Exercise 3

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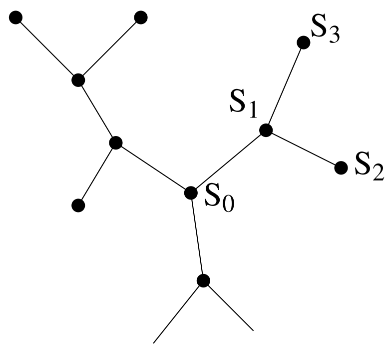
1. Consider the site-percolation problem on a Bethe lattice with coordination number 3:



a. Consider one of the bonds evolving from S_0 and calculate the probability Q_∞ that there is no path starting from S_0 along that bond which extends to infinity. Calculate from this the percolation concentration x_c, the percolation probability P_∞ and the exponent β.

b. Let T be the mean cluster size for a branch, i.e., the mean size of sites which are connected to the origin S of this branch and which belong to this branch (e.g. the branch which starts from S_0 and contains S_1), averaged over all possible origins S. On the average, each sub-branch of this branch (e.g., the one starting from S_1 and containing S_2) has the same mean cluster size T. Derive an equation for T by using this fact and determine the mean cluster size $S(x)$ and the exponent γ for this Bethe lattice.

c. The values of the exponents $\beta = \gamma = 1$ correspond to the mean-field values of the $Q \to 1$ Potts model. For a thermal phase transition the mean-field exponents are exact for infinite dimension. Make it clear that the Bethe lattice corresponds to that case. To do this, calculate the number V of „volume sites“ within a „sphere“ containing r „generations“ (example: The origin S_0 is circumvented by three sites of the first „generation“), as well as the number S of „surface sites“ belonging to the last of those r generations. Define the dimension d via

$$S \sim V^{(d-1)/d} \,. \qquad (1)$$

2. In the lecture the representation

$$S_k = \exp(2\pi i\, m_k/Q) \quad , \quad m_k = 0, 1, \ldots Q-1 \qquad (2)$$

for the variable S_k of the Q-Potts-model has been discussed. Show that the magnetic moment of the Potts model is given by

$$M = \sum_k \sum_{r=1}^{Q-1} \langle S_k^r \rangle \ . \qquad (3)$$