Spontaneous Symmetry Breaking and Field Theory
Elective course
SS 2015
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Exercise 2

22.05.2015

1. <u>entropy-elasticity</u>
Assume that a linear polymer molecule consists of N rigid monomers of respective length a connected among each other by joints at their respective end points. Each monomer may be aligned parallel or antiparallel to the direction of the polymer molecule. Assume that the segments do not have a mass and do not interact. The macromolecule is fixed at one end and loaded with a weight F_G at the other end. Calculate the length L of the macromolecule as function of F_G and the temperature T from the thermodynamic potential $G(T, F_G, N) = -kT \ln Y$ where Y is the corresponding partition function.

2. Calculate the percolation concentration x_c, the number n_s of s-clusters per lattice site, the mean cluster size $S(x)$ and the correlation length $\xi(x)$ for the one-dimensional site-percolation problem. Verify the relation

$$\sum_s n_s \cdot s + P_\infty \cdot x = x \tag{1}$$

for this case, where P_∞ is the percolation probability.

3. The mean cluster size of a site-percolation problem is defined as

$$S(x) = \sum_s n_s \cdot s^2 \Big/ \sum_s n_s \cdot s\,, \tag{2}$$

where n_s is the number of s-clusters per lattice site and s is the number of sites in the cluster.

Show by plausibility arguments that the magnetic analogy to $S(x)$ is the zero-field susceptibility $\chi(T)$. To do this, calculate for a $S = \frac{1}{2}$ Ising model with infinite exchange coupling between nearest neighbours the magnetization in an external magnetic field and therefrom $\chi(T)$. The relation between $S(x)$ and $\chi(T)$ may be determined by use of the equation (1) from exercise **2.**