# Performance of OFDM AF Relaying System with Subcarrier Mapping in the Presence of Phase Noise

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***Abstract*** —In this article we investigate the performance of Orthogonal Frequency Division Multiplexing (OFDM)-based dual-hop Amplify-and-Forward (AF) relay networks in the presence of Phase Noise (PN). A scenario with Rayleigh fading statistics on both hops is assumed. We prove the intercarrier interference is Gaussian random variable firstly, which is recognized by everyone but has not been proved, and obtain the form of Signal-to-Noise Ratios (SNR). Then we investigate the end-to-end SNR Cumulative Distribution Function (CDF) and probability density functions (PDF) with ordered subcarrier mapping schemes, i.e., Best-to-Worst (BTW) subcarrier mapping (SCM) and Best-to-Best (BTB) SCM, and get the accurate Closed-Form expressions for CDF and PDF of the OFDM AF system with PN in Rayleigh fading channel. Using the expressions above, we get outage probability. Closed-form asymptotic expressions of outage probability are then derived. With the help of end-to-end SNR PDF we have the Bit Error Rate (BER), closed-form asymptotic expressions of BER show that the BER of system in the presence of PN can not exceed a fixed level even when SNR is in high regime. Finally, we simulate the results and verify their accuracy. Conclusion analysis will provide a useful help in future application of the system.

***Index Terms***—Amplify-and-forward, OFDM, SCM, Phase noise

## I. Introduction

With the ability to increase coverage area and capacity of wireless communication, relay system is regarded as a promising solution to major problems in future wireless communication. The relay terminal has two main relaying methods: Amplify-and-Forward (AF) and Decode-and-Forward (DF). AF relay only retransmits and amplifies version of its received signals, which leads to lower-complexity relay transceivers and lower power consumption compared with DF as is shown in [1]. About single carrier relaying system, [2] already examined its performances in different channel conditions, as well as for different relaying strategies. Lately, multicarrier relaying system using Orthogonal Frequency Division Multiplexing (OFDM) is considered to be an attractive transmission technique.

Since [3] put forward the method that combines subcarrier mapping (SCM) with OFDM AF relaying system to improve efficiency, many literatures have discussed on implementation of the method. Reference [4] presented ergodic capacity comparison for different mapping schemes and relaying system. Reference [5] investigated capacity maximization subject to total power constraint; it shown that the proposed scheme achieves maximum capacity for various positions of relay in a dual-hop system. Reference [6] discuss Bit Error Rate (BER) minimization problems with total power and individual power constraints respectively. It transformed subcarrier pairing into a linear assignment problem and apply Jonker-Volgenant (JV) algorithm to deal with it. Also there are some performance analysis literatures about the OFDM AF relaying system with SCM, nevertheless, most assume in ideal channel, just like [7]-[10]. Surely ideal channel didn’t exist in reality, but the research methods can be referenced in studying imperfect systems. Reference [7] evaluated statistical performance for two-hop OFDM relay link with AF and derived closed-form expressions for the Probability Density Function (PDF) of the Signal-to-Noise Ratio (SNR) of the mapped subcarrier pair link. Reference [8] investigated the BER performance of the Best-to-Worst (BTW) SCM and the Best-to-Best (BTB) SCM schemes for the dual-hop OFDM AF relaying system. Closed-form BER expressions were obtained for DPSK-modulated OFDM AF relaying in both BTW and BTB SCM scenarios. Reference [9] analytically examined BER performance of DPSK OFDM AF with Fixed Gain (FG) relay systems with both BTB and BTW SCM. Reference [10] presented the analysis of BER and ergodic capacity for dual-hop OFDM AF relay system with SCM in Nakagami-m fading channel. Reference [11] introduced outage probability expressions for OFDM AF system with SCM under ideal condition. As we can see, system performances analysis in ideal condition is relatively complete.

For non-ideal case, there are some references about the system. Considering the Carrier Frequency Offset (CFO) in actual system, [12] derived generic closed-form expressions for outage probability and average Symbol Error Rate (SER). Also [13] analyzed outage and the average BER performance of the OFDM based full duplex Cognitive Radio (CR) relay network in the presence of Narrowband Interference (NBI).

In OFDM AF system with SCM, a small phase drift at the output of the local oscillator at each node can significantly limit the overall system BER and outage probability performance due to loss of orthogonality

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among subcarriers, but there is little investigation for it so far. Although [14] took PN into account in OFDM AF relaying system, characterized the performance of outage probability and derived an upper bound on the allowable PN level, still it has limitations such as the system doesn't merge SCM and its performance analysis only involved outage probability.

So in this paper, we investigate the OFDM AF system with SCM in the presence of PN. Firstly we prove that intercarrier interference is Gaussian random variable and investigate SNR of this system. Then end-to-end SNR Cumulative Distribution Function (CDF) and PDF with ordered subcarrier mapping schemes of OFDM AF system with PN in Rayleigh fading channel are analyzed. With the help of CDF we get the form of outage probability. Closed-form asymptotic expressions of outage probability are then obtained. BER expressions are derived later using PDF which show that the BER of system in the presence of PN can’t exceed a fixed level even when SNR in high regime. The analytical outage probability and BER expressions are verified by numerical simulations.

The rest of the paper is organized as follows. In Section 2 the system model is established and the characters of intercarrier interference are analyzed in Section 3. SNR and end-to-end SNR CDF and PDF expressions are derived in section 4. The performance analysis of outage provability and BER in the presence of PN is presented in Section 5. Numerical results and discussion are given in Section 6. Section 7 concludes the article.

## II. System Model

In this section, we introduce the system model with PN firstly and then the PN model is presented.

### A. System Model with PN

We assume a simple model of OFDM AF Dual-hop system shown in Fig. 1. The system is constituted by three single antennas represent source terminal $S$, half-duplex relay terminal $R$ and destination $D$ respectively.

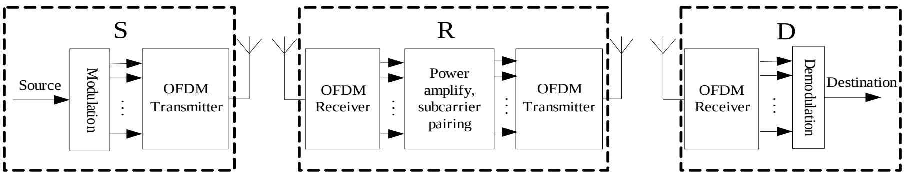


Fig. 1. Block diagram of OFDM AF relaying system with subcarrier mapping

Assuming that the distance between $S$ and $D$ is too far to transmit signals, so the signal transmission only exists in transmission line through relay. In dual-hop system the signals at source terminal are sent to relay after OFDM modulation during first hop. The received signals at relay node $\boldsymbol{Y}_R$ in frequency domain are represented as:

$$\boldsymbol{Y}_R = \boldsymbol{A}_R \otimes (\boldsymbol{H}_{SR}\boldsymbol{X}) + \boldsymbol{N}_R \tag{1}$$

where $\boldsymbol{X} = \left[X_0, \dots X_{N-1}\right]^T$ is the transmitting vector which is independent on all subcarriers, $(0 \cdots N-1)$ is the number of subcarriers, and $E\left\{\left|X_k\right|^2\right\} = E_s$, where $E\{\bullet\}$ denotes the expectation operator, $\boldsymbol{H}_{SR}$ is channel frequency response matrix between source terminal $S$ and relay terminal $R$, $\boldsymbol{A}_R$ is PN at $R$, $N_R$ is additive white Gaussian noise (AWGN) at relay, samples with $N_R \sim CN\left(0, \sigma_R^2\right)$, $\otimes$ denotes the convolution operator.

From (1), the form of $k$-th subcarrier signal, received at the relay node can be derived as:

$$Y_{R,k} = A_{R,0} H_{SR,k} X_k + \sum_{l=1}^{N-1} A_{R,l} H_{SR,k-l} X_{k-l} + N_{R,k} \tag{2}$$

Rewritten the second item on the formula as $ICI_{R,k} = \sum_{l=1}^{N-1} A_{R,l} H_{SR,k-l} X_{k-l}$, obviously $ICI_{R,k}$ is intercarrier interference (ICI) at relay node.

After the signals arrive at $R$, they will be amplified and transmitted to destination during the second hop. Assuming amplification factor is $G$, the signal received at destination can be expressed as:

$$\boldsymbol{Y}_D = \boldsymbol{A}_D \otimes (\boldsymbol{H}_{RD} G \boldsymbol{Y}_R) + \boldsymbol{N}_D \tag{3}$$

where $\boldsymbol{A}_D$ is PN at destination, $\boldsymbol{H}_{RD}$ is channel frequency response matrix between relay terminal $R$ and destination terminal $D$, $E\{|Y_R|^2\} = E_R$, $\boldsymbol{N}_D$ is AWGN, samples with $N_D \sim CN\left(0, \sigma_D^2\right)$.

We assume the channels frequency responses $H_{ij,k}, i, j \in \{S, R, D\}, (k = 0, \dots, N-1)$ are independent on all subcarriers with the average subcarrier symbol power $E\left\{\left|H_{ij,k}\right|^2\right\} = \sigma_{H_{ij}}^2$. Let relay station maps $k$ th subcarrier of first hop to $m$ th subcarrier of second hop, we have post-DFT signal on the $k$ th subcarrier received at $D$ as

$$\begin{aligned} Y_{D,m} &= GA_{D,0} H_{RD,m} Y_{R,k} + G\sum_{l=1}^{N-1} A_{D,l} H_{RD,m-l} Y_{R,k-l} + N_{D,m} \\ &= GA_{D,0} A_{R,0} H_{SR,k} H_{RD,m} X_k + GA_{D,0} H_{RD,m} N_{R,k} \\ &\quad + ICI_{1,m} + ICI_{2,m} + ICI_{3,m} + N_{D,m} \end{aligned} \tag{4}$$

where

$$\begin{aligned} ICI_{1,m} &= GA_{R,0} \sum_{l=1}^{N-1} A_{D,l} H_{RD,m-l} H_{SR,k-l} X_{k-l} \\ ICI_{2,m} &= G\sum_{l=0}^{N-1} A_{D,l} H_{RD,m-l} ICI_{R,k-l} \\ ICI_{3,m} &= G\sum_{l=1}^{N-1} A_{D,l} H_{RD,m-l} N_{R,k-l} \end{aligned} \tag{5}$$

### *B. Phase Noise Model*

There are many models of phase noise. Here we choose a commonly used model, Wiener model, whose accuracy is verified in many literatures. Modeling PN as Wiener process has mathematical trackability, and is suitable for evaluating the impact of PN on system performance by analytical methods. This paper only considers the phase noise at relay and destination. PN at receiver's (i.e., relay or destination) local oscillator introduces a random phase rotation of $e^{j\theta_k(n)}$ to the received signal in time domain. PN at sample instant $n$ is given by

$$\theta_k(n) = \theta_k(n+1) + \varepsilon \tag{6}$$

where $\varepsilon$ is a Gaussian random variable with zero mean and variance $2\pi\beta T_s$, where $T_s$ is the sampling interval and $\beta$ is the 3dB PN bandwidth, $n = 1, ..., N-1$ ,with $N$ is the number of subcarriers.

PN in frequency-domain is presented by

$$A_{t,l} = \frac{1}{N}\sum_{i=0}^{N-1}\exp(j\theta_{N_g+i})\exp(-j\frac{2\pi li}{N}) \qquad t \in \{R, D\} \tag{7}$$

where $N_g$ is the length of guard interval (GI). The existence of PN will mainly bring common phase error (CPE) and ICI, which will lead to the attenuation and rotation of desired signals.

Combine the symbols used in the formulas before and [15], we get PN variance expression as

$$E\left[\left|A_{t,l}\right|^2\right] = \frac{1}{N^2}\left[\frac{\rho_l^{N+1} - (N+1)\rho_l + N}{(\rho_l - 1)^2} - N\right] \tag{8}$$

where $\rho_l = \exp(j(2\pi l/N) - (\pi\beta_t T_s/N))$ . By substituting $l = 0$ in (8) variance of the CPE is obtained. If the variance of the PN is much less then unity (i.e., $\sigma_\theta^2 \approx 0$) which is the case in practice, then variance of the PN term can be simplified by approximating the exponential term with Taylor series and represented as

$$\begin{aligned} E\left[\left|A_{t,0}\right|^2\right] &= 1 - \frac{\pi\beta_t T_s}{3} = C_{t,0} \\ E\left[\sum_{l=1}^{N-1}\left|A_{t,l}\right|^2\right] &= 1 - E\left[\left|A_{t,0}\right|^2\right] = 1 - C_{t,0} \end{aligned} \tag{9}$$

From (9) we can see that when PN does not exist, $C_{t,0}$ equals to 1.

## III. Statistical Analysis of Intercarrier Interference

In this section we calculate the mean and variance of inter-carrier-interference and prove its Gaussianity.

### *A. Mean and Variance of Intercarrier Interference*

To make the analysis more clearly, we rewrite the conclusions obtained above as follows:

Transmitted data $\boldsymbol{X}$ on all subcarriers are independent and $E\left\{\left|X_k\right|^2\right\} = E_s$ . Similarly, the channels frequency responses $H_{ij,k}$ $i, j \in \{S, R, D\}, (k = 0, \ldots, N-1)$ are independent on all subcarriers with $E\left\{\left|H_{ij,k}\right|^2\right\} = \sigma_{H_{ij}}^2$ .

Assuming modulation in OFDM AF relaying system is M-QAM, so $X_k$ is zero mean. $H_{SR,k}$ and $X_k$ are independent of each other. $ICI_{R,k}$ in (2) is zero mean, the variance $\sigma_{ICI_R}^2$ of $ICI_{R,k}$ is defined as

$$\begin{aligned} \sigma_{ICI_R}^2 &= E\left\{\left|ICI_{R,k}\right|^2\right\} = E\left\{\left|\sum_{l=1}^{N-1} A_{R,l} H_{SR,k-l} X_{k-l}\right|^2\right\} \\ &= E_s \sigma_{H_{SR}}^2 \left(1 - C_{R,0}\right) \end{aligned} \tag{10}$$

### *B. Gaussianity of Intercarrier Interference*

In previous articles, intercarrier interference term is modeled as Gaussian random variable with zero mean and variance $\sigma_{ICI_R}^2$ , but there is no proof given so far to validate this assumption. So before analysis the variance, we firstly prove the Gaussianity of ICI. We restate Lyapunov's central limit theorem ([16]) here in a convenient form.

**Lemma** (Lyapunov): If $\Lambda_1, \ldots, \Lambda_N$ are independent random variables each with mean $\mu_i$ variance $\sigma_i^2$ and finite absolute third moment $\eta_i^3$ . And if $\lim_{N\to\infty}\frac{\eta}{\sigma} = 0$ where $\eta = \left(\sum_{i=0}^{N-1}\eta_i^3\right)^{1/3}, \sigma = \left(\sum_{i=0}^{N-1}\sigma_i^2\right)^{1/2}$ . Then $\sum_{i=0}^{N-1}\Lambda_i$ is asymptotical Gaussian.

**Proof** (Gaussianity of intercarrier interference): Take the intercarrier interference at relay, $ICI_{R,k}$ , for example to analysis the Gaussianity of ICI, we rewrite it as follow

$$ICI_{R,k} = \sum_{l=1}^{N-1}\Lambda_l \tag{11}$$

where $\Lambda_l = A_{R,l} H_{SR,k-l} X_{k-l}$ . Thus we only need to evaluate the mean, variance and third absolute moment of $\Lambda_l$ to proceed with the proof. As is mentioned above, the data symbols are independent of the channels and the channels are zero mean complex Gaussian random variables, mean $\mu_{\Lambda_l}$ variance $\sigma_{\Lambda_l}^2$ and finite absolute third moment $\eta_{\Lambda_l}^3$ can be calculated as

$$\begin{aligned} \mu_{\Lambda_l} &= E\left\{A_{R,l} H_{SR,k-l} X_{k-l}\right\} = 0 \\ \sigma_{\Lambda_l}^2 &= E\left\{\left|A_{R,l} H_{SR,k-l} X_{k-l}\right|^2\right\} = E_S \sigma_{H_{SR}}^2 \left|A_{R,l}\right|^2 \\ \eta_{\Lambda_l}^3 &= E\left\{\left|A_{R,l} H_{SR,k-l} X_{k-l}\right|^3\right\} = \left(E_S \sigma_{H_{SR}}^2\right)^{\frac{3}{2}} \left|A_{R,l}\right|^3 \end{aligned} \tag{12}$$

From (12), we can easily calculate that

$$\lim_{N\to\infty}\frac{\left(\sum_{l=1}^{N-1}\left(E_S\sigma_{H_{SR}}^2\right)^{\frac{3}{2}}\left|A_{R,l}\right|^3\right)^{1/3}}{\left(\sum_{l=1}^{N-1}E_S\sigma_{H_{SR}}^2\left|A_{R,l}\right|^2\right)^{1/2}} \le \lim_{N\to\infty}\frac{\left(\max\left\{\left|A_{R,l}\right|^3\right\}\right)^{1/3}}{(N-1)^{1/6}\left(\min\left\{\left|A_{R,l}\right|^2\right\}\right)^{1/2}} = 0 \tag{13}$$

Summing up (12) and (13), $ICI_{R,k}$ meets all the conditions of the lemma, thus $ICI_{R,k}$ is proved to be the Gaussian random variable. Similarly, we can derive $ICI_{1,m}$, $ICI_{2,m}$ and $ICI_{3,m}$ are Gaussian random variables, so they are all zero mean and the variances can be obtained as follows

$$\begin{aligned}
\sigma^2_{ICI_{1,m}} &= E\left\{\left|GA_{R,0}\sum_{l=1}^{N-1}A_{D,l}H_{RD,m-l}H_{SR,k-l}X_{k-l}\right|^2\right\} \\
&= G^2E_SC_{R,0}\left(1-C_{D,0}\right)\sigma^2_{H_{SR}}\sigma^2_{H_{RD}} \\
\sigma^2_{ICI_{2,m}} &= E\left\{\left|G\sum_{l=0}^{N-1}A_{D,l}H_{RD,m-l}Q_{R,k-l}\right|^2\right\} \\
&= G^2E_S\left(1-C_{R,0}\right)\sigma^2_{H_{SR}}\sigma^2_{H_{RD}} \\
\sigma^2_{ICI_{3,m}} &= E\left\{\left|G\sum_{l=1}^{N-1}A_{D,l}H_{RD,m-l}N_{R,k-l}\right|^2\right\} \\
&= G^2\left(1-C_{D,0}\right)\sigma^2_{H_{RD}}\sigma^2_R
\end{aligned} \tag{14}$$

## IV. SNR Expression

### A. The End-To-End SNR Expression

According to the system model, the item $GA_{D,0}A_{R,0}H_{SR,k}H_{RD,m}X_k$ in (4) is the desired signal at destination, while $ICI_{R,k}$ is ICI at relay node, $ICI_{1,m}+ICI_{2,m}+ICI_{3,m}$ is ICI at destination, $GA_{D,0}H_{RD,m}N_{R,k}$ is caused by AWGN at relay, and $N_{D,m}$ is AWGN at destination.

Using (4), the variance of ICI and AWGN at destination can be expressed as:

$$E_d = \left|ICI_{1,m}\right|^2+\left|ICI_{2,m}\right|^2+\left|ICI_{3,m}\right|^2+\left|N_{D,m}\right|^2 \tag{15}$$

We assume that the $S-R$ and $R-D$ channels are independent with Rayleigh fading among the subcarriers, Since the Rayleigh distribution for the dual-hop system means both hops are Gaussian channels and they are orthogonal. Two orthogonal Gaussian channels are independent as a matter of fact. So the SNR of $S-R$ and $R-D$ channels under constraint conditions above are given as

$$\begin{aligned}
\bar{\gamma}_{SR} &= E_S\,E\left\{\left|H_{SR}\right|^2\right\}\Big/\sigma^2_R \\
\bar{\gamma}_{RD} &= E_R\,E\left\{\left|H_{RD}\right|^2\right\}\Big/\sigma^2_D
\end{aligned} \tag{16}$$

where $E_S$ and $E_R$ respectively represent average symbol power per subcarrier, transmitted by node $S$ and $R$ respectively. Thus, the SNR of the system can be written and simplified as:

$$\begin{aligned}
\gamma_{PN} &= \frac{G^2\left|A_{D,0}A_{R,0}H_{SR,k}H_{RD,m}X_k\right|^2}{G^2\left|A_{D,0}H_{RD,m}N_{R,k}\right|^2+E_d} \\
&\approx \frac{\dfrac{G^2C_{D,0}C_{R,0}\left|H_{SR,k}\right|^2\left|H_{RD,m}\right|^2E_sE_R}{G^2\sigma^2_R\sigma^2_D}}{\dfrac{G^2E_RC_{D,0}\left|H_{RD,m}\right|^2\sigma^2_R}{G^2\sigma^2_R\sigma^2_D}+\dfrac{E_dE_R}{G^2\sigma^2_R\sigma^2_D}} \\
&= \frac{C_{D,0}C_{R,0}\gamma_{SR,k}\bullet\gamma_{RD,m}}{C_{D,0}\gamma_{RD,m}+\rho}
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\rho &= \frac{E_RE_d}{G^2\sigma^2_R\sigma^2_D} = \frac{E_R\left[\sigma^2_{ICI_{1,m}}+\sigma^2_{ICI_{2,m}}+\sigma^2_{ICI_{3,m}}+\sigma^2_D\right]}{G^2\sigma^2_R\sigma^2_D} \\
&= (1-C_{D,0}C_{R,0})\bar{\gamma}_{SR}\bullet\bar{\gamma}_{RD}+(1-C_{D,0})\bar{\gamma}_{RD}+\frac{E_R}{G^2\sigma^2_R}
\end{aligned} \tag{18}$$

### B. The End-To-End SNR CDF and PDF

The process of signal transmission in OFDM AF system is divided into two hops, during the first hop, signals transmit in $k$ th subcarrier of $S-R$ pathway. From [8] we can get the PDF and CDF of the SNR in each $S-R$ subchannel as $f_{SR}(x)=\lambda_{SR}\exp(-\lambda_{SR}x)$ and $F_{SR}(x)=1-\exp(-\lambda_{SR}x)$, where $\lambda_{SR}=1/\bar{\gamma}_{SR}$. During the second hop, signals are sent by the relay and transmit in $m$ th sub carrier of $R-D$ pathway. Similarly the PDF and CDF of the SNR in each $R-D$ subchannel are $f_{RD}(x)=\lambda_{RD}\exp(-\lambda_{RD}x)$ and $F_{RD}(x)=1-\exp(-\lambda_{RD}x)$, where $\lambda_{RD}=1/\bar{\gamma}_{RD}$.

We classify the signal transmission process into two cases. In the BTW SCM scheme, the strongest subcarrier of the first hop mapping to the weakest subcarrier of the second hop etc. Let $f^w_{k,SR}(x)$ denotes the PDF of the SNR of the $k$ th weakest subcarrier among all subcarriers in $S-R$ link, $f^w_{k,SR}(x)$ can be expressed by ([20]: Eq. (15))

$$\begin{aligned}
f^w_{k,SR}(x) &= N\binom{N-1}{k-1}f_{SR}(x)\left(F_{SR}(x)\right)^{k-1}\left(1-F_{SR}(x)\right)^{N-k} \\
&= \sum_{i=0}^{k-1}\lambda_{SR}\alpha_i\exp\left(-\beta_i\lambda_{SR}x\right)
\end{aligned} \tag{19}$$

where $\alpha_i$ and $\beta_i$ are given as

$$\begin{aligned}
\alpha_i &= (-1)^i\,N\binom{N-1}{k-1}\binom{k-1}{i} \\
\beta_i &= i+N-k+1
\end{aligned} \tag{20}$$

In the BTB SCM scheme, the strongest subcarrier of the first hop mapping to the strongest subcarrier of the second hop etc. The PDF of the $k$ th strongest subcarrier in the $R-D$ link can be written as ([20] Eq. (11))

$$\begin{aligned}
f^s_{k,RD}(x) &= N\binom{N-1}{k-1}f_{RD}(x)\left(F_{RD}(x)\right)^{N-k}\left(1-F_{RD}(x)\right)^{k-1} \\
&= \sum_{i=0}^{k-1}\lambda_{RD}p_i\exp\left(-q_i\lambda_{RD}x\right)
\end{aligned} \tag{21}$$

where $p_i$ and $q_i$ are

$$\begin{aligned}
p_i &= (-1)^i\,N\binom{N-1}{k-1}\binom{k-1}{i} \\
q_i &= i+N-k+1
\end{aligned} \tag{22}$$

Since we already get the expression of $\gamma_{PN}$ as (17), we know that the end-to-end CDF of SNR can be expressed as

$$F_{\gamma_k}(x) = \mathrm{P}\left(\gamma_{PN} < x\right) = P\left(\frac{C_{D,0}C_{R,0}\gamma_{SR,k}\bullet\gamma_{RD,m}}{C_{D,0}\gamma_{RD,m}+\rho} < x\right) \quad (23)$$

*C. BTW SCM Scheme*

In BTW SCM scheme, (23) can be rewritten as

$$F_{\gamma_k}^{BTW}(x) = \int_0^\infty P\left(\frac{C_{D,0}C_{R,0}\gamma_{SR,k}\bullet\gamma_{RD,m}}{C_{D,0}\gamma_{RD,m}+\rho} < x \middle| \gamma_{RD,m}\right) f_{k,RD}^{s}\left(\gamma_{RD,m}\right) d\gamma_{RD,m} \quad (24)$$

Based on the analysis of appendix A closed-form expression for CDF of SNR in BTW SCM scheme can be derived as (25) with the help of $\lambda_{SR} = 1/\bar{\gamma}_{SR}$ $\lambda_{RD} = 1/\bar{\gamma}_{RD}$, written at next page, where $K_1(\bullet)$ is first order modified Bessel function of the second kind.

$$F_{\gamma_k}^{BTW}(x) = 1 - \sum_{i=0}^{N-k}\frac{p_i}{q_i}\exp\left(-\frac{q_i x}{\bar{\gamma}_{RD}C_{D,0}}\right) + \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\bar{\gamma}_{RD}\beta_j}p_i\exp\left(-\frac{q_i x}{\bar{\gamma}_{RD}C_{D,0}}\right)\left[\frac{\bar{\gamma}_{RD}C_{D,0}}{q_i} - 2\sqrt{\frac{\rho\beta_j\bar{\gamma}_{RD}C_{D,0}x}{q_i C_{R,0}\bar{\gamma}_{SR}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right] \quad (25)$$

$$f_{\gamma_k}^{BTW}(x) = \frac{2}{C_{R,0}\bar{\gamma}_{SR}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\alpha_j p_i\exp\left(-\frac{\beta_j x}{C_{R,0}\bar{\gamma}_{RS}}\right)\left[\sqrt{\frac{\rho\beta_j x}{q_i C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) + \frac{\rho}{C_{D,0}\bar{\gamma}_{RD}}K_0\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right] \quad (26)$$

$$F_{\gamma_k}^{BTB}(x) = 1 - \sum_{i=0}^{N-k}\frac{p_i}{q_i}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right) + \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{p_j}{q_j}p_i\lambda_{RD}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right)\left[\frac{C_{D,0}}{q_i\lambda_{RD}} - 2\sqrt{\frac{\rho q_j\lambda_{SR}C_{D,0}x}{q_i C_{R,0}\lambda_{RD}}}K_1\left(2\sqrt{\frac{\rho q_j q_i\lambda_{RD}\lambda_{SR}x}{C_{R,0}C_{D,0}}}\right)\right] \quad (27)$$

$$f_{\gamma_k}^{BTB}(x) = \frac{2}{C_{R,0}\bar{\gamma}_{SR}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}p_j p_i\exp\left(-\frac{q_j x}{C_{R,0}\bar{\gamma}_{SR}}\right)\left[\sqrt{\frac{\rho q_j x}{q_i C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{\rho q_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) + \frac{\rho}{C_{D,0}\bar{\gamma}_{RD}}K_0\left(2\sqrt{\frac{\rho q_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right] \quad (28)$$

Using (25), we get the end-to-end PDF of SNR in BTW SCM scheme through derivations, shown in appendix B, as (26), where $K_1(\bullet)$ and $K_0(\bullet)$ are zero and first order modified Bessel functions of the second kind.

*D. BTB SCM Scheme*

Under BTB SCM condition we have end-to-end CDF expression as (27) through the same analysis method seen in appendix A.

Similarly, we have the end-to-end PDF of SNR in BTB SCM scheme as (28) according to appendix B.

## V. Performance Analysis

*A. Outage Probability*

Outage probability is another performance criterion of OFDM AF system operating over fading channels, which is defined as the probability that the instantaneous equivalent SNR, $\gamma_{PN}$, falls below a certain specified threshold $\gamma_{th}$, put it another way, it is equivalent to the end-to-end CDF of SNR when $x = \gamma_{th}$. Using $P_{out}$ represents outage probability, (29) is reduced on the foundation of (23).

$$P_{out} = \mathrm{P}\left(\gamma_{PN} < \gamma_{th}\right) = P\left(\frac{C_{D,0}C_{R,0}\gamma_{SR,k}\bullet\gamma_{RD,m}}{C_{D,0}\gamma_{RD,m}+\rho} < \gamma_{th}\right) \quad (29)$$

*B. Outage Probability for BTW SCM*

Based on the conclusions of 4.2 and relationship between outage probability and CDF, closed-form expression for outage probability of BTW SCM scheme can be written as (30).

$$P_{out}^{BTW} = 1 - \sum_{i=0}^{N-k}\frac{p_i}{q_i}\exp\left(-\frac{q_i\gamma_{th}}{\bar{\gamma}_{RD}C_{D,0}}\right) + \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\bar{\gamma}_{RD}\beta_j}p_i\exp\left(-\frac{q_i\gamma_{th}}{\bar{\gamma}_{RD}C_{D,0}}\right)\left[\frac{\bar{\gamma}_{RD}C_{D,0}}{q_i} - 2\sqrt{\frac{\rho\beta_j\bar{\gamma}_{RD}C_{D,0}\gamma_{th}}{q_i C_{R,0}\bar{\gamma}_{SR}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i\gamma_{th}}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right] \quad (30)$$

$$EP_{out}^{BTW} = 1 - \sum_{i=0}^{N-k}\frac{p_i}{q_i} + \sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\beta_j}p_i\left[\frac{1}{q_i} - 2\sqrt{\frac{(1-C_{D,0}C_{R,0})\beta_j\gamma_{th}}{q_i C_{D,0}C_{R,0}}}K_1\left(2\sqrt{\frac{(1-C_{D,0}C_{R,0})\beta_j q_i\gamma_{th}}{C_{R,0}C_{D,0}}}\right)\right] \quad (31)$$

$$P_{out}^{BTB} = 1 - \sum_{i=0}^{N-k}\frac{p_i}{q_i}\exp\left(-\frac{q_i\gamma_{th}}{\bar{\gamma}_{RD}C_{D,0}}\right) + \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{p_j}{\bar{\gamma}_{RD}q_j}p_i\exp\left(-\frac{q_i\gamma_{th}}{\bar{\gamma}_{RD}C_{D,0}}\right)\left[\frac{\bar{\gamma}_{RD}C_{D,0}}{q_i} - 2\sqrt{\frac{\rho q_j\bar{\gamma}_{RD}C_{D,0}\gamma_{th}}{q_i C_{R,0}\bar{\gamma}_{SR}}}K_1\left(2\sqrt{\frac{\rho q_j q_i\gamma_{th}}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right] \quad (32)$$

$$EP_{out}^{BTW} = 1 - \sum_{i=0}^{N-k}\frac{p_i}{q_i} + \sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{p_j}{q_j}p_i\left[\frac{1}{q_i} - 2\sqrt{\frac{(1-C_{D,0}C_{R,0})q_j\gamma_{th}}{q_i C_{D,0}C_{R,0}}}K_1\left(2\sqrt{\frac{(1-C_{D,0}C_{R,0})q_j q_i\gamma_{th}}{C_{R,0}C_{D,0}}}\right)\right] \quad (33)$$

Analyzing (30), we obtain that when in presence of PN, as $\bar{\gamma}_{SR} \to \infty$ and $\bar{\gamma}_{RD} \to \infty$, $P_{out}^{BTW} \to error\_\_floor$, which means an error floor is expected at high SNR regime due to PN. In order to derive the error floor at high SNR, we will simplify (30). Since the situation that $\bar{\gamma}_{SR} \to \infty$ and $\bar{\gamma}_{RD} \to \infty$ only exists in ideal state, we adopt an asymptotic analysis for the system performance. Considering that $\bar{\gamma}_{SR} \to \infty$ and $\bar{\gamma}_{RD} \to \infty$, then

$\exp\left(-q_i\gamma_{th}/(\bar{\gamma}_{RD}C_{D,0})\right)\to 1$, combine with (18) we have $\frac{\rho}{\bar{\gamma}_{SR}\bar{\gamma}_{RD}}=(1-C_{D,0}C_{R,0})+\frac{(1-C_{D,0})}{\bar{\gamma}_{SR}}+\frac{E_R}{G^2\sigma_R^2\bar{\gamma}_{SR}\bar{\gamma}_{RD}}\to(1-C_{D,0}C_{R,0})$. The expression of the error floor in BTW SCM scheme is simplified as (31), from which we can see that at high SNR, the influence of $C_{R,0}$ and $C_{D,0}$ on the error floor are same, i.e., when ICI at relay and destination change in the same ratio, outage probability will also changes in the same ratio.

### C. Outage Probability for BTB SCM

Using the analysis method above, outage probability expression of BTB SCM is presented as (32). And the expression of the error floor in BTB SCM scheme can be obtained similarly as (33).

### D. BER

In this section, we evaluate the BER of the OFDM AF system in presence of PN and derive closed-form expressions of the BER in two cases respectively.

### E. Average BER for BTW SCM

To obtain a closed-form approximate BER expression for M-QAM modulation, we choose a PDF-based approach. Using the theories in 4.2, the average BER for k-th subcarrier permutation in BTW scheme is given as (34), shown at next page. Let $P_{b|\gamma_k}$ represents BER for M-QAM modulation, from [8] it is generally modeled as $P_{b|\gamma}=\sum_{l=1}^{M}A_lQ\left(\sqrt{B_l\gamma}\right)$, where $A_l$ and $B_l$ are modulation dependent constants [18], $Q(\bullet)$ is Gaussian $Q$ function which can be expressed in terms of complementary error function $\mathrm{erfc}(\bullet)$ as $Q(x)=0.5\mathrm{erfc}\left(x/\sqrt{2}\right)$. The function $\mathrm{erfc}(\bullet)$ can be approximate expressed as $\mathrm{erfc}(x)\approx\frac{1}{6}\exp\left(-x^2\right)+\frac{1}{2}\exp\left(-\frac{4}{3}x^2\right)$. Using above results, (34) can be rewritten as (35).

$$P_{b,k}^{BTW}=\int_0^\infty P_{b|\gamma_k}f_{\gamma_{PN}}^{BTW}(\gamma)d\gamma \tag{34}$$

Combine (26), equations (6.614.4), (6.631.3) in [19] with equations (13.1.33), (13.6.28), (13.6.30), and (6.5.19) in [18], a closed-from integral in (35) can be drawn and BER expression is derived using more common exponential integral function as (36), where $T_{l,j}(1/2)=B_l/2+\beta_j/\bar{\gamma}_{SR}$, $T_{l,j}(2/3)=2B_l/3+\beta_j/\bar{\gamma}_{SR}$, $M_{j,i}=\beta_jq_i/(\bar{\gamma}_{RD}\bar{\gamma}_{SR})$ and $E_1(\bullet)$ is the exponential integral function defined in [18] (Eq. (5.1.1)). For OFDM AF system the total average BER is obtained through $P_b^{BTW}=(1/N)\sum_{k=1}^{N}P_{b,k}^{BTW}$.

Taking (36) into consider, we conclude that once there is no PN, $BER\to 0$ when $\bar{\gamma}_{SR}\to\infty$ and $\bar{\gamma}_{RD}\to\infty$, while in presence of PN, $BER\to error\_\_floor$. In order to derive error floor at high SNR regime, we will simplify (36). Same as the analysis for outage probability we suppose $\bar{\gamma}_{SR}\to\infty$ and $\bar{\gamma}_{RD}\to\infty$, then $T_{l,j}(1/2)\approx B_l/2$, $T_{l,j}(2/3)\approx 2B_l/3$, so using the hypothesis above, we get (37).

The expression of the error floor for M-QAM modulation in BTW SCM scheme can be simplified as (38), where $\Delta=(1-C_{R,0}C_{D,0})/(C_{R,0}C_{D,0})$. $\Delta$ indicates that at high SNR, the influences of $C_{R,0}$ and $C_{D,0}$ on the error floor are same, i.e., when ICI at relay and destination change in same ratio, BER will also changes in the same ratio.

$$P_{b,k}^{BTW}=\frac{1}{2}\sum_{l=1}^{M}\left[\frac{1}{6}\int_0^\infty A_l\exp\left(-\frac{B_l}{2}\gamma\right)f_{\gamma_k}^{BTW}(\gamma)d\gamma+\frac{1}{2}\int_0^\infty A_l\exp\left(-\frac{2B_l}{3}\gamma\right)f_{\gamma_k}^{BTW}(\gamma)d\gamma\right] \tag{35}$$

$$\begin{aligned}P_{b,k}^{BTW}=\frac{1}{2C_{R,0}\bar{\gamma}_{SR}}\sum_{l=1}^{M}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}A_l\alpha_jp_i&\left[\frac{1}{2q_i}\left(\frac{1}{T_{l,j}(1/2)}+\frac{1}{T_{l,j}(2/3)}\right)+\frac{\exp\left(\frac{\rho M_{j,i}}{C_{R,0}C_{D,0}T_{l,j}(1/2)}\right)}{6T_{l,j}(1/2)}E_1\left(\frac{\rho M_{j,i}}{C_{R,0}C_{D,0}T_{l,j}(1/2)}\right)\times\right.\\&\left.\left(\frac{\rho}{C_{D,0}\bar{\gamma}_{RD}}-\frac{\rho M_{j,i}}{2q_iC_{R,0}C_{D,0}T_{l,j}(1/2)}\right)+\frac{\exp\left(\frac{\rho M_{j,i}}{C_{R,0}C_{D,0}T_{l,j}(2/3)}\right)}{2T_{l,j}(2/3)}E_1\left(\frac{\rho M_{j,i}}{C_{R,0}C_{D,0}T_{l,j}(2/3)}\right)\left(\frac{\rho}{C_{D,0}\bar{\gamma}_{RD}}-\frac{\rho M_{j,i}}{2q_iC_{R,0}C_{D,0}T_{l,j}(2/3)}\right)\right]\end{aligned} \tag{36}$$

$$\rho M_{j,i}=\beta_jq_i\left(1-C_{R,0}C_{D,0}\right)+\beta_jq_i\left(1-C_{R,0}\right)/\bar{\gamma}_{SR}+E_R\,\beta_jq_i/\sigma_R^2G^2\bar{\gamma}_{SR}\bar{\gamma}_{RD}\approx\beta_jq_i\left[1-C_{R,0}C_{D,0}\right] \tag{37}$$

$$EP_{b,k}^{BTW}=\frac{1}{2C_{R,0}\bar{\gamma}_{SR}}\sum_{l=1}^{M}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}A_l\alpha_jp_i\left[\frac{7}{4q_iB_l}+\frac{\exp\left(\frac{2\beta_jq_i\Delta}{B_l}\right)}{3B_l}E_1\left(\frac{2\beta_jq_i\Delta}{B_l}\right)\left(-\frac{\beta_j\Delta}{B_l}\right)+\frac{3\exp\left(\frac{3\beta_jq_i\Delta}{2B_l}\right)}{4B_l}E_1\left(\frac{3\beta_jq_i\Delta}{2B_l}\right)\left(-\frac{3\beta_j\Delta}{4B_l}\right)\right] \tag{38}$$

$$P_{b,k}^{BTB}=\frac{1}{2C_{R,0}\bar{\gamma}_{SR}}\sum_{l=1}^{M}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}A_l p_j q_i\left[\frac{1}{2q_i}\left(\frac{1}{3T_{l,j}(1/2)}+\frac{1}{T_{l,j}(2/3)}\right)+\frac{\exp\left(\frac{\rho Z_{j,i}}{C_{R,0}C_{D,0}T_{l,j}(1/2)}\right)}{6T_{l,j}(1/2)}E_1\left(\frac{\rho Z_{j,i}}{C_{R,0}C_{D,0}T_{l,j}(1/2)}\right)\times\right.$$
$$\left.\left(\frac{\rho}{C_{D,0}\bar{\gamma}_{RD}}-\frac{\rho Z_{j,i}}{2q_iC_{R,0}C_{D,0}T_{l,j}(1/2)}\right)+\frac{\exp\left(\frac{\rho Z_{j,i}}{C_{R,0}C_{D,0}T_{l,j}(2/3)}\right)}{2T_{l,j}(2/3)}E_1\left(\frac{\rho Z_{j,i}}{C_{R,0}C_{D,0}T_{l,j}(2/3)}\right)\left(\frac{\rho}{C_{D,0}\bar{\gamma}_{RD}}-\frac{\rho Z_{j,i}}{2q_iC_{R,0}C_{D,0}T_{l,j}(2/3)}\right)\right] \tag{39}$$

$$EP_{b,k}^{BTB}=\frac{1}{2C_{R,0}\bar{\gamma}_{SR}}\sum_{l=1}^{M}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}A_l p_j p_i\left[\frac{7}{4q_iq_l}+\frac{\exp\left(\frac{2q_jq_i\Delta}{B_l}\right)}{3B_l}E_1\left(\frac{2q_jq_i\Delta}{B_l}\right)\left(-\frac{q_j\Delta}{B_l}\right)+\frac{3\exp\left(\frac{3q_jq_i\Delta}{2B_l}\right)}{4B_l}E_1\left(\frac{3q_jq_i\Delta}{2B_l}\right)\left(-\frac{3q_j\Delta}{4B_l}\right)\right] \tag{40}$$

$$G^2=E\left\{\frac{E_R}{E_s\left|H_{SR,k}\right|^2+\sigma_R^2}\right\} \tag{41}$$

### F. Average BER for BTB SCM

In the similar way, using (28) and (35), the average BER of the $k$-th subcarrier permutation in BTB SCM scheme is given as (39), where $Z_{j,i}=q_jq_i/(\bar{\gamma}_{SR}\bar{\gamma}_{RD})$. In OFDM AF system the total average BER of the BTB SCM scheme is obtained through $P_b^{BTB}=(1/N)\sum_{k=1}^{N}P_{b,k}^{BTB}$. Following the same steps above, an error floor in the BTB SCM is expressed as (40).

## VI. Numerical Result

This section presented analytical and simulation results when the OFDM AF relaying system with SCM is in the presence of PN in Rayleigh fading channels. Although the derived outage probability and BER expressions can compute arbitrary value of $\bar{\gamma}_{SR}$ and $\bar{\gamma}_{RD}$, we limit the discussion to the balance link case ($E_R=E_S,\sigma_R^2=\sigma_D^2,\bar{\gamma}_{SR}=\bar{\gamma}_{RD}$). Let the OFDM system has $N=16$ subcarriers and the relay Gain $G$ is calculated as rage fading power on $S-R$ link. We use $\beta$ to represent PN, $\beta$ is 3dB PN bandwidth. In frequency domain waveform of PN, we abandon the part whose value is less than $1/\sqrt{2}$, that is the part whose energy is not enough to affect the useful signals. Only the part whose value is more than $1/\sqrt{2}$ is useful, that is the 3dB PN bandwidth. Then the simulation figures are as follows.

In Fig. 2 and Fig. 3 we present the outage probability of OFDM AF relay system for BTW and BTB SCM with different phase noises, an excellent match between analytical results and simulation results can easily be seen.

Fig. 2 depicts that outage probability will keep a fixed level even when SNR is sufficiently large because of the presence of PN; error floor of outage probability is approximately $5\times10^{-4}$ when PN linewidthes at $R$ and $D$ are 1Hz whereas $6\times10^{-2}$ when 10Hz. Combining the two cases above, it shows that the outage probability performance is very sensitive to PN.

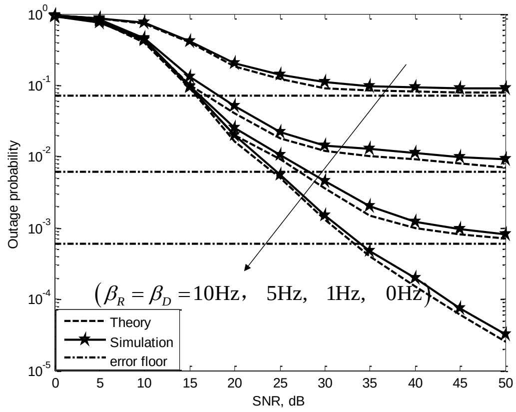


Fig. 2. Outage probability of OFDM AF relay system for BTW SCM with different Phase noise

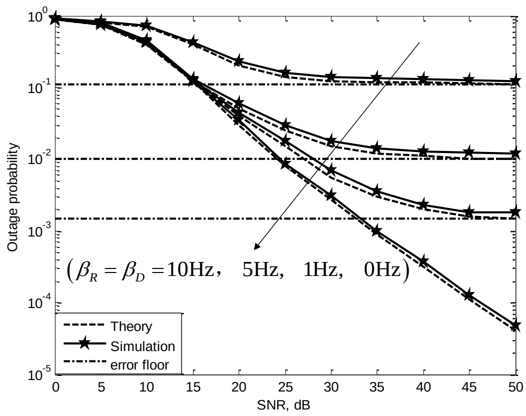


Fig. 3 Outage probability of OFDM AF relay system for BTB SCM with different Phase noise

In Fig. 3, BTB SCM situation is considered. Similarly with Fig. 2, the error floor of outage probability is approximately $2\times10^{-3}$ when PN linewidthes at relay and destination are 1Hz whereas $10^{-1}$ when 10Hz. Comparing Fig. 2 and Fig. 3, when PN for $R$ and $D$ are same, BTW SCM has a lower outage probability than BTB SCM.

In Fig. 4 and Fig. 5, the BER performances of 16QAM modulated OFDM AF relay system in presence of PN as a function of SNR are investigated over the BTW SCM scheme and the BTB SCM scheme under different PN respectively. A fairly good agreement between theoretical and simulated results is achieved, which confirms the validity of the performance analysis presented in this article. In Fig. 4, the error floor of BER is approximately $8.5\times10^{-3}$ when PN linewidthes at $R$ and $D$ are 1Hz whereas $3.2\times10^{-2}$ when 10Hz, compared with $9\times10^{-3}$ when 1Hz and $3.8\times10^{-2}$ when 10Hz in Fig. 5. Similarly with outage probability, BTW SCM has a lower BER than BTB SCM when PN for relay and destination are same as shown in Fig. 4 and Fig. 5. Since $\beta$ is 3dB PN bandwidth and has negative linear correlation with $C_{t,0}$ from Eq. (9). The BER for BTW SCM expression Eq. (36) reveals negative exponential relationship between $C_{t,0}$ and BER, which means when $C_{t,0}$ decreases BER will increase exponentially. Thus in Fig. 4 the $\beta_{\mathrm{R}}=\beta_{\mathrm{D}}$=10Hz line is quite different from other three lines.

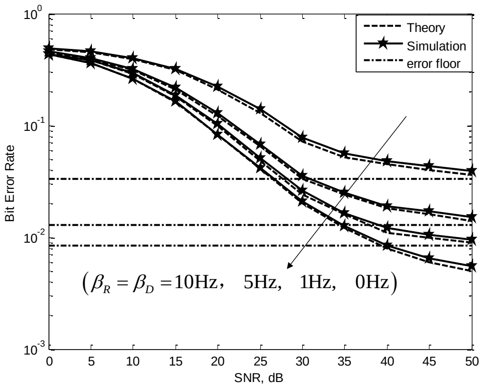


Fig. 4. The BER of OFDM AF system for BTW SCM. The constellation size is 16QAM

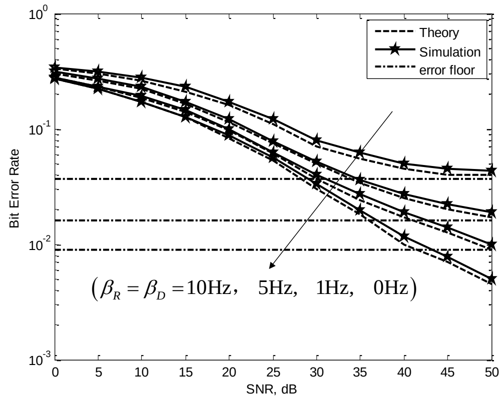


Fig. 5. The BER of OFDM AF system for BTB SCM. The constellation size is 16QAM

Based on the data analysis above and comprehensive observe Fig. 2-Fig. 5, we can conclude that: (1) For the low SNR regime, i.e. when SNR is between 0~5dB , the different effects of PN on outage probability and BER performance of OFDM AF relay system are not obvious. Because PN degrades outage probability and BER performance through introduces phase offset as well as ICI to useful signals and destroys subcarrier orthogonally. Under small transmitting power, little subcarrier power leaks to adjacent subcarrier. Therefore despite PN linewidthes ranging from 0Hz to 10Hz, system performance changes are not significantly. (2) The OFDM AF system becomes more sensitive on PN in high SNR regime, i.e. when SNR is larger than 5dB. For the high SNR regime with large PN, outage probability and BER performances of OFDM AF relay system tend to error floor rather than zero. Contrary to conclusion (1) above, when PN is large, the power of subcarrier leaks a lot to the adjacent subcarrier, leading to a qualitative change in performances. (3) Comparing Fig. 4 with Fig. 5, we get the difference BER performance between low SNR regime and high SNR regime. For low SNR regime, i.e. SNR is close to 0dB, BER for BTW SCM is about $3\times10^{-1}$ under different PN conditions comparing with $2\times10^{-1}$ for BTB SCM according to Fig. 4 and Fig. 5. Therefore in low SNR regime BTB SCM performance better than BTW SCM because when SNR is low, signal is greatly disturbed by noise which means some subcarriers can't transmit signal to destination itself even though they are mapped to better subcarriers in next hop. In contrary, BTB SCM ensures the better subcarriers for both hops help each other transmit to destination. For high SNR regime we have already get the data conclusion during the analysis of Fig. 4 and Fig. 5 above. From these data we can conclude that in high SNR regime BTW SCM performs better than BTB SCM in and BER under the same PN and SNR constraints. Since when SNR is high, basically all subcarriers can transmit signals successfully, BTW SCM can ensure that all channels are transmitted at a higher efficiency.

Fig. 6 and Fig. 7 give simulation results for M-QAM modulation for OFDM AF relay system implementing both BTW SCM and BTB SCM for different total numbers of subcarrier. The two figures depict that: (1) With the increasing of subcarrier number, the BER performance is descending. That because when the number of subcarrier increases, the frequency interval between subcarriers will be reduced, and frequency offset caused by Doppler channel expansion will lead to the increase of ICI, therefore the subcarrier will be more susceptible to ICI damages. (2) Comparing Fig. 4 and Fig. 7, it can be seen that the BER performances of 4QAM modulation and the BER performances of 16QAM modulation have little difference in BTB SCM scheme. In BTB SCM scheme the strongest subcarrier from first hop is mapped to the strongest subcarrier in second hop, etc, so the two hops both work in high SNR regime, thus according to the analysis above, 4QAM modulation and 16QAM modulation achieve almost the same BER performance.

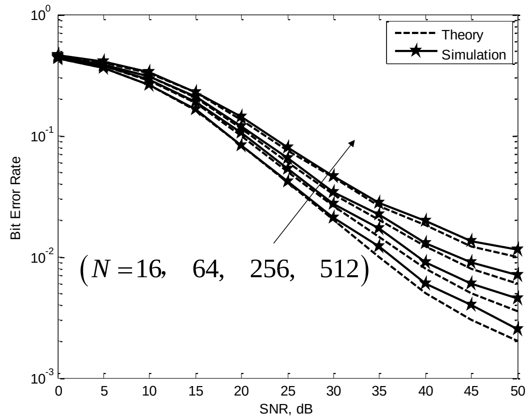


Fig. 6. The BER of OFDM AF system for BTW SCM for different numbers of subcarrier. The constellation size is 4QAM

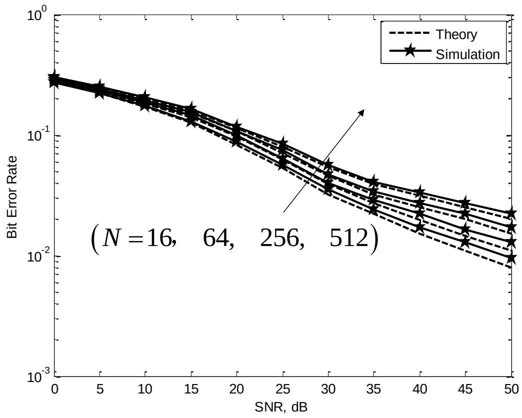


Fig. 7. The BER of OFDM AF system for BTB SCM for different numbers of subcarrier. The constellation size is 4QAM

## VII. Conclusions

In this paper, we have examined the performance of OFDM AF system with SCM over exponential correlated Rayleigh fading channels and uncorrelated Rician fading channels in presence of phase noise. A direct expression for post-processing SNR is derived with phase noise. Based on the end-to-end SNR CDF and PDF, we derived closed-form outage probability and BER of system for BTW SCM and BTB SCM. The simulation results demonstrated that a small phase noise has great impact on the outage probability and BER performances.


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## Appendix A: Proof of End-To-End SNR CDF Expressions

### A. BTW SCM

Equation (24) can be expressed as the sum of two integrals

$$F_{\gamma_k}^{BTW}(x) = \int_0^x P\left(C_{R,0}\gamma_{SR,k}\left(C_{D,0}\gamma_{RD,m}-x\right) \le \rho x\right) f_{k,RD}^{s}\left(\gamma_{RD,m}\right) d\gamma_{RD,m} + \int_x^\infty P\left(C_{R,0}\gamma_{SR,k}\left(C_{D,0}\gamma_{RD,m}-x\right) \le \rho x\right) f_{k,RD}^{s}\left(\gamma_{RD,m}\right) d\gamma_{RD,m} \tag{42}$$

It should be noted that the item $P\left(C_{R,0}\gamma_{SR,k}\left(C_{D,0}\gamma_{RD,m}-\gamma_{th}\right) \le \rho x\right)$ in the first integral of (42) equals to unity for $0 \le \gamma_{SR,k} \le x$, thus (42) can be simplified as

$$F_{\gamma_k}^{BTW}(x) = \int_0^x f_{k,RD}^{s}\left(\gamma_{RD,m}\right) d\gamma_{RD,m} + \int_x^\infty F_{k,SR}^{w}\left(\frac{\rho x}{C_{R,0}\left(C_{D,0}\gamma_{RD,m}-x\right)}\right) f_{k,RD}^{s}\left(\gamma_{RD,m}\right) d\gamma_{RD,m} \tag{43}$$

Since $\frac{dF_X(x)}{dx} = 1 - F_X(x)$ where $F_X(x) = \int_{-\infty}^{x} f_X(x) dx$, (43) can be rewritten as

$$F_{\gamma_k}^{BTW}(x) = 1 - \int_x^\infty F_{k,SR}^{w}{}'\left(\frac{\rho x}{C_{R,0}\left(C_{D,0}\gamma_{RD,m}-x\right)}\right) f_{k,RD}^{s}\left(\gamma_{RD,m}\right) d\gamma_{RD,m} \tag{44}$$

Let $y = C_{D,0}\gamma_{RD,m} - x$, then (44) can be transformed as

$$F_{\gamma_k}^{BTW}(x) = 1 - \int_0^\infty \frac{1}{C_{D,0}} f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right) dy + \int_0^\infty \frac{1}{C_{D,0}} F_{k,SR}^{w}\left(\frac{\rho x}{C_{R,0}y}\right) f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right) dy \tag{45}$$

Using (21), the item $f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right)$ in (45) is expressed as

$$f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right) = \sum_{i=0}^{N-k} \lambda_{RD} p_i \exp\left(-q_i \lambda_{RD} \frac{y+x}{C_{D,0}}\right) \tag{46}$$

So we have the first integral of $F_{\gamma_k}^{BTW}(x)$ in (45) as

$$\int_0^\infty \frac{1}{C_{D,0}} f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right) dy = \sum_{i=0}^{N-k} \frac{p_i}{q_i} \exp\left(-q_i \lambda_{RD} \frac{x}{C_{D,0}}\right) \tag{47}$$

Based on (19), the CDF $F_{k,SR}^{w}(x)$ of the SNR in each S-R subchannel can be derived by integral of the PDF function,

when the variable is $\frac{\rho x}{C_{R,0}y}$, $F_{k,SR}^{w}\left(\frac{\rho x}{C_{R,0}y}\right)$ can be derived as

$$F_{k,SR}^{w}\left(\frac{\rho x}{C_{R,0}y}\right) = \int_0^{\frac{\rho x}{C_{R,0}y}} \sum_{j=0}^{k-1} \lambda_{SR} \alpha_j \exp\left(-\beta_j \lambda_{SR} x\right) dx = \sum_{j=0}^{k-1} \frac{\alpha_j}{\beta_j}\left(1 - \exp\left(-\beta_j \lambda_{SR} \frac{\rho x}{C_{R,0}y}\right)\right) \tag{48}$$

Combine (46), (48) and equation (3.471.9) in [19] the second integral of $F_{\gamma_k}^{BTW}(x)$ in (45) can be rewritten as (49).

Substituting (47) and (49) into (45) allows us to write the closed-form expression of CDF for BTW SCM as (50), where $K_1(\bullet)$ is first order modified Bessel function of the second kind.

$$\begin{aligned}\int_0^\infty \frac{1}{C_{D,0}} F_{k,SR}^{w}\left(\frac{\rho x}{C_{R,0}y}\right) f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right)dy &= \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\beta_j} p_i\lambda_{RD}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right)\int_0^\infty\left(\exp\left(-\frac{q_i\lambda_{RD}y}{C_{D,0}}\right)-\exp\left(\frac{\rho\beta_j\lambda_{SR}x}{C_{R,0}y}-\frac{q_i\lambda_{RD}y}{C_{D,0}}\right)\right)dy \\ &= \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\beta_j} p_i\lambda_{RD}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right)\left[\frac{C_{D,0}}{q_i\lambda_{RD}}-2\sqrt{\frac{\rho\beta_j\lambda_{SR}xC_{D,0}}{q_iC_{R,0}\lambda_{RD}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i\lambda_{RD}\lambda_{SR}x}{C_{R,0}C_{D,0}}}\right)\right]\end{aligned} \tag{49}$$

$$F_{\gamma_k}^{BTW}(x) = 1-\sum_{i=0}^{N-k}\frac{p_i}{q_i}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right)+\frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\beta_j}p_i\lambda_{RD}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right)\left[\frac{C_{D,0}}{q_i\lambda_{RD}}-2\sqrt{\frac{\rho\beta_j\lambda_{SR}C_{D,0}x}{q_iC_{R,0}\lambda_{RD}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i\lambda_{RD}\lambda_{SR}x}{C_{R,0}C_{D,0}}}\right)\right] \tag{50}$$

$$F_{\gamma_k}^{BTB}(x) = 1-\int_0^\infty\frac{1}{C_{D,0}} f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right)dy+\int_0^\infty\frac{1}{C_{D,0}}F_{k,RD}^{s}\left(\frac{\rho x}{C_{R,0}y}\right)f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right)dy \tag{51}$$

$$\int_0^\infty\frac{1}{C_{D,0}}F_{k,RD}^{s}\left(\frac{\rho x}{C_{R,0}y}\right)f_{k,RD}^{s}\left(\frac{y+x}{C_{D,0}}\right)dy = \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{p_j}{q_j}p_i\lambda_{RD}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right)\left[\frac{C_{D,0}}{q_i\lambda_{RD}}-2\sqrt{\frac{\rho q_j\lambda_{SR}C_{D,0}x}{q_iC_{R,0}\lambda_{RD}}}K_1\left(2\sqrt{\frac{\rho q_j q_i\lambda_{RD}\lambda_{SR}x}{C_{R,0}C_{D,0}}}\right)\right] \tag{52}$$

$$F_{\gamma_k}^{BTB}(x) = 1-\sum_{i=0}^{N-k}\frac{p_i}{q_i}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right)+\frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{p_j}{q_j}p_i\lambda_{RD}\exp\left(-\frac{q_i\lambda_{RD}x}{C_{D,0}}\right)\left[\frac{C_{D,0}}{q_i\lambda_{RD}}-2\sqrt{\frac{\rho q_j\lambda_{SR}C_{D,0}x}{q_iC_{R,0}\lambda_{RD}}}K_1\left(2\sqrt{\frac{\rho q_j q_i\lambda_{RD}\lambda_{SR}x}{C_{R,0}C_{D,0}}}\right)\right] \tag{53}$$

*B. BTB SCM*

When it comes to BTB SCM scheme, we just need to replace $k$ th weakest subcarrier expression for the CDF with $k$ th strongest subcarrier, which means that using the expression (21) to replace (19), then we get the CDF for BTB SCM as (51).

In the derivation process similar to BTW, we can derived the second integral term of $F_{\gamma_k}^{BTB}(x)$ as (52).

Substituting (47) and (52) into (51) allows us to write the closed-form CDF expression in BTW SCM as (53).

## APPENDIX B: PROOF OF END-TO-END SNR PDF EXPRESSIONS

Using (25) and (27) the derivation for $F_{\gamma_k}^{BTW}(x)$ and $F_{\gamma_k}^{BTB}(x)$ of $x$ are as follows.

Divide (25) into three parts as $F_{\gamma_k}^{BTW}(x)= 1-T_1(x)-T_2(x)$, therefore the derivation of $F_{\gamma_k}^{BTW}(x)$ is

$$\frac{dF_{\gamma_k}^{BTW}(x)}{dx} = -\frac{dT_1(x)}{dx}-\frac{dT_2(x)}{dx} \tag{54}$$

Calculate $T_1(x)$ and $T_2(x)$ derivations respectively as follows:

$$\frac{dT_1(x)}{dx} = -\sum_{i=0}^{N-k}\frac{p_i}{C_{D,0}\bar{\gamma}_{RD}}\exp\left(-q_i\frac{x}{C_{D,0}\bar{\gamma}_{RD}}\right) \tag{55}$$

With the help of equation (804.91) in [21], $T_2(x)$ derivation can be presented as (57), where $T_3(x) = -\sum_{j=0}^{k-1}\frac{\alpha_j}{\beta_j}\frac{dT_1(x)}{dx}$.

According to (48), the CDF of weakest subcarrier in S-R link is

$$F_{k,SR}^{w}(x) = \int_0^x\sum_{j=0}^{k-1}\frac{\alpha_j}{\bar{\gamma}_{SR}}\exp\left(-\frac{\beta_j}{\bar{\gamma}_{SR}}x\right)dx-\sum_{j=0}^{k-1}\frac{\alpha_j}{\beta_j}\left(1-\exp\left(-\frac{\beta_j}{\bar{\gamma}_{SR}}x\right)\right) \tag{56}$$

From (56), $F_{k,SR}^{w}(0)=0$, $F_{k,SR}^{w}(\infty)=\sum_{0}^{k-1}\frac{\alpha_j}{\beta_j}$. Considering the nature of CDF, $F_{k,SR}^{w}(\infty)-F_{k,SR}^{w}(0)=1$ since the value of $x$ ranges from zero to infinity, so $\sum_{j=0}^{k-1}\frac{\alpha_j}{\beta_j}=1$ and then we can get $T_3(x)=-\frac{dT_1(x)}{dx}$.

Taking (55) and (57) into (54), the derivation of $F_{\gamma_k}^{BTW}(x)$ is finally obtained as (58).

Exchanging the PDF of hop1 and hop2, the further simplification of formula is (59).

Similarly, the PDF of BTB SCM can be derived as (60).

$$\frac{dT_2(x)}{dx} = -\frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\bar{\gamma}_{RD}\beta_j}p_i\exp\left(-\frac{q_i x}{\bar{\gamma}_{RD}C_{D,0}}\right) - \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\bar{\gamma}_{RD}\beta_j}p_i\exp\left(-\frac{q_i x}{\bar{\gamma}_{RD}C_{D,0}}\right)$$
$$+\frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\bar{\gamma}_{RD}\beta_j}p_i\exp\left(-\frac{q_i x}{\bar{\gamma}_{RD}C_{D,0}}\right)2\sqrt{\frac{\rho\beta_j x}{q_i C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)$$
$$\times\left(\sqrt{\frac{\rho\beta_j\bar{\gamma}_{RD}C_{D,0}}{q_i C_{R,0}x\bar{\gamma}_{SR}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) + 2\frac{\rho\beta_j}{C_{R,0}\bar{\gamma}_{SR}}\left(-K_0\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) - \frac{K_1\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)}{2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}}\right)\right) \quad (57)$$
$$= T_3(x) + \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\bar{\gamma}_{RD}\beta_j}p_i\exp\left(-\frac{q_i x}{C_{D,0}\bar{\gamma}_{RD}}\right)$$
$$\times\left(2\sqrt{\frac{\rho q_i\beta_j x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) + 2\frac{\rho\beta_j}{C_{R,0}\bar{\gamma}_{SR}}K_0\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right)$$

$$\frac{dF_{\gamma_k}^{BTW}(x)}{dx} = \frac{1}{C_{D,0}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\frac{\alpha_j}{\bar{\gamma}_{RD}\beta_j}p_i\exp\left(-\frac{q_i x}{C_{D,0}\bar{\gamma}_{RD}}\right)\cdot\left(2\sqrt{\frac{\rho q_i\beta_j x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) + 2\frac{\rho\beta_j}{C_{R,0}\bar{\gamma}_{SR}}K_0\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right) \quad (58)$$

$$f_{\gamma_k}^{BTW}(x) = \frac{2}{C_{R,0}\bar{\gamma}_{SR}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\alpha_j p_i\exp\left(-\frac{\beta_j x}{C_{R,0}\bar{\gamma}_{RS}}\right)\left[\sqrt{\frac{\rho\beta_j x}{q_i C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) + \frac{\rho}{C_{D,0}\bar{\gamma}_{RD}}K_0\left(2\sqrt{\frac{\rho\beta_j q_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right] \quad (59)$$

$$f_{\gamma_k}^{BTB}(x) = \frac{2}{C_{R,0}\bar{\gamma}_{SR}}\sum_{j=0}^{k-1}\sum_{i=0}^{N-k}\alpha_j\alpha_i\exp\left(-\frac{\beta_j x}{C_{R,0}\bar{\gamma}_{SR}}\right)\left[\sqrt{\frac{\rho\beta_j x}{\beta_i C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}K_1\left(2\sqrt{\frac{\rho\beta_j\beta_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right) + \frac{\rho}{C_{D,0}\bar{\gamma}_{RD}}K_0\left(2\sqrt{\frac{\rho\beta_j\beta_i x}{C_{R,0}C_{D,0}\bar{\gamma}_{SR}\bar{\gamma}_{RD}}}\right)\right] \quad (60)$$

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