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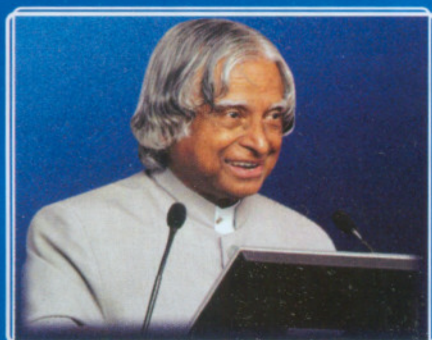

முடியும் என்றால் முயற்சி செய்,
முடியாதென்றால் பயிற்சி செய்.

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MATHEMATICS FORMULAE BOOK
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1. MATHEMATICS IMPORTANT FORMULAE

0.1 Trigonometry

1. $\sin^2\theta + \cos^2\theta = 1$

2. $1 + \tan^2\theta = \sec^2\theta$

3. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

4. $\sin(A+B) = \sin A\cos B + \cos A\sin B$

5. $\sin(A-B) = \sin A\cos B - \cos A\sin B$

6. $\cos(A+B) = \cos A\cos B - \sin A\sin B$

7. $\cos(A-B) = \cos A\cos B + \sin A\sin B$

8. $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

9. $\cos(A+B)\sin(A-B) = \cos^2 A - \sin^2 B$

10. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

11. $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

12. $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

13. $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

14. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

15. $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

16. $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

17. $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

18. $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$

19. $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

20. $\sin 2A=2\sin A\cos A$

21. $\cos 2A=\cos^2 A-\sin^2 A=1-2\sin^2 A=2\cos^2 A-1$

22. $\sin 2A=\dfrac{2\tan A}{1+\tan^2 A}$

23. $\cos 2A=\dfrac{1-\tan^2 A}{1+\tan^2 A}$

24. $\tan 2A=\dfrac{2\tan A}{1-\tan^2 A}$

25. $\cos^2 A=\dfrac{1+\cos 2A}{2}$

26. $\sin^2 A=\dfrac{1-\cos 2A}{2}$

27. $\sin A=2\sin\left(\dfrac{A}{2}\right)\cos\left(\dfrac{A}{2}\right)$

28. $\cos A=\cos^2\left(\dfrac{A}{2}\right)-\sin^2\left(\dfrac{A}{2}\right)=1-2\sin^2\left(\dfrac{A}{2}\right)=2\cos^2\left(\dfrac{A}{2}\right)-1$

29. $1+\cos A=2\cos^2\left(\dfrac{A}{2}\right)$

30. $1-\cos A=2\sin^2\left(\dfrac{A}{2}\right)$

31. $\tan A=\dfrac{2\tan\left(\dfrac{A}{2}\right)}{1-\tan^2\left(\dfrac{A}{2}\right)}$

32. $\sin A = \dfrac{2\tan\left(\dfrac{A}{2}\right)}{1+\tan^2\left(\dfrac{A}{2}\right)}$

33. $\cos A = \dfrac{1-\tan^2\left(\dfrac{A}{2}\right)}{1+\tan^2\left(\dfrac{A}{2}\right)}$

34. $\sin 3A = 3\sin A - 4\sin^3 A$

35. $\cos 3A = 4\cos^3 A - 3\cos A$

36. $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

37. Values of trigonometrically ratios for known angles

θ	0^0	30^0	45^0	60^0	90^0
Sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞
Cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
Sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

38. $\sin\left(\frac{n\pi}{2}\pm\theta\right)=\begin{cases}\pm\sin\theta, & if\, n\, is\, even\\ \pm\cos\theta, & if\, n\, is\, odd\end{cases}$

39. $\cos\left(\frac{n\pi}{2}\pm\theta\right)=\begin{cases}\pm\sin\theta, & if\ n\ is\ even\\ \pm\cos\theta, & if\ n\ is\ odd\end{cases}$

The sign $\pm$ is depending on the quadrant in which $\left(\frac{n\pi}{2}\pm\theta\right)$ lies.

40. $\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}$

41. $\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

0.2. Quadratic equations:

1. The general form of the quadratic equation is $ax^2+bx+c=0$

2. The solutions are $x_1=\frac{-b+\sqrt{b^2-4ac}}{2a}, x_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$

   If $b^2-4ac>0$, the roots are real and distinct.

   If $b^2-4ac=0$, the roots are real and equal.

   If $b^2-4ac<0$, the roots are imaginary.

3. If b^2-4ac is a perfect square, then the roots are real and rational.

4. For the equation $ax^2+bx+c=0$,

   Sum of the roots= $-\frac{b}{a}$, product of the roots = $\frac{c}{a}$

5. If α and β are the roots of the quadratic equation, then the equation

   x^2 −(Sum of the roots) $x+$ (product of the roots) = 0

   i.e., $x^2-(\alpha+\beta)x+\alpha\beta=0$

[redacted]

6. Extension:

If $\alpha_1, \alpha_2 ...., \alpha_n$ are the roots of the general equation of degree 'n' with integral coefficients

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_n = 0$$

Then,

$$\sum a_1 = -\frac{a_1}{a_0}$$

$$\sum a_1 a_2 = \frac{a_2}{a_0} \quad ,$$

$$\sum a_1 a_2 a_3 = -\frac{a_3}{a_0}$$

$$\sum a_1 a_2 a_3 ....a_n = \pm \frac{a_n}{a_0}$$

0.3 Logarithms

1. $y = e^x \Leftrightarrow x = \log y$
2. $\log 0 =$ Undefined
3. $\log 1 = 0$
4. $\log e = 1$
5. $\log \infty = \infty$
6. $\log uv = \log u + \log v$
7. $\log\left(\frac{u}{v}\right) = \log u - \log v$
8. $\log u^n = n \log u$
9. $\log_b a = \frac{\log a}{\log b}$
10. $\log_{10} x = \log_e x \times \log_{10} e$

0.4 Permutations and Combinations

1. 0! =1
2. $n! = 1.2.3..........n$
3. $n!=n(n-1)!$
4. $nP_r = \dfrac{n!}{(n-r)!}$
5. $nP_n = n!$
6. $nC_r = \dfrac{n!}{(n-r)!r!}$
7. $nC_0 = nC_n = 1$
8. $nC_1 = n$
9. $nC_r = nC_{n-r}$
10. $x+y=n \Rightarrow nC_x = nC_y$

0.5 Binomial theorem

1. If n is a natural number then

$$(x+a)^n = x^n + nC_1x^{n-1}a + nC_2x^{n-2}a^2 + ...+ nC_rx^{n-r}a^r + ... + a^n$$

2. $(x-a)^n = x^n - nC_1x^{n-1}a + nC_2x^{n-2}a^2 + ...+(-1)^r nC_rx^{n-r}a^r + ...+(-1)^n a^n$

3. The sum of the binomial coefficients= 2^n

4. Sum of the coefficients of even terms=sum of the coefficients of odd terms.

Binomial theorem for rational index

(i) If n is rational number and $-1 < x < 1$ then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{2!}x^3 + ...$$

(ii) $(1+x)^{-1} = \dfrac{1}{(1+x)} = 1 - x + x^2 - x^3 + x^4.....$

(iii) $(1-x)^{-1} = \dfrac{1}{(1-x)} = 1 + x + x^2 + x^3 + x^4.....$

0.6 Analytical Geometry

1. The distance between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
2. The point which divides the line joining the points (x_1, y_1) and (x_2, y_2) in the ratio $m:n$ internally is $\left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$
3. Point of the external division is $\left[\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right]$
4. The midpoint of the line joining the points (x_1, y_1) and (x_2, y_2) is $\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$
5. The centroid of the triangle formed by the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$
6. Any first degree equation in x and y will represent a straight line.
7. Slope of $ax+by+c=0$ is $m=-\frac{a}{b}=-\frac{coefficient\ of\ x}{coefficient\ of\ y}$
8. x-intercept of $ax+by+c=0$ is $-\frac{c}{a}$
9. y-intercept of $ax+by+c=0$ is $-\frac{c}{b}$
10. Equation of the line with slope m and y-intercept c is $y=mx+c$
11. Equation of the line with slope m and which passes through the point (x_1, y_1) is $y-y_1=m(x-x_1)$
12. Equation of the line which passes through the points (x_1, y_1) and (x_2, y_2) is $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

13. Slope of the line joining the points (x_1, y_1) and (x_2, y_2) is

$$\frac{y_2 - y_1}{x_2 - x_1} (or) \frac{y_1 - y_2}{x_1 - x_2}$$

14. If a straight line makes intercepts a and b on the axes then its equatio

is $\frac{x}{a} + \frac{y}{b} = 1$

15. The normal form of a straight line is $x\cos\alpha + y\sin\alpha = p$ where p is th length of the perpendicular from the origin to the line and α is the ang made by the normal with x-axis.

16. The distance form or symmetric form or parametric form of a straight line is $\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r.$ where r is the algebraic distance of any poi on the line from the fixed point (x_1, y_1) and θ is the angle made by the line with the x-axis.

17. (i) The equation of the x-axis is $y = 0$

(ii) The equation of the y-axis is $x = 0$

(iii) The slope of the x-axis is 0

(iv) The slope of the y-axis is ∞

(v) The equation of a line perpendicular to the x-axis (parallel to the y-axis) at a distance 'a' from the origin is $x = a$.

(vi) The equation of a straight line parallel to the x-axis (perpendicula to the y-axis) at a distance b from the origin is $y = b$.

(vii) The equation of a line having slope m and passing through the origins $y = mx$

(viii) The parametric representation of any point on the line

$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$ is $(x_1 + r\cos\theta, y_1 + r\sin\theta)$

18. The area of the triangle formed by the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

19. The condition for the three points $(x_1, y_1), (x_2, y_2)$ *and* (x_3, y_3) to be collinear is $\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right] = 0$

20. The perpendicular distance of the point (x_1, y_1) from the line $ax + by + c = 0$ is $\pm\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$

21. The length of the perpendicular from the origin to the line $ax + by + c = 0$ is $\pm\frac{c}{\sqrt{a^2 + b^2}}$

22. If θ is the angle between the lines $y = m_1x + c_1$ and $y = m_2x + c_2$ then θ is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

23. If $m_1 = m_2$ the lines are parallel and if $m_1m_2 = -1$, the lines are perpendicular.

24. Any line parallel to $ax + by + c = 0$ is of the form $ax + by + k = 0$.

25. Any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$ (or) $-bx + ay + k = 0$.

26. Any line through the intersection of the lines $L_1 = 0$ and $L_2 = 0$ is $L_1 + KL_2 = 0$

27. The distance between the parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

0.7 Pair of straight lines:

28. (i) If $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are the separate equations of two lines respectively, then the combined equation of the lines is $(a_1x+b_1y+c_1)(a_2x+b_2y+c_2)=0$.

    (ii) If $(a_1x+b_1y+c_1)=0$ *and* $(a_2x+b_2y+c_2)=0$ are two lines passing through the origin, the combined equation of the lines is $(a_1x+b_1y+c_1)(a_2x+b_2y+c_2)=0$ Which, is a homogeneous equation of degree 2 in x and y.

29. The general second degree homogenous equation $ax^2+2hxy+by^2=0$ always represents two lines passing through the origin.

30. If m_1 and m_2 are the slopes of the lines represented by $ax^2+2hxy+by^2=$ then $m_1+m_2=\frac{-2h}{b}$ and $m_1m_2=\frac{a}{b}$.

31. The angle between the lines $ax^2+2hxy+by^2=0$ is given by $\tan^{-1}\left|2\frac{\sqrt{h^2-ab}}{a+b}\right|$.

32. The line $ax^2+2hxy+by^2=0$ is parallel if $h^2-ab=0$ and perpendicular if a + b = 0.

33. The equation to the pair of bisectors of the angle betwe $ax^2+2hxy+by^2=0$ is $\frac{x^2-y^2}{a-b}=\frac{xy}{h}$.

34. The condition for the general second degree equation in x and $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a pair of lines $abc+2fgh-af^2-bg^2-ch^2=0$ (or). $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0$

35. The angle between the lines given by $ax^2+2hxy+by^2+2gx+2fy+c=0$ is given by $\tan^{-1}\left|2\frac{\sqrt{h^2-ab}}{a+b}\right|$

36. The combined equation of lines joining the origin to the point of intersection of a curve and a straight line is got by homogenizing the equation of the curve with the help of the line.

0.8 Circle:

37. The equation of the circle with centre (a, b) and radius r is $(x-a)^2+(y-b)^2=r^2$

38. The equation of the circle with centre as the origin and radius r is $x^2+y^2=r^2$.

39. The condition for the general second degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ to represent a circle is (i) h=0, (ii) a=b.

40. The general form of the equation of a circle is $x^2+y^2+2gx+2fy+c=0$ Its centre is $(-g,-f)$ and radius is $\sqrt{g^2+f^2-c}$

41. If the equation of the circle is given by $ax^2+by^2+2gx+2fy+c=0$ then its centre is $\left(\frac{-g}{a},\frac{-f}{a}\right)$ and radius is $\sqrt{\frac{g^2}{a^2}+\frac{f^2}{a^2}-\frac{c}{a}}$

42. The equation of the tangent to the circle $x^2+y^2=a^2$ is $xx_1+yy_1=a^2$.

43. The condition for the line $y=mx+c$ to be a tangent to the circle $x^2+y^2=a^2$ is $c^2=a^2(1+m^2)$.The length of the perpendicular from the centre to the line is equal to the radius of the circle.

0.9 Differentiation:

Important Limits

(i) $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ (ii) $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

(iii) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$ (iv) $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

Standard results:

SL.NO	FUNCTION	DERIVATIVE
1.	x^n	nx^{n-1}
2.	e^x	e^x
3.	$\log x$	$\frac{1}{x}$
4.	$\sin x$	$\cos x$
5.	$\cos x$	$-\sin x$
6.	$\tan x$	$\sec^2 x$
7.	$\sec x$	$\sec x \tan x$
8.	$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
9.	$\cot x$	$-\operatorname{cosec}^2 x$
10.	$\sin^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
11.	$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$
12.	$\tan^{-1} x$	$\frac{1}{1+x^2}$

13.	$\cot^{-1} x$	$\dfrac{-1}{1+x^2}$
14.	$\sec^{-1} x$	$\dfrac{1}{x\sqrt{x^2-1}}$
15.	$\text{cosec}^{-1} x$	$\dfrac{-1}{x\sqrt{x^2-1}}$
16.	a^x	$a^x \log a$
17.	$\log_a x$	$\dfrac{1}{x}\log_a e$

Rules:

1. (i) If $y = k$ is a constant then $\dfrac{dy}{dx}=0$

   (ii) If $y = ku$ where k is a constant and u is a function of x then

   $$\frac{dy}{dx} = k\frac{du}{dx}$$

2. If u and v are functions of x then

   (i) $\dfrac{d}{dx}(u \pm v) = \dfrac{du}{dx} \pm \dfrac{dv}{dx}$

   (ii) $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

   (iii) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

3. (i) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

   (ii) $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$

0.10. Integral Calculus

Standard results:

SL.NO	FUNCTION $f(x)$	$\int f(x)dx$
1.	x^n	$\frac{x^{n+1}}{n+1}+c, n \neq -1$
2.	$\frac{1}{x}$	$\log x + c$
3.	e^x	$e^x + c$
4.	$\sin x$	$-\cos x + c$
5.	$\cos x$	$\sin x + c$
6.	$\tan x$	$\log \sec x + c$
7.	$\sec x$	$\log(\sec x + \tan x) + c$ **(or)** $\left(\log \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+c\right)$
8.	$\operatorname{cosec} x$	$-\log(\operatorname{cosec} x + \cot x) + c$ **(or)** $\log\left(\tan\frac{x}{2}\right)+c$
9.	$\cot x$	$\log \sin x + c$
10.	$\sec^2 x$	$\tan x + c$
11.	$\operatorname{cosec}^2 x$	$-\cot x + c$
12.	$\sec x \tan x$	$\sec x + c$
13.	$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x + c$

14. $\sqrt{1-x^2}$	$\sin^{-1} x + c$
15. $\frac{1}{1+x^2}$	$\tan^{-1} x + c$
16. $\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1} x + c$
17. $\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
18. $\frac{1}{a^2-x^2}$	$\frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$
19. $\frac{1}{x^2-a^2}$	$\frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$
20. $\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
21. $\frac{1}{\sqrt{x^2+a^2}}$	$\log\left(x+\sqrt{x^2+a^2}\right)+c$
22. $\frac{1}{\sqrt{x^2-a^2}}$	$\log\left(x+\sqrt{x^2-a^2}\right)+c$
23. $\sqrt{a^2-x^2}$	$\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+c$
24. $\sqrt{x^2+a^2}$	$\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left(x+\sqrt{x^2-a^2}\right)+c$
25. $\sqrt{x^2-a^2}$	$\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left(x+\sqrt{x^2+a^2}\right)+c$
26. $e^{ax}\sin bx$	$\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)+c$

27. $e^{ax}\cos bx$ — $\frac{e^{ax}}{a^2+b^2}(a\cos bx+b\sin bx)+c$

28. $(ax+b)^n$ — $\frac{(ax+b)^{n+1}}{a(n+1)}+c, n\neq -1$

29. $\frac{1}{(ax+b)}$ — $\frac{\log(ax+b)}{a}+c$

30. e^{ax+b} — $\frac{e^{ax+b}}{a}+c$

31. $\sin(ax+b)$ — $-\frac{\cos(ax+b)}{a}+c$

32. $\cos(ax+b)$ — $\frac{\sin(ax+b)}{a}+c$

33. $\sec^2(ax+b)$ — $\frac{\tan(ax+b)}{a}+c$

34. $\operatorname{cosec}^2(ax+b)$ — $-\frac{\cot(ax+b)}{a}+c$

35. $\sec(ax+b)\tan(ax+b)$ — $\frac{\sec(ax+b)}{a}+c$

36. $\operatorname{cosec}(ax+b)\cot(ax+b)$ — $-\frac{\operatorname{cosec}(ax+b)}{a}+c$

37. Integration by parts:

$$\int u\,dv = uv - \int v\,du$$

Properties of definite integrals

1. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

2. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

3. If $f(x)$ is even, then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

If $f(x)$ is odd, then $\int_{-a}^{a} f(x)dx = 0$

4. $\int_{a}^{b} f(x)dx = \int_{a}^{c} f(x)dx + \int_{c}^{b} f(x)dx$

5. $\int_{0}^{2a} f(x)dx = 2\int_{0}^{a} f(x)dx, when f(x) = f(2a-x)$

6. $$\int_{0}^{\frac{\pi}{2}} \sin^n xdx = \begin{cases} \frac{n-1}{n}.\frac{n-3}{n-2}.................\frac{2}{3}, n\,is\,odd \\ \frac{n-1}{n}.\frac{n-3}{n-2}.................\frac{1}{2}.\frac{\pi}{2}, n\,is\,even \end{cases}$$

7. $$\int_{0}^{\frac{\pi}{2}} \cos^n xdx = \begin{cases} \frac{n-1}{n}.\frac{n-3}{n-2}.................\frac{2}{3}, n\,is\,odd \\ \frac{n-1}{n}.\frac{n-3}{n-2}.................\frac{1}{2}.\frac{\pi}{2}, n\,is\,even \end{cases}$$

0.11. Probability

1. If the outcome of an experiment cannot be predicted with certainly, it is called a random experiment
2. The set of all possible outcomes of a random experiment is called a sample space.
3. Any subset of the sample space is called an event.
4. Two events are said to be mutually exclusive if $A \cap B = \phi$

5. Axioms of probability

(i) $0 \le P(A) \le 1$

(ii) $P(S) = 1$

(iii) $A \cap B = \phi$ then $P[A \cup B] = P(A) + P(B)$

6. If there is a total of n possible outcomes and m cases are favourable to the happening of the event A, then the probability of the happening of A is defined as $P(A) = \frac{m}{n}$

7. $P(\phi) = 0$

8. $P(\overline{A}) = 1 - P(A)$

9. If A and B are any two events then $P[A \cup B] = P(A) + P(B) - P(A \cap B)$

10. The probability of an event A given that B has already occurred is denoted by A/B called the conditional event. The Probability of A given B has already occurred is $P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$.

In the same way $P(B/A) = \frac{P(A \cap B)}{P(A)}, P(A) \neq 0$.

11. Two events A and B are said to be independent if $P(A/B) = P(A)$

12. If A and B are independent then $P(A \cap B) = P(A)P(B)$

13. $P(A \cap B) = P(A)P(B/A)(or)P(B)P(A/B)$

2. MATRICES AND DETERMINANTS

1. Adjoint Matrix:

Let A be a square matrix of order n then

(i) $adj\ A = Co\,factor\,of\ A^T$

(ii) $|adj\ A| = A^{-1}$

(iii) $adj(KA) = K\ adj(A)$

(iv) $|adj\ (adj\ A)| = |A|^{n-2}.A$

(v) $A\,(adj\ A) = (adj\ A)A = |A|\,I_n$

2. Inverse Matrix:

Let A be a square of order n such that $|A| \neq 0$, then

(i) $A^{-1} = \frac{1}{|A|}(adj\ A)$

(ii) $|A^{-1}| = \frac{1}{|A|}$

(iii) $(KA)^{-1} = \frac{1}{K}A^{-1}$

(iv) $(A^{-1})^{-1} = A$

(iv) $AA^{-1} = A^{-1}A = I_n$

(vi) If $AX = B$ then $X = A^{-1}B$

3. Reversal Laws

(i) $(AB)^T = B^T A^T$

(ii) $adj(AB) = adjB.adjA$

(iii) $(AB)^{-1} = B^{-1}A^{-1}$

4. Rank of Matrix

(i) The highest order of non-vanishing minor of a matrix A is the rank A and is denoted by $\rho(A)$

(ii) Rank of zero matrixes is 0

(iii) If $O(A) = m \times n$ then $\rho(A) \leq \min[m, n]$

(iv) Matrices of the same order and same rank are called equivale matrices.

(v) The rank of a matrix is echelon form is equal to the number of no zero rows of the matrix.

5. Elementary Transformation

(i) Interchanging of two rows (columns).

(ii) Multiplication of a row (column) by a non-zero scalar.

(iii) Addition of a scalar multiple of a row (column) to any row (colum

6. Consistency of a system of linear equations

(i) A system is said to be consistent if it has solution.

(ii) A system can have

(a) a unique solution (or)

(b) Infinitely many solutions (or)

(c) no solution

(iii) If a system of homogeneous linear equations has more number of unknowns than the number of equations, then it will have infinitely many solutions.

(iv) If a system of non-homogeneous linear equations has more numb of unknowns than the number of equations is consistent then it wi have infinitely many solutions.

7. Notations

For the system of equations:

$$a_1x+b_1y+c_1z=d_1,$$

$$a_2x+b_2y+c_2z=d_2,$$

$$a_3x+b_3y+c_3z=d_3$$

(i) $\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (ii) $\Delta_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

(iii) $\Delta_y=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$ (iv) $\Delta_z=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

(v) $A=\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}, X=\begin{pmatrix} x \\ y \\ z \end{pmatrix}, B=\begin{pmatrix} d_1 \\ d_2 \\ d_2 \end{pmatrix}, [A,B]=\begin{pmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{pmatrix}$

8. Solution by determinant methods

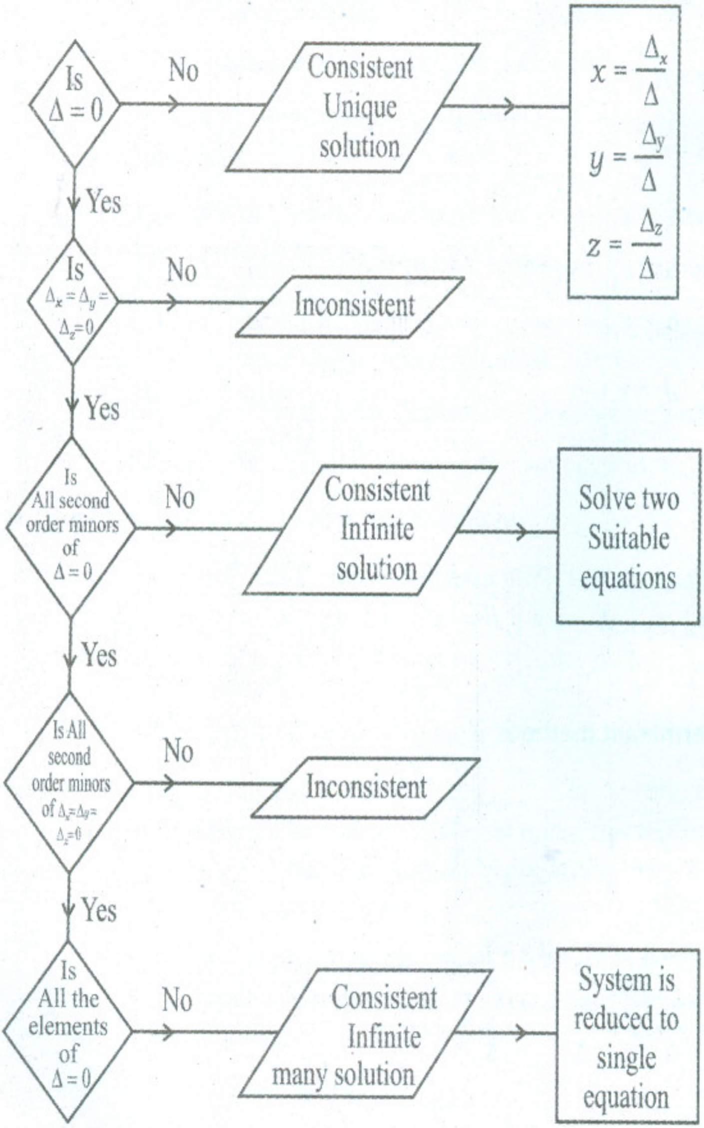
(i) Cramer's rule

$$\frac{x}{\Delta_x}=\frac{y}{\Delta_y}=\frac{z}{\Delta_z}=\frac{1}{\Delta}$$

This is meaningful only if $\Delta\neq 0$

Hence $x=\frac{\Delta_x}{\Delta}, y=\frac{\Delta_y}{\Delta}, z=\frac{\Delta_z}{\Delta}$ if $\Delta\neq 0$.

(ii) Test of Consistency



9. Solution by rank method

(i) If $\rho(A) = \rho(A,B)$ =n then the system is consistent with unique solution.

(ii) If $\rho(A) = \rho(A,B)$ <n then the system is consistent with infinitely many solutions.

(iii) If $\rho(A) \neq \rho(A,B)$ then the system is inconsistent.

Here n denotes the number of unknowns in the system.

10. A system of homogeneous linear equations

Consider a system of homogeneous linear equations.

$$a_1x + b_1y + c_1z = 0$$

$$a_2x + b_2y + c_2z = 0$$

$$a_3x + b_3y + c_3z = 0$$

(i) It is always consistent.

(ii) $x = 0, y = 0, z = 0$ is certainly a solution. It is known as trivial solution.

(iii) If $\Delta \neq 0$ (i.e.) $\rho(A)$ =3, then the system has only trivial solution.

(v) If $\Delta = 0$ (i.e.) $\rho(A)$<3, then the system has infinitely many solutions, both trivial and non-trivial.

3. VECTOR ALGEBRA

Preliminary results

1. $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ then $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$
2. $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$
3. For any vector $\vec{a} = |\vec{a}|\,\hat{a}$
4. Two vectors are parallel if one can be expressed as the scalar multiple of the other i.e., $\vec{b} = \lambda\vec{a}$ where $\lambda \neq 0$
5. If three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar then one vector is the linear combination of the other two.

   i.e., if $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $\vec{a} = x\vec{b} + y\vec{c}$ where x, y, are not simultaneously zero

Dot product or scalar product

6. $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$
7. If $\vec{a}$ and $\vec{b}$ are perpendicular then $\vec{a}.\vec{b} = 0$
8. If $\vec{a}.\vec{b} = 0$ then either $\vec{a} = 0$ (or) $\vec{b} = 0$ (or) a and b are perpendicular.
9. If $\vec{a}$ and $\vec{b}$ are parallel then $\vec{a}.\vec{b} = \pm|\vec{a}||\vec{b}|$
10. $\vec{a}.\vec{b}$ is negative if θ is obtuse.
11. $\cos\theta = \dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$
12. $\vec{a}^2 = |\vec{a}|^2 = a^2$

13. If m is a scalar then $(m\vec{a}).\vec{b} = \vec{a}.(m\vec{b}) = m(\vec{a}.\vec{b})$

14. If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ then $\vec{a}.\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

15. $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

16. $\vec{a}.(\vec{b} - \vec{c}) = \vec{a}.\vec{b} - \vec{a}.\vec{c}$

17. $(\vec{a} + \vec{b})^2 = \vec{a}^2 + \vec{b}^2 + 2\vec{a}.\vec{b}$

18. $(a - b)^2 = \vec{a}^2 + \vec{b}^2 - 2\vec{a}.\vec{b}$

19. $(\vec{a} + \vec{b})(\vec{a} - \vec{b}) = a^2 - b^2$

20. $(\vec{a}.\vec{b}) = |\vec{a}|$.Projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}.\vec{b}}{|\vec{a}|}$

21: Work done= $\vec{F}.\vec{d}$

22. If several forces act on the body then work done $= (\vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n).\vec{d}$

23. If $\vec{a}$ and $\vec{b}$ are unit vectors then

(i) $\sin(\theta/2) = \dfrac{1}{2}|\vec{a} - \vec{b}|$

(ii) $\cos(\theta/2) = \dfrac{1}{2}|\vec{a} + \vec{b}|$

24. In ΔABC, $a^2 = b^2 + c^2 - 2bc\cos A$

25. The diagonals of a rhombus are at right angles.

26. The angle between any two diagonals of a cube is $\cos^{-1}(1/3)$

27. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then $\vec{a}$ and $\vec{b}$ are perpendicular.

28. $\overline{a} = (\vec{a}.\vec{i})\vec{i} + (\vec{a}.\vec{j})\vec{j} + (\vec{a}.\vec{k})\vec{k}$

29. In ΔABC, $a = b\cos C + c\cos B$

30. In ΔABC, if D is the mid-point of BC then, $AB^2 + AC^2 = 2(AD^2 + BD^2)$

31. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular, vectors of equal magnitude then $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to each of $\vec{a}, \vec{b}, \vec{c}$ by $\cos^{-1}(1/\sqrt{3})$.

Vector product

32. $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\, \hat{n}$

33. $\hat{n} = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

34. $\sin\theta = \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

35. $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

36. If $\vec{a}$ and $\vec{b}$ are parallel then $\vec{a} \times \vec{b} = \vec{0}$

37. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

38. If $\vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}$ then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_2 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

39. If $\vec{a} \times \vec{b} = \vec{0}$ then either $\vec{a} = 0$ (or) $\vec{b} = 0$ (or) $\vec{a}$ and $\vec{b}$ are parallel.

40. $(m\vec{a}) \times \vec{b} = \vec{a} \times (m\vec{b}) = m(\vec{a} \times \vec{b})$ where $m \neq 0$ is a scalar.

41. If $\vec{a}$ and $\vec{b}$ represent two adjacent sides of a parallelogram

then $|\vec{a}\times\vec{b}|$ represents the area of the parallelogram, $\vec{a}\times\vec{b}$ represents a vector area of the parallelogram.

42. If $\vec{a}$ and $\vec{b}$ are any two sides of a triangle then area of the triangle= $\frac{1}{2}|\vec{a}\times\vec{b}|$

43. $(\vec{a}\times\vec{b})^2+(\vec{a}.\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$

44. $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times(\vec{c}+\vec{a})+\vec{c}\times(\vec{a}+\vec{b})=\vec{0}$

45. Area of quadrilateral ABCD $=\frac{1}{2}|\overrightarrow{AC}\times\overrightarrow{BD}|$

46. In ΔABC, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

47. $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2(\vec{a}\times\vec{b})$

48. If $\vec{a}\times\vec{b}=\vec{c}\times\vec{d}$ and $\vec{a}\times\vec{c}=\vec{b}\times\vec{d}$ then $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ are parallel.

49. (i) The area of the triangle with position vectors $\vec{a},\vec{b},\vec{c}$ is

$$\frac{1}{2}(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})$$

(ii) If the three points are collinear then

$$(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a})=0$$

50. The moment of the force $\vec{F}$ this acts through the point P about the point O is $\vec{r}\times\vec{F}$ where $\vec{r}=OP$ ($\vec{r}$ = at the point – about the point)

51. (i) When finding the angle between the two vectors $\vec{a}$ and $\vec{b}$ if we use vector product we get only the acute angle between the vectors. Hence, the use of dot product is preferable. Since it specifies the position of the angle θ.

(ii) Twice the area of the parallelogram is equal to the area of another parallelogram formed by taking as its adjacent sides the diagonals of the former parallelogram.

Scalar triple product

52. $\vec{a}.(\vec{b}\times\vec{c})$ is defined as the scalar triple product. It is denoted by $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$. It is also called as the box product.

53. In box product, dot and cross can be interchanged.

54. In box product if any two vectors are equal or parallel then its value is zero.

55. In box product, if the vectors are interchanged cyclically, the value is not altered.

56. In box product, if any two vectors are interchanged then the sign of the box product changes.

57. $\left[\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}\right]=2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

58. $\left[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}\right]=2\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$

59. $\left[\vec{i}\,\vec{j}\,\vec{k}\right]=1$

60. $\left[\vec{i}+\vec{j},\vec{j}+\vec{k},\vec{k}+\vec{i}\right]=2$

61. $\left[\vec{i}\times\vec{j},\vec{j}\times\vec{k},\vec{k}\times\vec{i}\right]=1$

62. If $\vec{a},\vec{b},\vec{c}$ represent the coterminous edges of a rectangular parallelepiped, then $\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$ = volume of the parallelepiped.

63. Volume of the tetrahedron = $\frac{1}{6}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

64. If $\vec{a}=a_1\vec{i}+a_2\vec{j}+a_3\vec{k}$, $\vec{b}=b_1\vec{i}+b_2\vec{j}+b_3\vec{k}$ and $\vec{c}=c_1\vec{i}+c_2\vec{j}+c_3\vec{k}$

$$\left[\vec{a}\,\vec{b}\,\vec{c}\right]=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

65. If $\vec{a},\vec{b},\vec{c}$ are coplanar then, $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$

66. If $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$ then

(i) Any one vector is zero (or)

(ii) Any two vectors are parallel (or)

(iii) All the three are coplanar.

67. $$\begin{vmatrix} \vec{p}.\vec{a} & \vec{p}.\vec{b} & \vec{p}.\vec{c} \\ \vec{q}.\vec{a} & \vec{q}.\vec{b} & \vec{q}.\vec{c} \\ \vec{r}.\vec{a} & \vec{r}.\vec{b} & \vec{r}.\vec{c} \end{vmatrix}=\left[\vec{p}\quad\vec{q}\quad\vec{r}\right]\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$$

68. If $\vec{a},\vec{b},\vec{c}$ are a right handed triad of mutually perpendicular

vectors of magnitudes a, b, c respectively, then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=abc$

69. If $x.\vec{a}=0, x.\vec{b}=0, x.\vec{c}=0$ and $\vec{x}\neq\vec{0}$ then $\left[\vec{a}\,\vec{b}\,\vec{c}\right]=0$

70. $\left[\vec{a}-\vec{b},\vec{b}-\vec{c},\vec{c}-\vec{a}\right]=0$

Vector triple product

71. $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}.\vec{c})\,\vec{b}-(\vec{c}.\vec{b})\vec{a}$

72. $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\,\vec{b}-(\vec{a}.\vec{b})\vec{c}$

73. $\vec{a}\times(\vec{b}\times\vec{c})$ lies in the plane of $\vec{b}$ and $\vec{c}$

$(\vec{a}\times\vec{b})\times\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$

Hence $\vec{a}\times(\vec{b}\times\vec{c})\neq(\vec{a}\times\vec{b})\times\vec{c}$

i.e., Vector product is not associative.

74. If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$ then $\vec{a}$ and $\vec{c}$ are parallel or $(\vec{c}\times\vec{a})\times\vec{b}=\vec{0}$

75. $\vec{i}\times(\vec{a}\times\vec{i})+\vec{j}\times(\vec{a}\times\vec{j})+\vec{k}\times(\vec{a}\times\vec{k})=2\vec{a}$

Scalar product of four vectors

76. $(\vec{a}\times\vec{b})(\vec{c}\times\vec{d})=\begin{vmatrix}\vec{a}.\vec{c} & \vec{a}.\vec{d}\\ \vec{b}.\vec{c} & \vec{b}.\vec{d}\end{vmatrix}$

77. $(\vec{a}\times\vec{b})(\vec{c}\times\vec{d})+(\vec{b}\times\vec{c})(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a})(\vec{b}\times\vec{d})=0$

Vector product of four vectors

78. $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\left[\vec{a}\,\vec{b}\,\vec{d}\right]\vec{c}-\left[\vec{a}\,\vec{b}\,\vec{c}\right]\vec{d}$

$=\left[\vec{c}\,\vec{d}\,\vec{a}\right]\vec{b}-\left[\vec{c}\,\vec{d}\,\vec{b}\right]\vec{a}$

79. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are coplanar then $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$

80. $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=0$

81. If $\vec{a}$ and $\vec{c}$ are parallel then $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$

82. If α,β,γ are the angles made by the line with the co-ordinate axis then,

$\cos\alpha,\cos\beta,\cos\gamma$ are the direction cosines.

83. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

84. $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

85. If l_1,m_1,n_1 and l_2,m_2,n_2 are the direction cosines of two lines and if θ is the angle between the lines then $\cos\theta=l_1l_2+m_1m_2+n_1n_2$

86. Any three quantities proportional to the direction cosines are the direction ratios.

87. Any vector with direction cosines a, b, c is $\vec{r} = a\vec{i} + b\vec{j} + c\vec{k}$

88. Vector equation of a line passing through a point A whose position vector $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + t\vec{b}$

Its cartesian form is $\dfrac{x-x_1}{l} = \dfrac{y-y_1}{m} = \dfrac{z-z_1}{n}$

89. Vector equation of a line passing through two points A and B whose position vectors are $\vec{a}$ and $\vec{b}$; is $\vec{r} = (1-t)\vec{a} + t\vec{b}$

Its cartesian form is $\dfrac{x-x_1}{x_2-x_1} = \dfrac{y-y_1}{y_2-y_1} = \dfrac{z-z_1}{z_2-z_1}$

90. Angle between the lines $\vec{r} = \vec{a} + t\vec{u}$ and $\vec{r} = \vec{a} + t\vec{v}$ is $\cos^{-1}\left(\dfrac{\vec{u}.\vec{v}}{|\vec{u}||\vec{v}|}\right)$

91. Vector equation of a plane passing through a point A and whose position vector $\vec{a}$ and parallel to two vectors $\vec{b}$ and $\vec{c}$ is $\vec{r} = \vec{a} + t\vec{b} + s\vec{c}$.

Its cartesian form is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

92. The vector equation of the plane passing through two points A and B whose position vector are $\vec{a}$ and $\vec{b}$ and parallel to $\vec{c}$ is $\vec{r} = (1-t)\vec{a} + t\vec{b} + s\vec{c}$.

Its cartesian form is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l & m & n \end{vmatrix} = 0$

93. Vector equation of the plane passing through the non-collinear points are A, B and C whose position vectors are $\vec{a}, \vec{b}, \vec{c}$ is $\vec{r} = (1-t-s)\vec{a} + t\vec{b} + s\vec{c}$.

Its Cartesian form is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

94. Equation of the plane in the intercept form is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

95. Normal form is $\vec{r}.\vec{n}=p$ (or) $lx+my+nz=p$ where $\hat{n}$ is the unit normal vector and p is the perpendicular distance of the plane from the origin.

96. If $\vec{n}$ is not a unit normal vector then $\vec{r}.\vec{n}=q$ where $p=\frac{q}{|\vec{n}|}$

97. Perpendicular distance of the plane $\vec{r}.\vec{n}=q$ from the origin is $\frac{q}{|\vec{n}|}$

98. Vector equation of the plane passing through a given point $\vec{a}$ and perpendicular to $\vec{n}$ is $(\vec{r}-\vec{a}).\vec{n}=0$ or $\vec{r}.\vec{n}=\vec{a}.\vec{n}$

Its Cartesian form is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

99. The angle between the planes $\vec{r}.\vec{n}_1=q_1$ and $\vec{r}.\vec{n}_2=q_2$ is θ

Where $\cos\theta=\frac{n_1n_2}{|n_1||n_2|}$

100. Angle between the line $\vec{r}=\vec{a}+t\vec{b}$ and the plane $\vec{r}.\vec{n}=q$ is θ

Where $\sin\theta=\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}$

101. The distance of the point $\vec{a}$ from the plane $\vec{r}.\hat{n}=q$ is $|p-\vec{a}.\hat{n}|$

102. Lines on the same plane are called coplanar lines.

103. Two lines in space which are either intersecting or parallel are coplanar.

104. Two lines in space which are not coplanar are called skew lines.

105. The distance between two parallel lines $\vec{r}=\vec{a}_1+t\vec{u}$ and

$\vec{r}=\vec{a}_2+s\vec{v}$ is $d=\dfrac{|\vec{u}\times(\vec{a}_2-\vec{a}_1)|}{|\vec{u}|}$

106. The distance between the skew lines $\vec{r}=\vec{a}_1+t\vec{u}$ and

$\vec{r}=\vec{a}_2+s\vec{v}$ is $d=\dfrac{|[(\vec{a}_2-\vec{a}_1)\vec{u},\vec{v}]|}{|\vec{u}\times\vec{v}|}$

107. The conditions for $\vec{r}=\vec{a}_1+t\vec{u}$ and $\vec{r}=\vec{a}_2+s\vec{v}$ to intersect $[\vec{a}_2-\vec{a}_1,\vec{u},\vec{v}]=0$.

108. The conditions for $\dfrac{x-x_1}{l_1}=\dfrac{y-y_1}{m_1}=\dfrac{z-z_1}{n_1}$, *and* $\dfrac{x-x_2}{l_2}=\dfrac{y-y_2}{m_2}=\dfrac{z-z_2}{n_2}$ to

intersect is $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$

109. The vector equation of the sphere with centre at c and radius a is $|\vec{r}-\vec{c}|=a$

110. The Cartesian equation of the sphere with centre at(a, b, c) and radius r units is $(x-a)^2+(y-b)^2+(z-c)^2=r^2$

111. The general equation of a sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$

With (i) Centre= (-u, -v, -w)

(ii) Radius= $\sqrt{u^2+v^2+w^2-d}$

112. The vector equation of the sphere with $\vec{a}$ and $\vec{b}$ as extremities of a diameter $(\vec{r}-\vec{a}).(\vec{r}-\vec{b})=0$.

4. COMPLEX NUMBERS

1. Any number of the form $x+iy$ where x and y are real numbers is called complex number. x is the real part any y is the imaginary part of the complex number. If the real part is zero, the number is purely imaginary and if the imaginary part is zero, the number is purely real.
2. If $z=x+iy$ where $i=\sqrt{-1}$ the real part is denoted by Re(z) and the imaginary part is denoted by Im(z).
3. If $z=x+iy$, then the conjugate of z denoted by $\bar{z}$ is defined by $\bar{z}=x-$
4. If $z=x+iy$, then $-z=-x-iy$.
5. If z is real then $\bar{z}=z$.
6. $\bar{\bar{z}}=z$
7. If $z=x+iy$ then $\text{Re}(z)=\frac{z+\bar{z}}{2}$; $\text{Im}(z)=\frac{z-\bar{z}}{2i}$
8. If $z_1=x_1+iy_1$, $z_2=x_2+iy_2$ then
   (i) $z_1+z_2=(x_1+x_2)+i(y_1+y_2)$
   (ii) $z_1-z_2=(x_1-x_2)+i(y_1-y_2)$
   (iii) $z_1z_2=(x_1x_2-y_1y_2)+i(x_1y_2+x_2y_1)$
   (iv) $\frac{z_1}{z_2}=\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+i\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}$
9. If $z_1=(x_1,y_1)$ $z_2=(x_2,y_2)$ then
   (i) $z_1+z_2=(x_1+x_2,y_1+y_2)$
   (ii) $z_1-z_2=(x_1-x_2,y_1-y_2)$
   (iii) $z_1.z_2=(x_1x_2-y_1y_2,x_1y_2+x_2y_1)$
   (iv) $\frac{z_1}{z_2}=\left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2},\frac{x_2y_1-x_1y_2}{x_2^2+y_2^2}\right)$

18. **Properties**

For any three complex numbers z_1, z_2, z_3 we have

(i) $z_1 + z_2 = z_2 + z_1$

(ii) $z_1.z_2 = z_2.z_1$

(iii) $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

(iv) $(z_1 z_2) z_3 = z_1 (z_2 z_3)$

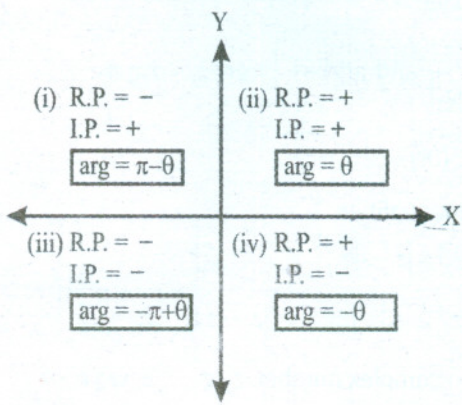
(v) $z_1 + (z_2 + z_3) = z_1 z_2 + z_1 z_3$

Geometrical representation

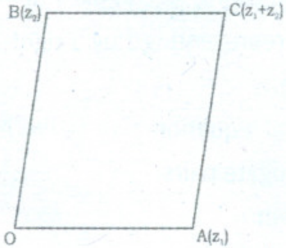
19. Any complex number $z = x + iy$ can be represented as a point(x, y) i argand plane.

20. If $z = x + iy$ is any complex number then the correct argument is fou using the following procedure.

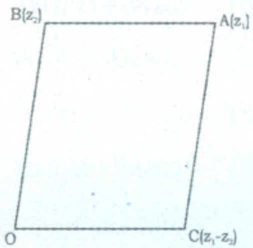
Define $\theta = \tan^{-1}\left|\frac{y}{x}\right|$ and an acute. Care must take that in the calculat of θ, the sign of the real and imaginary parts must be suppressed.



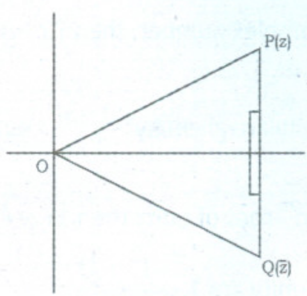
21. If A and B represent the complex numbers z_1 and z_2 respectively and if OABC is the parallelogram formed by OA and OB as adjacent sides then, C represents $z_1 + z_2$.



22. If A and B represents the complex numbers z_1 and z_2 respectively and if OABC is the parallelogram formed by OA as one of the diagonals and OB as one of the sides then C represent $z_1 - z_2$



23. If P represents the complex numbers z then the conjugates of z is the reflection of P on the real axis.



24. The effect of multiplying a complex number by I is equivalent to the point representing z by a right angle about the origin, anticlockwise direction.

25. The effect of multiplying a complex number by –i is equiva rotating the point representing z by a right angle about the origi clockwise direction.

26. For any polynomial equation f(x) =0 with real coefficient, im roots occur in conjugate pairs.

27. **DeMoivre's theorem :**

If n is an integer then $(\cos\theta + i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$.If n is r number then $(\cos n\theta + i\sin n\theta)$ is one of the values $(\cos\theta + i\sin\theta)^n$

28. If n is a real number then

(i) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

(ii) $(\cos\theta + i\sin\theta)^{-n} = \cos n\theta - i\sin n\theta$

(iii) $(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$

(iv) $(\cos\theta - i\sin\theta)^{-n} = \cos n\theta + i\sin n\theta$

29. **Euler's Formula :**

(i) $e^{i\theta} = (\cos\theta + i\sin\theta)$,

(ii) $e^{-i\theta} = (\cos\theta - i\sin\theta)$

30. *If* $z = x + iy$ *is a* complex number, then $(x+iy)^{p/q}$ has q values.

31. The n^{th} roots of unity are given by $e^{i\frac{2k\pi}{n}}$ where k=0,1,2,.....n-1

32. If ω is a complex n^{th} root of unity then $1+\omega+\omega^2+\dots\omega^{n-1}=0$

33. The cube roots of unity are 1, $\frac{-1\pm i\sqrt{3}}{2}$

34. All the cube roots of unity lie on the circumference of the unit circle and they form the vertices of an equilateral triangle.

35. The fourth root of unity are ± 1, $\pm i$.

36. The fourth root of unity form the vertices of a square all lying on the unit circle.

37. $(i)\ (1+i)^n - (1-i)^n = 2^{\frac{n+2}{2}} \sin\frac{n\pi}{4}$

$(ii)\ (1+i)^n + (1-i)^n = 2^{\frac{n+2}{2}} \cos\frac{n\pi}{4}$

38. $(i)\ (1+i\sqrt{3})^n + (1-i\sqrt{3})^n = 2^{n+2} \cos\frac{n\pi}{3}$,

$(ii)\ (1+i\sqrt{3})^n - (1-i\sqrt{3})^n = 2^{n+2} i \sin\frac{n\pi}{3}$

39. $(i)\ (\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1} \cos\frac{n\pi}{6}$,

$(ii)\ (\sqrt{3}+i)^n - (\sqrt{3}-i)^n = 2^{n+1} i \sin\frac{n\pi}{6}$

40. $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos n\left(\frac{\pi}{2}-\theta\right) + i\sin n\left(\frac{\pi}{2}-\theta\right)$

41. $\left(1+\cos\theta+i\sin\theta\right)^n + \left(1+\cos\theta-i\sin\theta\right)^n = 2^{n-1}\cos\frac{n\theta}{2}\cos\frac{n\theta}{2}$

42. $\left(\frac{\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^8 = 1 \left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^8 = 1$

43. If $\cos\alpha+\cos\beta+\cos\gamma = \sin\alpha+\sin\beta+\sin\gamma = 0$ then

$\sum\cos 3\alpha = 3\cos(\alpha+\beta+\gamma)$

$\sum\sin 3\alpha = 3\sin(\alpha+\beta+\gamma)$

5.ANALYTICAL GEOMETRY

Conic Sections

1. The curves obtained slicing a cone with a plane not passing throu vertex are called conics.

   They are (i) Circle

   (ii) Ellipse

   (iii) Parabola

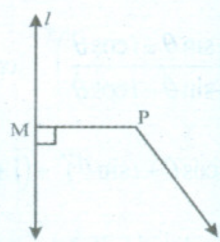
   (iv) Hyperbola

2. A conic is the locus of a point which moves in a plane so that its di from a fixed point bears a constant ratio to its distance from a straight line.

   The fixed point is called focus, the fixed straight line is called di and the constant ratio is called eccentricity.

   Focus is denoted F, directrix by *l* an eccentricity by e.

   Let P be the varying point of a conic then $\frac{FP}{PM} = e$ where M is the foot of perpendicular drawn from P to *l*.



3. Eccentricities :

   (i) e=0 for circle

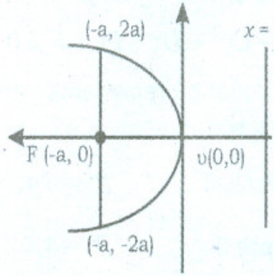
   (ii) 0<e<1 for ellipse

   (iii) e=1 for parabola

   (iv) e>1 for hyperbola

   (v) e=$\sqrt{2}$ for rectangular hyperbola

II. For the parabola $y^2 = -4ax$



(i) Focus : (-a, 0)

(ii) Vertex : (0, 0)

(iii) Axis : $y = 0$

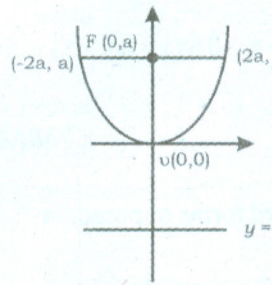
(iv) Tangent at the vertex : $x = 0$

(v) Directrix : $x = a$

(vi) Latus rectum = 4a

(vii) Ends of the latus rectum : (-a, -2a), (-a, -2a)

III. For the parabola $x^2 = 4ay$



(i) Focus : (0, a)

(ii) Vertex : (0, 0)

(iii) Axis : $x = 0$

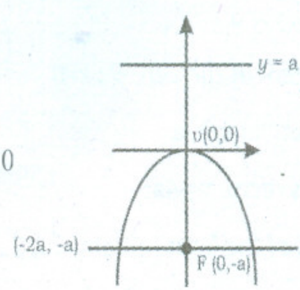
(iv) Tangent at the vertex : $y = 0$

(v) Directrix : $y = -a$

(vi) Latus rectum = 4a

(vii) Ends of the latus rectum : (2a, a), (-2a, a)

IV. For the parabola $x^2 = -4ay$



(i) Focus : (0, -a)

(ii) Vertex : (0, 0)

(iii) Axis : $x = 0$

(iv) Tangent at the vertex: y = 0

(v) Directrix : y = a

(vi) Latus rectum = 4a

(vii) Ends of the latus rectum : (2a, -a), (-2a, -a)

9. If the vertex of the parabola is (h, K) then the parabola which opens to right is of the form $(y-k)^2 = 4a(x-h)$.

Its axis is y=k

Focus is (h+a, k)

The directrix is x=h-a

The other forms are $(y-k)^2 = -4a(x-h)$

$$(x-h)^2 = 4a(y-k)$$

$$(x-h)^2 = -4a(y-k)$$

Standard result with respect to the parabola $y^2 = 4ax$

10. The equation to the chord joining (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{4a}{y_1 + y_2}(x - x_1)$$

11. The equation of the tangent at (x_1, y_1) is $yy_1 = 2a(x + x_1)$

12. The condition for y = mx+c to be a tangent to $y^2 = 4ax$ is $c = \frac{a}{m}$

13. The general form of the equation of the tangent to the parabola $y^2 = 4ax$ is $y = mx + \frac{a}{m}$

14. The point of contact of the tangent $y = mx + \frac{a}{m}$ with the parabola $y^2 = 4ax$ is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

15. Two tangents can be drawn from any point (x_1, y_1) to the parabola $y^2 = 4ax$ .is $(y^2 - 4ax)(y_1^2 - 4ax_1) = \left[yy_1 - 2a(x + x_1)\right]^2$

16. Equation of the normal at (x_1, y_1) is $xy_1 + 2ay = x_1y_1 + 2ay_1$

17. $(at^2, 2at)$ are the parametric coordinates of $y^2 = 4ax$ denoted by 't'.

18. The Equation of the chord joining t_1 and t_2 on the parabola $y^2 = 4a$

$y(t_1 + t_2) = 2x + 2at_1t_2$.

19. The equation of the tangent at 't' is $y = \frac{x}{t}$ (or) $yt = x + at^2$

20. The equation of the normal at 't' is $y + xt = 2at + at^3$.

21. The slope of the tangent at 't' is $m = \frac{1}{t}$.

22. The equation of the normal in terms of its slope is $y = mx - 2am - am$

23. Three normals can be drawn from any point to the parabola.

24. The equation to the chord of contact of tangents drawn from (x_1, y_1) t

the parabola $y^2 = 4ax$ is $yy_1 - 2a(x + x_1)$

25. The point of intersection of the tangents at t_1 and t_2 to the parab

$y^2 = 4ax$ at $[at_1t_2, a(t_1 + t_2)]$

26. The locus of the foot of the perpendicular from the focus on any tange to the parabola is the tangent at the vertex.

27. The locus of the point of intersection of perpendicular tangents to t parabola is the directrix.

28. If t_1, t_2 are the extremities of any focal chord then $t_1t_2 = -1$.

29. The tangents at the ends of any focal chord intersect at right angles on t directrix.

30. The condition for the line $lx + my + n = 0$ to be a normal to the parab

$y^2 = 4ax$ is $al^3 + 2alm^2 + m^2n = 0$

31. If the normal t_1 on the parabola $y^2 = 4ax$ meets the parabola again at t_2 then $t_2 = -t_1 - \frac{2}{t_1}$

32. The tangent and normal at a point P meets the axis at T and G respectively and F is the focus. Then FT=FG

33. The chord of contact of tangents from any point on the directix passes through the focus of the parabola.

34. If the chord of the parabola $y^2 = 4ax$ subtends a right angle at the vertex then the tangents at its extremities meet on the line x+4a=0

II. ELLIPSE

The Standard form of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a<b)

(i) Length of the major axis =2a

(ii) Length of the minor axis =2b

(iii) $b^2 = a^2(1-e^2)$

(iv) $e = \sqrt{\frac{a^2 - b^2}{a^2}}$ (e<1)

(v) The ellipse meets the x-axis at A(a, 0) , A'(-a,0). It meets the y- axis at B(0,b), B'(0,-b) A. A', B, B' are the vertices of the ellipse.

(vi) Foci are at (ae, 0); (-ae, 0)

(vii) The directrices are $x = \frac{a}{e}$, $x = \frac{-a}{e}$

(viii) Equation of the major axis is $y = 0$ (x-axis)

(ix) Equation of the minor axis $x=0$ (y-axis)

(x) The major and minor axes intersect at the centre (0, 0)

(xi) Ellipse is a central conic.

(xii) Length of the latus rectum $= \frac{2b^2}{a}$

(xiii) Ends of the latus rectum are $\left(ae, \pm\frac{b^2}{a}\right), \left(-ae, \pm\frac{b^2}{a}\right)$

(xiv) The foci lie on the major axis and the directrices are perpendicular to the major axis (or) parallel to the minor axis.

The other forms of ellipses.

$$(i).\ \frac{x^2}{b^2}+\frac{y^2}{a^2}=1, (\mathrm{a}>\mathrm{b})$$

$$(ii).\ \frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1\ (\mathrm{a}<\mathrm{b})$$

$$(iii).\ \frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1\ (\mathrm{a}>\mathrm{b})$$

1. The equation of the tangent at (x_1, y_1) on the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ *is* $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$

2. Equation of the normal at (x_1, y_1) is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$.

3. Condition for the line $y = mx+c$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

$c^2=a^2m^2+b^2$ (or) $c=\pm\sqrt{a^2m^2+b^2}$. The point of contact is $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$

4. For all real values of m, $y = mx \pm \sqrt{a^2m^2 + b^2}$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its point of contact is $\left(\frac{-a^2m}{\pm\sqrt{a^2m^2+b^2}}, \frac{b^2}{\pm\sqrt{a^2m^2+b^2}}\right)$

5. For The equation to the chord of contact of tangents from (x_1, y_1) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

6. The parametric equations of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $x = a\cos\theta, y = b\sin\theta$

7. The point $(a\cos\theta, b\sin\theta)$ is denoted by θ

8. The equation of the tangent at θ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

9. The equation of the chord at θ and ϕ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{x}{a}\cos\frac{\theta+\phi}{2} + \frac{y}{b}\sin\frac{\theta+\phi}{2} = \cos\frac{\theta-\phi}{2}$$

10. The equation of the normal at θ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$$

11. The Condition the line *lx+my+n*=0 to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $a^2l^2 + b^2m^2 = n^2$

12. Two tangents can be drawn from any point to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

13. The condition for the line $lx+my+n=0$ to be a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)^2}{n^2}$
14. Through any point in the plane of the ellipse, four normal can be drawn to it.The locus of the foot of the perpendicular from the focus to a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2+y^2=a^2+b^2$. This is called the auxiliary circle of the ellipse. It is the circle on the major axls as the diameter.
15. The locus of the point of intersection of perpendicular tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x^2+y^2=a^2+b^2$. This circle is called the director circle.
16. If F_1Y and F_2Y' are perpendiculars from the foci S and S' of a_1 ellipse on the tangent at any point then $F_1Y.F_2Y'=b^2$
17. The chord of contact of tangents drawn from any point on the directix of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through the corresponding focus.
18. The Sum of the focal distances of any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the length of its major axis. i.e., If S and S' and the foci and P is any point on the ellipse then $F_1P+F_2P=2a$.
19. The locus of a point such that the sum of its distances from two fixed points is a constant in an ellipse.

III.HYPERBOLA

1. The Standard form of the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

   (i) Length of the transverse axis =2a

   (ii) Length of the conjugate axis =2b

   (iii) $b^2=a^2(e^2-1)$

   (iv) $e=\sqrt{\frac{a^2+b^2}{a^2}}$ (e>1)

   (v) The vertices of the hyperbola are A(a, 0) , A'(-a,0).

   (vi) Foci are at (ae, 0), (-ae, 0)

   (vii) The directrices are $x=\frac{a}{e}$, $x=\frac{-a}{e}$

   (viii) Equation of the transverse axis is y=0 (x-axis)

   (ix) Equation of the conjugate axis x=0 (y-axis)

   (x) Transverse and conjugate axes intersect at the centre (0, 0)

   (xi) Hyperbola is a central conic.

   (xii) Latus rectum = $\frac{2b^2}{a}$

   (xiii) Ends of the latus rectum are $\left(ae,\pm\frac{b^2}{a}\right)\left(-ae,\pm\frac{b^2}{a}\right)$

   (xiv) The foci lie on the transverse axis and the directrices are perpendicular to the major axis

2. **The other forms of Hyperbola**

   (i) $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$

   (ii) $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

   (iii) $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$

3. The equation of the tangent at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

4. Equation of the normal at (x_1, y_1) is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$.

5. Condition for the line y=mx+c to be a tangent to the hyperb $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $c^2 = a^2m^2 - b^2$ (or) $c = \pm\sqrt{a^2m^2 - b^2}$. The point of cont is $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$

6. For all real values of m, $y = mx \pm \sqrt{a^2m^2 - b^2}$ is a tangent to the Hyperbo $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

7. The equation to the chord of contact of tangent from (x_1, y_1) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

8. The parametric equations of the Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $x = a\sec\theta, y = b\tan\theta$.

9. The point $(a\sec\theta, b\tan\theta)$ is denoted by θ

10. The equation of the tangent at θ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$

11. The equation of the normal at θ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$$\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2$$

12. The chord of contact of tangents from any point on the directrix of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ Passes through the corresponding focus.

13. The equation of the chord joining 'θ' and ϕ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{x}{a}\cos\left(\frac{\theta-\phi}{2}\right)-\frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right)=\cos\left(\frac{\theta+\phi}{2}\right)$

14. Asymptote is a straight line touching the curve at infinity but does not lie altogether at infinity.

15. The conditions for $ax^2+bx+c=0$ to have both roots infinite are a=0 and b=0.

16. The equation to the asymptotes of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ are $\frac{x}{a}+\frac{y}{b}=0$ and $\frac{x}{a}-\frac{y}{b}=0$.

17. The combined equation of the asymptotes to the hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

18. The asymptotes pass through the centre of the hyperbola.

19. The slope of the asymptotes are b/a and –b/a.

20. The asymptotes are equally inclined, to the transverse axis.

21. The transverses and the conjugate axes bisect the angles between the asymptotes.

22. The angle between the asymptotes is $2\tan^{-1}\left(\frac{b}{a}\right)$ or $2\sec^{-1}(e)$

23. The combined equation of the asymptotes and the hyperbola differ only by a constant.

24. If L=0 and L'=0 are the separate equations to the asymptotes then the equations to the asymptotes then the equation of the hyperbola is LL'=K .where K is a non-zero constant.
25. If the transverse and conjugate axes of a hyperbola are equal then it is a rectangular hyperbola.
26. The equation to the rectangular hyperbola is $xy = c^2$.
27. The locus of a point such that the difference of its distances from two fixed points is a constant is a hyperbola.
28. The difference of the focal distances of any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is equal to the length of the transverse axis. i.e., If F_1 and F_2 the foci and P is any point on the hyperbola then $F_1P - F_2P = 2a$
29. The tangent at any point P on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with centre C meets the transverse axis at T and PN is perpendicular on the transverse axis then CN.CT=a^3
30. The normal at the end of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ intersects the transverse axis at G. Then CG=a e^3.
31. The locus of the foot of the perpendicular from a focus on an asymptotes is the corresponding directrix.
32. The product of the perpendicular from any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes $1 \Big/ \frac{1}{a^2} + \frac{1}{b^2}$.

33. The tangent at any point P on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets the tangent at A. A' at L and M respectively. Then AL.A'M= b^2

34. The locus of the foot of the perpendicular from the focus on any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is the auxiliary circle $x^2+y^2=a^2$.

35. The locus of the point of intersection of perpendicular tangent of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is the director circle $x^2+y^2=a^2-b^2$.

36. The condition that the line lx+my+n=0 is a normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is t $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)^2}{n}$

37. P is a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. The ordinate at P meets the asymptotes in Q and Q' then QP.Q'P= b^2

38. The area of triangle formed by asymptotes and tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is a constant equal to ab square unit.

39. A chord PQ of the hyperbola intersect the asymptotes in P' and Q' then PP'=QQ'

40. The eccentricity of the rectangular hyperbola is $\sqrt{2}$.

41. The standard equation of the rectangular hyperbola referred to the coordinate axes as asymptotes is $xy=c^2$, where $c^2=\frac{a^2}{2}$, a is semi transverse axis.

42. If the centre of the rectangular hyperbola is (h , k) and the asymptotes are parallel to x and y axis then equation of the R.H is $(x-h)(y-k)=c^2$

43. The equation of the tangent at (x_1, y_1) to the Rectangular hype

$xy = c^2$ is $xy_1 + yx_1 = 2c^2$

44. The equation of the normal at (x_1, y_1) is $xx_1 - yy_1 = x_1^2 - y_1^2$.

45. Equation of the chord of contact of tangent to the rectangular hyper

$xy = c^2$ drawn from (x_1, y_1) is $xy_1 + yx_1 = 2c^2$

46. Parametric equation of the rectangular hyperbola are x=ct , y=c/t

47. The point (ct, c/t) is represented by 't'.

48. Equation of the tangent at 't' on $xy = c^2$ is $x + yt^2 = 2ct$.

49. Equation of the normal at 't' to the rectangular hyperbola $xy = c^2$ is

$$y - xt^2 = \frac{c}{t} - ct^3$$

50. Four normal can be drawn from any point to the rectangular hyperbola

$xy = c^2$.

51. If the normal at t_1 to the rectangular hyperbola $xy = c^2$ meets the curv

again at t_2 then $t_1^3 t_2 = -1$.

52. The area of the triangle formed by the tangent at any point of the R.H with

asymptotes is $2c^2$.

53. The condition for the line lx+my+n=0 to be a tangent to the rectangular

hyperbola $xy = c^2$ is $4c^2 lm = n^2$.

54. The tangent at any point of the rectangular hyperbola $xy = c^2$ makes

intercepts a,b and the normal at the point makes intercepts p, q on the

axes. Then ap + bq = 0.

asure:

dy moves x distance in time t along a straight line, then

(i) $v=\frac{dx}{dt}$ (ii) acceleration $\frac{d^2x}{dt^2}=\frac{dv}{dt}$

: (i) initial means $t=0$ (ii) Rest means $v=0$

nd y be two time variables such that $y=f(x)$ then $\frac{dy}{dt}=f(x)\frac{dx}{dt}$

nd Normal

y_1) be any point on the curve $y=f(x)$ and $m=f(x_1)$ then

ope of tangent at $(x_1,y_1)=m$

ope of normal at $(x_{1,}y_1)=-\frac{1}{m}$

uation of tangent at $(x_1,y_1):(y-y_1)=m(x-x_1)$

uation of normal at $(x_1,y_1):(y-y_1)=-\frac{1}{m}(x-x_1)$

angent at (x_1,y_1) is horizontal then,

pe of the tangent $=0$

uation of the tangent is $y=y_1$

uation of the normal is $x=x_1$

nt at (x_1,y_1) is vertical then,

pe of normal $=0$

uation of the tangent is $x=x_1$

uation of the normal is $y=y_1$

tween two curves:

etween the tangents at a point of intersection of two curves is same angle between the curves at the point of intersection.

6. DIFFERENTIAL CALCULUS-APPLICATIONS- I

I. Rate Measure:

1. If a body moves x distance in time t along a straight line, then

   (i) $v = \frac{dx}{dt}$ (ii) acceleration $\frac{d^2x}{dt^2} = \frac{dv}{dt}$

   Note: (i) initial means $t = 0$ (ii) Rest means $v = 0$

2. Let x and y be two time variables such that $y = f(x)$ then $\frac{dy}{dt} = f(x)\frac{dx}{dt}$

II. Tangent and Normal

1. Let (x_1, y_1) be any point on the curve $y = f(x)$ and $m = f(x_1)$ then

   (i) Slope of tangent at $(x_1, y_1) = m$

   (ii) Slope of normal at $(x_{1,}y_1) = -\frac{1}{m}$

   (iii) Equation of tangent at $(x_1, y_1) : (y - y_1) = m(x - x_1)$

   (iv) Equation of normal at $(x_1, y_1) : (y - y_1) = -\frac{1}{m}(x - x_1)$

2. If the tangent at (x_1, y_1) is horizontal then,

   (i) Slope of the tangent $= 0$

   (ii) Equation of the tangent is $y = y_1$

   (iii) Equation of the normal is $x = x_1$

3. If tangent at (x_1, y_1) is vertical then,

   (i) Slope of normal $= 0$

   (ii) Equation of the tangent is $x = x_1$

   (iii) Equation of the normal is $y = y_1$

III. Angle between two curves:

1. Angle between the tangents at a point of intersection of two curves is same as the angle between the curves at the point of intersection.

2. If m_1 and m_2 be the slopes of tangents at a point of intersectio curves $y=f_1(x)$ and $y=f_2(x)$ then the angle between the curve: by $\theta=\tan^{-1}\left(\frac{m_1-m_2}{1+m_1m_2}\right)$

The above formula is relevant only if m_1 and m_2 are real numbers.

3. If the slope be m and ∞ then $\theta=\tan^{-1}\left|\frac{1}{m}\right|$
4. If the slope be $-\infty$ and ∞ then $\theta=0$
5. Two curves are said to be orthogonal if they intersect at right angle.
6. If $m_1m_2=-1$ then the curves are orthogonal.
7. If $m_1=m_2$ then the angle between the curves is 0 and the curve: common tangent at the point of intersection. Hence, they touch other.

IV. Mean value theorems:

1. Rolle's theorem:

   Let $f(x)$ to be a real valued function.

   (i) Continuous in $[a,b]$

   (ii) Differentiable in (a,b)

   (iii) $f(a)=f(b)$

   then $f^{1}(c)=0$ for at least one value of c in between a and b.

2. Lagrange theorem:

   Let $f(x)$ to be a real valued function.

   (i) Continuous in $[a,b]$.

   (ii) Differentiable in (a,b)

   then $f'(c)=\frac{f(b)-f(a)}{b-a}$ for at least one value of c in between a and b.

3. Taylor series:

$$f(a+h)=f(a)+\frac{h}{1!}f'(a)+\frac{h^2}{1!}f''(a)+.....\infty$$

4. Maclaurin's series:

$$f(x)=f(0)+\frac{h}{1!}f'(0)+\frac{h^2}{1!}f''(0)+.....\infty$$

5. L-Hospital rule:

   If $f(a)=0$ and $g(a)=0$ then $\lim\limits_{x\to a}\frac{f(x)}{g(x)}=\lim\limits_{x\to a}\frac{f'(x)}{g'(x)}$

6. Composite function theorem:

   If $\lim\limits_{x\to a} g(x)=b$ and f is continuous at b, then $\lim\limits_{x\to a} f(g(x))=f(\lim\limits_{x\to a} g(x))$

V. Monotonic function:

1. $f(x)$ is said to be increasing if $f(x)$ increases as *x* increases.
2. $f(x)$ is said to be decreasing if $f(x)$ decreases as *x* increases.
3. A function that is increasing everywhere or decreasing every-where is called a monotonic function
4. Conditions:

   (i) For increasing function: $x_1 < x_2 \Rightarrow f(x_1) \le f(x_2)$

   Hence $f'(x) \ge 0$ (if $f'(x)$ exists)

   (ii) For strictly increasing function: $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

   Hence $f'(x) > 0$

   (iii) For decreasing function : $x_1 < x_2 \Rightarrow f(x_1) \ge f(x_2)$

   Hence $f'(x) \le 0$

   (iv) For strictly decreasing function: $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

   Hence $f'(x) < 0$

Maxima and Minima:

(i) A function f has an absolute maximum at c if

$f(c) > f(x)$ for all *x* in the domain of f.

(ii) A function f has an absolute maximum at c if

$f(c) \leq f(x)$ for all *x* in the domain of f.

(iii) A function f has a local maximum at $x = c$ if $f(c) \geq f(x)$ for all the value of x in a neighbourhood of c.

(iv) A function f has a local minimum at $x = c$, if $f(c) \leq f(x)$ for all the value of x in a neighbourhood of c.

(v) A point of local maximum(or) local minimum is called turning point of the curve $y = f(x)$. No function will attain local maximum or local minimum at the end points of its domain.

(vi) Maximum and minimum are commonly known as extremum.

(vii) A critical number of a function is a number c in the domain of f such that $f'(c) = 0$ or $f'(c)$ does not exist.

(viii) A point on the curve $y = f(x)$ is said to be stationary if $f'(x) = 0$ at the point.

(ix) **The extreme value problem:**

If *f* is continuous on a closed interval then f attains an absolute maximum and an absolute minimum in the interval.

(x) **First derivative test.**

a) As x increases through a local maximum $f'(c)$ changes from positive to negative.

b) As x increases through a local minimum $f'(c)$ changes from negative to positive.

(Xi) **Fermi's theorem:**

If $f(x)$ has a local extremum at c then $f'(c)=0$ if $f(x)$ exists at $x=c$.

(Xii) **Second derivative test:**

a) $f(x)$ is local maximum at $x=c$ if $f'(c)=0$ and $f'(c)<0$

b) $f(x)$ is local minimum at $x=c$ if $f'(c)=0$ and $f'(c)>0$

(Xiii) If $f(x)$ is a continuous function on a closed interval $[a,b]$ then

a) Absolute maximum $= Max\{f(a), f(b), criticalvalues\}$

b) Absolute minimum $= Min\{f(a), f(b), criticalvalues\}$

VI. Concavity, Convexity and Points of inflexion:

1. A curve that lies above its tangents is called 'concave'.
2. A curve that lies below its tangents is called 'convex'.
3. Test for concavity and convexity:
   (i) If $f'(x)>0$ then $y=f(x)$ is concave.
   (ii) If $f'(x)<0$ then $y=f(x)$ is convex.
4. Concave is also known as concave upwards (or) convex downwards.
5. Convex is also known as concave downwards (or) convex upwards.
6. A point on the curve that separates a concave and convex part is called a point of inflexion.
7. Test for point of inflexion:
   (i) A point of inflexion exists where $f'(x)$ changes its sign.
   (ii) A point of inflexion exists at $x=c$ if $f'(c)=0$ and $f''(c)\neq 0$

1. Let $y = f(x)$ be a differential function then we define that

   (i) $dx = \Delta x$ if $\Delta x \to 0$

   (ii) $dy = \underset{\Delta y \to 0}{Lt} \Delta y$

2. dx and dy are known as differentials,

3. $dy = \underset{\Delta y \to 0}{Lt} \Delta y = \underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} \Delta x = \frac{dy}{dx} dx = f'(x) dx$

4. Approximation: Δ is very small then $\Delta y = dy$

   $\therefore f(x+\Delta x) - f(x) = f(x) dx$

   Hence $f(x+\Delta x) = f(x) + f(x) dx$

5. Errors:

   (i) Absolute error $= \Delta f$

   (ii) Relative error $= \frac{\Delta f}{f}$

   (iii) Percentage error $= \frac{\Delta f}{f} \times 100$

I. Curve tracing:

1. Any first degree equation on x and y represents straight line.

2. Any second degree equation on x and y represents a conic.

3. Symmetry: Let $f(x, y) = 0$ represent a curve, then the curve is Symmetric about

   (i) x axis if $f(x, -y) = f(x, y)$

   (ii) y axis if $f(-x, y) = f(x, y)$

   (iii) Origin if $f(-x, -y) = f(x, y)$

(iv) $y=x$ if $f(y,x)=f(x,y)$

(v) $y=x$ if $f(-y,-x)=f(x,y)$

4. **Region:** The domain and range of $y=f(x)$ decides the region of existence of the curve.

5. **Nature:** By nature of a curve we mean the nature of

   (i) Openness of the curve

   (ii) Increasing or decreasing

   (iii) Concave or convex in the domain of curve

6. **Special point:** The points of importance of a curve are

   (i) Origin

   (ii) intercepts

   (iii) turning points

   (iv) points of inflexion

7. **Asymptotes:**

   (i) If $\lim_{x \to a} y=\infty$ then $x=a$ is an asymptote.

   (ii) If $\lim_{x \to \infty} a$ then $y=a$ is an asymptote.

8. Symmetry, region, nature, special points and asymptotes are important to trace a curve.

II. Partial differentiation:

1. If $u=f(x,y)$

   (i) $\dfrac{\partial u}{\partial x}=\underset{\Delta x \to 0}{Lt}\dfrac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$

   (ii) $\dfrac{\partial u}{\partial y}=\underset{\Delta y \to 0}{Lt}\dfrac{f(x,y+\Delta y)-f(x,y)}{\Delta y}$

2. (i) $\frac{\partial^2 u}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$ (ii) $\frac{\partial^2 u}{\partial x.\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$

(iii) $\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)$ (iv) $\frac{\partial^2 u}{\partial x.\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)$

3. **Homogeneous function:**

If $f(tx,ty)=t^n f(x,y)$ then f is said to be a homogeneous function of degree n of x and y.

4. **Euler's theorem:**

If $f(tx,ty)=t^n f(x,y)$ then $x\frac{\partial u}{\partial x.}+y\frac{\partial u}{\partial y}=nf$

5. If $f(tx,ty)=t^n f(x,y)$ then (i) $x\frac{\partial^2 u}{\partial x^2.}+y\frac{\partial^2 u}{\partial y.\partial x}=(n-1)\frac{\partial u}{\partial x}$

(ii) $x\frac{\partial^2 u}{\partial x.\partial y}+y\frac{\partial^2 u}{\partial y^2}=(n-1)\frac{\partial u}{\partial y}$

6. **Chain Rules:**

(i) If $u=f(x,y), x=g(t), y=h(t)$, then $\frac{du}{dx}=\frac{\partial u}{\partial x}.\frac{dx}{dt}+\frac{\partial u}{\partial y}.\frac{dy}{dt}$

(ii) If $u=f(x,y,z), x=g(t), y=h(t)$ and $z=s(t)$ then

$$\frac{du}{dt}=\frac{\partial u}{\partial x}.\frac{dx}{dt}+\frac{\partial u}{\partial y}.\frac{dy}{dt}+\frac{\partial u}{\partial z}.\frac{dz}{dt}$$

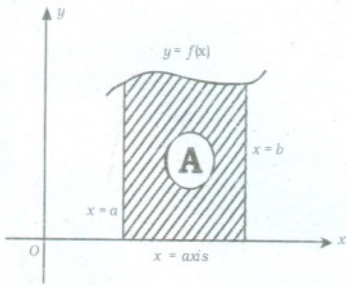
(iii) If $w=f(u,v), u=g(x,y), v=h(x,y)$ then

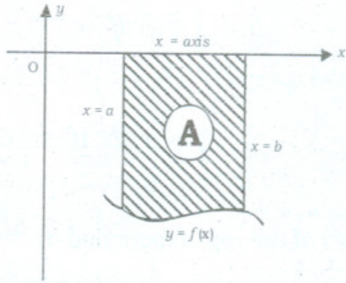
(a) $\frac{dw}{dt}=\frac{\partial w}{\partial x}.\frac{dx}{dt}+\frac{\partial w}{\partial y}.\frac{dy}{dt}$

(b) $\frac{dw}{dy}=\frac{\partial w}{\partial u}.\frac{du}{dy}+\frac{\partial w}{\partial v}.\frac{dv}{dy}$

8. APPLICATION OF INTEGRAL CALCULUS

I. Area:

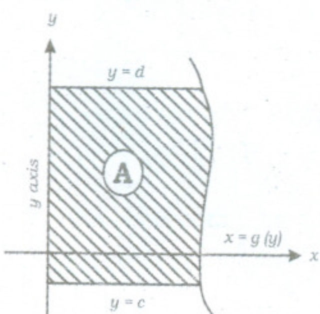


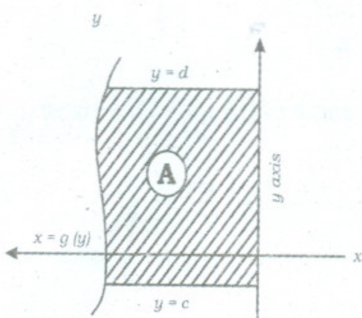


The area of the region bounded by the curve $y = f\left(x\right)$, X-axis $x = a$ and $x = b$ is given by $A = \int_{x=a}^{x=b} |y| dx$

$$A = \begin{cases} \int_{x=a}^{x=b} y\,dx \text{ if } y \geq 0 \\ \int_{x=a}^{x=b} (-y)\,dx \text{ if } y \leq 0 \end{cases}$$

The area of the region bounded by the curve $x = g\left(y\right)$, Y-axis $y = c$ and $y = d$ is given by



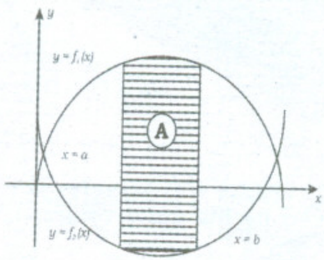


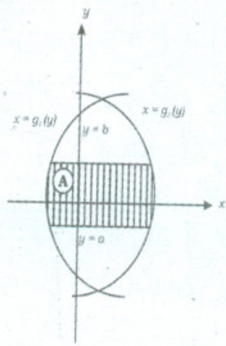
$$A = \int_{y=d}^{y=c} |x| dx$$

$$A = \begin{cases} \int_{y=c}^{y=d} x\,dx \text{ if } x \geq 0 \\ \int_{y=c}^{y=d} (-x)\,dx \text{ if } x \leq 0 \end{cases}$$

2. The area of the region bounded by the curve $y = f_1(x)$ and $f_2(x)$ in between $x = a$ and $x = b$ is given by

   The area of the region bounded by the curve $x = g_1(x), x = g_2(x)$ in between $y = c$, $y = d$ is given by $A = \int_{x=c}^{x=d} |g_1(y) - g_2(y)| dy$





II. Length of Arc:

1. The length of the arc of the curve $y = f(x)$ in between $x = a$ and $x = b$ is given $S = \int_{y=a}^{y=b} \sqrt{1 + \left\{\frac{dy}{dx}\right\}^2} dx$

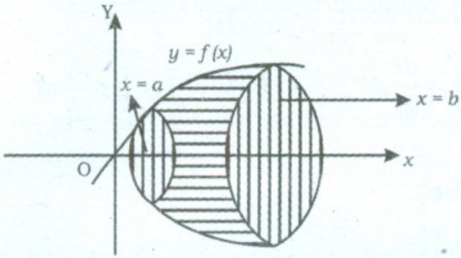
2. The length of the arc of the curve $x = g(y)$ in between $y = c$

and $y = d$ is given by $S = \int_{y=c}^{y=d} \sqrt{1+\left\{\frac{dy}{dx}\right\}^2}\, dy$

3. The length of the arc of the curve $x = \phi(t)$ and $y = \psi(t)$ in between $t = t_1$

and $t = t_2$ is given by $S = \int_{t=t_1}^{t=t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

II. Volume and Surface area:



1. When the region bounded by y-axis, and on the sides by the lines $x = a$ and $x = b$ revolves about x-axis solid is generated. The volume and the surface area of the solid area given by

$$V = \int_{x=a}^{x=b} \pi y^2 dx$$

$$S = \int_{y=c}^{y=b} 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

2. The volume and the surface area generated by resolving the region bounded by $y = g(x)$,y-axis y = c and y = d about y-axis are given by

$$V=\int_{x=c}^{x=d}\pi y^2 dy$$

$$S=\int_{y=c}^{y=d}2\pi x\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dy$$

3. For parametric form

$$V=\int_{t=t_1}^{t=t_2}\pi y^2\frac{dx}{dt}dt \quad \text{(or)} \quad V=\int_{t=t_1}^{t=t_2}\pi x^2\frac{dy}{dt}dt$$

$$S=\int_{t=t_1}^{t=t_2}2\pi y\sqrt{\left(\frac{dx}{dy}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt \quad \text{(or)} \quad S=\int_{t=t_1}^{t=t_2}2\pi x\sqrt{\left(\frac{dx}{dy}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$A=\int_{t=t_1}^{t=t_2}y\frac{dx}{dt}dt \quad \text{(or)} \quad A=\int_{t=t_1}^{t=t_2}x\frac{dy}{dt}dt$$

Trigonometric values:

$$\cos\pi=-1,\cos 2\pi=1,\cos 3\pi=-1,\cos 4\pi=1,\ldots\ldots..$$

$$\cos n\pi=(-1)^n\rightarrow\cos 2n\pi=1$$

$$\sin\pi=\sin 2\pi=\sin 3\pi.........\sin n\pi=0$$

$$\sin n\pi=0,\rightarrow\sin 2n\pi=1$$

Bernoulli's formula

$$\int uvdx=uv_1-u'v_2+u''v_3.....$$

9. DIFFERENTIAL EQUATIONS

1. Ordinary differential equation is a differential equation involving derivatives with respect to single independent variable.

2. A partial differential equation is a differential equation involving partial derivatives with respect to two or more independent variables.

3. Order of the differential equation is the order of the highest order derivative appearing in it.

4. The degree of a differential equation is the degree of the highest derivative, provided the derivatives are made free from radicals and fractional or negative indices.

5. The general solution of a differential equation is a solution in which the number of arbitrary constants is the same as the order of the differential equations.

6. An equation of the form $f_1(x)g_1(y)dx + f_2(x)g_2(y)dy = 0$ is of the variables separable type.

7. Equations of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ where $f(x,y)$ and $g(x,y)$ are homogeneous functions of the same degree in x and y is called a homogeneous first order differential equation.

8. If we put $y = vx$, a homogeneous first order differential equation will be reduced to variables separable type in v and x.

9. $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ is a homogeneous differential equation.

10. A differential equation is said to be linear when the dependent vari and its derivatives occur only in the first degree and no product of t occur.

11. Differential equations of the form $\frac{dy}{dx}+Py=Q$ where P and Q are function of $\underline{x}$ only.

    $I.F=e^{\int pdx}$: Solution: $y(I.F)=\int Q(I.F)dx+c$

12. The general form of a second order linear differential equation is $(aD^2+bD+c)y=Q(x)$. It is usually denoted by $f(D)y=Q(x)$.

13. The characteristic equation is $ap^2+bp+c=0$

    (i) If the roots of the A.E. are real and different say p_1, p_2 then $C.F=Ae^{p_1x}+Be^{p_2x}$.

    (ii)If the roots of the A.E are equal say P then $C.F=e^{px}(Ax+B)$.

    (iii) If the roots of the A.E are imaginary say $\alpha\pm\beta i$ then

    $C.F=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

14. $P.I=\frac{Q(x)}{aD^2+bD+c}$

15. **Type- I**

    Let $Q(x)=e^{\alpha x}$

    (i) If α is not equal to any of the roots of the A.E. (i.e.) $\alpha\neq p_1\neq p_2$ then

    $$P.I=\frac{e^{\alpha x}}{a\alpha^2+b\alpha+c}$$

    (ii) If α is equal to any of the roots of the A.E. then put x in the numerator for the factor which vanishes when $D=\alpha$ and put $D=\alpha$ in the other factor. $\frac{1}{D-\alpha}e^{\alpha x}=xe^{\alpha x}$.

(iii) If α is equal to both of the roots of the A.E. then put $\frac{x^2}{2}$ in the numerator for the factors which vanish when $D=\alpha$.

(i.e.) $\frac{1}{D^2-\alpha^2}e^{\alpha x}=\frac{x^2}{2}e^{\alpha x}$

16. Alternative method: If $f(D)y=e^{\alpha x}$ then

$$P.I=\begin{cases}\frac{e^{\alpha x}}{f(\alpha)}, f'(\alpha)\neq 0\\ \frac{xe^{\alpha x}}{f(\alpha)}, f(\alpha)=0, f'(\alpha)\neq 0\\ \frac{x^2e^{\alpha x}}{2}, f(\alpha)=0, f'(\alpha)=0\end{cases}$$

17. **Type-II:**

(i) If $Q(x)=\sin\alpha x(or)\cos\alpha x$ then

$$P.I=\frac{\sin\alpha x}{aD^2+bD+c}, D^2=-\alpha^2 \text{ (or) } \frac{\cos\alpha x}{aD^2+bD+c}, D^2=-\alpha^2$$

(ii) If the D.E is of the form $(D^2+\alpha^2)y=\sin\alpha x(or)\cos\alpha x$

then $P.I=\frac{\sin\alpha x}{D^2+\alpha^2}$

$$=I.P\frac{e^{i\alpha x}}{(D^2+\alpha i)(D-\alpha i)}=\frac{x}{2\alpha}\cos\alpha x$$

$\therefore P.I=\frac{\sin\alpha x}{D^2+\alpha^2}$

$$=R.P\frac{e^{i\alpha x}}{(D+\alpha i)(D-\alpha i)}=\frac{x}{2\alpha}\sin\alpha x$$

18. Alternative method: If $f(D^2)y = \sin\alpha x (or) \cos\alpha x$ then

$$P.I = \begin{cases} \dfrac{\sin\alpha x (or) \cos\alpha x}{D^2+\alpha^2}, f(\alpha^2) \neq 0 \\ \dfrac{x}{2\alpha}\int \sin\alpha x (or) \cos\alpha x dx, f(\alpha^2) = 0 \end{cases}$$

19. **Type III:**

Method-I: Let $Q(x) = x(or)x^2$

$$P.I = \frac{Q(x)}{f(D)} = \frac{Q(x)}{K[1 \pm g(D)]} = \frac{1}{K}[1 \pm g(D)]^{-1} Q(x)$$

Use Binomial expansion for $[1 \pm g(D)]^{-1}$ and operate it on $Q(x)$ just 2 3 terms is enough.

Method-II: Let $Q(x) = x$

$P.I = c_0 + c_1(x)$ is also a solution.

Let $Q(x) = x^2$

$P.I = c_0 + c_1(x) + c_2(x^2)$ is also a solution.

Notation: If z = f(x, y) then we denote $\frac{\partial z}{\partial x}$ *as* p, $\frac{\partial z}{\partial y}$ *as* q, $\frac{\partial^2 z}{\partial x^2}$ *as* r, $\frac{\partial^2 z}{\partial x \partial y}$ *as* s, $\frac{\partial^2 z}{\partial y^2}$ *as* t.

10. DISCRETE MATHEMATICS

1. N - Set of all natural numbers = {1, 2, 3 ...}

   W – Set of all whole numbers = {0,1,2,3, ...}

   Z – Set of all integers = {0, $\pm$ 1, $\pm$ 2, ...}

   Z^+ - Set of all positive integers

   Q – The set of all rationales

   Q^+ - The set of all irrationals

   R - The set of all real numbers

   R' - The set of all non-zero real numbers

   R^+ - The set of all positive reels

   C - The set of all complex numbers

   C^+ - The set of all non-zero complex numbers

2. **Closure axiom:**

   Binary operation on G is a map from $G \times G$ into G .If * is a binary operation then: $\forall$ a, b$\in G$, a*b$\in G$ This axiom can also be called as the closure property of G with respect to $*$.

3. **Associative axiom** (a*b)*c=a*(b*c), $\forall$ a, b$\in G$,

4. **Identity axiom**: If there exits an element of the form G , such that a*e=a=e*a $\forall$ a$\in G$ then e is called identity.

5. **Commutative axiom:** * is commutative if a*b=b*a $\forall$ a,b$\in G$

6. **Semi group:** Closure + associative

7. **Monoid:** Closure + associative + Existence of identity

8. **Group:** Closure +associative + identity + Inverse

9. **Abelian group:** Group + commutativity

10. (i) (N, +) is a semi group but not a monoid.

(ii) (N, .)is a monoid but not a Group.

(iii) (2Z, +) is a group

(iv) Set of odd integers is not a group wart. +

(v) (Z, +) is an abelian group of infinite order.

(vi) The order of a group is the number of elements present in the gr

vii) Every group is a monoid

(viii) Every group is a semigroup

(ix) Every monoid is a semigroup

(x) The converse of the above three results are false

(xi) $(Q,+)$ is an abelian group.

(xii) $(Q^*,.)$ is an abelian group.

(xiii) $(R,+)$ is an abelian group

(xiv) $(R^*,+)$ is an abelian group

(xv) $(C,+)$ is an abelian group

(xvi) $(C^*,.)$ is an abelian group

(xvii) $(Z,.)$ is not a group

(xviii) The set of all 2x2 nonsingular matrices form a non abelian group of infinite order with respect to matrix multiplication

(xix) $(Z_n, +_n)$ is an abelian group $\forall$ n.

(xx) $(Z_n, *)$ is not always a group.

(xxi) $(Z_n, +_n)$ is a group if n is a prime number

(xxii) The set of all 2×2 matrices M of the form $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$, $x \in R - \{0\}$ is an abelian group of infinite order with respect to matrix multiplication.

11. Every group of prime order is abelian .

12. Every group of order 4 is abelian.

13. The least non abelian group is of order 0.

14. The least positive integer n satisfying a^n =e is called the order of a, denoted by O(a)

15. Let G be a group and O(a)=n. For any integer $ma^m = e \Leftrightarrow$ n divides m

16. O(a) = O(a^{-1})

17. The identity is the only element with order 1.

18. If $a * a = a$ then $a = e$.

19. In a group G $\forall\, a, b \in G, (a*b)^2 = a^2 * b^2$ if G is abelian.

20. If $a^2 = e, \forall\, a \in G$, then every element has its own inverse.

21. If every element of a group G is of order 2(except the identity) then G is abelian.

22. If every element of a group G is its own inverse, then G is abelian.

23. In a group of even order there exist at least one elementa different such that $a^2 = e$

24. The identity element of group is unique.

25. The inverse of every element is unique.

26. If a, b, c are elements of a group (G, *) then

$a*b = a*c \Rightarrow b = c(\text{left cancellation})$

$b*a = c*a \Rightarrow b = c(\text{Right cancellation})$

27. In a group $a*a = a \Rightarrow a = e$

28. In a group the equation $a*x = b$ and $y*a = b$ have unique solution. The solution are $x = a^{-1}*b$ and $y = b*a^{-1}$

29. $(a*b)^{-1} = b^{-1}*a^{-1}$

30. $(a^{-1})^{-1} = a$

31. The order of every element of a finite group is a divisor of the order of the group.

32. Hence if $(G,*)$ is a group of order m where m is Prime number O(A)=m for all $a \neq e, a \in G$

Mathematical Logic

33. A statement or a proposition is a sentence which is either true or false but not both.

34. The truth or falsity of a statement is called the truth value.

35. If a statement is true, the truth value is T. If it is false, the truth value is F.

36. A statement is said to be simple if it cannot be broken into two or more statements.

37. If a statement is a combination of two or more simple Statement then it is called a compound statement.

38. If two simple statement p and q are connected by the word and denoted by '$\wedge$' the resulting compound Statement $p \wedge q$ is called a conjunction.

39. If two simple statement p and q are connected by the word and denoted by 'V' the resulting compound statement p Vq is called a disjunction.

40. The negation of a simple statement p is denoted by $\neg p$

41. A table that shows the relation between the truth values of a compound statement and the truth values of its sub statements is called 'Truth table'.

42. If the compound statement is made up of n sub statements, then its truth table will contain 2^n rows.

43. Two compound statement A and B are said to be logically equivalent if they have identical last columns in their truth tables.

44. The statement of the form "if p then q" are called conditional statements. If is denoted by $p \rightarrow q$

45. The compound statement $(p \rightarrow q) \wedge (q \rightarrow p)$ is called a bi-conditional statement.

46. A statement is said to be a tautology if the last column of its truth table contains only T.

47. A tautology is a statement; it is true for all logical possibilities.

48. A statement is said to be contradiction if the last column of its truth table contain only F.

49. A contradiction is false for all logical possibilities.

50. A statement which can be both true and false is called a paradox.

11. PROBABILITY

1. Random variable is a real valued function from the Sample Space S
   $(X : S \rightarrow R) \in (-\infty, \infty)$

2. A Random variable x is said to be discrete if it takes a finite number values (or) countably infinite number of values.

3. If a random variable which take uncountable infinite values or all va an interval is called a continuous

4. Let X be a discrete random variable x_1 which take $x_1, x_2, x_3, \ldots$
   Let $P[X - x_i] = P(x_i)$ the probability of x_i, then the function P is called the probability mass function of X if the numbers $p(x)$ satisfy the conditions

   $$(i)\ P(x_i) \geq 0 \quad \forall i$$

   $$(ii)\ \sum_{0=1}^{\infty} P(x_i) = 1$$

   If $x_1 < x_2 < x_3 < \ldots < x_i$ then

   $$P[X = x_i] = P[x = x_1] + P[x = x_2] + P[x = x_3] + \ldots$$

   $$P[x > x_i] = 1 - P[x \leq x_i]$$

   $$P[x \leq x_i] = 1 - P[x > x_i]$$

5. A function f defined for all is called the p.m. of a continuous random variable

   $$(i)\ f(x) \geq 0\ \forall x \in (-\infty, \infty)$$

   $$(ii) \int_{-\infty}^{\infty} f(x)dx = 1$$

6. If x is continuous then P(X=x) =0 for all x.

7. If X is continuous P $(a \leq X < b)$ =P $(a \leq X \leq b)$ =P $(a < X \leq b)$

8. If X is continuous then P $(a<X\leq b) = \int_a^b f(x)dx$

9. A function F(x) defined as $F(x)=P(X\leq x)$ is called the distribution function of X.

10. (i) If X is discrete $F(x) = \sum_{-\infty}^{x} P(x_i)$

(ii) If X is continuous then $f(x) = \int_{-\infty}^{x} f(x)dx$

(iii) F (b)-F (a) =P $(a\leq x\leq b)$

11. (i) F(x) is non-decreasing function of X

(ii) $0\leq F(x) \leq 1, -\infty < x < \infty$

(iii) $F(-\infty) = \infty \rightarrow \lim_{x\rightarrow-\infty} F(x) = 0$

$F(\infty) = 1 \rightarrow \lim_{x\rightarrow-\infty} F(x) = 1$

(iv) F′(x) = f(x)

12. If X is discrete then $E(x) = \sum_{i=1}^{\infty} x_i p(x_i)$

If X is continuous then $E(x) = \int_{-\infty}^{\infty} xf(x)dx$

13. If X is discrete $E(\phi(x_i)) = \sum_{i=1}^{n} \phi(x_i)\, p(x_i)$

14. If X is continuous then $E(\phi(x)) = \int_{-\infty}^{\infty} \phi(x) f(x)dx$

15. $E(X^r) = \sum x_i^r\, p(x_i)$ if X is discrete

$\int_{-\infty}^{\infty} x^2 f(x)dx$ if x is continuous

16. $E(x^r)$ is the r^{th} moment about the origin. It is also denoted by μ_r'

17. r^{th} Central moment about the mean is $\mu_r = E(x-\mu)^r$

18. $E(X-\overline{X})=0$

19. Var(x) = μ_2 = μ_2^1 - $(\mu_1)^2$ = $E(X^2)-[E(X)]^2$

20. E(X+Y) =E(X) +E(Y)

21. E(C) =C when C is constant.

22. E(X) = $\overline{X}$

23. E (aX$\pm$ b)=aE(X)$\pm$ b

24. Var(X)= E(X^2)-$[E(X)]^2$

25. Var (aX) = a^2 var(X)

26. Var $(ax \pm b)$ = a^2 $Var(X)$

27. Var(C)=0

Binomial distribution:

28. A random experiment whose outcomes can be classified into two categories usually called success and failure is called a Bernoulli's trial.

29. We may choose to define either one as a success.

30. P(X = x)= $nC_x P^x q^{n-x}$, x = 0, 1, 2, 3...n

31. The parameters are n and P

32. The moment generating function $M_x(t)=(q+pe^t)^n$

    The mean = np

    Variance = npq

33. The probabilities of the random variables taking Values x= 0, 1, 2...n are given by the terms in the Binomial expansion of $(q+p)^r$.

34. p > 0, q > 0 and p +q=1

35. If X is the R.V. having binomial distribution with Parameters n & p we denote $X \sim B(n,p)$

36. Variance of the binomial distribution is always less than the mean.

37. $\mathrm{S.D} = \sqrt{npq}$

Poisson distribution:

38. Poisson distribution is a limiting case of the binomial Distribution under
    i) n the number of trails is indefinitely large i.e. $n \to \infty$
    ii) $p \to 0$
    iii) np= λ is finite.
39. Poisson distribution was discovered by the French mathematician and physicist Simeon Denis Poisson (1781-1804) who published it in 1837.
40. Poisson distribution is discrete distribution.
41. $p(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}$,x=0, 1, 2...
42. λ is the parameter of the distribution. The moments Generating function $M_X(t)=e^{-\lambda(e^t-1)}$
43. Mean = λ, variance= λ, S.D = $\sqrt{\lambda}$
44. For Poisson distribution , $E(X^2)=\lambda^2+\lambda$
45. Poisson distribution is related to rare events.

Normal distribution:

46. Discovered by the English mathematician DeMoivre (1667-1754) in 1733.
47. It is a limiting case of the binomial distribution.
48. French Mathematician Laplace (1749-1827) applied it in Natural and Social Sciences.
49. Also known as Gaussian distribution in honors of Karl Friedrich Gauss
50. A continuous random variable X is said to follow Normal distribution. With mean μ and standard deviation σ, if its probability density function is given by $f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ ....(1) the mean μ and σ the standard deviation are called the parameters of normal distribution.

    A random variable X with mean and variance σ^2 and the following normal law (i)is expressed by $X \sim N(\sigma^2, \mu)$

Properties of normal distribution:

51. (i) The normal curve is bell shaped and symmetric about the line $x=\mu$

(ii) Mean, Median and mode of the distribution coincide. Thus Mean=Median=Mode= μ

(iii) As x increases numerically (x) decreases rapidly. The maximum probability occurs at the point $x=\mu$ and is given by $[p(x)]_{\max}=\frac{1}{\sigma\sqrt{2\pi}}$

(iv) X-axis is an asymptote to the curve.

(v) It has only one mode at $x=\mu$

(vi) Since the curve is symmetrical, skewness is zero

(vii) The points of inflection of the normal curve are at $x=\mu\pm\sigma$

(viii) Area property

$$P(\mu-\sigma<X<\mu+\sigma)=0.6826$$

$$P(\mu-2\sigma<X<\mu+2\sigma)=0.9545$$

$$P(\mu-3\sigma<X<\mu+3\sigma)=0.9973$$

Standard Normal Distribution:

52. A random variable x is called a standard random variable if its mean is zero and standard deviation is unity.

The Probability density function of the standard normal variety Z is given by $\phi(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}} : -\infty<z<\infty \text{ where } z=\frac{x-\mu}{\sigma}$

53. The standard normal distribution is usually denoted by $Z\sim N(0,1)$

54. The total area under the normal probability curve is unity. That is,

$$\int_{-\infty}^{\infty} f(x)\,dx=\int_{-\infty}^{\infty}\phi(z)\,dz=1$$

$$\int_{-\infty}^{0}\phi(z)\,dz=\int_{0}^{\infty}\phi(z)\,dz=0.5$$

55. Normal distribution is a limiting form of the binomial Distribution under the following conditions.

(a) n, the number trails is indefinitely large $n \to \infty$

(b) Neither p are q is very small.

56. Normal distribution can also be obtained as a limiting form of Poisson distribution with parameters $\lambda \to \infty$

Probability:

- The conditional probability of an event E, given that occurrence of the event F is given by

$$P(E/F) = \frac{P(E \cap F)}{P(F)}, P(F) \neq 0$$

- $0 \leq P(E/F) \leq 1, P(E/F) = 1 - \overline{P(E/F)}$

$$P(E \cup F/G) = P(E/G) + P(F/G) - P(E \cap F/G)$$

- $P(E \cap F) = P(E)P(F/E) \cdot P(E) \neq 0$

$$P(E \cap F) = P(F)P(E/F) \cdot P(E) \neq 0$$

- Theorem of total probability

Let $\{E_1, E_2, ..., E_n\}$ be a partition of a space and suppose that each of $E_1, E_2, ..., E_n$ has non zero Probability. Let A be any event associated with S.

Then $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + ... P(E_n)P(A/E_n)$

Baye's Theorem:

If $E_1, E_2, ..., E_n$ are events which constitute a partition of a sample space. If $E_1, E_2, ..., E_n$ are Pair wise disjoint $E_1 \cup E_2 \cup ... \cup E_n = S$ and A be any event with non-zero probability, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_j)(A/E_j)}$$

12. STATISTICS

- Range Quartile deviation, mean deviation, variance, Standard deviation are measures of dispersion.
- Range=Maximum Value - Minimum value Mean deviation for ungrouped data

$$M.D.(\bar{x}) = \frac{\sum (x_i - \bar{x})}{n},$$

$$M.D(\mathrm{M}) = \frac{\sum (x_i - M)}{n}$$

- Mean deviation for grouped data

$$M.D.(\bar{x}) = \frac{\sum \mathrm{f}_i (x_i - \bar{x})}{N}, \qquad where\ N = \sum \mathrm{f}_i$$

$$M.D(\mathrm{M}) = \frac{\sum \mathrm{f}_i (x_i - M)}{N}.$$

- Variance and Standard deviation for ungrouped data

$$\sigma^2 = \frac{1}{n} \sum (x_i - \bar{x})^2$$

$$\sigma = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$$

- Variance and Standard deviation of a discrete frequency distribution.

$$\sigma^2 = \frac{1}{n} \sum \mathrm{f}_i (x_i - \bar{x})^2$$

$$\sigma = \sqrt{\frac{1}{n} \sum \mathrm{f}_i (x_i - \bar{x})^2}$$

- Shortcut method to find variance and standard deviation

$\sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i y_j{}^2 - (\sum f_i y_j{}^2)\right]$ and $\sigma = \frac{h}{N}\left[\sqrt{N\sum f_i y_j{}^2 - (\sum f_i y_j{}^2)}\right]$

Co-efficient of variation (C.V) $\frac{\sigma}{x} \times 100.\bar{x} \neq 0$

13. RELATIONS AND FUNCTIONS

- **Ordered pair:** A pair of elements grouped together in a particular order.
- **Cartesian Product:** $A \times B$ of two sets A and B is given by

$$A \times B = (a,b), a \in A, b \in B$$

- In particular $R \times R = \{(x,y) : x, y \in R\}$ and $R \times R \times R = \{(x,y,z) : x,y,z \in R\}$
- If $(a, b) = (x, y)$, then a $= x$ and $b = y$
- If n(A) = p and n(B) = q then $n(A \times B) = pq$
- $A \times \phi = \phi$
- If general $A \times B \neq B \times A$
- **Relation :** A relation R from a set A to a set B is a subset of the Cartesian product $A \times B$ obtained by describing a relationship between the first element x and he second element y of the ordered pairs $A \times B$
- **Image:** The image of an element x under a relation is given by y where $(x,y) \in R$
- **Domain:** The domain of R is the set of all first elements of the ordered pairs in a relations R
- **Function:** A function f from the set A to a set B is a specific type of relation for which every element in set A has one and only one image *y* in set B. We write $f : A \to B$ where $f(x) = y$
- **Range:** The range of the relation R is the set of all second elements of the ordered pairs is a relation R
- A is the domain and B is the co-domain of f
- The range of the function is the set of images.
- A real function has the set of real numbers or the set of real numbers or one of its subsets of both as its domain and as its range.
- Empty relation is a relation R in x given by $R = \phi \subset X \times X$

- Universal relation is a relation R in x given by $R = X \times X$
- Reflexive relation is a relation R in X is a relation with $(a,a) \in R, \forall\, a \in R$
- Symmetric relation R in X is a relation satisfying $(a,b) \in R$ implies $(b,a) \in R$
- Transitive relation R in x is a relation satisfying $(a,b) \in R$ and $(b,c) \in R$ implies that $(a,c) \in R$
- Equivalence relation R in x is a relation which is Reflexive, symmetric and transitive.
- A function $f : X \to Y$ in one – one (or) injective if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$, $x_1, x_2 \in X$
- A function $f : X \to Y$ is onto (or) surjective if given by any $x \in X$, $y \in Y$ such that f(x)=y
- A function $f : X \to Y$ is one-one and onto (or) bijective $f^{-1}(y) = x$ if $\forall\, x \in X,\ y \in Y$
- The composition of functions $f : A \to B$ and $g : B \to C$ is the function $h : A \to C$ given by $h(x) = g(f(x))$
- A function $f : X \to Y$ is invertible if $g : Y \to X$ such that $g \cdot f = I_x$ and $f \cdot g = I_y$
- A function $f : X \to Y$ is invertible if and only if f is one to one and onto.
- A binary operation * on a set A is a function from $A \times A$ to A
- An element $e \in X$ is the identity element for binary operation $f : X \times X \to X$, if a*e = a = e*a, $\forall a \in X$
- An element $e \in X$ is invertible for binary operation $* : X \times X \to X$, if there exists $b \in X$ such that a*b=e= b*a where e is the identity for binary operation. The element b is called inverse of a and is denoted by a^{-1}
- An operation * on *x* is commutative if a*b = b*a, a, b in *x*
- An operation * on *x* is associative if (a*b)*c=a*(b*c), $\forall$ a, b, c is *x*

14. SETS

- A set is well-defined collection of objects.
- A set which does not contain any element is called empty set
- A set which consist of a finite number of element is called finite set, otherwise, the set is called infinite set.
- Two sets A and B are said to be equal if they have exactly the same elements.
- A set A is said to be subset of a set B, if every element of A is also an element of B. Intervals are subset of R
- A power set of a set A is collection of all subsets of A. It is denoted by P(A).
- The union of two sets A and B is the set of all those elements which are either in A or in B.
- The intersection of two sets is the set of all elements which are common. The difference of two sets A and B in this order is the set of elements which belongs to A but not to B
- The complement of a subset A of universal set U of the set of all elements of U which are not the element of S
- For any two sets A and B, $(A \cup B) = A' \cap B'$ and $(A \cap B) = A' \cup B'$
- If A and B are finite sets such $A \cap B = \phi$ that then $n(A \cup B) = n(A) + n(B)$
- If $A \cap B \neq \phi$ then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Principle of Mathematical induction:

Suppose there is a given statement P(n) involving the natural number n such that

(i) The statement is true for n=1i.e. (1) is true

(ii) If the statement is true for n=k (where k is some positive Integer), then the statement is also true from n=k+1.(i. e) true P(k) of implies the truth of P(k+1)

Then P(n) is true for all natural numbers n.

Linear inequalities:

Two real numbers (or) two algebraic expressions related by the symbols <, >, ≤ and ≥ from an inequality.

- Equal members may be added (or subtracted) to both sides of an inequality.
- Both sides of an inequality can be multiplied (or divided) by the same positive number. But both sides of an inequality are multiplied (or divided) by a negative number, then the inequality is reversed.

Sequence and series

- An arrangement of numbers $(x_1, x_2, x_3, ..., x_n)$ according to defined rule (or) set of rules is called a sequence. For example:

(1) The numbers (1, 4 , 9, 15 ,...) represent a sequence written according to the rule $x_n = n^2, n \in N$

(2) The numbers $\left(\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}\right)$ represents a sequence written according to the rule $x_n = \frac{n}{n+1}, n \in N$

- Sequence containing finite number of terms is called a finite sequence and it is an infinite sequence if contains infinite number of terms.
- If $(x_n) = (x_1, x_2, x_3, ...,)$ is a sequence, then the Expression $x_1 + x_2 + x_3 + ...$ is called the series. A Series is called finite series if it has finite number of terms
- The general term of the n^{th} term of the A.P is given by $a_n = a + (n-1)d$
- The sum of the first n terms of an A.P is given by $S_n = \frac{n}{2}[(2a + n - 1)d] = \frac{n}{2}[a + l]$
- The arithmetic mean A of any two numbers a and b Is given by $\frac{a+b}{2}$
- The general term of the n^{th} terms of G.P is given by $a_n = ar^{n-1}$
- The sum of the first n terms of G.P is given by

  $$S_n = \frac{a(r^n - 1)}{r-1} \quad (or) \frac{a(1-r^n)}{r-1} \quad if\ r \neq 1$$

- The G.P of any two positive numbers a and b is given by $\sqrt{ab}$

Operations on Real functions:

Let $f : X \to R\ and\ g : X \to R$ be the two real functions, then

(i) $(f + g)(x) = f(x) + g(x) \quad \forall x \in X$

(ii) $(f - g)(x) = f(x) - g(x) \quad \forall x \in X$

(iii) $(f \cdot g)(x) = f(x) \cdot g(x) \quad \forall x \in X$

(iv) $(kf)(x) = kf(x) \quad \forall x \in X$

(V) $\frac{f}{g}(x) = \frac{f(x)}{g(x)} \quad \forall x \in X, g(x) \neq 0$

Relation between Degree and Radian

- If in a circle radius r, an arc of length l subtends an angle of θ radians, at the centre then $\theta = \frac{1}{r}$
- Radian measure $\frac{\pi}{180} \times Degree$
- Degree measure $\frac{\pi}{180} \times Radian$

Solutions of trigonometric equations:

- If $\sin\theta = 0$ $then\ \theta = n\pi$
- If $\cos\theta = 0$ $then\ \theta = (2n+1)\frac{\pi}{2}$
- If $\tan\theta = 0$ $then\ \theta = n\pi$
- If $\sin\theta = \sin\alpha$ $then\ \theta = n\pi + (-1)^n\alpha$
- If $\cos\theta = \cos\alpha$ $then\ \theta = 2n\pi \pm \alpha$
- If $\tan\theta = \tan\alpha$ $then\ \theta = n\pi + \alpha$
- If $\sin\theta = \sin^2\alpha, \cos^2\alpha = \cos^2\alpha = \cos^2\alpha, \tan^2\alpha = \tan^2\alpha$ $then = n\pi \pm \alpha$(Here α is in radians and $n \in 1$)

Inverse trigonometric Function

Function	Domain	Range
$y = \sin^{-1}\theta$	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1}\theta$	$[-1,1]$	$[0,\pi]$
$y = \tan^{-1}\theta$	$[-\infty,\infty]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos ec^{-1}\theta$	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1}\theta$	$[-1,1]$	$[0,\pi] - \{\frac{\pi}{2}\}$
$y = \cot^{-1}\theta$	$[-\infty,\infty]$	$[0,\pi]$

Properties on inverse Circular Functions:

$\sin^{-1}(\sin\theta)=\theta$ if $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$

$\sin(\sin^{-1}\theta)=\theta$ if $-1\le\theta\le1$

$\cos^{-1}(\cos\theta)=\theta$ if $0\le\theta\le\pi$

$\cos(\cos^{-1}\theta)=\theta$ if $-1\le\theta\le1$

$\tan^{-1}(\tan\theta)=\theta$ if $-\frac{\pi}{2}\le\theta\le\frac{\pi}{2}$

$\sin(\sin^{-1}\theta)=\theta$ if $-\infty\le\theta\le\infty$

$\sin^{-1}x=\operatorname{cosec}^{-1}\frac{1}{x}$ $\qquad$ $\sin^{-1}(-x)=-\sin^{-1}(x)$

$\cos^{-1}x=\sec^{-1}\frac{1}{x}$ $\qquad$ $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$

$\tan^{-1}x=\cot^{-1}\frac{1}{x}$ $\qquad$ $\tan^{-1}(-x)=-\tan^{-1}(x)$

$\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ $\qquad$ $2\tan^{-1}(x)=\tan^{-1}\frac{2x}{1-x^2}$

$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$ $\qquad$ $2\cos^{-1}(x)=\cos^{-1}(2x^2-1)$

$\sec^{-1}x+\operatorname{cosec}^{-1}x=\frac{\pi}{2}$ $\qquad$ $2\sin^{-1}(x)=\sin^{-1}(2x\sqrt{1-x^2})$

$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$

$\tan^{-1}x-\tan^{-1}y=\tan^{-1}\frac{x-y}{1+xy}$

$2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

- Limits of a function at a point is the common value of the left and right hand limits, if they coincide.
- For a function and a real number, a, $\lim_{x\to a} f(x)$ and $f(a)$ may be same
- For function f and g the following holds.

$$\lim_{x\to a}[f(x)\pm g(x)]=\lim_{x\to a} f(x)\pm\lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)\cdot g(x)]=\lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

Following are some of the standard limits.

$$\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)=na^{n-1},$$

$$\lim_{x\to 0}\frac{\sin x}{x}=0,$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

- The derivation of a function f at a is defined by $f(\mathrm{a})=\lim_{f\to a}\frac{f(a+b)-f(a)}{b}$

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