

## Towards a Tomographic Index of Systemic Risk Measures\*

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**Abstract.** Due to the recent financial crisis, several systemic risk measures have been proposed in the literature for quantifying financial system-wide distress. In this note we propose an aggregated Index for financial systemic risk measurement based on EOF and ICA analyses on the several systemic risk measures released in the recent literature. We use this index to further identify the states of the market as suggested in Kouontchou et al. [7]. We show, by characterizing markets conditions with a robust Kohonen Self-Organizing Maps algorithm that this measure is directly linked to crises markets states and there is a strong link between return and systemic risk.

### 1 Introduction

Following the experience of the recent 2007-2009 financial crisis, a special attention has been paid to the “macroprudential” regulation, *i.e.* the prevention of a financial system-wide distress that can adversely impact the real economy. For this purpose, several systemic risk measures have been developed since 2010. Some of the most important described in Bisias et al. [2], and Giglio et al. [4] include the Conditional Value-at-Risk (CoVaR), the Delta Conditional Value-at-Risk ( $\Delta$ CoVaR) and the Marginal Expected Shortfall (MES). Other important

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measures are the SRISK from Brownlees and Engle [3] and the Component Expected Shortfall (CES) from Banulescu and Dumitrescu [1].

However, these measures are focused on different aspects of financial risk, such as leverage, kurtosis or skewness, and thereby the comparison of two firms may present dissimilar results according to the chosen measure. In this paper, following the intuition of Giglio et al. [4], we aim to create an aggregated index able to identify the main systemic risk factors through the study of 16 systemic risk measures applied to American securities. We will first try to identify the link between this index and the market states as suggested in Kouontchou et al. [7] by using the neural network classification algorithm corresponding to the Self-Organizing Maps (SOM) in its robust version of Guinot et al. [5], known as R-SOM. We will then apply an Empirical Orthogonal Function Analysis on the selected measures, which will allow us to establish an index based on the so-called Empirical Orthogonal Functions and their respective weights. Then, following the same intuition, we construct the aggregated index by applying an Independent Component Analysis. Our results confirm the findings in Giglio et al. [5] that when the measures are studied as a whole, they contain important predictive information about future macroeconomic outcomes.

## 2 Identifying the Link between Markets States and the Systemic Risk Measures

The classification by robust Kohonen maps aims to identify the correlation between the Systemic Risk Measures and the shape of the financial market.

### 2.1 R-SOM Algorithm and American Stock Market Index

In this part, we analyze a data set composed of 95 financial institutions in American Stocks Market, each of which are measured by 6 systemic risk measures (VaR, CoVaR, MES,  $\Delta$ CoVaR, SRISK, CES) at the individual firm level, evaluated on a daily basis from the 28th of March, 2003 to the 28th of June 2014.

We use for understanding the markets states the R-SOM algorithm proposed by Guinot et al. [5]. This approach provides a two-step stochastic method based on a bootstrap process to increase the reliability of the underlying neighbourhood structure. The increase in robustness is relative to the sensitivities of the output to the sampling method (see Guinot et al. [5], Kouontchou et al. [7], Olteanu et al. [8] and Sorjamaa et al. [9]). Figure 1 shows the codevectors of the [4x4] classes of the R-SOM algorithm, and the average line of value of each class in order to compare the values of these codevectors. Each class represents different dates for systemic risk measures of financials institutions.

In figure 2, we compute the performance of the MSCI US Index on the periods corresponding to the different systemic risk classes obtained with the R-SOM algorithm. In order to have a relevant idea of the evolution of the MSCI US, we only consider classes which contain at least 5% of the initial samples (*i.e.* classes

that contain more than 86 samples dates).

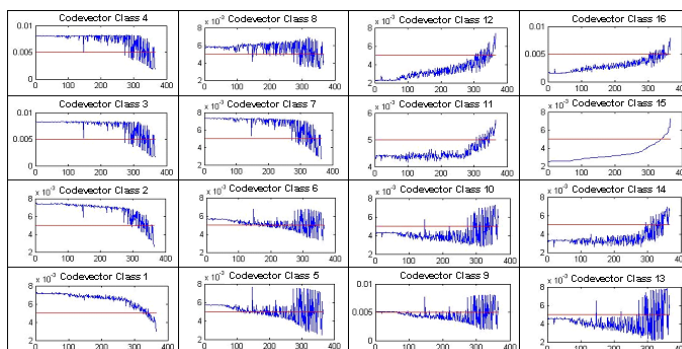


Fig. 1: Codevectors of the systemic risk classes obtained with the R-SOM algorithm.

Figure 2 confirms that the MSCI US Index is significantly decreasing over the periods of classes 14 and 15, whereas it is increasing over the other periods. If we compare these results to the former figure, we note that classes 14 and 15 codevectors are lower than those of classes 3, 4, 9 and 16; this comparison thus underlines the link between the systemic risk measures and the evolution of the MSCI: a fall of the MSCI Index is associated with a lowering of the codevectors of the R-SOM algorithm.

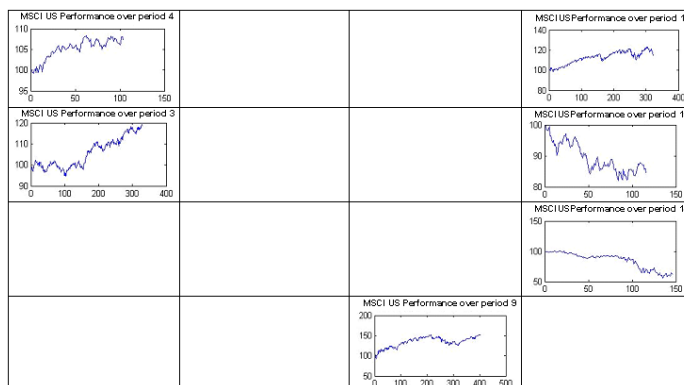


Fig. 2: MSCI US performance over the periods corresponding to the systemic risk classes of the R-SOM algorithm.

## 2.2 Characterizing Market Conditions with the Systemic Risk Measures

We now focus on the link between the systemic risk measures through the R-SOM algorithm, and the market states defined above. With the same principle

as in Figure 2, we represent on Figure 3 the part of each Market State for the 6 classes of the R-SOM algorithm that contain more than 5% of the initial samples.

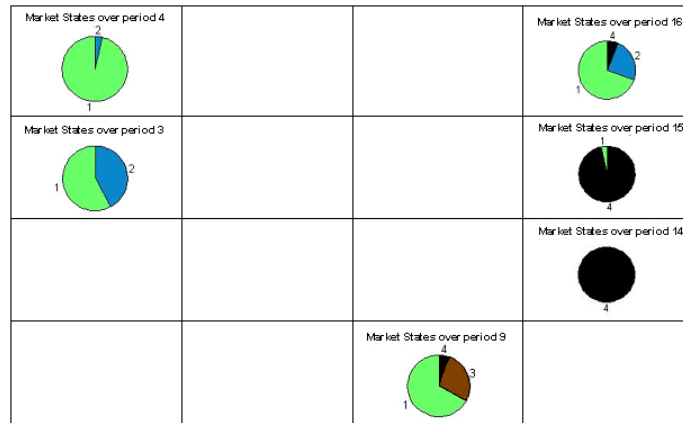


Fig. 3: Market states over the periods associated with the systemic risk classes.

In Figure 3, we clearly note that classes 14 and 15 are exclusively composed of samples which dates are associated with market state 4, whereas the other classes match more with States 1, 2 and 3 which stand for periods of growth of the market. Figure 2 and 3 thus confirm the correlation between the systemic risk measures and the state of the financial market: a decrease of the values of the codevectors is associated with a decrease of the MSCI World, and an increase of the Market State. This correlation is also described in the following section with the construction of the Index of Systemic Risk Measures (ISRM).

### 3 Empirical Orthogonal Functions

The Empirical Orthogonal Function (EOF) analysis is a decomposition of a data set in terms of orthogonal basis functions which are determined from the data. The EOF method is thus used to analyse the variability of a single field, and can allow afterwards to denoise the initial data. If we consider a  $(n, p)$  Matrix  $M$  of initial data, corresponding in our case to  $p$  time series of systemic risk measures, the algorithm is constructed as follows: after removing the mean value from each of the time series (*i.e.* each column of  $M$ ), The covariance matrix  $C$  of  $M$  is formed. The next step consists then in solving the eigenvalue problem:

$$CR = R\Lambda, \tag{1}$$

where  $R$  is a matrix of eigenvectors of covariance matrix  $C$ , and  $\Lambda$  is a diagonal matrix containing the eigenvalues of  $C$ . Assuming that  $\Lambda$  is ordered according to the size of the eigenvalues, the eigenvectors associated to those eigenvalues are the EOF we are looking for.

Since the EOF methodology is based on second order moments only, it lacks information on higher order statistics. A different technique for data analysis called Independent Component Analysis (ICA) takes into account higher order moments and exploits inherently non-Gaussian features of the data. The goal of ICA is to minimize the statistical dependence between the basis vectors. This can be written as:

$$M = As, \quad (2)$$

where  $A$  is an unknown matrix called the mixing matrix and  $M$ ,  $s$  are the two vectors representing the observed signals and source signals respectively. The objective is to recover the original signals,  $s$ , from only the observed vector of zero mean  $M$ . We obtain estimates for the sources by first obtaining the “unmixing matrix”  $W$ , where,  $W = A^{-1}$ . This enables an estimate, of the independent sources to be obtained:

$$s = WM, \quad (3)$$

We thus construct an aggregated Index, which we will name Index of Systemic Risk Measures (ISRM). In a first step, we calculate this index based on a EOF Analysis, where each orthogonal function is weighted by the part of variance that it represents, *i.e.* by the percentage of the eigenvalue associated to the orthogonal function. In a second step, following the same intuition, we calculate the index based on an Independent Component Analysis using the algorithm FastICA by Hyvärinen and Oja [6]. Note that since there is no order of magnitude in the ICA methodology, the independent components are equally weighted for the construction of the index. Figure 4 displays the results obtained with the two methodologies, along with the Market Crises (state 4 of the market).

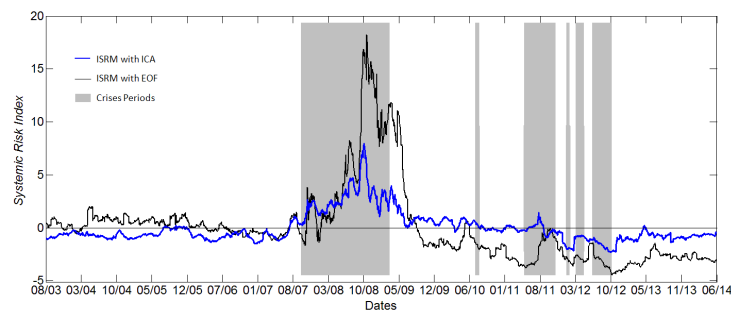


Fig. 4: Index of Systemic Risk Measures and crisis periods.

Figure 4 clearly shows a corresponding increase of the ISRM using both methodologies during the crisis periods.

## 4 Conclusion

Through the analysis of different Systemic Risk Measures computed for different firms, we have been able to build an Index based on mathematical algorithms relying on the variance and higher order moments between these different measures, which ties in the evolution of the financial market. This has been confirmed by the simultaneity between the significant rise of our Index and the Market State 4, corresponding to crisis periods. The comparison of the variations of the systemic risk measures and of the MSCI US also shows that they are globally opposite, which confirms the relevance of the Index towards the evolution of the financial market.

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