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W. Li

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Wenzhou Li‡

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720, U.S.A.

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‡on leave from Department of Physics, Zhejiang University, Hangzhou, China

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Wenzhou Li*

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University of California
Berkeley, California 94720 U.S.A.

The harmonic superspace formalism has been used to construct the consistent chiral anomaly in $N = 1$, $d = 6$ supersymmetric Yang-Mills theory. The expressions of the gauge anomaly Δ_s^ϕ and of the supersymmetric anomaly Δ_{SUSY}^ϕ are given together with the consistent condition.

The consistent chiral anomaly in $N = 1, d = 4$ supersymmetric Yang-Mills theory has been studied intensively. For higher dimension, due to the difficulties to get an unconstrained superspace formulation, it has been studied only in the Wess-Zumino gauge by Itoyama *et al* [1], who have found an $N = 1, d = 6$ nontrivial supersymmetric anomaly. The aim of this work is to get the $N = 1$, $d = 6$ supersymmetric consistent anomalies by using the harmonic superspace formulation[2),3)].

For six dimensions, the use of the $d = 6$ $SU(2)$ Majorana-Weyl spinors [4),5)] leads to a manifestly $SU(2)$ covariant formulation, which is naturally incorporated into the harmonic superspace formalism.

The $N = 1, d = 6$ harmonic superspace is parametrized by

$$\{z^M, u_i^\pm\} \tag{1}$$

in the central basis, where $z^M = (x^m, \theta_\alpha^i), (m = 0, \cdots 5, i = 1, 2, \alpha = 1, \cdots 4)$ parametrize the ordinary superspace, $u_i^\pm$, the harmonic variables, parametrize the coset space.

The SUSY transformation is given by

$$\begin{aligned} \delta x^m &= \frac{i}{2}\epsilon_i^\alpha \Sigma^m_{\alpha\beta}\theta^{\beta i} \\ \delta\theta_{\alpha i} &= \epsilon_{\alpha i} \\ \delta u_i^\pm &= 0 \end{aligned} \tag{2}$$

where $\Sigma^m_{\alpha\beta}$ are 4×4 antisymmetric matrices.

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We can pass from the central basis $\{z^M, u_i^\pm\}$ to the analytic basis, defined by

$$A.B. : \{z_A^M = (x_A^m, \theta_\alpha^+, \theta_\alpha^-), u_i^\pm\}, \tag{3}$$

where the new independent variables are related to the old ones as follows

$$\begin{aligned} x_A^m &= (x^m + \frac{i}{2}\theta^\alpha_{(i}\Sigma_{\alpha\beta}\theta^\beta_{j)}u^{+i}u^{-j}), \\ \theta^{\pm\alpha} &= u^{\pm i}\theta_i^\alpha \end{aligned} \tag{4}$$

In the analytic basis, the $N = 1, d = 6$ SUSY is realized as

$$\begin{aligned} \delta x_A^m = \frac{i}{2}(\epsilon_i^\alpha\Sigma_{\alpha\beta}\theta^{+\beta})u^{-i} \quad \delta\theta_\alpha^+ = \epsilon_\alpha^i u_i^+ \\ \delta\theta_\alpha^- = \epsilon_\alpha^i u_i^- \quad \delta u_i^\pm = 0. \end{aligned} \tag{5}$$

Since $x_A^m, \theta_\alpha^+, u_i^\pm$, form a subset closed under $N = 1, d = 6$ SUSY transformations,we get the analytic subspace,

$$(\varsigma_A^M = (x_A^m, \theta_\alpha^+), u_i^\pm). \tag{6}$$

In the analytic basis, the harmonic derivative D^{++} becomes

$$D^{++} = u^{i+}\frac{\partial}{\partial u^{i-}} = \frac{\partial}{\partial u^{i-}} + \frac{i}{2}\theta^{+\alpha}\Sigma^m_{\alpha\beta}\theta^{+\beta}\frac{\partial}{\partial x^m} + \theta_\alpha^+\frac{\partial}{\partial\theta_\alpha^-} \tag{7}$$

and the spinor derivatives are decomposed into $D^\pm$ parts:

$$D_\alpha^+ = \frac{\partial}{\partial\theta^{-\alpha}}, \quad D_\alpha^- = \frac{\partial}{\partial\theta^{+\alpha}} + i\Sigma^m_{\alpha\beta}\theta^{-\beta}\frac{\partial}{\partial x_A^m} \tag{8}$$

The fact that D_α^+ are reduced to simple derivatives with respect to θ^- reflects the existence of the invariant analytic subspace. Therefore in the analytic basis, we can define superfields which do not depend on θ^- and are analytic automatically,

$$F^{(q)}(x_A,\theta^+,u^\pm), \quad D^+_\alpha F^{(q)} \equiv 0. \qquad (9)$$

The unconstrained off-shell formulation just relies on the use of the real analytic superfields.

The exterior differential d in the harmonic superspace can be split as follows

$$d = \hat{d} + D + \hat{D}$$

where $\hat{d} = e^{\alpha\beta}\partial_{\alpha\beta}, D = e^{\alpha i}D_{\alpha i}, \hat{D} = e^a D_a$, with $D_a = (D++, D--)$. The rigid vielbeins e^A are 1-superforms, with the property

$$de^A = t^A_{BC}e^C e^B$$

$$t^{\tau\tau'}_{\alpha_i\beta_j} = -i\epsilon_{ij}\delta^{\tau\tau'}_{[\alpha\beta]}.$$

From the constraints $F^{++}_{\alpha\beta} = 0$, $F^{++,--} = 0$ we can solve for φ^+_α and $\varphi^{++}, \varphi^{--}$

$$\varphi^+_\alpha = e^{-U}D^+_\alpha e^U$$

$$\varphi^{++} = e^{-V}D^{++}e^V$$

$$\varphi^{--} = e^{-V}D^{--}e^V$$

From $F^{+\,++}_\alpha = 0$ we can get the important relation

$$D^+_\alpha V^{++} = 0 \qquad (10)$$

$$V^{++} = e^U e^{-V}D^{++}(e^V e^{-U})$$

As φ_A have been obtained, the gauge transformations can be expressed as

$$e^U \to e^X e^U e^K, \quad D^+_\alpha X = 0, \qquad (11)$$

$$e^V \to e^Y e^V e^K, \quad D^{++}Y = 0. \qquad (12)$$

where K is the gauge transform parameter. By using a K gauge transformation the V can be set to zero, $V = 0$, and hence $\varphi^{++} = \varphi^{--} = 0$. Now (10) becomes $V^{++} = e^U D^{++}e^{-U}$ which can be reduced to

$$(D^{++} + V^{++})e^U = 0 \qquad (13)$$

Equation (13) can be solved to express the prepotential U in terms of the unconstrained analyltic superfield V^{++}.

The gauge transformations can be expressed by the exterior differential operator in gauge group space[6),7)]

$$s = \sum_i dt_i \frac{\partial}{\partial t^i} \qquad (14)$$

where t^i are all parameters the gauge group elements may depend on.

The following notations are introduced to express the gauge transformed quantities

$$e^v = e^X e^V e^K$$

$$e^u = e^X e^U e^K$$

$$\phi = e^{-K}\varphi e^K + e^{-K}de^K$$

$$\mathcal{F} = d\phi + \phi\phi$$

We denote $c = e^{-K}se^K$, $c' = e^X se^{-X}$ where c, c' are $\mathcal{G}$-valued 1-forms in t space.

We can get

$$se^v = s(e^X e^V e^K) = -c'e^v + e^v c \qquad (15)$$

$$se^u = s(e^X e^U e^K) = -c'e^u + e^u c \qquad (16)$$

$$s\phi = [\phi, c] - dc$$

$$sc = -c^2 \qquad (17)$$

which coincide with B.R.S. transformations.

The transgression formula [6)] is therefore still true, and we consider the following for $d = 6$,

$$Tr(\mathcal{F}\mathcal{F}\mathcal{F}\mathcal{F}) = (d+s)\Omega \qquad (18)$$

$$\Omega = \int_0^1 dt$$

$$str((\phi+c), \hat{\mathcal{F}}_t, \hat{\mathcal{F}}_t, \hat{\mathcal{F}}_t) = t(d+s)(\phi+c) + t^2(\phi+c)(\phi+c) \qquad (19)$$

The operator S_{SUSY} corresponding to rigid supersymmetry is defined by acting on superfields ψ integrated over space-time,

$$S_{SUSY}\int_x \psi = \int_x \epsilon^A D_A \psi = \int_x (\epsilon^{\alpha i}D_{\alpha i})\psi \qquad (20)$$

where $\epsilon^A = (\epsilon^{\tau\tau'}, \epsilon^{\alpha i}, \epsilon^a) \equiv (0, \epsilon^{\alpha i}, 0)$ are parameters with the same commuting properties as dz^A, i.e., the spinor like $\epsilon^{\alpha i}$ are commuting among themselves. The reason we put $\epsilon^a = 0$ is that supersymmetry is realized with $\delta u^\pm_i = 0$; also we may set the space-time like $\epsilon^{\tau\tau'} = 0$ because they occur under space-time integration. Due to the commuting property of $\epsilon^{\alpha i}$, S_{SUSY} is nilpotent when acting on space-time integrals of superfields,

$$S^2_{SUSY} = 0 \qquad (21)$$

To construct consistent SUSY anomaly, we start from the transgression formula (19). The 7-superform

Ω can be split according to the powers of c, and by the number of factors of the vielbeins e^A with different dimensions, i.e., $q_1, q_2, (7-p) - q_1, -q_2$, which denote the number of the factors of the spinor-like vielbeins, $e^{\alpha i}$, the harmonic-like vielbeins e^a, and the space-time like vielbein $e^{\tau\tau'}$ respectively. Therefore, equation (18) contains different sets of identities, one for each sector. We depict the following,

$$p = 0, q_1 = 2, q_2 = 0:$$
$$str(\mathcal{FFFF})_{6,2,0} = d\omega^0_{5,2,0} + [D\omega^0_{6,1,0}]_{6,2,0} \quad (22)$$

$$p = 1, q_1 = 0, q_2 = 0:$$
$$0 = s\omega^0_{6,1,0} + \hat{d}\omega^1_{5,1,0} + [D\omega^1_{6,0,0}]_{6,1,0} \quad (23)$$

$$p = 2, q_1 = 0, q_2 = 0 : 0 = s\omega^1_{6,0,0} + \hat{d}\omega^2_{5,0,0} \quad (24)$$

After some calculation, we can get the final result,

$$s\Delta^\phi_S = 0 \quad (25)$$

$$S_{SUSY}\Delta^\phi_{SUSY} = 0 \quad (26)$$

$$s\Delta^\phi_{SUSY} + S_{SUSY}\Delta^\phi_S = 0 \quad (27)$$

where

$$\Delta^\phi_S = \int_s \omega^1_{6,0,0}$$

$$\Delta^\phi_{SUSY} = \int_s i_\epsilon \hat{\omega}_{6,1,0}$$

The notation $i_\epsilon \chi_m$ denotes the interior product of the m-superforms χ_m with respect to ϵ^M, where

$$\epsilon^M = \epsilon^A e^M_A \quad (28)$$

and e^M_A are the inverse vielbeins,

$$i_\epsilon \chi_{m=m} \epsilon^{A_1} e^{A_2} \cdots e^{A_m} \chi_{A_m} \cdots A_1$$

Equations (25), (26) and (27) are the consistency conditions and $\Delta^\phi_S, \Delta^\phi_{SUSY}$ are the gauge anomaly and supersymmetric anomaly respectively. Δ^ϕ_S, Δ^ϕ_{SUSY} can be expressed by

$$\Delta^\phi_S = 12 \int_s \int_0^1 dt(1-t)[str(cd\underline{\phi}\underline{\mathcal{I}}t\underline{\mathcal{I}}t)]_{\theta=0} \quad (29)$$

$$\Delta^\phi_{SUSY} = -12 \int_s \int_0^1 dt$$

$$str\{\epsilon^{\alpha}[\ \underline{\phi}(\mathcal{F}_t)_{\underline{\alpha}_i}\underline{\mathcal{I}}_t\underline{\mathcal{I}}_t - \frac{1}{3}\phi_{\underline{\alpha}}\underline{\mathcal{I}}_t\underline{\mathcal{I}}_t\underline{\mathcal{I}}_t$$
$$- \frac{i}{64}\ (\mathcal{F}_{\underline{\alpha}_i}\underline{\mathcal{I}}\mathcal{F}_{\underline{\alpha}_2 i_2}\epsilon^{i_1 i_2}\delta^{[\alpha_2,\alpha_1]}\mathcal{F}_{\underline{\alpha}_1 i_1})]\}_{\theta=0} \quad (30)$$

where

$$\underline{\phi} = e^{\tau\tau'}\phi_{\tau\tau'},$$
$$\mathcal{F}_{\underline{\alpha}_i} = e^{\tau\tau'}\mathcal{F}_{\underline{\alpha}_i\tau\tau'},$$
$$\underline{\mathcal{I}} = e^{(\tau\tau')_1}e^{(\tau\tau')_2}\mathcal{F}_{(\tau\tau')_2(\tau\tau')_1}$$
$$\underline{\delta}^{[\alpha_2,\alpha_1]} = e^{\tau\tau'}\delta^{[\alpha_2,\alpha_1]}_{\tau\tau'}. \quad (31)$$

and similar definitions for $\mathcal{F}_t$ and $(\mathcal{F}_t)_{\underline{\alpha}i}$.

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LAWRENCE BERKELEY LABORATORY
TECHNICAL INFORMATION DEPARTMENT
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720