A New Method of 3-D Motion Analysis Using A Concept of Projective Geometry

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ABSTRACT

This paper describes a method of interpreting three dimensional motion of an object by making use of rigidity assumption and orientation of its edge. We employ a vanishing point to determine the orientation. We propose a new idea of using cross ratio, i.e. one of the most fundamental concepts in projective geometry to find a vanishing point of a line. This allows to calculate the location of a vanishing point with a known sequence of points on it without another parallel line required by the conventional method.

1. Introduction

These years a number of researchers workinq on the computer vision come to be interested in the analysis of three dimentional motions of an object for the application to improving industrial robots and some intelligent systems. Three major approaches have been taken for this problem, optical flow analysis, algebraic geometry and projective geometry. These methods have their own advantages.

Some robots, for example mobile robots, require visual systems providing a standard (not long focus) lens for getting a rather wide scope. Perspective distortions appear in the image, since projection through such a lens yields an image of central projection. Perspective geometry is useful to deal with the image. The most advantageous point of it is that it enables to calculate relative distance between the camera and arbitary points on a line could be calculated by a simple ratio of the one (or two) ' dimensional distances between the images of the points and the vanishing point of the line. The difficulty in the procedure is to find the vanishing point by means of image processing. Hence it has been given a priori in most of the scene analysis using vanishing points. In case of utilizing image processing it has been determined an intersection of several edges in an image assumed to be parallel in the orginal space. However we often encounter the cases to analyze images which involve no parallel edges or lines.

In this paper we propose a method which does not require the existence of parallel lines for finding the vanishing point of a line. It is expected to extend application areas of projective geometry in the field of computer vision and scene analysis. We describe an application to interpreting the 3-D motion of an object moving freely in space.

2. Preliminary

In this paper the vanishing point is used to know the orientation of an object and the rigidity assumption is also used to calculate position of the object at each instance of the motion. Below will be presented several fundamental properties of algebraic geometry which are necessary to the discussion.

2.1 Coordinates and Central projection

Cartesian coordinates to express three dimensional location of an object are fixed to a camera. The origin is set on the eyepoint, i.e.the center of the lens and the z-axis is aligned to the optical axis. A focal distance is denoted by f.

An image plane with coordinates (ξ, η) is defined by an equation z=f, that is, to be parallel to the xy plane at a distance f in front of the eyepoint such that the z-axis pierces the coordinate axes origin. As illustrated in Fig.1, ξ and η axes on the image plane are defined to be parallel to the x and y axes, respectively. Since a point P(x,y,z) is projected to a point $p(\xi, \eta)$ on

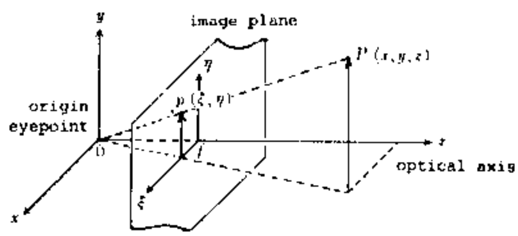


Fig. 1 Coordinates and Central Projection

the image plane by the central projection, the following equations hold.

$$\xi = \frac{x}{z} f, \qquad \eta = \frac{y}{z} f \tag{1}$$

When a line with a directional cosine (n_1, n_2, n_3) is projected onto the image plane, the coordinates $(\xi_\infty, \eta_\infty)$ of its vanishing point is represented by the following equations.

$$\xi_\infty = \frac{n_1}{n_3} f, \qquad \eta_\infty = \frac{n_2}{n_3} f \tag{2}$$

Inversely, a line with the vanishing point $(\xi_\infty, \eta_\infty)$ on the image plane is pointing in the direction (n_1, n_2, n_3) in the 3-D space, where

$$n_1 = k\xi_\infty, \quad n_2 = k\eta_\infty, \quad n_3 = k \cdot f \quad \text{and} \quad k = \frac{1}{\sqrt{\xi_\infty^2 + \eta_\infty^2 + f^2}} \tag{3}$$

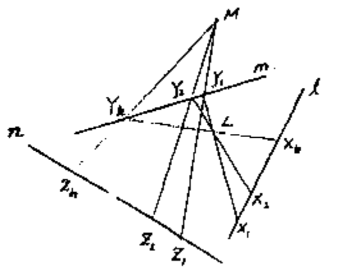


Fig. 5 Total Edge Images of the Sequence

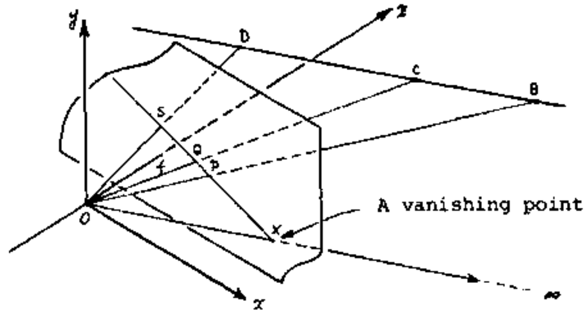


Fig. 3 A Central Projection

2.2 Rigidity-assumption and central projection

Let r denote the distance between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ on an object and (n_1, n_2, n_3) denote the orientation of the line P_1P_2 running on these points. We get the following relations.

$$\left.\begin{aligned} x_2 &= x_1 + r n_1 \\ y_2 &= y_1 + r n_2 \\ z_2 &= z_1 + r n_3 \end{aligned}\right\} \qquad (4)$$

If these points change the position to $P_1'(x_1', y_1', z_1')$ and $P_2'(x_2', y_2', z_2')$ after a motion of the object, the distance between them is invariant from the rigidity-assumption of the object. Thus the similar relations to eq.(4) hold as follows.

$$\left.\begin{aligned} x_2' &= x_1' + r n_1' \\ y_2' &= y_1' + r n_2' \\ z_2' &= z_1' + r n_3' \end{aligned}\right\} \qquad (5)$$

where (n_1', n_2', n_3') also represents the orientation of the line $P_1'P_2'$.

On the other hand, suppose that these points P_1, P_2, P_1' and P_2' are projected to the points $p_1(\xi_1, \eta_1)$, $p_2(\xi_2, \eta_2)$, $p_1'(\xi_1', \eta_1')$ and $p_2'(\xi_2', \eta_2')$ on the image plane, respectively. Then, the following equations are derived from the equations (1), (4) and (5).

$$\left.\begin{aligned} z_1' &= \frac{\xi_2' n_3' - f n_1'}{\xi_2 n_3 - f n_1} \cdot \frac{\xi_1 - \xi_2}{\xi_1' - \xi_2'} \\ &= \frac{\eta_2' n_3' - f n_2'}{\eta_2 n_3 - f n_2} \cdot \frac{\eta_1 - \eta_2}{\eta_1' - \eta_2'} \cdot z_1 \end{aligned}\right\} \qquad (6)$$

When the value of z is given, the coordinates of P_1' can be calculated with eq.'s (6) and (1) and those of P_2' as well. In this process it is assumed that the orientations of the lines $\overline{P_1P_2}$ and $\overline{P_1'P_2'}$ are also given. In the next section we propose a new approach to know the vanishing point of a line from its image.

3. Projective Geometry

Equations (3) shows that a vanishing point is a key to know the orientation of a line from its image. Conventional methods to find vanishing points have utilized parallelpiped structures of objects. However, there are not always parallels in the scene and moreover even if there are any, we do not have means how to verify them now. This is the greatest problem in employing vanishing points to recover 3-D structures. Consequently if we could obtain an algorithm to find the vanishing point of a line without using other lines parallel to it, we could extend applicable fields of the method using vanishing points to much more scenes as well as reduce the load of image processing.

3.1 Cross ratio of a range of points

We have solved this problem by introducing the cross ratio, to calculate the position of the vanishing point of a line with the locations of a sequence of points.

We describe several definitions and properties about the concept of the cross ratio below.

[Def.1] The cross ratio R_{ABCD} of the ordered four points A,B,C and D on a line is defined by the following equation.

$$R_{ABCD} = \frac{\overline{AB}}{\overline{AD}} : \frac{\overline{CB}}{\overline{CD}} = \frac{\overline{AB}}{\overline{AD}} \cdot \frac{\overline{CD}}{\overline{CB}} \qquad (7)$$

where XY denotes the line segment directing from X to Y.

[Prop.1] The cross ratio is invariant through projective transformation of a range of points.

[Prop.2] If a point A in a range of points A,B,C and D locates at the infinite (∞) and the others do not, the following equation holds.

$$R_{\infty BCD} = \frac{\overline{CD}}{\overline{CB}} \qquad (8)$$

By the central projection, a point located at the infinite in the 3-D world is projected to a vanishing point on an image plane. Suppose that a range of points ∞,B,C and D in the 3-D world is projected to a range of points X,P,Q and S on an image plane, and that their cross ratios are denoted by $R_{\infty BCD}$ and R_{XPQS}, respectively.

Then from [Prop.1] and [Prop.2] we can readily obtain the following relation.

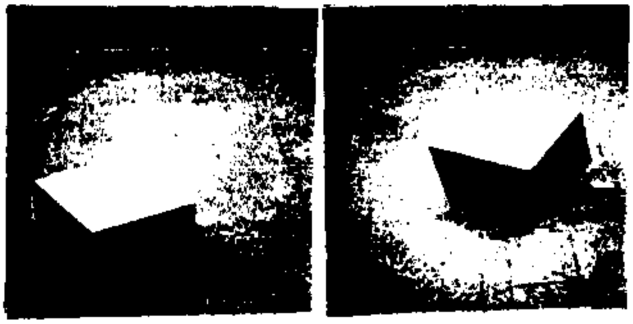
$$\frac{CD}{CB} = \frac{XP}{XS} \cdot \frac{QS}{QP} \qquad (9)$$

The relation (9) is immediately interpreted as a relation among the coordinates of the points in the 3-D world as follows.

$$\frac{\zeta_X - \zeta_P}{\zeta_X - \zeta_S} \cdot \frac{\zeta_Q - \zeta_S}{\zeta_Q - \zeta_P} = \frac{k_C - k_D}{k_C - k_B} \qquad (10)$$

,where $\begin{cases} \zeta = \xi \text{ or } \eta \\ k = x \text{ or } y \end{cases}$ (ordered), $\begin{cases} U = (\xi_U, \eta_U) & U = X, P, Q, S \\ V = (x_V, y_V, z_V) & V = B, C, D \end{cases}$

This is the relation that we have pursued for the purpose of calculating the location of a vanishing point without the assumption of an existence of parallel edges and detection of them.



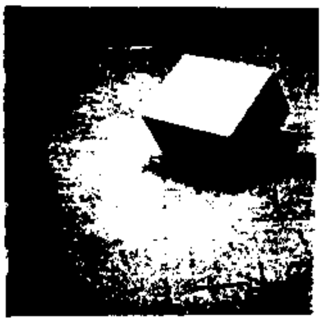


Fig. 4
A Picture Sequence of a Twisted Motion of an Object

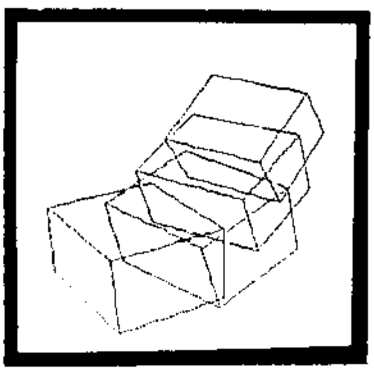


Fig. 5
Total Edge Images of the Sequence

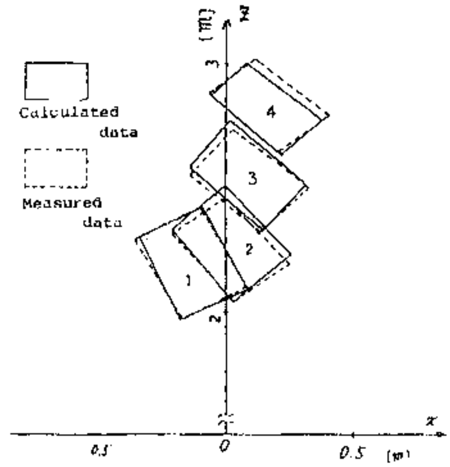


Fig. 6 Results of Experiment compared with Measured Data

4. Experiment

The concept of the cross ratio enables us to calculate a location of a vanishing point with a simple formula by imposing rather easy restrictions satisfied in most of the applications as described in the previous section. Thus together with the discussion in chapter 2 now we have a new method for the analysis of free 3-D motion of an object.

We have experimentally applied this method to an analysis of a motion of a simple shaped object, i.e. a carton. We move it 4 feet from 7 feet in front of a fixed camera tilting gradually to realize taken, although pictures in Fig.3 are displayed 128x128 in size and 8 levels in intensity for convenience.

The requirement of the proposed method is satisfied by three points on a diagonal of the top-surface of the box, that is both end-points of the diagonal and the intersection of diagonals. Since a pair of facing edges of the top-surface could be assumed parallel, the intersection is the middle point of each diagonal. In this case and the cross ratio becomes as simple as equal to -1.

The computed results of the experiment, are shown in Fig.6 and the measured data are added for comparison. The relative errors in the position are lower than 2%. This is well acceptable to most of the actual applications.

5. Conclusion

In this paper, we have proposed a new method to analyze non-restricted 3-D motions by introducing a simple but fundamental concept useful to deal with actual problems. Results of the experiments proved its validity for a variety of" applications. Our method is also rolevent to the recovery of three dimensional structures of static scenes although it may fail to analyze structures of small objects. The accuracy for the estimation should be well considered in such applications.

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