PROCESS RECOVERY

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ABSTRACT

In many disciplines, the scientist uses the shape of an entity to infer processes that have acted upon the entity. We develop inference rules by which the curvature extrema of a shape can be used to infer the precise trajectories of processes that have created the shape. A formal grammar is also elaborated by which, given two views of the same entity at two developmental stages, the scientist can infer the processes that acted in between the two stages.

I. INTRODUCTION.

In many disciplines, such as astrophysics, developmental biology, medicine, geography, meteorology, etc., the shape of some entity, such as an embryo, tumor, or cloud, is used by the scientist to infer processes that have acted upon the entity. In this paper, we present a set of inference rules by which the curvature extrema of a shape can be used to infer the significant processes that have determined the shape. We also develop a formal grammar by which a scientist, who has two views of an entity at two developmental stages, can infer the processes that produced the second stage from the first.

0. TWO BASIC RULES

We begin by presenting two simple rules by which the curvature extrema of an *individual* shape can be used to infer significant processes that have acted upon that shape. Throughout the paper, we will assume that the input to the rules are shapes represented in the form of smooth planar outlines. However, an equivalent analysis applies to three-dimensional input, as shown in Leyton (1987c).

The inference, from curvature extrema to processes, will be seen as requiring two stages:

curvature extrema -»symmetry axes -»processes

Section ELI deals with the first stage, and section II.2 deals with the second stage.

IL1. Curvature Extrema -»Symmetry Axes

As just stated, central to the development of our process inference rules will be the use of a symmetry analysis.

Observe first, however, that, given two segments of a curve, it is rarely the case that a straight axis reflects one segment on to the other. Nevertheless, it may be possible to define symmetry in a *differential* sense. Consider the two bold curves c, and c_2, in Fig Ia. Although there is no mirror that reflects one curve onto the other, a mirror along line QO reflects the *tangent line,* at A, onto the *tangent line,* at B. It turns out that the existence of such a symmetry is equivalent to the existence of a circle that is tangential at both A and B. (The mirror will contain the circle center O.)

Now drag the circle along the two curves, while always maintaining the double-touching property. As in Leyton (1987b), one can define a *differential symmetry axis* to be the trajectory of some *midpoint* associated with the circle. For example, the *Symmetric Axis Transform (SAT)* of Blum (1973) defines the symmetry axis to be the locus of circle-centers O. This will give some curved axis running between the two curves c_1 and c_2. Again, the *Smooth Local Symmetry (SLS)* of Brady (1983) defines the symmetry axis to be the locus of chord midpoints P. (In the present case, this will produce a slightly different axis from the SAT.) Alternatively, we shall propose here a new symmetry analysis in which the symmetry axis is the trajectory of the midpoint Q of the arc AB, as the circle moves. As shown in Leyton (1987c), this analysis has remarkably different properties from the other two analyses and these properties make the analysis particularly appropriate for the inference of processes. Because of its appropriateness to process-inference, the new analysis will be called *Process Inferring Symmetry Analysis (P1SA)* We shall see several examples of applying PISA, later.

It is now possible to state a result that is crucial to the entire paper. It is a theorem that was proposed and proved in Leyton (1987b), and it relates the curvature extrema of a smooth planar curve to the curve's symmetry structure:

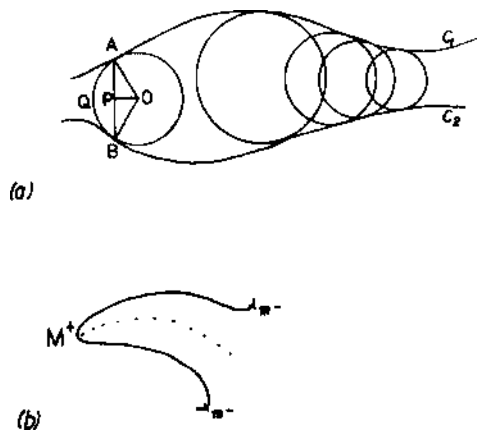

Fig 1. (a) The definition of differential symmetry, Ib) An illustration of the Symmetry-Curvature Duality Theorem.

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SYMMETRY-CURVATURE DUALITY THEOREM (Leyton, 1987b): *Any segment of a smooth planar curve, bounded by two consecutive curvature extrema of the same type (either both maxima or both minima) has a unique differential symmetry axis (under SAT, SLS, or PISA) and this axis terminates at the curvature extremum of the opposite type (minimum or maximum, respectively).*

Fig Ib illustrates the theorem. The two points labeled m" are two consecutive extrema of the same type (ie. they bend in the same direction relative to the curve). The theorem states that, under any of the alternative differential symmetry analyses, there is one and only one symmetry axis. Furthermore, this axis terminates at the extremum *M :* of the opposite type (this extremum bends in the opposite direction).

The theorem will be our first inference rule: It assigns, to each extremum, a unique symmetry axis that terminates at that extremum.

II2. Symmetry Axes -»Processes

The value, to our argument, of identifying symmetry axes, is given by the following crucial principle which was proposed and extensively corroborated in Ley ton (1984, 1985, 1986a, 1986b, 1986c, 1987a)

INTERACTION PRINCIPLE (Leyton, 1984): *The symmetry axes of a perceptual organization are interpreted as the principal directions along which processes are most likely to act or have acted.*

The basis for this proposal is as follows: It was argued in Leyton (1986b) that if a transformation, acting on an organization is one in which symmetry axes become invariant lines (eigenspaces) under the transformation, then the transformation will tend to preserve the symmetries; i.e. be *structure-preserving* on the organization. Two further stages of argument are then needed to obtain the above principle: (I) invariant lines are interpreted as principal directions of action, and (2) transformations that are most structure-preserving tend to be understood as most likely. The above principle has been psychologically validated on simple and complex shape, as well as motion perception (Leyton, 1985, 1986b, 1986c, 1987a).

The Interaction Principle can be regarded as an inference rule. It claims that a symmetry axis is interpreted as the principal direction of a hypothesized process. In fact, since we are concerned, in the present paper, with the inference of processes that have already taken place, the principle implies that a symmetry axis is interpreted as a *record* of a process.

n.3. Curvature Extrema -» Processes

Putting together the two inference rules given thus far, we obtain this conclusion:

Each curvature extremum implies a process
whose trace is the unique symmetry axis
associated with, and terminating at, that extremum

To see that our two inference rules yield highly appropriate process-analyses, let us consider all possible shapes that have eight extrema or less. Drawings of all such shapes have been provided by Richards, Koenderink & Hoffman (1985). In Fig 2. we have taken all these drawings and applied our two inference rules. The lines with arrows represent the process-records inferred by our rules. That is, the lines are the directions of growth, indentation, squashing, etc, along which the deformations are supposed to have happened. As can readily be seen, these process-records correspond strongly with one's intuition as to how the shapes were formed.

Three comments are in order

(1) The letters on each shape denote the curvature extrema. as follows: Let *M* and m denote a local maximum and local minimum respectively, and + and - denote positive and negative curvature repectively. Thus there are four types of extrema: *M :* m , m : and *M* .

(2) The symmetry analysis, used in Fig 2, was PISA, the new analysis introduced in section II.I. This is because, as shown in Leyton (1987c), the SAT and SLS cannot correctly infer squashing and indentation, whereas PISA can.

(3) Fig 2 is stratified into levels according to number of extrema. The only three levels that can exist, with up to eight extrema, are: Level 1, shapes with 4 extrema; Level II, shapes with 6 extrema; Level III, shapes with 8 extrema.

III. THE PROCESS GRAMMAR

The above analysis presented inference rules by which a *single* shape can be given a process-description. However, in many observational situations, the scientist has *two* views of an object at two developmental stages. The scientist attempts to infer the processes that produced the second stage from the first. It is this problem that we now attempt to formalize and solve. We shall develop a *process-grammar* such that, given two shapes, one shape is expressed as the extrapolation of processes inferred in the other by the above inference rules.

The operations of the grammar are specified purely in terms of modifications at extrema, but such that these modifications correspond to process-extrapolations. Physically natural extrapolations have two forms, *continuations* and *bifurcations,* which we now consider in turn.

Continuations.

The continuation of a process at a *M* or m extremum does not change the extremum type, as can be seen by looking at any of the *M* and *m* extrema in Fig 2. However, continuation at a m or *M* does change the type, as we shall now see:

(1) Continuation at m (labeled C m) yields m, as is illustrated by continuing the upward m^+ process at the bottom of shape PI (Fig 2) and thus obtaining shape P2. This form of extrapolation is fully specified by the following rewrite rule expressed purely in terms of the curvature extrema along the bottoms of the respective shapes:

Cm':m -Om *O*

(where each 0 represents a curvature zero, indicated by a dot on the curve in P2).

(2) Continuation at M (labeled CM) yields M \ as is illustrated by continuing the upward *M* process in shape T3 (Fig 2) and thus obtaining the top protruding M^4 process in T4. The rule is specified as:

CM *:M~* **OM*0*

Bifurcations:

A bifurcation has the effect of taking an extremum (maximum or minimum) and pulling it apart into two copies of itself. Between the copies, an extremum of the opposite type (minimum or maximum, respectively) is unavoidably introduced. In the smallest such non-trivial change, the introduced extremum has the same sign as the splitting extremum. Logically, there are four such primitive bifurcations, as follows. All other bifurcations are derivable from these together with the above two continuation operations.

(3) Bifurcation at M^+ (labeled BM^+) must be given by the re-write rule

$$BM^+ : M^+ \to M^+ m^+ M^+$$

An illustration is the following. Consider the protruding process at the top of T4 (in Fig 2). If the process bifurcates, one branch goes to the left and the other to the right The resulting shape has been drawn on its side as Q6 in Fig 2.

(4) Bifurcation at m^- (labeled Bm^-) must be given by the re-write rule

$$Bm^- : m^- \to m^- M^- m^-$$

An illustration is the following. Consider the indenting process at the top of T2 (in Fig 2). If the process bifurcates, one branch goes to the left and the other to the right. The resulting shape is Q2 (in Fig 2).

(5) Bifurcation at m^+ (labeled Bm^+) must be given by the re-write rule

$$Bm^+ : m^+ \to m^+ M^+ m^+$$

and is simply the introduction of a protrusion. An illustration is the transition from P1 to T1 (in Fig 2), i.e. a protrusion has been introduced at the top of T1.

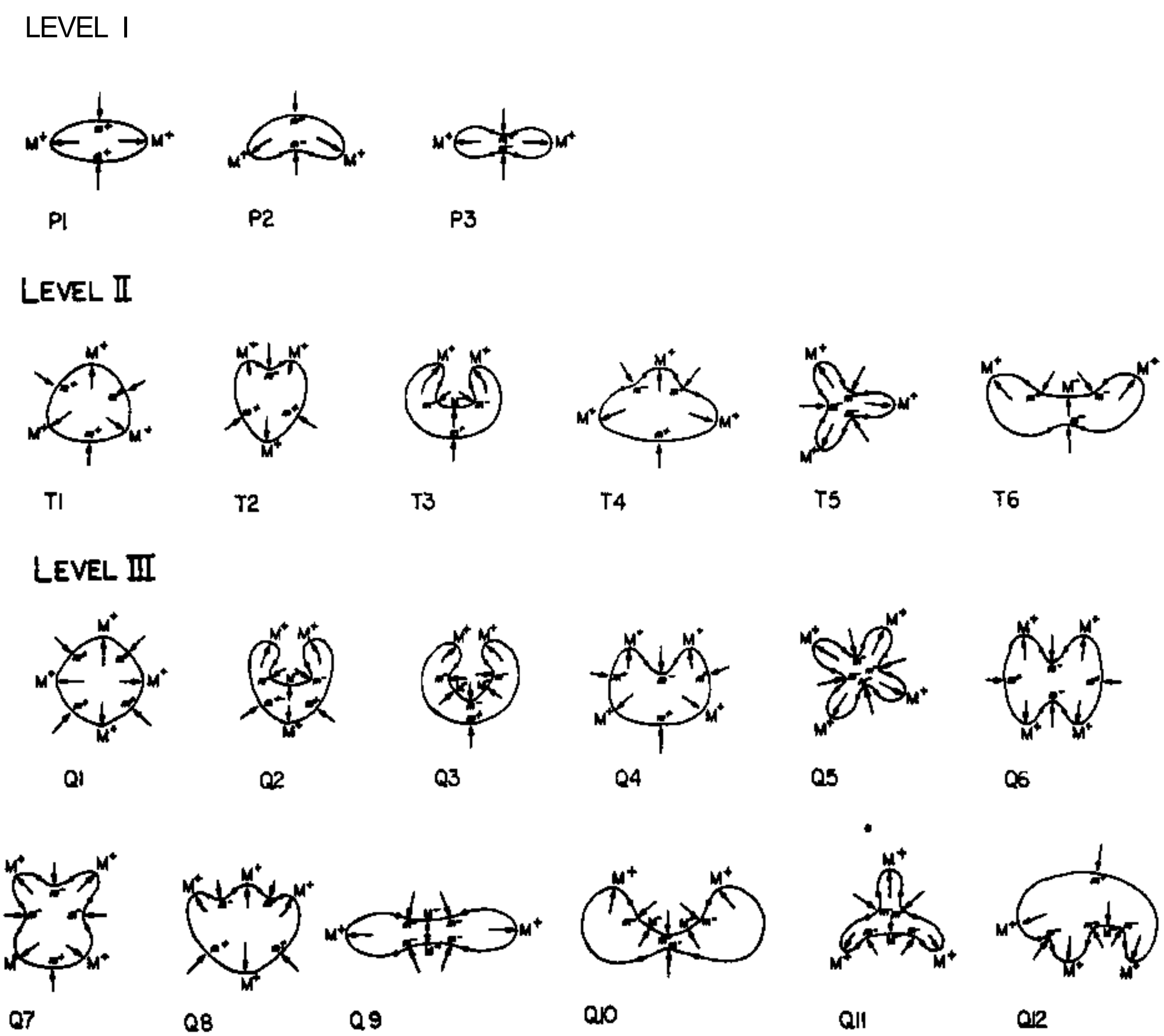

Fig 2. The inference rules applied to the shapes of Richards, Koenderink and Hoffman, 1985.

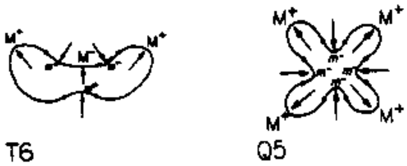

Fig 3. Two shapes to be related by the grammar.

(6) Bifurcation at M (labeled BM) must be given by the re-write rule

$$BM^- : M^- \rightarrow M^- m^- M^-$$

and is simply the introduction of an indentation. An illustration is the transition from T3 to Q3 (in Fig 2>, i*. an extra indentation has been introduced in the bottom of the lagoon in Q3.

IV. THE POWER OF THE GRAMMAR

The first remarkable thing to emerge from the above analysis is that a grammar of only six operations generates all process extrapolations; and, furthermore, that the grammar successfully encodes extrapolations purely in terms of re-write rules on extrema.

Let us illustrate also the intuitive power of the grammar to yield process-explanations. Let us choose two arbitrary shapes, for example, the pair shown in Fig 3. The assumption is that the two shapes are two stages in the development of the same object; e.g. a tumor, cloud, island, embryo, etc. We now show that the process-grammar gives a compelling account of the intervening development.

Using a blurring heuristic, described in Ley ton (1987c), one can identify the intervening succession of *shape-outlines.* Let us suppose that the succession is T6 -» T5 -»Q7 -» Q5, as shown in Fig 4. However, a succession of outlines is not a process-explanation. One requires the *grammar* to provide the intervening process-history. The grammar does this by the successive transformation of process-diagrams. In particular, for Fig 4, the grammar generates the successive process-structures by the sequence of operations, CM^- • BM^+ • Cm \ The operation-sequence is the *process explanation* for the intervening development The explanation can be expressed as follows:

(1) One particular process turns out to be crucial to the entire development. It is the internal process represented by the bold upward arrow in the first shape of Fig 4 (the arrow terminating at *M*).

(2) This continues upward and creates the protrusion in the second shape of Fig 4.

(3) This same process then bifurcates, creating the lobe in the third shape of Fig 4, where a downward squashing process has also been introduced from above.

(4) The new squashing process continues, creating the top indentation shown in the fourth shape of Fig 4.

As the reader can see, the grammar, although derived formally, provides an intuitively powerful process explanation.

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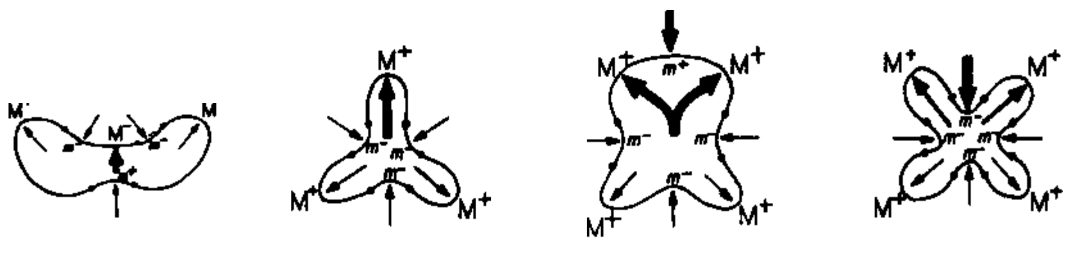

T6 T5 Q7 Q5

Fig 4. The history intervening between the two shapes in Fig 3.