Distance Hedonic Games

Extended Abstract

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ABSTRACT

In this paper we consider Distance Hedonic Games, a class of non-transferable utility coalition formation games that properly generalizes previously existing models, like Social Distance Games and Fractional Hedonic Games. In particular, in Distance Hedonic Games we assume the existence of a scoring vector α, in which the i-th coefficient α_i expresses the extent to which x contributes to the utility of y if they are at distance i.

We focus on Nash stable outcomes and consider two natural scenarios for the scoring vector: monotonically decreasing and monotonically increasing coefficients. In both cases we give NP-hardness and inapproximability results for the problems of finding a social optimum and a best Nash stable outcome. Moreover, we characterize the topologies of coalitions with high social welfare and give bounds on the Price of Anarchy and on the Price of Stability.

KEYWORDS

Coalition Formation, Hedonic Games, Nash stability, Price of Anarchy, Price of Stability

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1 INTRODUCTION

Coalition Formation Games (CFGs) model multi-agent systems where selfish agents form coalitions and have preferences over the coalitions they could belong to. More specifically, an outcome of a CFG is a partition of the set of agents into disjoint coalitions, referred to as a *coalition structure*. The utility of an agent for a given outcome represents her satisfaction with the outcome. When such a utility only depends on the coalition she belongs to, and not on how the other agents are aggregated, we are in the class of Hedonic Games (HGs) [17]. HGs form one of the most important classes of CFGs and have been widely studied in the literature [4].

One of the main goals in CFGs is to understand the stable outcomes that the selfish behavior of the agents leads to, both from an existential and an algorithmic perspective. To this aim, several stability concepts have been defined. Since we focus on individual deviations, we consider the fundamental notion defined in this setting, that is, Nash stability. A coalition structure is *Nash stable* (NS), if no agent can improve her utility by unilaterally moving to a different coalition. In order to evaluate the efficiency of NS outcomes, we resort to the classical measures of *price of anarchy* (PoA) [23, 25] and *price of stability* (PoS) [1]. The PoA and the PoS are defined as the worst/best-case ratio between the highest achievable social welfare (i.e., the sum of the agents' utilities) of a coalition structure and that of an NS one, respectively.

In many natural scenarios the relations between agents can be modeled as a graph, and the utilities that agents receive from their coalitions strongly depend on the graph structure. Classical examples are Additively Separable Hedonic games [12], Fractional Hedonic Games (FHGs) [2] and Social Distance Games (SDGs) [14]. In the literature, HGs have been studied w.r.t. both individual [7, 12, 19, 21] and group deviations [11, 18, 20, 21]. The same holds for FHGs [2, 3, 8–10, 13, 24] and SDGs [5, 6, 14, 22].

In this work, we introduce a class of games termed *Distance Hedonic Games* (DHGs), that generalizes both unweighted FHGs and SDGs, and provides a unifying framework for hedonic models based on distances. In particular, in SDGs each agent x contributes to the utility of another agent y in her coalition in an inversely proportional fashion with respect to their distance, while in FHGs only if they are neighbors. In DHGs we assume the existence of a scoring vector in which the i-th coefficient expresses the extent to which x contributes to the utility of y if they are at distance i. Such a vector is assumed to be the same for all the agents. We focus on the most natural types of scoring vectors whose coefficients have a monotone growth (decreasing and increasing). In the case of decreasing vectors, if the distance between two agents increases, the interest of being together decreases. Thus, these vectors model situations in which agents want to aggregate into coalitions in which they are close together. Increasing vectors are suitable for modeling influence spreading, collecting information, or the case where edges express competition between agents. Note that this does not violate the hedonic nature of the game: the graph structure does not necessarily represent friendship, but it can represent, e.g., physical distance. Thus, the idea of increasing vectors is wishing to reach far away nodes while relying only on internal communication.

2 MODEL AND PRELIMINARIES

Given an undirected graph $G = (V, E)$, a *coalition structure* $C = (C_1, \dots, C_k)$ is a partition of V into coalitions.

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Definition 2.1. A *Distance Hedonic Game* (DHG) is defined by an undirected graph $G = (V, E)$ and a scoring vector $\alpha \in \mathbb{R}^{n-1}$ where (i) V is a set of n agents and (ii) the *utility* of an agent $x \in V$ in a given coalition structure C for being in her coalition $C(x) \in C$ is $u_x^{\alpha}(C(x)) = \frac{1}{|C(x)|} \sum_{y \in C(x) \setminus \{x\}} \alpha_{d_{C(x)}(x,y)}$, where $d_{C(x)}(x, y)$ is the distance induced by the subgraph $G_{C(x)} = (C(x), E_{C(x)})$, with $E_{C(x)} = \{(x, y) \in E \ : y \in C(x)\}$. If x and y are disconnected in coalition C, then $d_C(x, y) = \infty$ and $\alpha_\infty = 0$.

We define a scoring vector α to be *decreasing* if $\alpha_i \geq \alpha_{i+1}$ and *increasing* if $\alpha_i \leq \alpha_{i+1}, \forall i \in [n-2]$. An increasing or decreasing scoring vector α is *normalized* if $\alpha_1 = 1$.

Given a game instance $\langle G, \alpha \rangle$, we denote by $\mathsf{NS}_{\langle G, \alpha \rangle}$ the set of its NS outcomes. If for some α there exists a graph G in which the social optimum has strictly positive social welfare and $C \in \mathsf{NS}_{\langle G, \alpha \rangle}$ s.t. $\mathrm{SW}(C) = 0$, we say that $\mathrm{PoA}(\alpha) = \infty$. Moreover, if $\forall C \in \mathsf{NS}_{\langle G, \alpha \rangle}, \mathrm{SW}(C) = 0$, we say that $\mathrm{PoS}(\alpha) = \infty$.

Model Discussion. Both symmetric unweighted FHGs and SDGs are a subclass of DHGs, with scoring vector $\alpha = (1, 0, \dots, 0)$ and $\alpha = (1, \frac{1}{2}, \dots, \frac{1}{n-1})$, respectively. We also observe that these vectors induce utilities that are proportional to the degree and to the harmonic centrality measure of nodes in coalitions. Also further centrality indices fit into the DHG model, such as the Dangalchev centrality measure [15, 16].

Because of the hedonic character of the game, and as is traditional in clustering settings, agents' utilities depend only on the distances induced by the coalitions they belong to and not on the distances in the original graph. Similarly, we set $\alpha_\infty = 0$, both for increasing and decreasing vectors, as we wish the agents to achieve positive utility only from the nodes they are connected to.

3 OUR RESULTS

First we discard vectors that contain negative coefficients, not only because they do not represent natural scenarios, but also because of their low efficiency of NS outcomes. Indeed, it is possible to show that given a general scoring vector α that has negative components, both $\mathrm{PoA}(\alpha)$ and $\mathrm{PoS}(\alpha)$ are unbounded. Thus, in what follows we focus on vectors with non-negative components, i.e. $\alpha_j \geq 0$ for each $j \in [n-1]$. In particular, for these vectors an NS outcome always exists, since the grand coalition is NS. Unfortunately, it is possible to show inefficiency of equilibra also for non-negative vectors (see Table 2). These negative results are due to vectors with $\alpha_1 = 0$. Thus, we further assume that $\alpha_1 > 0$ and that, w.l.o.g., α is normalized, i.e. $\alpha_1 = 1$. Furthermore, we will assume that the scoring vector components are monotone, decreasing and increasing. While other scoring vector classes can still be of theoretical interest, the ones we focus on seem also to be the most natural choices.

We first provide hardness results concerning the determination of optimal and best NS outcomes (w.r.t. the social welfare), and then move on to the analysis of the PoA and the PoS.

3.1 Hardness Results

We show intractability or inapproximability results for the two problems defined below, examining separately the case of decreasing and increasing vectors. Our results are summarized in Table 1.

Scoring Vector	MSW	MSW-S
Decreasing normalized	NP-hard [3],[14]	NP-hard for any $\alpha_2 < \frac{1}{2}$
Increasing with $\alpha_1 = 0$	no poly-time w. approx. > 0	no poly-time w. approx. > 0
Increasing normalized	no poly-time w. approx. $< \frac{2^{n+1}}{n^2+1}$	

Table 1: Hardness and inapproximability results for finding the optimal (MSW) and best NS (MSW-S) outcomes.

Scoring Vector	PoA	PoS
General	∞	∞
Non-negative	∞	$\Theta(n)$
Normalized	$\leq M_\alpha(n-1)$	$\leq \min\{\frac{M_\alpha}{m_\alpha}, \frac{n}{2} \cdot M_\alpha\}$
Constant	1	1
Decreasing normalized with $\alpha_2 \leq \frac{1}{2}$, girth ≥ 5	$n-1$	$\leq n \cdot \frac{\alpha_2 + \sqrt{\alpha_2^2 + (1-\alpha_2)^2}}{1 + 2\alpha_2 \cdot (n-1)}$
Increasing normalized	$\Theta\left(\frac{\mathrm{SW}(\mathsf{P}_n)}{n}\right)$	$\Theta\left(\frac{\mathrm{SW}(\mathsf{P}_n)}{n}\right)$

Table 2: PoA and PoS, where m_α and M_α denote the min and the max component of the scoring vector α, respectively. $\mathrm{SW}(\mathsf{P}_n)$ is the social welfare achieved by a path of n nodes.

Definition 3.1. MSW (resp. MSW-S) is the problem of computing, given a DHG instance $\langle G, \alpha \rangle$, a coalition structure C^* of maximum social welfare (resp. a NS coalition structure of maximum social welfare).

For decreasing vectors the NP-hardness of MSW is already implied by the previous results on FHGs [3] and SDGs [14]. Even though in this work we focus on normalized vectors, we also present a nice inapproximability result for increasing vectors with $\alpha_1 = 0$.

3.2 Price of Anarchy and Price of Stability

In the previous subsection we reported that the best (NS) outcomes cannot be easily computed. As the next step, we want to estimate the quality of $\mathsf{NS}_{\langle G, \alpha \rangle}$ solutions by providing bounds on their PoA and PoS. We focus on non-negative and normalized scoring vectors, showing that in this case the PoS and the PoA are indeed bounded. We provide improved bounds for decreasing and increasing vectors by examining the topologies of coalitions with high social welfare, namely star partitions and paths for decreasing and increasing scoring vectors, respectively. Table 2 summarizes our main results.

The refined PoS bound for decreasing vectors is better than the general bound of $\frac{n}{2}$ for normalized scoring vectors if and only if $\alpha_2 \geq \frac{\sqrt{4n^2-12n+6}-n}{2((n-2)^2-2)} \approx \frac{1}{2n}$ and this quantity tends to 0 as n increases.

This result generalizes the one for SDGs obtained in [5]. That is, $\mathrm{PoS}((1, \frac{1}{2}, \dots)) \leq \frac{1}{2} + \frac{1}{\sqrt{2}}$.

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