The Temporary Exchange Problem

Extended Abstract

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ABSTRACT

We consider an allocation model under ordinal preferences that is more general than the well-studied Shapley-Scarf housing market. In the model, the agents do not just care which house or resource they get but also care about who gets their own resource. This assumption is especially important when considering temporary exchanges in which each resource is eventually returned to the owner. We show that several positive axiomatic and computational results that hold for housing markets do not extend to the more general setting. We then identify natural restrictions on the preferences of agents for which several positive results do hold. One of our central results is a general class of algorithms that return any allocation that is individually rational and Pareto optimal with respect to the responsive set extension.

KEYWORDS

Matching under preferences, economic paradigms

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1 INTRODUCTION

The Shapley-Scarf housing market is a well-studied formal model for barter markets where the goods can be dormitory rooms or kidneys [10]. In the market, each agent owns a single good referred to as a house. The goal is to redistribute the houses to the agents in the most desirable fashion. Shapley and Scarf [9] showed that under strict preferences, a simple yet elegant mechanism called *Gale's Top Trading Cycle (TTC)* is polynomial-time, strategyproof and finds an allocation that is Pareto optimal and core stable. Even if the preferences are not strict, the algorithm can be suitably generalized while not losing any of the properties (see e.g., [1, 2, 4, 8]). There has also been work where agents have multi-unit demand and endowments (see e.g., [5, 7, 11]). In this paper we focus on single-unit demands.

In the Shapley-Scarf market, agents only have preferences over houses. This is a reasonable assumption especially when the exchange is irrevocable. However, if the exchange is temporary, and the original house of an agent will be returned to her, the agent may care as to who temporarily used her house. In order to capture this additional issue, we consider the *temporary exchange problem* that is a generalisation of the Shapley-Scarf housing market. In this generalisation, an agent has preferences over outcomes that take into account both what house the agent gets and also who gets her own house. The assumption of the temporary exchange also makes sense when for example a kidney patient not only cares about getting a suitable kidney but also has preference over who should get his or her donor's kidney. For this more general setting, we want to study fundamental questions as follows: does a core stable allocation exist and what is the complexity of finding it? What is the complexity of finding a Pareto optimal allocation?

Contributions. We consider an exchange market setting that is more general than the well-studied Shapley-Scarf market. It models several scenarios where agents are performing a temporary exchange or they care about who gets their resource.

We first focus on core allocations in such settings and show that the core can be empty and it is NP-hard to check whether a core stable allocation exists. We also prove that finding a Pareto optimal allocation is NP-hard and testing Pareto optimality and weak Pareto optimality is coNP-complete.

We circumvent the negative computational results by considering more structured preferences. We first show that a Pareto optimal allocation can be computed in polynomial time if the preferences are strict. We then consider a weakening of Pareto optimality called Pareto optimality with respect to the responsive set extension. For this particular concept, we propose a general class of polynomial-time algorithms that return an allocation that is individually rational and Pareto optimal with respect to the responsive set extension. The algorithm can be used to characterize the set of all such allocations because for each such allocation p, there is a run of the algorithm that finds the allocation.

We also consider strategic aspects and present two key impossibility results. Firstly, there exists no core-consistent and strategyproof mechanism. Secondly, there exists no individually rational, Pareto optimal, and strategyproof mechanism. We then identify restrictions on the preferences in particular house-predominant and tenant-predominant preferences under which we regain the positive axiomatic and computational results that hold for the Shapley-Scarf market.

Independent of our work, Lesca and Todo [6] considered the same model in the context of a 'service exchange' application. Gupta et al. [3] considered a different model which has a similar interpretation that agents care about who gets their resource. The model and problem focus is different in several respects.

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2 TEMPORARY EXCHANGE PROBLEM

An instance of *Temporary Exchange Problem* is a tuple $(N, H, e, \succsim)$ where

- $N = \{1, \ldots, n\}$ is the set of agents.
- $H = \{h_1, \ldots, h_n\}$ is the set of houses.
- Endowment function $e : N \to H$ maps each agent to a house. Each agent i owns exactly one house $e(i)$. We will denote $\bigcup_{i \in S} e(i)$ by $e(S)$.
- $\succsim = (\succsim_1, \ldots, \succsim_n)$ is the preference profile that specifies for each agent $i \in N$, the weak order preference relation $\succsim_i$ over $H \times N$.[1] The symbol $\succsim_i$ denotes "prefer at least as much", $\succ_i$ denotes "strictly more prefer", and $\sim_i$ denotes indifference.

A feasible outcome for the setting is an allocation of the houses to the agents. An allocation is a one-to-one mapping from N to H. If p is the allocation, we will denote the house given to agent i by $p(i)$. We will denote by $p^{-1}(h)$ the agent who gets house h. Each agent cares about the combination of two things: which house she gets and who gets her own house. We will refer to this combination as the *outcome* for the agent. For an agent i, the outcome $(e(k), j)$ represents the scenario where i gets house $e(k)$ and gives house $e(i)$ to agent j. For an agent i, the outcome $(e(i), i)$ represents the situation where i keeps her own house. The outcome $(e(j), j)$ represents the situation where i swaps her house with j. When we write that $(e(j), k) \succ_i (h_\ell, m)$, it means that i prefers outcome $(e(j), k)$ to (h_ℓ, m).

Therefore for any allocations p and q, an agent compares them only from the point of view of what house she gets and who gets her house: $p \succsim_i q \iff (p(i), p^{-1}(e(i))) \succsim_i (q(i), q^{-1}(e(i)))$. Note that an agent i will be interested in the outcome (h_j, k) only if is it more preferred by her than $(e(i), i)$. Any outcome that is less preferred than $(e(i), i)$ is *unacceptable* to agent i. Otherwise agent i would rather not be part of the exchange. Note that there could be multiple allocation for which the *outcome* for an agent is the same.

Properties of allocations and mechanisms. We consider the standard properties in market design: (i) *Pareto optimality*: there should be no allocation in which each agent is at least as happy and at least one agent is strictly happier (ii) *individual rationality (IR)*: no agent should have an incentive to leave the allocation program (iii) *strategyproofness*: no agent should have an incentive to misreport her preferences; and (iv) *core stability*: an allocation should be such that no set of agents can form a coalition where they just exchange among themselves to get a better outcome than the allocation. We define these properties as follows.

An allocation p is *Pareto optimal (PO)* if there exists no other allocation q such that $q \succsim_i p$ for all $i \in N$ and $q \succ_i p$ for some $i \in N$. An allocation p is *weakly Pareto optimal* if there exists no other allocation q such that $q \succ_i p$ for all $i \in N$. An allocation p is *individually rational (IR)* if for all $i \in N$, it is the case that $(p(i), p^{-1}(e(i))) \succsim_i (i, e(i))$.

A coalition $S \subseteq N$ *blocks* an allocation p on N if there exists an allocation q on S such that for all $i \in S$, it is the case that $q(i) \in e(S)$ and $q(i) \succ_i p(i)$. An allocation is *core stable* if it admits no blocking coalition.

[1]Note that in the standard housing market, the preferences are simply over the set of houses. The temporary exchange model allows for more complex preferences.

A mechanism takes as input an instance $(N, H, e, \succsim)$ and returns an allocation. An allocation algorithm / mechanism is *strategyproof* if no agent can misreport and get a better outcome.

3 CORE STABILITY

We first show that unlike the Shapley-Scarf housing market, the Temporary Exchange market may not admit a core stable allocationa and checking existence is NP-complete.

Theorem 3.1. *Checking whether there exists a core stable allocation is NP-hard if there are indifferences in the preferences and even if each agent has at most 6 acceptable outcome pairs.*

4 PARETO OPTIMALITY

Since a PO allocation is guaranteed to exist, we focus on *computing* such an allocation.

Theorem 4.1. *Checking whether there exists an allocation that is most preferred for each agent is NP-complete if we allow indifferences in the preferences and even if each agent has at most 4 acceptable outcome pairs.*[2]

Theorem 4.2. *Finding a PO allocation is NP-hard if there are indifferences in the preferences even if each agent has at most 4 acceptable outcome pairs.*

Theorem 4.3. *Checking whether a given allocation is weakly PO is coNP-complete even if preferences are strict and even if each agent has at most 4 acceptable outcome pairs.*

We note that if the preference are strict, then finding a PO (but not necessarily IR) allocation is polynomial-time solvable.

Theorem 4.4. *If preferences are strict, a PO allocation can be computed in polynomial time.*

In certain scenarios, an agent may have underlying preferences $\succsim_i^H$ over houses and over tenants $\succsim_i^N$. Her preferences over the combinations of houses and tenants may depend naturally on their underlying preferences. In particular, we study the situation where the preferences are based on the *responsive set extension.* We say that agent i's preferences $\succsim_i$ over $H \times N$ are *responsive* if for any $j, j' \in N \setminus \{i\}$ and $h, h' \in H$,

$$(h \succsim_i^H h') \wedge (j \succsim_i^N j') \iff (h, j) \succsim_i^{RS} (h', j').$$

We say that allocation p is RS-PO (PO with respect to the responsive set extension) if there exists no other allocation q such that $q(i) \succsim_i^{RS} p(i)$ for all $i \in N$ and $q(i) \succ_i^{RS} p(i)$ for some $i \in N$. Informally speaking, if an allocation is not RS-PO, it admits an unambiguous improvement for the agents. We say that an allocation p is RS-IR (IR with respect to the responsive set extension) if for all $i \in N, p(i) \succsim_i^{RS} ((e(i), i)$.

Theorem 4.5. *There exists a polynomial-time algorithm that returns an RS-IR and RS-PO allocation.*

Finally, we prove the following.

Theorem 4.6. *There exists no IR, PO, and strategyproof mechanism.*

[2]Note that the problem is trivial is each agent has a unique most preferred outcome.

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