Restricted Domains of Dichotomous Preferences with Possibly Incomplete Information

Extended Abstract

Zoi Terzopoulou
Institute for Logic, Language and Computation
Amsterdam, The Netherlands
z.terzopoulou@uva.nl

Alexander Karpov
National Research University Higher School of Economics,
Institute of Control Sciences of Russian Academy of Sciences
Moscow, Russia
akarpov@hse.ru

Svetlana Obraztsova
Nanyang Technological University
Singapore
lana@ntu.edu.sg

ABSTRACT

Restricted domains have been extensively studied within computational social choice, initially for voters' preferences that are total orders over the set of alternatives and subsequently for preferences that are dichotomous—i.e., that correspond to approved and disapproved alternatives. We contribute to the latter stream of work. We obtain forbidden subprofile characterisations for various important dichotomous domains, and we also study profiles with incomplete information about the voters' preferences. Specifically, we design polynomial algorithms to determine whether such incomplete profiles admit completions within certain restricted domains.

ACM Reference Format:
Zoi Terzopoulou, Alexander Karpov, and Svetlana Obraztsova. 2020. Restricted Domains of Dichotomous Preferences with Possibly Incomplete Information. In *Proc. of the 19th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2020), Auckland, New Zealand, May 9–13, 2020,* IFAAMAS, 3 pages.

1 BACKGROUND

Preferences and their aggregation constitute a central element of AI research [5]. But well-behaved aggregation mechanisms are not always easy to find, notably because determining the outcome of the aggregation is often an intractable task [14]. Luckily, good news come to light under the assumption that the voters' preferences conform to a certain structure, also known as a *domain restriction* [9]. On a conceptual level, restricted domains represent structures that arise as natural preference models in many real-life settings.

Restricted domains of preferences that are total orders over the set of alternatives are well-studied. On the other hand, domains of dichotomous preferences—although very natural—were developed only recently by Elkind and Lackner [8]. Suppose that a voter v_j either approves or disapproves an alternative a_i ($p_{i,j} = 1$ or 0, respectively). The dichotomous preferences of all voters are captured by a *profile* P—an $m \times n$ binary matrix for n voters and m alternatives. Elkind and Lackner showed that the following structures of dichotomous preferences—performing some ordering of the voters—admit polynomial algorithms in the context of two popular approval-based multiwinner rules for which determining the winning committee is known to be NP-hard, namely Proportional Approval Voting (PAV) [10] and Maximin Approval Voting (MAV) [3]:[1]

$$\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 1 & 1\end{pmatrix}$$

(a) Vote Interval (VI): for every alternative, voters approving it form an interval of the ordering.

$$\begin{pmatrix}1 & 1 & 0\\ 1 & 0 & 0\\ 0 & 1 & 1\end{pmatrix}$$

(b) Vote External Interval (VEI): for every alternative, voters approving it are a prefix or a suffix of the ordering.

$$\begin{pmatrix}1 & 1 & 0\\ 1 & 0 & 0\\ 1 & 1 & 0\end{pmatrix}$$

(c) Single-sided VEI (SVEI): for every alternative, voters approving it are a prefix of the ordering.

We provide characterisation theorems for the above domains (and straightforwardly for their dual ones concerning orderings of the alternatives instead of the voters), by identifying the patterns that prevent a profile from exhibiting a certain structure. The literature on domains of total orders contains characterization results using forbidden patterns, reminiscent to ours [1, 4, 13]. It is also worth stressing here that, for any domain characterised by a finite number of forbidden patterns, it is computationally easy to check whether a preference profile conforms to it. Bartholdi III and Trick [2] designed the original algorithm for detecting whether a profile is single-peaked, while Elkind et al. [7] and Bredereck et al. [4] solved the same exercise for single-crossing preferences.

However, one important aspect has not been considered in the literature to date: our information about the dichotomous preferences of the voters will often be *incomplete*. Either because it is costly for the voters to report all their preferences, or because they have not yet formed full preferences when they are asked to express them, we may have no access to the complete preference profile. Let $\mathcal{M}_{m\times n}$ be the set of all *complete* $m \times n$ matrices with entries "0" or "1", and $\mathcal{I}_{m\times n}$ the set of all *incomplete* matrices with entries "0", "1", or "?". Given matrices $X \in \mathcal{I}_{m\times n}$ and $Y \in \mathcal{M}_{m\times n}$, we say that Y is a *completion* of X if every cell of known value in X has the same

[1] Although not studied by Elkind and Lackner [8], SVEI is logically stronger than VEI, and hence also allows for efficiency regarding both PAV and MAV. SVEI embodies a reasonable restriction in voting scenarios: Assume that the alternatives constitute the candidates of a specific political party. We may then order the voters from the most loyal party supporter to the most adversary one so that each candidate firstly wins the support of the most loyal voter, then possibly also gets a vote from the next most loyal voter (depending on how convincing she is), and so on.

Proc. of the 19th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2020), B. An, N. Yorke-Smith, A. El Fallah Seghrouchni, G. Sukthankar (eds.), May 9–13, 2020, Auckland, New Zealand.

value in Y. It is important to know whether a certain structure can *potentially* be manifested in a given incomplete profile, for instance to understand whether an aggregation method has the chance to be efficiently applied. We design two polynomial algorithms that perform this task, and also subsume the analogous algorithms for the complete case by Elkind and Lackner [8].[2]

2 COMPLETE PROFILES

For two matrices $X, Y \in \mathcal{I}_{m\times n}$, we say that X and Y are *equivalent* if X equals Y after some permutation of rows and columns. The matrix X *occurs as a pattern* in the matrix Y if for some submatrix $Z \in \mathcal{I}_{k\times \ell}$ of Y it is the case that X is equivalent to Z. If X does not occur as a pattern in Y, we say that Y *avoids* X.

Tucker [15] obtained a combinatorial characterisation for VI with infinitely many forbidden patterns. We present original characterisations for VEI and SVEI. The former uses Tucker's result, while the latter relies on a lemma showing that SVEI is satisfied by a profile if and only if its *consecutive order graph* is acyclic.[3]

Proposition 1. *A profile $P \in \mathcal{M}_{m\times n}$ satisfies the VEI property if and only if it avoids the patterns* $Z_1 = \begin{pmatrix} 0&1&1\\1&0&1\\1&1&0 \end{pmatrix}$, $Z_2 = \begin{pmatrix} 1&0&0\\0&1&0\\0&0&1 \end{pmatrix}$, $Z_3 = \begin{pmatrix} 1&1&0\\1&0&1\\1&0&0 \end{pmatrix}$, $Z_4 = \begin{pmatrix} 0&0&1\\0&1&0\\0&1&1 \end{pmatrix}$, *and* $Z_5 = \begin{pmatrix} 1&1&0&0\\1&0&1&0 \end{pmatrix}$.

Proposition 2. *A profile $P \in \mathcal{M}_{m\times n}$ satisfies the SVEI property if and only if it avoids the pattern* $X = \begin{pmatrix} 1&0\\0&1 \end{pmatrix}$.

The presented characterisation results are also important in that they guarantee the detection in polynomial time of whether a given incomplete profile will *necessarily* conform to a certain restriction.

3 INCOMPLETE PROFILES

Given an incomplete profile $P \in \mathcal{I}_{m\times n}$, can we detect easily whether P admits a completion that conforms to a specific structure? And if such a completion exists, can we find it?

We know that the problem of detecting whether an incomplete profile can be completed in a way such that VI is satisfied is NP-complete [11]. In this work we design polynomial algorithms that answer our question for SVEI and VEI.

SVEI. The algorithm `SVEI-INCOMPLETE` is based on the sub-algorithm `FILLING`, described below (see Figure 2 for an example).

`FILLING`(P): For an arbitrary cell of unknown value in P, check whether it needs to be filled with "0" or "1" in order to avoid the forbidden pattern X for SVEI. If both values need to be filled, announce "invalid" and exit; if only one does, then fill it; if neither does, continue to the next cell. Return the filled profile P_f.

`SVEI-INCOMPLETE`(P): Apply `FILLING`(P). If "invalid" is announced, exit with failure. Otherwise, extend the the consecutive order graph of P to a linear order L and obtain an ordered profile using L.

Proposition 3. `SVEI-INCOMPLETE` *detects in polynomial (in $m \times n$) time whether a profile of dichotomous preferences possibly satisfies SVEI. If it does,* `SVEI-INCOMPLETE` *also finds an appropriate order of the voters that we can easily complete in polynomial time.*

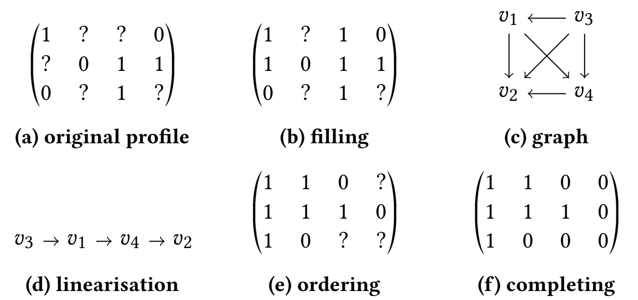

Figure 2: `SVEI-INCOMPLETE`: An example.

VEI. The key intuition regarding the property of VEI is that a profile P satisfies it if and only if it also satisfies SVEI after having all values of certain rows flipped. But how can we know which values may need to be flipped? This problem is solved by the algorithm `VEI-INCOMPLETE`, which we informally explain here.

First, for any two rows that *contradict* each other, i.e., contain the pattern $\begin{pmatrix} 1&0\\0&1 \end{pmatrix}$, we know that exactly one of them needs to be flipped. Similarly, if two rows *match*, i.e., contain the pattern $\begin{pmatrix} 1&0\\1&0 \end{pmatrix}$, then either both or neither of them should be flipped.

Starting from an arbitrary row i, the algorithm `EXPANDING` gathers together those rows that match i and—after flipping them—those that contradict i. Next, the same process is repeated for rows that are not related to i in any of the aforementioned ways, until a partition of all rows is obtained.

The algorithm `COARSENING` applies the same idea, but on sets of rows instead of single ones: Two such sets X and Y are merged if some row from X matches some row from Y; and if some row from X contradicts some row from Y, then X is merged with all rows in Y after their values are flipped. Continuing this way, the coarsest possible partition of the rows of a given profile is formed. If at any point two rows in the same set contradict each other, then we exit with failure. Otherwise, we can apply `SVEI-INCOMPLETE`.

Proposition 4. `VEI-INCOMPLETE` *detects in polynomial (in $m \times n$) time whether a profile of dichotomous preferences possibly satisfies VEI. If it does,* `VEI-INCOMPLETE` *also finds an appropriate order of the voters that we can easily complete in polynomial time.*

4 CONCLUSION

We have initiated the study of restricted domains of dichotomous preferences under settings of incomplete information. We have characterised two important such domains via forbidden patterns and have designed polynomial algorithms for detecting whether a profile's completion will possibly belong to a given domain. These results are potentially interesting for scenarios of judgment aggregation as well, where analogous restrictions to the ones investigated in this paper guarantee majority-consistent outcomes.

ACKNOWLEDGMENTS

Support from the Basic Research Program of the National Research University Higher School of Economics is gratefully acknowledged.

[2] Regarding incomplete profiles of total orders, Lackner [12] and Elkind et al. [6] were among the first to address the problem of extending partial preferences to full preferences that respect a given restriction.

[3] The consecutive order graph of a profile P consists of n nodes, one for each voter, and of directed edges from v_j to v_ℓ whenever there exists alternative a_i such that v_j approves a_i and v_ℓ disapproves a_i.

REFERENCES

[1] Miguel A Ballester and Guillaume Haeringer. 2011. A characterization of the single-peaked domain. *Social Choice and Welfare* 36, 2 (2011), 305–322.

[2] John Bartholdi III and Michael A Trick. 1986. Stable matching with preferences derived from a psychological model. *Operations Research Letters* 5, 4 (1986), 165–169.

[3] Steven J Brams, D Marc Kilgour, and M Remzi Sanver. 2007. A minimax procedure for electing committees. *Public Choice* 132, 3-4 (2007), 401–420.

[4] Robert Bredereck, Jiehua Chen, and Gerhard J Woeginger. 2013. A characterization of the single-crossing domain. *Social Choice and Welfare* 41, 4 (2013), 989–998.

[5] Carmel Domshlak, Eyke Hüllermeier, Souhila Kaci, and Henri Prade. 2011. Preferences in AI: An overview. *Journal of Artificial Intelligence Research* 19, 4 (2011), 1037–1052.

[6] Edith Elkind, Piotr Faliszewski, Martin Lackner, and Svetlana Obraztsova. 2015. The complexity of recognizing incomplete single-crossing preferences. In *Proc. of the 29th AAAI Conference on Artificial Intelligence (AAAI)*. 865–871.

[7] Edith Elkind, Piotr Faliszewski, and Arkadii M. Slinko. 2012. Clone structures in voters' preferences. In *Proc. of the 13th ACM Conference on Electronic Commerce (EC)*. 496–513.

[8] Edith Elkind and Martin Lackner. 2015. Structure in dichotomous preferences. In *Proc. of the 24th International Joint Conference on Artificial Intelligence (IJCAI)*. 2019–2025.

[9] Edith Elkind, Martin Lackner, and Dominik Peters. 2017. Structured preferences. In *Trends in Computational Social Choice*, Ulle Endriss (Ed.). AI Access, 187–207.

[10] D Marc Kilgour and Erica Marshall. 2012. Approval balloting for fixed-size committees. In *Electoral Systems*. Springer, 305–326.

[11] Bettina Klinz, Rüdiger Rudolf, and Gerhard J Woeginger. 1995. Permuting matrices to avoid forbidden submatrices. *Discrete applied mathematics* 60, 1-3 (1995), 223–248.

[12] Martin Lackner. 2014. Incomplete preferences in single-peaked electorates. In *Proc. of the 28th AAAI Conference on Artificial Intelligence (AAAI)*. 742–748.

[13] Dominik Peters and Martin Lackner. 2017. Preferences single-peaked on a circle. In *Proc. of the 31st AAAI Conference on Artificial Intelligence (AAAI)*. 649–655.

[14] Ariel D Procaccia, Jeffrey S Rosenschein, and Aviv Zohar. 2007. Multi-Winner Elections: Complexity of Manipulation, Control and Winner-Determination.. In *Proc. of the 20th International Joint Conference on Artificial Intelligence (IJCAI)*. 1476–1481.

[15] Alan Tucker. 1972. A structure theorem for the consecutive 1's property. *Journal of Combinatorial Theory, Series B* 12, 2 (1972), 153–162.