Parameterized Algorithms for Kidney Exchange

Extended Abstract

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ABSTRACT

In kidney exchange programs, multiple patient-donor pairs each of whom are otherwise incompatible, exchange their donors to receive compatible kidneys. The Kidney Exchange problem is typically modelled as a directed graph where every vertex is either an altruistic donor or a pair of patient and donor; directed edges are added from a donor to its compatible patients. The computational task is to find if there exists a collection of disjoint cycles and paths starting from altruistic donor vertices of length at most ℓ_c and ℓ_p respectively that covers at least some specific number t of non-altruistic vertices (patients). We study parameterized algorithms for the kidney exchange problem in this paper. Specifically, we design FPT algorithms parameterized by each of the following parameters: (1) the number of patients who receive kidney, (2) treewidth of the input graph + $\max\{\ell_p, \ell_c\}$, and (3) the number of vertex types in the input graph when $\ell_p \leq \ell_c$. We also present interesting algorithmic and hardness results on the kernelization complexity of the problem. Finally, we present an approximation algorithm for an important special case of Kidney Exchange.

KEYWORDS

Fixed-Parameter Tractability; Kidney Exchange; Vertex Type; Kernelization; Altruistic donors; Compatibility graph; Trading-cycle; Trading-chain

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1 INTRODUCTION

Patients having acute renal failures are typically treated either with dialysis or with kidney transplantation. However, the quality of life on dialysis is comparitively lower and also the average life span of the patients on dialysis is around 10 years [3]. For this reason, most patients prefer a kidney transplantation over periodic dialysis. However, the gap between the demand and supply of kidneys, which can be obtained either from a deceased person or from a living donor, is so large that the average waiting time varies from 2 to 5 years at most centers [1, 2]. Moreover, even if a patient is able to find a donor, there could be many medical reasons (like blood group or tissue mismatch) due to which the donor could not donate his/her kidney to the patient.

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The *Kidney Paired Donation (KPD)*, a.k.a *Kidney Exchange* program, allows donors to donate their kidneys to compatible other patients with the understanding that their patients will also receive medically compatible kidneys thereby forming some kind of barter market [4, 7, 10]. Since its inception in [25], an increasing amount of people register in the kidney exchange program since, this way, patients not only have a better opportunity to receive compatible kidneys, but also can get medically better matched kidneys which last longer [26]. The central problem in any kidney exchange program, also known as the *clearing problem*, is how to transplant kidneys among various patients and donors so that a maximum number of patients receive kidneys.

Related Work. To the best of our knowledge, Rapaport was the first person to introduce the idea of kidney exchange [25]. Since then, many variants and properties have been explored [8, 9, 26]. For example, a line of research allow only cycles [13, 18, 27] while others allow kidney exchange along both cycles and chains [16, 24, 28]

Krivelevich et al. and Abraham et al. showed that the basic kidney exchange problem along with its various incarnations are NP-hard [4, 19]. Krivelevich et al. and Jia et al. developed approximation algorithms for the kidney exchange problem by exploiting interesting connection with the set packing problem [17, 19]. Practical heuristics and integer linear programming based algorithms haven been extensively explored for the kidney exchange problem [12, 15, 16, 18, 21, 24]. Dickerson et al. introduced the notion of "vertex type" and showed its usefulness as a graph parameter in real-world kidney exchange instances. Two vertices is said to have the same vertex type if their in-neighbourhood and out-neighbourhood are the same.

The closest predecessor of our work is by Xiao and Wang [28]. They proposed an exact algorithm with running time $O(2^n n^3)$ where n is the number of vertices in the underlying graph. They also presented a fixed parameter tractable algorithm for the kidney exchange problem parameterized by the number of vertex types if we do not have any restriction on the length of cycles and chains. Lin et al. [22] studied the version of the kidney exchange problem which allows only cycles and developed a randomized parameterized algorithm with respect to the parameter being (number of patients receiving a kidney, maximum allowed length of any cycle).

2 PRELIMINARIES

A kidney exchange problem is formally represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ which is known as the *compatibility graph*. A subset $\mathcal{B} \subseteq \mathcal{V}$ of vertices denotes *altruistic donors* (also called *non-directed donors*); the other set $\mathcal{V} \setminus \mathcal{B}$ of vertices denote a patient-donor pair who wish to participate in the kidney exchange program. We have a directed edge $(u, v) \in \mathcal{A}$ if the donor of the vertex

$u \in \mathcal{V}$ has a kidney compatible with the patient of the vertex $v \in \mathcal{V} \setminus \mathcal{B}$. Kidney exchange happens either (i) along a *trading-cycle* $u_1, u_2, \ldots, u_k$ where the patient of the vertex $u_i \in \mathcal{V} \setminus \mathcal{B}$ receives a kidney from the donor of the vertex $u_{i-1} \in \mathcal{V} \setminus \mathcal{B}$ for every $2 \leq i \leq k$ and the patient of the vertex u_1 receives the kidney from the donor of the vertex u_k, or (ii) along a *trading-chain* $u_1, u_2, \ldots, u_k$ where $u_1 \in \mathcal{B}, u_i \in \mathcal{V} \setminus \mathcal{B}$ for $2 \leq i \leq k$ and the patient of the vertex u_j receives a kidney from the donor of the vertex u_{j-1} for $2 \leq j \leq k$. Due to operational reasons, all the kidney transplants along a trading-cycle or a trading-chain should be performed simultaneously. This puts an upper bound on the length ℓ of feasible trading-cycles and trading-chains. We define the length of a path or cycle as the number of edges in it. The kidney exchange clearing problem is to find a collection of feasible trading-cycles and trading-chains which maximizes the number of patients who receive a kidney. Formally it is defined as follows.

*Definition 2.1 (*Kidney Exchange*).* Given a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{A})$ with no self-loops, an altruistic vertex set $\mathcal{B} \subset \mathcal{V}$, two integers ℓ_p and ℓ_c denoting the maximum length of respectively paths and cycles allowed, and a target t, compute if there exists a collection C of disjoint cycles of length at most ℓ_c and paths with starting from altruistic vertices only each of length at most ℓ_p which cover at least t non-altruistic vertices. We denote an arbitrary instance of Kidney Exchange by $(\mathcal{G}, \mathcal{B}, \ell_p, \ell_c, t)$.

3 RESULTS

Designing exact algorithms for Kidney Exchange has been a research focus in algorithmic game theory. We contribute to this line of research in this paper. We present our results in this section. We refer the reader to the full version of our paper [23] for the proofs. We begin with presenting an FPT result for the Kidney Exchange problem parameterized by the number of patients who receive a kidney. We use the technique of color coding [5, 6] to design our algorithm.

Theorem 3.1. *There is a algorithm for the* Kidney Exchange *problem which runs in time $O^*(2^{O(t)})$.*

We now consider the parameter treewidth to design an FPT algorithm. Towards that, we first show the following.

Theorem 3.2. *The optimization version of the* Kidney Exchange *problem is a linear extended monadic second-order (EMS) extremum problem when* $\max\{\ell_p, \ell_c\} = O(1)$.

It follows immediately from Theorem 3.2 and [11] that Kidney Exchange is FPT parameterized by τ (treewidth) and $\ell = \max\{\ell_p, \ell_c\}$. However, the running time that we get is not practically useful. Next an efficient dynamic programming based algorithm with running time $O^*(\ell^{O(\tau)}\tau^{O(\tau)})$ helps us establish the following.

Theorem 3.3. *There is an algorithm for the* Kidney Exchange *problem which runs in time $O^*(\ell^{O(\tau)}\tau^{O(\tau)})$.*

Let θ denote the number of vertex types in a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$. [28] presented an FPT algorithm parameterized by θ when $\ell_p = \ell_c = |\mathcal{V}|$. We now improve the result by presenting our FPT algorithm parameterized by θ when $\ell_p \leq \ell_c$. Towards that, we first present an important lemma on the structure of an optimal solution.

Lemma 3.4. *In every* Kidney Exchange *problem instance when $\ell_p \leq \ell_c$, there exists an optimal solution where the length of every path and cycle in that solution is at most $\theta + 3$.*

We use the following useful result by Lenstra to design our FPT algorithm in Theorem 3.6.

Lemma 3.5 (Lenstra's Theorem [20]). *There is an algorithm for computing a feasible as well as an optimal solution of an integer linear program which is fixed parameter tractable parameterized by the number of variables.*

We now use Lemma 3.4 to construct an ILP for Kidney Exchange problem whose number of variables is bounded by a function of θ and then apply Lemma 3.5 to obtain the following.

Theorem 3.6. *There exists an* FPT *algorithm for the* Kidney Exchange *problem parameterized by θ when $\ell_p \leq \ell_c$.*

We now present our result on kernelization. We show that the Kidney Exchange problem admits a polynomial kernel for the parameter $t + \Delta$ for every constant ℓ_p and ℓ_c. Formally we show the following result.

Theorem 3.7. *For the* Kidney Exchange *problem, there exists a vertex kernel of size $O(t\Delta^{\max\{\ell_p, \ell_c\}})$.*

Theorem 3.7 raises the following question: does the Kidney Exchange problem admit a polynomial kernel parameterized by $t + \Delta + \ell_p + \ell_c$? We answer this question negatively in Theorem 3.8

Theorem 3.8. Kidney Exchange *does not admit any polynomial kernel with respect to the parameter $t + \Delta + \ell_p + \ell_c$ unless* NP $\subseteq$ co − NP*/poly.*

We now present our approximation result for the Kidney Exchange problem when only cycles of length at most 3 are allowed; no path is allowed. [12] studied this problem with the name Max Size ≤3-Way Exchange and proved APX-hardness. A trivial extension from the result on MAX CYCLE WEIGHT≤k-WAY EXCHANGE in [12] leads to a $2+\varepsilon$ approximation algorithm for Max Size ≤3-Way Exchange. Now towards designing the approximation algorithm, we use the 3-Set Packing problem which is defined as follows.

*Definition 3.9 (*3-Set Packing*).* Given a universe $\mathcal{U}$, a family $\mathcal{F} \subseteq 2^{\mathcal{U}}$ of sets of size at most 3, and an integer k, compute if there exists a subfamily $\mathcal{W} \subseteq \mathcal{F}$ of pairwise disjoint sets such that $|\mathcal{W}| \geq k$. We denote any instance of 3-Set Packing by $(\mathcal{U}, \mathcal{F}, k)$.

The following result is due to [14].

Lemma 3.10. *For every $\varepsilon > 0$, there is $(4/3 + \varepsilon)$-approximation algorithm for the* 3-Set Packing *problem for optimizing k.*

The following result relates Max Size ≤3-Way Exchange to 3-Set Packing.

Theorem 3.11. *If there is a α-approximation algorithm for* 3-Set Packing *problem, then there is a $\frac{4\alpha}{3}$-approximation algorithm for* Max Size ≤3-Way Exchange.

Theorem 3.11 immediately gives us the following corollary.

Corollary 3.12. *For every $\varepsilon > 0$, there is a $(\frac{16}{9} + \varepsilon)$-approximation algorithm for* Kidney Exchange *if only cycles of length at most* 3 *are allowed (and no paths are allowed).*

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