of the control system and the control system is equilibrium stable.

4 Simulation result

Consider a sampled-data system which is obtained by sampling the motor-driven inverted pendulum

$$\frac{dx}{dt} = \begin{bmatrix} x_2 \\ \delta \sin x_1 - x_2 + 10u \end{bmatrix} \tag{6}$$

with a sampling rate of 0.05 second. Here, $x \in \Re^2$ is the state vector, $u \in \Re$ is the input, and $\delta \in [1.8,\ 2.2]$ is the uncertain parameter of the system. The approximate difference equation for the system (6) with a constant input at each time step can be simply written using Euler's formula as

$$x(k+1) = \begin{bmatrix} x_1(k) + 0.05 x_2(k) \\ x_1(k) + 0.05(\delta \sin x_1(k) - x_2(k) + 10u(k)) \end{bmatrix}. \tag{7}$$

By following the same procedure as in Example 1, we obtain the model (1b) of the

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system (7) as

$$x(k+1) = \left(\begin{bmatrix} 1.00 & 0.05 \\ 0.10 & 0.95 \end{bmatrix} + \alpha_1 \begin{bmatrix} 0.00 & 0.00 \\ 0.10 & 0.00 \end{bmatrix} \right) x(k) + \begin{bmatrix} 0.01 \\ 0.50 \end{bmatrix} u(k)$$

where $|\alpha_1| \leq 0.1$ and the trained weights and related matrices are found to be

$$W_1 = \begin{bmatrix} -0.50868560 & -0.00453348 \\ -0.54005599 & 0.04906967 \\ -0.46211462 & -0.07214932 \\ 0.29284129 & 0.33525182 \end{bmatrix},$$

$$W_2 = [0.48524009 \quad 0.34312202 \quad 0.64917501 \quad -0.99552114],$$

$$P = \begin{bmatrix} 44.82832736 & 4.20470253 \\ 4.20470253 & 3.50589685 \end{bmatrix},$$

$$q = \begin{bmatrix} 0.13633992 & -0.55178551 & 0.64501033 & 0.41911450 \\ -0.06838212 & 0.35885360 & -0.40765282 & -0.42026085 \end{bmatrix},$$

and

$$\Gamma = \begin{bmatrix} -0.40952492 & 0.93229043 & 0.69814220 & 0.51542845 \\ 0.98742458 & 0.23033026 & -0.51450794 & 0.77640411 \\ -0.83442536 & -0.58978048 & -0.55640354 & 0.61207556 \\ -0.48901040 & 0.97337314 & -1.01047819 & 0.05625793 \end{bmatrix}.$$

In Fig. 3, the solid line represents the contour of dV for the existing Lyapunov function, whereas the dashdot line corresponds to the other Lyapunov function of the case without uncertainty. The lines separate the positive and negative regions of dV. From this figure, we observe that the Euclidean balls of radius 5.8 and 3.6, which lie within the regions where $dV < 0$, are estimates of the regions of stability for the corresponding NN control systems. The stability region, when employing the robust stability condition (4), is noticeably larger than that of the other. The comparison of the system responses between two NN control systems and the reference model is shown in Fig. 2

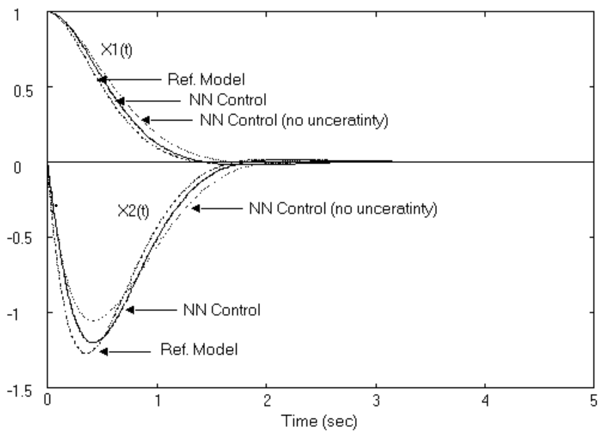

Fig. 2 : Control responses, $x_1(t)$ and $x_2(t)$.

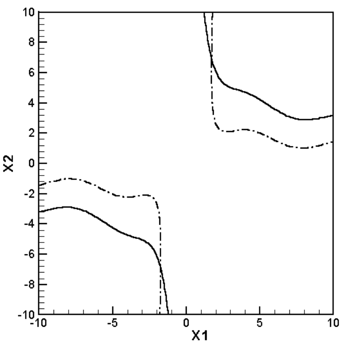

Fig. 3 : Contour plot of dV.

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5 Conclusion

Robust stability conditions for neural network control of uncertain sampled-data nonlinear systems have been derived in this paper. A modified backpropagation algorithm imposed the derived stability conditions as the training constraints are used to adjust the weights of the NNCs. The robust stability is achieved by showing the existence of a Lyapunov function of the closed loop systems. By evaluating the existing Lyapunov functions to obtain the finite region of stability, we found that the stability regions, when employing our new robust stability conditions, are visibly larger than when using the other stability conditions that do not include the uncertainties. The simulation results show satisfactory control responses and are consistent with the derived conditions.

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