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# Strategic Management of Workforce Diversity: An Evolutionary Game Theory Approach as a Foundation for AI-Driven Systems

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**Abstract:** In the complex organisational landscape, managing workforce diversity effectively has become crucial due to rapid technological advancements and shifting societal values. This study explores strategic workforce management through the novel methodological framework consisting of the evolutionary game theory concept integrated with replicator dynamics and traditional game theory, addressing a notable gap in the literature and suggesting an evolutionarily stable workforce structure. Key findings indicate that targeted rewards for the most *Enthusiastic* employee type can reduce overall costs and enhance workforce efficiency, although managing a diverse team remains complex. The study reveals that while short-term incentives boost immediate productivity, long-term rewards facilitate favourable behavioural changes, which are crucial for sustaining organisational performance. Additionally, the role of artificial intelligence (AI) is highlighted, emphasising its potential to integrate with these theoretical models, thereby enhancing decision-making processes. The study underscores the importance of strategic leadership in navigating these dynamics, suggesting that leaders must tailor their approaches to balance short-term incentives and long-term rewards to maintain an optimal workforce structure.

**Keywords:** leadership; game theory; evolutionary game theory; replicator dynamic; strategic decisions; employee groups; optimisation; artificial intelligence



**Citation:** Talajić, M.; Vrankić, I.; Pejić Bach, M. Strategic Management of Workforce Diversity: An Evolutionary Game Theory Approach as a Foundation for AI-Driven Systems. *Information* **2024**, *15*, 366. https://doi.org/10.3390/info15060366

Academic Editors: Martin Wynn and Jose Irizar

Received: 27 May 2024
Revised: 15 June 2024
Accepted: 17 June 2024
Published: 20 June 2024



## 1. Introduction

Rapid advancements in technology and shifts in human behaviour have profoundly transformed societal values and priorities in the modern era. Today's organisational leaders face the challenge of managing a workforce that spans diverse generational perspectives, each with unique motivational drivers and engagement preferences. The task is further complicated by the need to cater to both individuals who thrive under close supervision and those who prefer autonomy and independence in their work.

Effective leadership now requires an understanding of these varied motivational drivers to tailor strategies that engage and maximise the potential of all team members. Employee engagement, a critical driver of business success, hinges on this understanding. Studies consistently link higher levels of employee engagement to increased profitability, underscoring the importance of fostering a work environment that accommodates diverse needs and preferences. Excellent leadership represents a key element that differentiates outstanding organisations from good ones. This study—which builds on previous work by Talajić, Vrankić, and Kopal [1]—explores strategic workforce management through the novel methodological framework consisting of the evolutionary game theory concept integrated with replicator dynamics and traditional game theory.

Related to traditional game theory, Kopal and Korkut [2] describe it as a strategic interplay in which the result of an individual's decision is dependent on the decisions made by others. Similarly, Dixit and Skeath [3] provide a comparable explanation, stating that game theory addresses scenarios in which multiple players base their decisions on the anticipated actions of their counterparts.

Jehly and Renny [4] define a strategic game as a pair in which one part of the pair is the set of all strategies available to each of the players (the so-called pure strategies). In contrast, the other part of the pair is a function (the so-called payment function) that calculates the payment for a chosen strategy of one player, taking into account the strategic choices of all other players. The same authors define a mixed strategy game as a probability distribution over pure strategies that each player has.

Additionally, related to traditional game theory, the concept of Nash equilibrium is highlighted. It states that no player can benefit by changing their strategy while the other players keep theirs unchanged. It represents a state of mutual best responses, where each player's strategy is optimal given the strategies of all other players. Essentially, it is a situation where every participant is doing the best they can, given the choices of their opponents [5].

When a game has a single Nash equilibrium, the choices become clear for rational players. However, the theory faces challenges when multiple Nash equilibriums are present. The dilemma then arises: Which equilibrium should a player choose, especially if not all players behave rationally? In such scenarios, evolutionary game theory becomes relevant. As an extension of the traditional principles of game theory, evolutionary game theory examines how equilibriums are achieved through players' learning processes, informed by their experiences of trial and error.

Traditional game theory has been utilised to model the principal–agent relationship, focusing on the conflicts and incentives between leaders and followers. However, significant numerical models in this area are limited. Bierman and Fernandez [6] explored leadership through a principal–agent game theory model, but their approach did not account for the heterogeneity of employees. Research done by Talajić, Vrankić, and Kopal [1] expanded on this by considering three distinct employee types: *Enthusiast*, *Worker*, and *Parasite*; and using traditional game theory combined with replicator dynamics to optimise workforce structure for better financial outcomes.

Smith and Price [7] developed a concept through a simulation of interactions among animals utilising five strategies: Mouse, Hawk, Bully, Retaliator, and Prober Retaliator. Their research considered various animal species with differing abilities. Their simulation demonstrated that a dominant strategy emerges over others, signifying the animal species that ultimately prevails. In this context, pure strategies represent specific traits found within individuals of a population, whereas mixed strategies indicate the portion of the population exhibiting a certain trait. Utility is defined as the number of offspring an individual is expected to produce if the offspring adopt the same pure strategy (trait) as their parent. The concept of Nash equilibrium, as outlined in traditional game theory, does not align well with evolutionary games. This discrepancy arises because the evolutionary approach, rooted in biology (animals), does not take rationality as a key factor in achieving equilibrium.

Hence, the concept of an evolutionarily stable strategy (ESS) was formulated. ESS examines the composition of the population (the distribution of each trait within the population) in a way that is resilient to the potential emergence of mutants, where a mutant is defined as a player who adopts a different pure strategy. A strategy is considered stable when the mutant strategy yields a lower utility compared to the original strategy, indicating that the mutant strategy will not prevail within the population. When applied to humans, this refers to a scenario in which a minority of individuals attempts to alter the prevailing strategy of the population but achieves lower utility than the strategy currently in place. The determination of ESS is influenced not only by the specific power of the species (including humans) but also by the frequency of each type within the population.

The collection of pure strategies (various types or characteristics within the population) is determined by $S = \{s_1, s_2, \dots s_n\}$. A mixed strategy signifies the composition of the population, reflecting the proportion of each type within the population, and is defined by

$P = \{p_1, p_2, \ldots, p_n | p_1 + p_2 + \ldots + p_n = 1\}$ where $p_i$ is the percentage of type with pure strategy $s_i$.

Baron [8] introduced a pivotal concept in evolutionary game theory—the ESS—which is a strategy that, if adopted by a population, cannot be invaded by any alternative strategy that is initially rare. The ESS theorem, a cornerstone of evolutionary game theory, states that if a strategy is evolutionarily stable, it must be a Nash equilibrium. However, not all Nash equilibria are evolutionarily stable. The theorem establishes the necessary conditions for a strategy to be considered an ESS:

1. Strategy Superiority: The strategy must yield a higher fitness (or payoff) against itself than any mutant (alternative) strategy does against it. This ensures that within a population using the ESS, any mutant strategy will be less successful. In other words, for a strategy $S$ to be an ESS, it must have a higher fitness against a mutant strategy $T$ than $T$ has against itself, whenever $T$ is rare. This means that $S$ can resist invasion by any small number of mutants adopting a different strategy $T$.
2. Stability Condition: If there is a tie in the first condition, meaning the ESS and the mutant strategy yield the same fitness against the ESS, the ESS must perform better against the mutant strategy than the mutant does against itself. In other words, if the fitness of $S$ against $T$ is equal to the fitness of $T$ against itself (indicating a neutral stability), then $S$ must have a higher fitness against itself than $T$ does against $S$, to ensure that $S$ remains stable and cannot be replaced by $T$.

These conditions ensure that an ESS is not only a Nash equilibrium but also robust against invasion by alternative strategies, providing a stable state in the evolutionary dynamics of populations.

While the application of evolutionary game theory in leadership studies is less prevalent, it has shown promise in analysing strategic interactions and stability within organisations. Studies by Maussa Perez et al. [9], and Szolnoki and Perc [10] have demonstrated the utility of evolutionary game theory in evaluating strategic behaviours and social dilemmas. Ref. [9] specifically highlighted the role of evolutionary strategies in promoting cooperative behaviour within entrepreneurial activities. Ref. [10] proposed that in evolutionary social dilemmas, effective leaders should diverge from conformist behaviour, emphasising the importance of non-conformist strategies in achieving a competitive advantage.

Han, Albrecht, and Woolridge [11] investigated the mechanisms of the emergence and evolution of collective behaviours in multi-agent systems (MAS) through the lens of evolutionary game theory and agent-based modelling. Their work emphasises the importance of cognitive and emotional mechanisms in promoting prosocial behaviours within organisations. Similarly, Cun [12] developed a business intelligence simulation model aimed at enhancing leadership and crisis management through game theory techniques and machine learning models, demonstrating the potential of artificial intelligence (AI) in optimising leadership strategies.

Zhang et al. [13] investigated the impact of anxiety on cooperative behaviour using a network evolutionary game theory approach. Their study introduced an anxiety threshold to model the tendency of anxious players to change strategies, highlighting the significant influence of peer pressure and individual anxiety on cooperative behaviour. Ma et al. [14] examined the interplay between fiscal policy, the stability of farmers' cooperatives, and environmentally friendly digital management through an evolutionary game theory-based study; revealing critical insights into the factors influencing cooperative stability and pro-environmental behaviour.

Dong et al. [15] presented an innovative study on promoting cooperation in multi-agent systems using evolutionary game dynamics with a focus on agents utilising local information and the introduction of the "hide" strategy. This approach is designed to reduce defection and enhance stable payoffs among agents. Cheng et al. [16] proposed a consortium blockchain-based leasing platform that facilitates information sharing between small and medium-sized enterprises and leasing firms, using evolutionary game theory to model the dynamics of contract compliance. Li et al. [17] investigated the role of oxytocin in promoting fairness and cooperation in heterogeneous networks through data-driven evolutionary game models. Their study revealed that oxytocin enhances prosocial behaviours by increasing inequality aversion, providing insights into the mechanisms underpinning fairness and cooperation in human networks.

Taylor and Jonker [18] were at the forefront of developing the replicator dynamic equation. This model captures the evolution of strategies in both continuous and discrete settings using non-linear first-order differential equations and non-linear difference equations, respectively. This model provided a basis for understanding stability and asymptotic behaviour. They applied this model to a population of haploid organisms, each committed to a single pure strategy throughout their lifespan, with the assumption that these strategies are passed down to their offspring.

The change in the population's mix strategy is influenced by the rate at which users of each strategy reproduce. The change in the ratio $p_i$ of individuals adopting a specific strategy $s_i$ corresponds to the difference between the fitness of said strategy $E_i$ and the mean fitness of the entire population $M$. This concept is encapsulated in the form of a replicator dynamic equation: $\dot{p}_i = p_i * (E_i - M)$.

Despite all these advancements, there remains a notable gap in the literature regarding the integration of traditional game theory, evolutionary game theory, and replicator dynamics into a comprehensive framework for workforce management. This study seeks to address this gap by proposing a novel model that leverages these three theoretical approaches to optimise employee engagement and organisational performance.

Building on the foundational work of [1], this paper introduces a "cockpit" approach for dynamic employee management, allowing for real-time strategy adjustments. The integration of evolutionary game theory adds a significant layer of novelty by identifying ESS, which contributes to system stability.

The primary aim of this paper is to define a comprehensive theoretical framework that incorporates traditional game theory, evolutionary game theory, and replicator dynamics. This framework offers a new perspective on leadership and principal–agent relationships, addressing a notable gap in the current literature. Specifically, this study seeks to answer the following research questions:

1. Can the model developed by [1] be fine-tuned by utilising parameter variation to determine the optimal ratio of employee types for effective workforce management?
2. How can the concept of evolutionary game theory provide additional insights into this field of research?
3. Is there a structured theoretical framework that can serve as an effective leadership tool for managing people and achieving better organisational outcomes?

Furthermore, the increasing development and application of AI in business processes highlights the potential for integrating these theoretical models into AI-powered systems. Such integration can enhance decision-making processes, improve the quality of decisions, and accelerate digital transformation within organisations.

This study involves several key steps to achieve its objectives. Firstly, the existing model by [1] is extended by integrating evolutionary game theory to identify a stable and efficient workforce structure. This involves mathematical modelling and analysis using game theory and replicator dynamics. The next step is the development of the "cockpit" approach for real-time strategy adjustments. This model is then tested and validated through simulations and case studies to assess its practical applicability and effectiveness. Finally, the implications of integrating AI into these models are highlighted to provide a comprehensive framework for future applications in workforce management.

This paper contributes to the existing literature by proposing a novel, integrative approach to workforce management. It combines traditional game theory, evolutionary game theory, and replicator dynamics to create a comprehensive framework that addresses the complex dynamics of employee engagement and organisational performance. Through this framework, leaders can better understand and manage their workforce, ultimately driving improved business outcomes.

## 2. Literature Review

### 2.1. The Role of Leadership in Modern Organisation

According to Hogan and Kaiser [19], leadership is a real and vastly consequential phenomenon that significantly impacts team and organisational performance. They argue that personality is an important predictor of leadership effectiveness, which can be used to select future leaders or improve current ones.

According to Ulrich and Smallwood [20], leadership involves both individual and organisational components. They argue that individual leaders play critical roles in shaping strategy, executing decisions, managing talent, developing future talent, and acting with personal proficiency. However, they emphasise that organisational leadership—which involves building a cadre of future leaders capable of shaping the organisation's culture and creating patterns of success—is even more important. They propose an outside/in view of leadership, focusing on business values rather than psychological principles. Ulrich and Smallwood [20] outline four key principles of effective leadership: clarifying why leadership matters, nailing the basics, creating a leadership brand, and ensuring leadership sustainability.

Strategic leadership research has seen significant growth and diversification over the past few decades, addressing numerous themes and perspectives [21] according to which the field of strategic leadership is inherently tied to digital transformation and innovation, highlighting the importance of top management teams in driving organisational success. Comprehensive bibliometric–temporal analysis in [21] underscores the evolving nature of strategic leadership research, identifying key trends and future research directions that are critical for scholars and practitioners alike.

Digitalisation requires leaders to adopt new skills and characteristics to effectively navigate the challenges and opportunities presented by digital transformation [22]. According to this, successful digital leadership involves being visionary and customer-centred while embracing change and promoting teamwork and collaboration. The review highlights the importance of flat hierarchies, employee empowerment, digital savviness, and engagement in partnerships and ecosystems for leaders aiming to succeed in digitalisation initiatives.

Gilli, Lettner and Guettel [23] argue that as digitalisation intensifies within organisations, the role of business leaders is evolving to emphasise traditional leadership virtues alongside new digital skills. Leaders now need to manage relationships actively, oversee social processes within their teams, and navigate change processes effectively.

According to Amalia and Prayekti [24], the work environment plays a crucial role in shaping employee morale and productivity. They highlight that transformational leadership can significantly enhance employee motivation and engagement. Furthermore, their study demonstrates that incentives are a powerful tool in boosting employees' morale and overall job performance. They argue that effective transformational leadership can lead to improved organisational performance by increasing employee morale. The research indicates that a well-designed incentive programme can positively impact employee motivation and performance. Furthermore, it shows that integrating transformational leadership practices within an organisation can lead to a more motivated and high-performing workforce.

The study by Feng, Zhang, and Zhang [25] provides a comprehensive analysis of the role of compensation and incentives in facilitating digital transformation within organisations. It highlights that there is a nonlinear relationship between monetary compensation and digital transformation outcomes, suggesting an optimal level of compensation to maximise effectiveness. Additionally, the research discusses the concept of managerial myopia, where short-term goals may conflict with long-term digitalisation objectives. The authors emphasise the importance of equity-based incentives as a more effective strategy for promoting digital transformation; offering valuable insights for managers, investors, and policymakers aiming to navigate the complexities of digital transformation in the modern knowledge economy.

Zhu and Xie [26] highlight the crucial role of compensation incentives in human resource management, emphasising their significance in attracting and retaining talent, motivating employees, and enhancing performance. They argue that a well-designed compensation system not only fosters a positive workplace culture but also aligns employee objectives with organisational goals, thus driving overall company performance.

### 2.2. Game Theory and Leadership

One of the ways in which traditional game theory views leadership can be seen through the idea of the principal–agent (leader–follower) theory. The theory is described as an approach to addressing issues related to the assignment of duties from principals to agents and from leadership to followers; especially when there is a conflict of interest between the two groups [27]. Similarly, agency theory is explained as an economic concept that considers a company to be a collection of contracts between self-interested parties [28]. In his analysis of corporate governance within the banking sector, Tan [29] characterises agency theory as the study of a firm's behaviour through the lens of contracts between various stakeholders.

Stankova and Olsder [30] present the adverse-selection principal–agent framework as an inverted Stackelberg game. The study argues that an agent consistently endeavours to optimise their utility by adopting the identity of a different agent category. It outlines an ideal strategy for the principal based on the assumption that the agent's goal is to maximise their earnings. This model accounts for only a single agent type. While the authors hinted at the possibility of broadening the research scope to encompass multiple agents in subsequent studies, no such advancements have been reported thus far. The interaction between the principal and the agent employs incentive theory as a strategic tool for the principal to shape the agent's behaviour.

Shapiro and Stiglitz [31] investigated the concept of involuntary unemployment by examining the framework of the principal–agent relationship. They illustrated how the principal's lack of ability to monitor an employee's effort without incurring costs leads to involuntary unemployment as a feature of equilibrium. They argued that the efficacy of the threat of dismissal hinges on the extent of the loss experienced by the *Worker* upon termination. If a dismissed employee can promptly secure another position with a comparable salary, the threat of dismissal fails to incentivise effective work performance. An organisation might derive benefits from enforced unemployment if it leads to a decrease in the expected utility for the laid-off *Workers*. The unemployment rate at equilibrium needs to be sufficiently high to ensure that a *Worker* is more motivated to perform diligently rather than to shirk.

Bierman and Fernandez [6] researched the topic of involuntary unemployment through a moral hazard scenario, applying the principal–agent framework from game theory. Their examination focused on scenarios of involuntary unemployment, characterised by an employer's decision not to employ a person despite their readiness to work for the current market salary. They illustrated a condition where involuntary unemployment exists in equilibrium. The game initiates with an employment offer from the employer, followed by the employee's decision to accept the offer and the level of effort they intend to contribute. The employer cannot directly observe the employee's effort, and it is left to "nature" to determine the employee's effectiveness. Initially, the employee's von Neumann utility function is established—along with the employer's revenue function—which depends on the quantity of effective work produced by the employee (assuming uniform effectiveness across all employees). Utilising these definitions, a dynamic principal–agent game was constructed. Through the optimisation of the employee's expected utility, the optimal amount of effective work was derived, taking into consideration both the offered wage and the reservation wage (the employee's response to the offered wage) [32]. The employer's profit function was then formulated based on the employee's reaction curve. It was demonstrated that in certain scenarios, the employer compensates the employee with double the reservation wage to incentivise higher productivity (thus reducing the level of moral hazard). At equilibrium, all employers offer the same wage, which must surpass the market-clearing wage. Otherwise, any employee dismissed from one job would immediately secure another. Ultimately, it was concluded that no employer benefits from paying a new employee less than what is paid to existing employees within the organisation. The marginal value product of an employee falls below their wage (if the wage is higher than the reservation wage). Since employees cannot guarantee and commit to a certain level of efficient work, companies compensate with a higher wage (efficient wage) that exceeds the equilibrium wage produced by the standard supply and demand mechanism, aiming to achieve a greater level of efficient work.

Ref. [6] takes a step forward by describing leadership followers through a principal–agent game theory model. The model does not recognise different agents (followers), assuming that all followers are of the same type, and that is its main drawback.

No relevant studies exist on replicator dynamics related to modelling leadership from this theoretical perspective. Some literature suggests that principles could be applied to the leadership–follower concept.

In their study, Carrera and Pavlinović [33] explored how the concept of place attachment (player identity) plays a crucial role in environmental preservation. In their work, the authors used the ESS concept in combination with the replicator dynamic. The use of a combination of those theories in their article was an inspiration for this paper. They identified two categories of participants: those with a strong connection to the place and those without such connection. Participants had the option to either contribute to enhancing environmental quality or not doing so. A notable aspect of their research was the observation of individuals deeply connected to their environment who, nonetheless, chose not to contribute to a better-quality environment. This suggests that an increase in the number of individuals who feel a strong connection to a place does not necessarily lead to improved environmental conditions.

Conversely, they discovered that individuals lacking a strong sense of place identity might still take actions to improve environmental quality, indicating that environmental conditions could improve even with an increase in the number of such individuals. Hence, assessing the dynamics of change among these groups to identify stable equilibriums is critical. They employed the replicator dynamic equation for pinpointing potential equilibrium states and utilised a geometrical method (phase portraits) for evaluating stability. Their findings highlighted that the approach of individuals who are connected to their place and actively work towards environmental sustainability is evolutionarily stable. Another key insight was that a population solely comprising strongly attached individuals who do not contribute to environmental enhancement is not viable, as it reduces the benefits for those committed to environmental quality. Furthermore, they observed that, over time, those with a strong identity who do not invest in environmental quality could coexist with those who lack a strong identity and do not invest; eventually leading to a shift where the former group loses their strong sense of identity.

Vrankić, Herceg, and Pejić Bach [34] introduced various business tactics within a duopoly; categorising them as dominant, reactive, cooperative, and tit-for-tat strategies. Over time, a firm's approach evolves in response to the strategy adopted by its competitor; a process accurately captured through replicator dynamics. The initial set of strategies employed heavily influences the eventual structure of the industry. Their work incorporated the use of phase portrait methodology for the analysis of the system's dynamics.

In their research, the authors of [1] considered the heterogeneity of employees, avoiding the assumption that all employees are of the same type, as the authors of [6] did. The principal–agent concept was applied to three distinct groups of employees (agents): *Enthusiasts*, *Workers*, and *Parasites*. This approach represented a further development beyond [6].

The *Enthusiastic* type is a proactive employee who enjoys attractive and innovative tasks, is highly engaged, and likes to work.

The *Worker* group is a type of employee who is not ambitious and prefers repetitive tasks. Having a secure salary is important to this group, and it plays a vital role in the organisation's operational activities.

The *Parasite*, on the other hand, is the type who slacks off and dislikes work.

For this reason, it is assumed that the *Enthusiast* type works 100% of the time, giving them an effort level of 1. The *Parasite* type, who dislikes working, works 0% of the time and has an effort level of 0. The *Worker* group works between 0% and 100% of the time and has an effort level between 0 and 1.

The researchers examined employees in teams of two and calculated the payoffs (total costs) through all possible pair combinations.

The results indicate that, in the short run, it is impossible to eliminate the *Parasite* type, and the optimal combination is 59% of the *Worker* type, 24% of the *Enthusiast* type, and 17% of the *Parasite* type. However, in the long run, there would be about 71% *Workers* and 29% *Enthusiasts*.

It is assumed that, in the long run, *Parasites* will act like *Workers* to avoid being caught not working. This scenario was specifically examined by the authors. To calculate the optimal and stable proportions of groups in the population, traditional game theory combined with replicator dynamics is used. Both numerical and geometrical (phase portrait) methods are employed to describe the direction in which the population moves to achieve stability. This work is among the few that describe the leadership–follower relationship through these two theories, and its results and findings will form the basis of this paper.

Their research does not use evolutionary game theory and the ESS concept. This paper will extend their model by adding evolutionary game theory. The integration of these three theories into one model represents a significant scientific contribution, as people management and leadership have not previously been viewed through the lens of these three theories.

Graser et al. [35] apply game theory to the hiring process by modelling candidate selection as a strategic game between the HR manager and job candidates. The utility function evaluates candidates' professional and soft skills, aiming to maximise the benefit to the company. This approach highlights the competitive nature of recruitment, where both the employer and candidates' strategies to achieve their best outcomes. By viewing the selection process through the lens of game theory, the study provides insights into optimal decision-making strategies, balancing the interests of both the employer and potential employees.

Drouvelis and Pearce [36] examine the impact of intelligence on decision-making in infinitely repeated sequential public goods games. Their research indicates that leaders with higher intelligence are less likely to engage in free-riding behaviour, leading to increased group contributions and higher overall profitability. From a game theory perspective, this study underscores how intelligence can influence strategic choices, particularly in contexts requiring long-term cooperation and trust. These findings suggest practical implications for organisational policies aimed at promoting teamwork and group success by leveraging cognitive skills.

Zhang, Liang, and Wang [37] explore the optimal reinsurance and investment strategies for insurers and reinsurers using a mixed leadership game framework. In this model, the insurer acts as a leader in determining investment in risky assets and as a follower in setting reinsurance retention levels. Conversely, the reinsurer leads in setting reinsurance premiums but follows in investment decisions. By solving the Hamilton–Jacobi–Bellman (HJB) equations, they derive a Stackelberg–Nash equilibrium, providing theoretical and numerical insights into the economic implications of this mixed leadership dynamic.

In the context of multi-agent interactions, Khan and Fridovich-Keil [38] introduce an iterative algorithm for solving dynamic Stackelberg games with nonlinear dynamics and nonquadratic costs, demonstrating consistent convergence. They also propose the Stackelberg Leadership Filter (SLF): an online method for identifying leaders in two-agent games based on observed behaviours. This research enhances understanding of leadership inference in dynamic and complex environments, applying game theory to predict and interpret strategic interactions among agents, particularly in simulated driving scenarios.

Wang et al. [39] address the leader–follower consensus problem in hybrid multi-agent systems using game theory. Their study models the competitive behaviour of agents as a multiplayer game, designing cost functions for each agent based on game rules. They analyse the necessary and sufficient conditions for achieving consensus in systems with both continuous and discrete-time agents, providing a game-theoretic framework to understand the dynamics of cooperation and competition among multiple agents. This research highlights the application of game theory to control systems, demonstrating how strategic interactions can lead to optimised collective behaviour.

### *2.3. Evolutionary Game Theory and Leadership*

Han, Albrecht, and Woolridge [11] investigate the mechanisms of emergence and evolution of collective behaviours in multi-agent systems (MAS) through the lens of evolutionary game theory (EGT) and agent-based modelling (ABM). They explore various strategies; from incorporating cognitive and emotional mechanisms to promoting prosocial behaviours, emphasising the importance of these approaches in understanding and engineering MAS.

Cun [12] explores the development of a business intelligence simulation model aimed at enhancing leadership and crisis management through game theory techniques and machine learning models. The study highlights the use of a variational recurrent Boltzmann learning model (VRBL) for crisis management and leadership analysis, demonstrating the advantages of machine learning in prediction accuracy over traditional econometric models. The research also emphasises the importance of cloud network modelling in improving execution time, network performance, data optimisation, and reducing end-to-end delay. Cun's findings suggest that the final fusion model outperforms traditional voting and averaging techniques in terms of prediction performance, making it a valuable tool for crisis management. The study underlines the potential of AI together with game theory to revolutionise business practices by providing advanced solutions for risk assessment and proactive crisis response; thus enhancing the stability and efficiency of organisational operations.

Zhang et al. [13] investigate the impact of anxiety on cooperative behaviour using a network evolutionary game theory approach. The study introduces an anxiety threshold to model the tendency of anxious players to change strategies, considering both endogenous anxiety and peer pressure as sources. The findings highlight that higher anxiety leads to greater strategy shifts, and a combination of peer pressure and individual anxiety significantly influences cooperative behaviour within networks. The research also incorporates a collapse threshold to explore factors influencing the proportion of overanxious individuals, offering insights into how these dynamics affect group cooperation. Simulation results demonstrate that focusing on past game records or possessing strong resilience can lead to stable cooperation. At the same time, overanxious individuals can destabilise group dynamics unless moderated by peer pressure and strategic adjustments.

Ma et al. [14] conducted an evolutionary game theory-based study to examine the interplay between fiscal policy, the stability of farmers' cooperatives, and environmentally friendly digital management. Their research highlights the critical role of fiscal policy support, brand influence, and market share in determining the stability and contract selection within cooperative associations. The study also reveals that input costs and breaches of contracts by residents significantly impact the stability and pro-environmental behaviour of these associations.

Dong et al. [15] present an innovative study on promoting cooperation in multi-agent systems (MAS) using evolutionary game dynamics, with a focus on agents utilising local information and the introduction of the "hide" strategy. This approach is designed to reduce defection and enhance stable payoffs among agents. By considering only their own and non-defective neighbours' payoffs, agents can make more informed decisions, leading to stable equilibrium states under various conditions. The study's findings underscore the utility of the hidden strategy in promoting cooperation, providing valuable insights for autonomous decision-making in MAS. The introduction of the hidden strategy is particularly effective in scenarios where agents face high risks, such as autonomous unmanned systems

(AUS) operating under poor communication conditions. The research highlights the importance of local information in decision-making processes, suggesting that non-defective neighbour information can significantly influence agent behaviour and system stability. This study provides a practical reference for the development of cooperation promotion mechanisms in MAS, particularly in high-stakes environments such as military operations or disaster response.

Cheng et al. [16] propose a consortium blockchain-based leasing platform (CBLP) that facilitates information sharing between small and medium-sized enterprises (SMEs) and leasing firms (LFs) using evolutionary game theory (EGT) to model the dynamics of contract compliance. Their study reveals that both SMEs and LFs can achieve a "win-win" strategy by complying with contracts and adopting blockchain technology (BCT), particularly when incentives and penalties are appropriately balanced. The findings emphasise that large residual values of leased assets and the maintenance outsourcing model significantly influence the likelihood of SMEs defaulting; suggesting that blockchain integration could mitigate these risks by enhancing transparency and trust. The article highlights the potential of blockchain technology in addressing information asymmetry and default risks in the leasing sector. By implementing a consortium blockchain, SMEs and LFs can share critical data more efficiently, thereby improving credit assessments and asset management. The research also underscores the importance of designing effective incentive mechanisms to encourage SMEs to comply with lease contracts and participate in the blockchain network, ultimately leading to a more sustainable and trustworthy leasing environment. Furthermore, the study suggests that the evolutionary game model can guide policymakers in optimising these mechanisms to foster cooperative behaviour among all stakeholders involved in the leasing process.

Li et al. [17] investigate the role of oxytocin in promoting fairness and cooperation in heterogeneous networks through data-driven evolutionary game models. Their study reveals that oxytocin enhances prosocial behaviours by increasing inequality aversion, which, when combined with network heterogeneity, leads to the amplification and diffusion of fairness and cooperation. This effect is particularly pronounced in the ultimatum game and the two-stage prisoner's dilemma game with punishment. In contrast, the trust game shows that trust enhanced by oxytocin remains locally confined and does not promote broader prosociality. These findings provide insights into the mechanisms underpinning fairness and cooperation in human networks and highlight the differential impact of oxytocin across various social games. The research underscores the significant impact of oxytocin on promoting fairness and cooperation in social networks. By integrating oxytocin with evolutionary game models, the study demonstrates that central nodes in heterogeneous networks can amplify and spread prosocial behaviours initiated by inequality aversion. The distinct outcomes across different social games suggest that the mechanism of punishment, rather than reward, is more effective in fostering network-wide cooperation. These insights are crucial for understanding the complex interplay between biological factors and social network structures in driving human cooperative behaviour. The study's findings also suggest potential applications in designing interventions to enhance fairness and cooperation in various social contexts.

The perspective of leadership through the concept of evolutionary game theory can be found in the literature, though not as extensively as one might expect given the theory's potential. What follows is an overview of a few papers whose content may inspire further research in this area.

Maussa Perez et al. [9] demonstrated the significance of evolutionary game theory in evaluating and enhancing entrepreneurial activities. Szolnoki and Perc [10] proposed that in the context of evolutionary social dilemmas, effective leaders should diverge from conformist behaviour. Contrary to the traditional belief in evolutionary game theory that individuals will naturally gravitate towards strategies yielding the highest payoffs, their research highlighted situations where individuals opt for the most adopted strategies due to a herd mentality or collective behaviour; this underscores that outcomes favoured by the majority are often the most desirable. They argued that, especially in evolutionary social dilemmas, ideal leaders are those who resist conforming to majority behaviours, instead aiming to rally substantial groups around prevalent strategies to compete in the marketplace effectively. The methodology proposed by the authors presents a novel and compelling approach.

In their study, Silveira and Vasconcelos [40] investigated the strategic choices of retail companies within a duopoly market, focusing on two main strategies: maximising profits and maximising revenues. They sought to determine whether profit maximisation consistently represents an ESS. Employing evolutionary game theory and the replicator dynamic, they constructed an agent-based model and revealed instances where revenue maximisation emerged as an ESS, offering higher payoffs than the conventionally favoured profit maximisation strategy, as discussed in [32]. Their findings indicated that in a Cournot duopoly scenario, the choice between revenue and profit maximisation (disregarding production costs) depends on the market's size and the company's operational efficiency. They demonstrated that companies could alter their strategic approach over time through dynamic adaptation and learning. The viability of revenue maximisation as an ESS is contingent upon the specific business strategies of a company, such as the emphasis on expanding market share, wherein revenue growth assumes greater significance.

In the evolving digital economy, strategic management of workforce diversity has become critical for fostering innovation and maintaining competitive advantage. An evolutionary game theory approach provides a robust framework for analysing the interactions between diverse workforce strategies and organisational outcomes.

Vrankić [41] explores the dynamics of corporate social responsibility (CSR) in duopolies, highlighting how firms' investments in socially responsible activities can significantly influence market shares and profitability. His findings emphasise that strategic CSR investments, particularly under competitive pressures, can enhance an organisation's market position and drive social welfare improvements.

By integrating his insights into the evolutionary game theory approach, it can be better explained how strategic management of workforce diversity not only contributes to ethical and socially responsible business practices, but also catalyses achieving competitive advantage in a digitally driven market. The alignment of CSR initiatives with workforce diversity strategies can thus be seen as a vital element in the broader context of digital transformation and economic sustainability.

In the realm of strategic management of workforce diversity, employing an evolutionary game theory approach can provide robust insights into the dynamics of organisational behaviour and decision-making. Georgiou [42] analysed investment strategies in high-intensity R&D entities and constructed a game theory matrix using empirical data to evaluate how managerial decisions regarding the capitalisation of development costs influence investor behaviour. This method highlights the importance of strategic financial decision-making under conditions of uncertainty and risk, which parallels the complexities faced in managing a diverse workforce in AI-driven systems.

### *2.4. Game Theory and AI*

Game theory has increasingly intersected with deep learning, offering a rich framework for modelling and solving complex problems. Hazra and Anjaria [43] provide an extensive survey on the applications of game theory in deep learning, highlighting its potential to enhance model performance. They discuss how game-theoretic concepts underpin various deep learning architectures, including generative adversarial networks (GANs). GANs—framed as two-player zero-sum games—benefit significantly from game theory in training generative and discriminative models, leading to advancements in image generation and classification tasks. Their review underscores the symbiotic relationship between game theory and deep learning, showcasing its utility in addressing intricate problems in AI. Their study also details how game theory aids in reinforcement learning and other deep learning architectures; providing valuable insights, challenges, and future research directions in these fields.

Harré [44] discusses the role of a cognitive representation of others' unobserved causal states, which is central to the psychology of social interactions. This concept, known as the "Theory of Mind", is crucial for understanding how agents simplify the task of predicting others' behaviour. In his examination of introspection and theory of mind, the author highlights the economic analysis of game theory and its significance in modelling interpersonal relationships for both biological and artificial agents. He suggests that, while game theory is instrumental, it is not entirely sufficient and psychological refinements are necessary. The coordination challenges between artificial agents and humans are also addressed, emphasising the need to understand the cognitive processes that enable intelligent agents to make decisions in the presence of other intelligent agents. He points out that such understanding is vital for enhancing AI's ability to support human endeavours.

He et al. [45] introduce a novel approach to integrating generative artificial intelligence (GAI) with game theory to address complex networking optimisation problems in mobile networking. They highlight the inherent complexities of game theory-based solutions, which traditionally require extensive human expertise and experience. However, the authors propose that the advanced reasoning and generation capabilities of GAI can significantly enhance the design and optimisation processes in mobile networking. They discuss the synergistic benefits of combining these two fields, noting how GAI can mitigate some of the limitations of traditional game theory applications. The authors present a game theory framework enabled by large language models (LLMs) and demonstrate its effectiveness through a case study focused on secured UAV networks. This innovative approach could pave the way for more efficient and effective solutions to complex networking challenges.

Schelble et al. [46] conducted an empirical study examining how different reinforcement learning algorithms and game theory scenarios influence cooperation in human–AI teams. Their findings indicated that the Deep-Q Network (DQN) algorithm facilitated higher levels of cooperation compared to other algorithms; and the Hawk Dove scenario resulted in more significant cooperation than the Prisoner's Dilemma scenario. The study emphasised the importance of task and social framing in human–AI systems, demonstrating that the chosen reinforcement learning model and game theory scenario significantly impact the cooperation levels within these systems. Moreover, the research highlighted the potential of game theory as a valuable tool for evaluating and enhancing human–AI interaction, by fostering cooperative behaviours. The implications of this study suggest that a deeper understanding and careful design of human–AI systems can lead to more effective and harmonious cooperation between humans and AI.

Li and Lee [47] explore the intricate dynamics of goal alignment within human–AI teams. They present a dynamic game-theoretical framework that integrates the drift-diffusion model to simulate the interdependency between humans and AI. Their study highlights that situational structures and strategic behaviours significantly influence the process of goal alignment, with findings suggesting that teaming with altruistic agents in competitive situations can lead to the highest team performance. This work underscores the complexity of modelling goal alignment and provides a foundation for designing more effective human–AI teaming systems.

Tennenholtz [48] found the relationship between game theory and artificial intelligence. These two fields have evolved separately, yet share common origins; and the author emphasises the importance of bridging the gap between these disciplines by exploring foundational issues in representation, reasoning, and learning that intersect both areas.

Shen et al. [49] examine the convergence of AI and game theory in the context of next-generation communication networks, highlighting the limitations of traditional mathematical methods in addressing user behaviour. They propose a novel framework that combines machine learning and game theory to enhance network management, specifically addressing the network selection problem in 5G ultra-dense and heterogeneous networks. Their simulation results demonstrate the framework's effectiveness in reducing user delay.

Wang, Wan, and Wang [50] explore the integration of human values into ethical AI using experimental game theory. They propose a mathematical framework that links moral philosophy concepts—such as Kantian and utilitarian ethics—with industry standards for ethical AI, such as the IEEE P7000 [51] series. Their study demonstrates how values derived from experimental game theory, particularly trust game experiments, can inform the development of ethical AI systems. Additionally, they discuss using the iterated Prisoner's Dilemma game to test the ethical behaviour of AI algorithms, highlighting the advantages of their approach over existing methods.

Wang, Fu, and Chen [52] discuss the intersection of evolutionary game theory (EGT) and AI, highlighting the potential for cross-fertilisation between these fields to advance multi-agent learning systems. EGT focuses on the evolution of strategies within a population, where individuals adapt based on social learning. At the same time, AI— particularly in multi-agent environments—involves intelligent agents adjusting their strategies based on feedback and experience. This intersection is crucial for developing collective, cooperative intelligence, which bridges evolutionary dynamics and multi-agent reinforcement learning; addressing real-world problems through learning, adaptation, cooperation, competition, robustness, and stability.

Xing et al. [53] present an innovative approach to optimising path planning for robots in a human–robot collaboration environment typical of Industry 4.0. The study addresses the challenges faced by robots in navigating complex, dynamic environments shared with human *Workers*. Traditional deep reinforcement learning algorithms struggle with slow convergence in such scenarios. To overcome this, the authors propose a hybrid method that integrates deep reinforcement learning with game theory. Their approach involves a heuristic method to assess collision risks, employing deep reinforcement learning for path optimisation when no risk is detected and formulating a cooperative game to resolve collision concerns when risks are present. Numerical results demonstrate the superiority of their proposed algorithm over state-of-the-art solutions in distributed path planning for robots.

Yang et al. [54] used the integration of generative AI with game theory to overcome traditional limitations in handling large-scale and dynamic strategic interactions. The study identifies the challenges posed by traditional non-AI and discriminative AI approaches in deriving solutions and optimising performance in network games. By leveraging the superior data analysis and generation capabilities of generative AI, the authors propose a framework that enhances model formulation, solution derivation, and strategy improvement in game theory applications. The effectiveness of this integration is demonstrated through a case study on optimising machine learning model performance against false data injection attacks. The paper concludes with future research directions, highlighting the potential advancements in generative AI-enabled game theory.

In their study, Djehiche and Tembine [55] articulate that the results produced by generative AI, such as those by BloombergGPT, align precisely with Nash equilibria in non-potential games. Their work is about the convergence properties of deep neural networks, elucidating how these AI systems achieve equilibrium states. Additionally, they extend their analysis to federated learning systems, offering a comprehensive view of how neural network architectures can be optimised and stabilised through game-theoretic principles.

## 3. Methodology

The main theoretical frameworks to be used are traditional game theory, evolutionary game theory, and replicator dynamics.

The paper will use the concept of a Nash equilibrium in terms of traditional game theory. It plays a critical role in all decision-making processes and explains the Nash equilibrium and the methods by which players reach these equilibriums.

For checking an ESS, the reformulation of the main ESS theorem given by Baron (2013) is used [8]. The strategy is an ESS if and only if:

$$\begin{gathered} u(p^*, p^*) > u(p, p^*),\ \forall \mathrm{p_I} \in [0,1] \text{ and } p \neq p^* \\ \text{or} \\ u(p^*, p^*) = u(p, p^*) \Rightarrow u(p^*, p) > u(p, p),\ \forall p \neq p^* \end{gathered} \tag{1}$$

where $u$ is a fitness for the entire population; $p = (p_1,\ p_2,\ p_3 \ldots)$ is any mix strategy combination in a population (all probability distribution over the different entities in the population), and $p^* = (p_1{}^*, p_2{}^*, p_3{}^* \ldots)$ is an equilibrium point.

The third theory that will be used is the replicator dynamic theory. As mentioned above, the emphasis of this theory lies on the evolution of the population; specifically, on the shift in strategies over time through the comparison of payoffs, as discussed in this paper. It is presumed that strategies yielding higher payoffs are considered superior.

The variation in the population's mixed strategy distribution is determined by the reproduction rate of individuals following each strategy. The adjustment in the proportion $p_i$ of the population utilising a particular strategy $s_i$ is related to the discrepancy between the fitness of that strategy $E_i$ and the average fitness $M$ of the population. This principle is captured through the replicator dynamic equation:

$$\dot{p_i} = p_i * (E_i - M) \tag{2}$$

"Fitness of the strategy $s_i$ is equal to the expected payment of playing strategy $s_i$ whereby payments while playing that strategy are weighted with the relative frequencies of the individuals with whom this strategy is faced", as defined in [32] (p. 62).

Ref. [32] (p. 63) further defines that "the average fitness of the population is equal to the expected payment of the population in which the fitness of a particular strategy is weighted with the relative frequencies of individuals (who play a particular strategy) in the population".

The fitness of strategy $s_i$ corresponds to the anticipated payoff from engaging in strategy $s_i$, with payoffs during its execution being adjusted based on the proportions of the counterparts. The population's mean fitness is the aggregate expected payoff, where the effectiveness of a specific strategy is adjusted by the proportional representation of individuals (engaging in that strategy) within the population:

$$M = \sum_{i=1}^{n} p_i * E_i \tag{3}$$

Ref. [32] (p. 63) defines that "to find points where the system tends to move in the long and short run and its stability, those points where the replicator dynamics equation equals zero ($\dot{p_i} = 0$) should be found". Sometimes, these points are called stationary points. Simon and Blum [56] presented a fundamental stability theorem with the rules for checking point stability. The steps mentioned above will be used in this paper as well.

To identify the equilibrium points towards which the system gravitates and to ascertain its equilibrium stability, the points at which $\dot{p_i} = p_i * (E_i - M) = 0$ will be located and referred to as stationary points. Simon and Blum (1994) [56] provide a foundational stability theorem that outlines the criteria for evaluating the stability of these points. This theorem will be used as well.

The results in [8] will be used to describe the relationship between identifying stationary points via the replicator dynamic equation and the notion of evolutionarily stable strategies. An evolutionarily stable strategy is an asymptotically stable stationary point in the context of $\dot{p_i} = 0$. Furthermore, the interplay among evolutionarily stable strategies, the replicator dynamics, Nash equilibria, and their stability is described as follows and used in this paper:

- A point acting as a Nash equilibrium (within a symmetric game setting) concurrently satisfies the condition of $\dot{p}_i = 0$;
- If a point is recognised as a strict Nash equilibrium, it is locally asymptotically stable;
- A point that stands as a locally asymptotically stable solution of $\dot{p}_i = 0$ qualifies as a Nash equilibrium.

When determining the stability of critical points through numerical methods proves challenging, a geometrical approach will be used. This technique is known as phase portrait analysis, which will be used in this paper as well, particularly in the context of non-linear systems. Phase portraits provide a comprehensive view and insight into the dynamics of systems. Typically, a combination of both numerical and geometrical methods will be applied.

Direction Fields and Phase Portraits are presented as techniques to visualise the solutions of differential equations over time, providing perspectives on system stability via graphical interpretation. This strategy is applicable to nonlinear systems, characterised by equations with several variables, illustrating the movement of particles and the dynamics of the system through Vector Fields and Phase Diagrams. The method and examples are provided by [56].

## 4. Modelling

### *4.1. Previous Results Used for Modelling*

For the development of this model, results obtained from [1] will be used, which are listed below. The foundation for modelling has been taken from [32].

First, the employee (agent type) will be categorised as one of three types, as already described above: *Enthusiast*, *Worker*, or *Parasite*.

Utility function for the *Worker* type is defined as:

$$u_2 = \begin{cases} w(1-e_2) + s * \frac{e+e_2}{2} \text{ with given probability } \frac{e+e_2}{2} \\ w \quad \text{with given probability } 1 - \frac{e+e_2}{2} \end{cases} \tag{4}$$

where $e$ is an effort of any other agent type (including the *Worker* type as well); $e_2$ is an effort of *Worker* type; $s$ is the incentive factor (the principal stimulates the agent to work with the incentive factor). Calculated pair-wise outcomes two tables will be used:

Table 1 represents the payment matrix with respect to the total payment for row players for all team combinations. (*Parasite* acts as a *Worker*).

**Table 1.** Total payment matrix (*Parasite* is in the role of a *Worker*).

| | $p_1$ | $p_2$ | $p_3 = 1 - p_1 - p_2$ |
|---|---|---|---|
| **Employee Type** | ***E*** | ***W*** | ***P*** |
| *E* | 1 | $2 - \frac{\sqrt{2}}{4}$ | 1 |
| *W* | $1 + \frac{\sqrt{2}}{4}$ | $\frac{3}{2}$ | 1 |
| *P* | 1 | $\frac{3}{2}$ | $\frac{3}{2}$ |

Source: [1].

Table 1 displays the payment matrix detailing the total payment for row players across all team configurations. (*Parasite* is in the role of a *Worker*).

Table 2 shows the payment matrix, which includes two figures for each team pairing. The first figures in every cell denote the total expected costs (TEC), while the second figures reflect the effective work output (EW) of the team in question:

**Table 2.** TEC and EW strategic matrix.

| | $p_1$ | $p_2$ | $p_3 = 1 - p_1 - p_2$ |
|---|---|---|---|
| **Employee Type** | ***E*** | ***W*** | ***P*** |
| *E* | 2; 2 | $\left(4 - \frac{\sqrt{2}}{2}\right); \sqrt{2}$ | 2; 1 |
| *W* | $\left(4 - \frac{\sqrt{2}}{2}\right); \sqrt{2}$ | 5; 2 | 3; 1 |
| *P* | 2; 1 | 3; 1 | 5; 2 |

Source: [1].

The overall expected payment for the *Enthusiast* is:

$$E_1 = p_1 * 1 + p_2 * \left(2 - \frac{\sqrt{2}}{4}\right) + (1 - p_1 - p_2) * 1 = 1 + p_2 * \left(1 - \frac{\sqrt{2}}{4}\right) \tag{5}$$

The overall expected payment for the *Worker* is:

$$E_2 = p_1 * \left(1 + \frac{\sqrt{2}}{4}\right) + p_2 * \frac{3}{2} + (1 - p_1 - p_2) * 1 = 1 + p_1 * \frac{\sqrt{2}}{4} + p_2 * \frac{1}{2} \tag{6}$$

The overall expected payment for the *Parasite* is:

$$E_3 = 1 * p_1 + \frac{3}{2} * p_2 + (1 - p_1 - p_2) * \frac{3}{2} = \frac{3}{2} - \frac{1}{2} p_1 \tag{7}$$

The overall expected payment for all is:

$$E_{TOTAL} = \frac{3}{2} - p_1 - \frac{1}{2} * p_2 + \frac{1}{2} * {p_1}^2 + \frac{1}{2} * {p_2}^2 + \frac{3}{2} * p_1 * p_2 \tag{8}$$

The aggregate mean expected cost of effective work (total AEC) is given by:

$$\text{Total AEC} = \frac{2 * {p_1}^2 + \left(4 - \frac{\sqrt{2}}{2}\right) * 2 * p_1 * p_2 + 4 * p_1 * p_3 + 5 * {p_2}^2 + 6 * p_2 * p_3 + 5 * {p_3}^2}{2 * {p_1}^2 + \sqrt{2} * 2 * p_1 * p_2 + 2 * p_1 * p_3 + 2 * {p_2}^2 + 2 * p_2 * p_3 + 2 * {p_3}^2} \tag{9}$$

In all the above terms, $p_1$ represent the proportion of *Enthusiasts* in the population, $p_2$ the proportion of *Workers* and $p_3$ proportion of *Parasites* ($p_1 + p_2 + p_3 = 1$). After presenting all the necessary results of the model developed by Talajić, Vrankić, and Kopal [1], the model will be extended by incorporating additional parameters and integrating evolutionary game theory.

### 4.2. Extension of the Model with Reward Parameter for Enthusiast

Ref. [32] (p. 106) define that the parameter "*reward*" is incorporated into the model by rewarding only the *Enthusiast type.* The value of the reward parameter for the *Enthusiast* is denoted by *a*. Using this in Table 1, the payment for the *Enthusiast* should be adjusted by the reward parameter (see Table 3).

**Table 3.** Total payment matrix adjusted for reward parameter.

| | $p_1$ | $p_2$ | $p_3 = 1 - p_1 - p_2$ |
|---|---|---|---|
| **Employee Type** | ***E*** | ***W*** | ***P*** |
| *E* | $1 + a$ | $2 + a - \frac{\sqrt{2}}{4}$ | $1 + a$ |
| *W* | $1 + \frac{\sqrt{2}}{4}$ | $\frac{3}{2}$ | 1 |
| *P* | 1 | $\frac{3}{2}$ | $\frac{3}{2}$ |

In the model, the reward parameter is applied exclusively to the *Enthusiast* type by providing them with a reward. The magnitude of this reward for the *Enthusiast* is indicated by the variable $a$. This adjustment is applied in Table 1, where the *Enthusiast's* payment is modified according to the reward parameter (Table 3).

Expected payment for the *Enthusiast* is recalculated transforming (5); and using the fact that *Parasite* is in the role of *Worker* implies that $p_3 = 0 \Rightarrow p_2 = 1 - p_1$

$$E_1 = 2 + a - \frac{\sqrt{2}}{4} - p_1\left(1 - \frac{\sqrt{2}}{4}\right) \tag{10}$$

For the *Worker* $(E_2)$ and the *Parasite* $(E_3)$ the expected payments remain the same, but $E_2$ will be adjusted using the fact that $p_3 = 0$.

$$E_2 = 1 + p_1 * \frac{\sqrt{2}}{4} + p_2 * \frac{1}{2} = \frac{3}{2} - p_1\left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) \tag{11}$$

For $a \geq \frac{\sqrt{2}}{4}$ then $E_1 > E_2$ as soon as $p_2 > 0$, leading to conclusion that for the equilibrium to be $(E_1 = E_2)p_2$ should be zero. In this case, this is the pure Nash equilibrium $(p_1 = 1;\ p_2 = 0; p_3 = 0)$.

For $0 < a < \frac{\sqrt{2}}{4}$ the Nash equilibrium (using (10) and (11)) is calculated as:

$$\begin{array}{l} E_1 = E_2 \\ 2 + a - \frac{\sqrt{2}}{4} - p_1\left(1 - \frac{\sqrt{2}}{4}\right) = \frac{3}{2} - p_1\left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) \\ p_1 = 1 + 2a - \frac{\sqrt{2}}{2} \ \Rightarrow\ p_2 = 1 - p_1 = \frac{\sqrt{2}}{2} - 2a \end{array}$$

First it should be checked if $(p_1 = 1 + 2a - \frac{\sqrt{2}}{2},\ p_2 = \frac{\sqrt{2}}{2} - 2a, p_3 = 0)$ is really the Nash equilibrium. To prove that it must be checked whether in this point $E_3 < E_1$ and $E_3 < E_2$.

$E_3 < E_1 \Rightarrow$ (using (7) and (10) and given mixed strategy equilibrium):

$$\begin{array}{l} \frac{3}{2} - \frac{1}{2} * p_1 < 2 + a - \frac{\sqrt{2}}{4} - p_1\left(1 - \frac{\sqrt{2}}{4}\right) \\ \frac{3}{2} - \frac{1}{2} * \left(1 + 2a - \frac{\sqrt{2}}{2}\right) < 2 + a - \frac{\sqrt{2}}{4} - \left(1 + 2a - \frac{\sqrt{2}}{2}\right)\left(1 - \frac{\sqrt{2}}{4}\right) \\ a > \frac{\sqrt{2}-2}{4} \approx -0.59 \end{array}$$

For all $a$ that $0 < a < \frac{\sqrt{2}}{4}$ the above inequality always holds true.

The same is checked for $E_3 < E_2$ using (11) and (7):

$$\begin{array}{l} \frac{3}{2} - \frac{1}{2} * p_1 < \frac{3}{2} - p_1\left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) \\ \frac{3}{2} - \frac{1}{2} * \left(1 + 2a - \frac{\sqrt{2}}{2}\right) < \frac{3}{2} - \left(1 + 2a - \frac{\sqrt{2}}{2}\right)\left(\frac{1}{2} - \frac{\sqrt{2}}{4}\right) \\ a > \frac{\sqrt{2}-2}{4} \approx -0.15 \end{array}$$

For all $a$ that $0 < a < \frac{\sqrt{2}}{4}$ the above inequality always holds true.

With these calculations, it has been proved that $(p_1{}^* = 1 + 2a - \frac{\sqrt{2}}{2},\ p_2{}^* = \frac{\sqrt{2}}{2} - 2a, p_3{}^* = 0)$ is really the Nash equilibrium.

#### 4.2.1. Introduction of Evolutionary Game Theory in the Model

The next step is checking the ESS. Using (7), (10), and (11), it follows:

$$E_{TOTAL} = p_1E_1 + p_2E_2 + p_3E_3 = \frac{3}{2} - p_1 - \frac{1}{2} * p_2 + \frac{1}{2} * p_1{}^2 + \frac{1}{2} * p_2{}^2 + \frac{3}{2} * p_1 * p_2 + p_1a \tag{12}$$

$$u(p,p) = E_{TOTAL} = \frac{3}{2} - p_1 - \frac{1}{2} * p_2 + \frac{1}{2} * {p_1}^2 + \frac{1}{2} * {p_2}^2 + \frac{3}{2} * p_1 * p_2 + p_1 a$$

From $p_2 = 1 - p_1$ implies $u(p,p) = \frac{3}{2} + ap_1 - \frac{1}{2}{p_1}^2$.
From (10) and (11):

$$u(p^*,p) = {p_1}^* * E_1 + {p_2}^* * E_2 + {p_3}^* * E_3$$

$$u(p^*,p) = a\left[2 + 2a - \sqrt{2} - \mathrm{p}_1\right] + \frac{9}{4} + - \frac{\sqrt{2}}{2} - \mathrm{p}_1\left(1 - \frac{\sqrt{2}}{2}\right)$$

From (1) being an ESS the inequality $u(p^*,p) - u(p,p) \geq 0$ should be satisfied.

$$u(p^*,p) - u(p,p) = \frac{1}{2}\left[{p_1}^2 - 2\mathrm{p}_1\left(1 - \frac{\sqrt{2}}{2}\right) + \frac{3}{2} - \sqrt{2}\right] + a\left[2 + 2a - \sqrt{2} - \mathrm{p}_1\right] - a\mathrm{p}_1$$

$$u(p^*,p) - u(p,p) = \frac{1}{2}[\mathrm{p}_1 - \left(1 + 2a - \frac{\sqrt{2}}{2}\right)]^2 \geq 0$$

From the above point, $\mathrm{p}_1 = 1 + 2a - \frac{\sqrt{2}}{2}$, $\mathrm{p}_2 = \frac{\sqrt{2}}{2} - 2a, \mathrm{p}_3 = 0$ is an ESS.

For point $(\mathrm{p}_1 = 1;\ \mathrm{p}_2 = 0; \mathrm{p}_3 = 0)$ from Table 3 and the fact $a > \frac{\sqrt{2}}{4}$ implies that this point is the strict Nash equilibrium, so this is ESS.

With the fact that $(\mathrm{p}_1 = 1;\ \mathrm{p}_2 = 0; \mathrm{p}_3 = 0)$ is the Nash equilibrium considering the second player's decision, the optimal response of the first player is a convex mix of the *Enthusiast* and *Worker* strategies, incorporating calculations for $a = \frac{\sqrt{2}}{4}$. The ESS holds when $a = \frac{\sqrt{2}}{4}$.

In this case, it has been simulated that the *Enthusiast* receives a reward; whereas the other two types do not, indicating a certain bias towards the *Enthusiast*. Points $(\mathrm{p}_1 = 1 + 2a - \frac{\sqrt{2}}{2},\ \mathrm{p}_2 = \frac{\sqrt{2}}{2} - 2a, \mathrm{p}_3 = 0)$ and $(\mathrm{p}_1 = 1,\ \mathrm{p}_2 = 0, \mathrm{p}_3 = 0)$ are asymptotic stable ESS.

#### 4.2.2. Optimal Reward Calculation

To calculate the optimal reward ($a$), one should find an a that reduces total AEC to its minimum. Using the equilibrium point $(\mathrm{p}_1 = 1 + 2a - \frac{\sqrt{2}}{2},\ \mathrm{p}_2 = \frac{\sqrt{2}}{2} - 2a, \mathrm{p}_3 = 0)$ and (9), total AEC is given by:

$$\text{Total AEC} = \frac{2 * \left(1 + 2a - \frac{\sqrt{2}}{2}\right)^2 + \left(4 - \frac{\sqrt{2}}{2}\right) * 2 * \left(1 + 2a - \frac{\sqrt{2}}{2}\right) * \left(\frac{\sqrt{2}}{2} - 2a\right) + 4 * \left(1 + 2a - \frac{\sqrt{2}}{2}\right) * 0 + 5 * \left(\frac{\sqrt{2}}{2} - 2a\right)^2 + 6\left(\frac{\sqrt{2}}{2} - 2a\right) * 0 + 5 * 0^2}{2 * \left(1 + 2a - \frac{\sqrt{2}}{2}\right)^2 + \sqrt{2} * 2 * \left(1 + 2a - \frac{\sqrt{2}}{2}\right) * \left(\frac{\sqrt{2}}{2} - 2a\right) + 2 * \left(1 + 2a - \frac{\sqrt{2}}{2}\right) * 0 + 2 * \left(\frac{\sqrt{2}}{2} - 2a\right)^2 + 2\left(\frac{\sqrt{2}}{2} - 2a\right) * 0 + 2 * 0^2}$$

$$\text{Total AEC} = \frac{1 + 5\sqrt{2} - 2a\left(10 - 3\sqrt{2}\right) + 8\sqrt{2}a^2}{2\left[3\left(2 - \sqrt{2}\right) + 4a\left(4 - 3\sqrt{2}\right) + 8a^2\left(2 - \sqrt{2}\right)\right]}$$

Considering that, for the mixed strategy point, the range for $a$ fall within an interval $[0, \frac{\sqrt{2}}{4})$ (see Figure 1). For $a = \frac{\sqrt{2}}{4}$ it is point $(\mathrm{p}_1 = 1,\ \mathrm{p}_2 = 0, \mathrm{p}_3 = 0)$ that represents the pure Nash equilibrium, which is also an ESS. The total AEC function is established over a closed interval $\left[0, \frac{\sqrt{2}}{4}\right]$ due to the relevance for those identified as ESS.

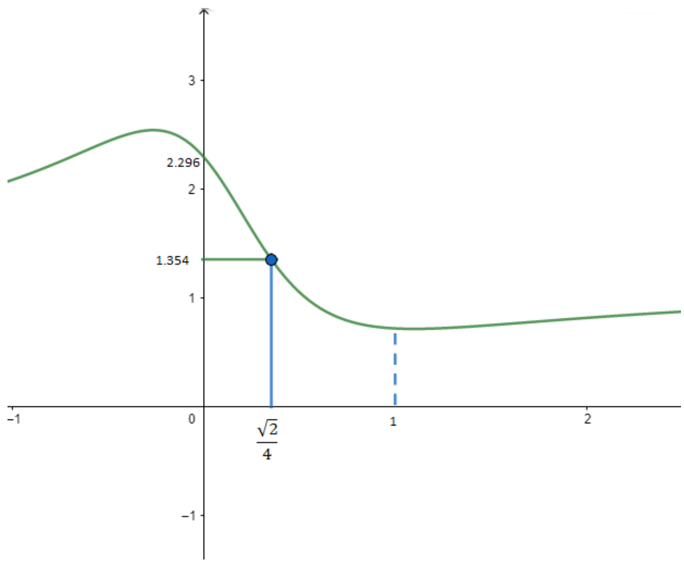


**Figure 1.** Total AEC when the *Enthusiast* receives a reward.

If $a > \frac{\sqrt{2}}{4}$ then $\left(\mathrm{p}_1 = 1,\ \mathrm{p}_2 = 0, \mathrm{p}_3 = 0\right)$ is an ESS, eliminating the necessity to raise the cost; thus the optimal value is $a = \frac{\sqrt{2}}{4}$.

This is true if the principal aims solely for the lowest total AEC at a point where all are of type *Enthusiast*, without considering the appeal of that specific point.

### 4.2.3. Replicator Dynamic and Phase Portrait

For $a = \frac{\sqrt{2}}{4}$ Table 3 is being converted into Table 4.

**Table 4.** Total payment matrix: *Enthusiast* reward is a = $\frac{\sqrt{2}}{4}$.

| | $p_1$ | $p_2$ | $p_3 = 1 - p_1 - p_2$ |
|---|---|---|---|
| **Employee Type** | ***E*** | ***W*** | ***P*** |
| *E* | $1 + \frac{\sqrt{2}}{4}$ | 2 | $1 + \frac{\sqrt{2}}{4}$ |
| *W* | $1 + \frac{\sqrt{2}}{4}$ | $\frac{3}{2}$ | 1 |
| *P* | 1 | $\frac{3}{2}$ | $\frac{3}{2}$ |

For $a = \frac{\sqrt{2}}{4}$, *Enthusiast's* expected payment (5) is transforming to:

$$\mathrm{E}_1 = 1 + \mathrm{p}_2 * \left(1 - \frac{\sqrt{2}}{4}\right) + \frac{\sqrt{2}}{4} \tag{13}$$

For *Worker* $(E_2)$ and *Parasite* $(E_3)$ the expected payments are the same as in (6) and (7). $E_{TOTAL} = p_1 E_1 + p_2 E_2 + (1 - p_1 - p_2) E_3 \Rightarrow$ from (6), (7) and (13)

$$E_{TOTAL} = p_1\left(1 + \mathrm{p}_2 * \left(1 - \frac{\sqrt{2}}{4}\right) + \frac{\sqrt{2}}{4}\right) + p_2\left(1 + \mathrm{p}_1 * \frac{\sqrt{2}}{4} + \mathrm{p}_2 * \frac{1}{2}\right)$$
$$+ (1 - p_1 - p_2)\left(\frac{3}{2} - \frac{1}{2} * p_1\right)$$

after transforming the term:

$$E_{TOTAL} = \frac{3}{2} - p_1\left(1 - \frac{\sqrt{2}}{4}\right) - \frac{1}{2}p_2 + \frac{3}{2}\mathrm{p}_1\mathrm{p}_2 + \frac{1}{2}{p_1}^2 + \frac{1}{2}{p_2}^2 \tag{14}$$

Ref. [32] (p. 115) indicates that "using the replicator dynamics tools (see (2)), the probability (percentage) growth rate is measured as a difference between the fitness of an observed agent's expected payment and the total expected payment".

Utilising tools of replicator dynamics (refer to (2)), the growth rate in terms of probability (percentage) is calculated by the difference between the fitness of a particular agent's expected payment and the overall expected payment:

$$\frac{\dot{\mathrm{p}}_1}{\mathrm{p}_1} = E_1 - E_{TOTAL} = \frac{1}{4} * [\sqrt{2} - 2 + p_1 * \left(4 - \sqrt{2}\right) + \left(6 - \sqrt{2}\right)p_2 - 2{p_1}^2 - 2{p_2}^2 - 6p_1p_2] \quad (15)$$

$$\frac{\dot{\mathrm{p}}_2}{\mathrm{p}_2} = E_2 - E_{TOTAL} = \frac{1}{2} * [-1 + 2p_1 + 2p_2 - {p_1}^2 - {p_2}^2 - 3p_1p_2] \quad (16)$$

Equilibrium points:

(a) For (15)

$\mathrm{p}_1 = 0$;

or

$\frac{1}{4} * \left[\sqrt{2} - 2 + p_1\left(4 - \sqrt{2}\right) + \left(6 - \sqrt{2}\right)p_2 - 2{p_1}^2 - 2{p_2}^2 - 6p_1p_2\right] = 0 \Leftrightarrow$

$$2{p_2}^2 + \left(6p_1 - 6 + \sqrt{2}\right)p_2 + 2{p_1}^2 - \left(4 - \sqrt{2}\right)p_1 + 2 - \sqrt{2} = 0 \quad (17)$$

Using (17), this should calculate $p_2$ as a function of $p_1$ and draw this dependence. It will be the first isocline. The second is $p_1 = 0$.

Determinant of (17) is calculated:

$$D = 36{p_1}^2 + 38 - 72p_1 + 12\sqrt{2}p_1 - 12\sqrt{2} - 16{p_1}^2 + 8p_1\left(4 - \sqrt{2}\right) - 16 + 8\sqrt{2}$$

$$D = 20{p_1}^2 - 40p_1 + 4\sqrt{2}p_1 + 22 - 4\sqrt{2} \quad (18)$$

using (17) and (18):

$$(p_2)_{1,2} = \frac{6 - 6p_1 - \sqrt{2} \pm \sqrt{D}}{4} = \frac{6 - 6p_1 - \sqrt{2} \pm \sqrt{20{p_1}^2 - 40p_1 + 4\sqrt{2}p_1 + 22 - 4\sqrt{2}}}{4} \quad (19)$$

In Figure 2, isoclines are drawn (dashed lines).

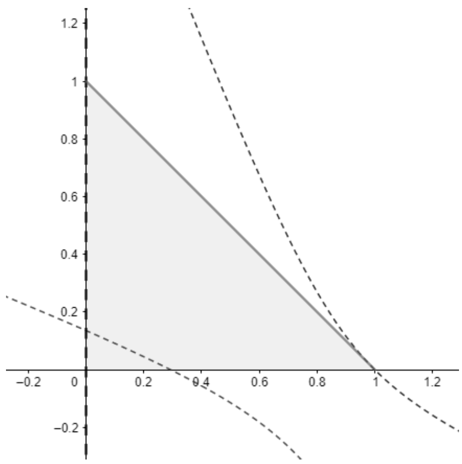


**Figure 2.** Isoclines for *Enthusiast*.

The area of interest is located within the shaded triangle. In Figure 3, the vector field is represented.

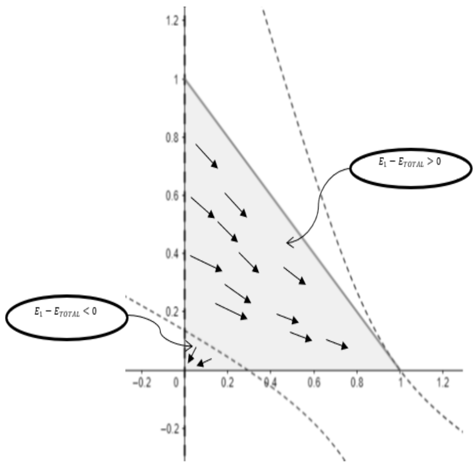


**Figure 3.** Vector field for *Enthusiast*.

(b) For (16)

$p_2 = 0$;
or
$\frac{1}{2} * \left[-1 + 2p_1 + 2p_2 - {p_1}^2 - {p_2}^2 - 3p_1p_2\right] = 0 \Leftrightarrow$

$$ {p_2}^2 + (3p_1 - 2)p_2 + {p_1}^2 - 2p_1 + 1 = 0 \quad (20) $$

Again, this should represent $p_2$ as a function of $p_1$ using (20) and draw this dependence. It will be the first isocline. The second is $p_2 = 0$.

Determinant of (20) is calculated:

$$ D = 9{p_1}^2 - 12p_1 + 4 - 4{p_1}^2 + 8p_1 - 4 = 5{p_1}^2 - 4p_1 \quad (21) $$

Using (20) and (21):

$$ (p_2)_{1,2} = \frac{2 - 3p_1 \pm \sqrt{D}}{2} = \frac{2 - 3p_1 \pm \sqrt{5{p_1}^2 - 4p_1}}{2} \quad (22) $$

The isocline crosses the shaded region (see Figure 4) exclusively at points $(1, 0)$ and $(0, 1)$. Within the shaded part $E_2 - E_{TOTAL} < 0$, the vector field appears as shown in Figure 5.

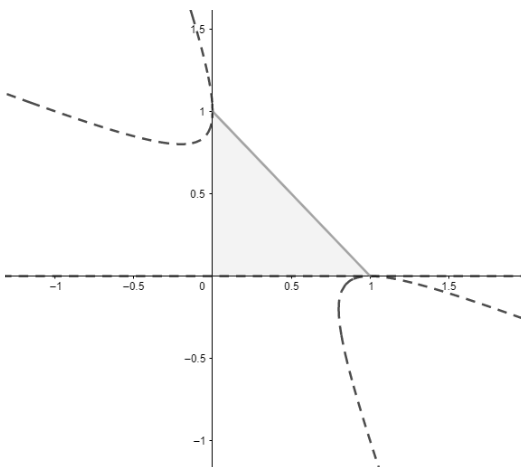

**Figure 4.** Isocline for *Workers*.

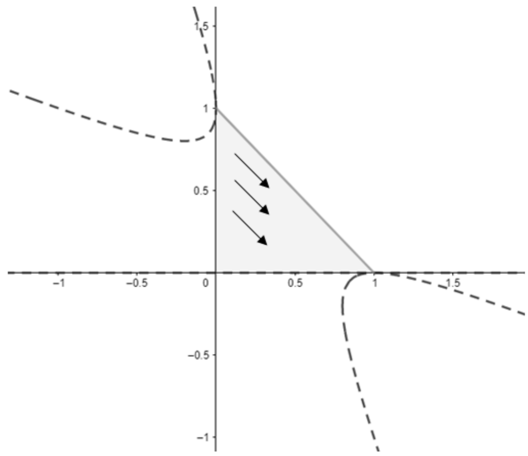


**Figure 5.** Vector field for *Workers*.

Now all isoclines will be drawn on the same figure and the orbits will be sketched using the above-described vector fields (see Figure 6). Isoclines will be plotted on a single diagram, and the orbits will be outlined based on the previously detailed vector fields (see Figure 6).

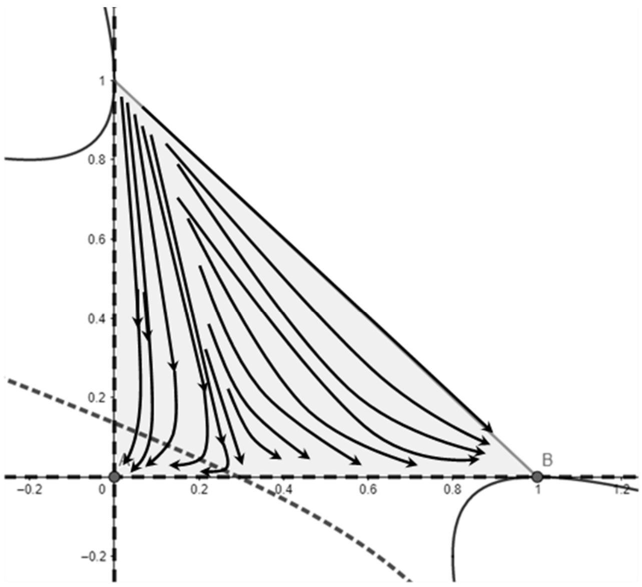


**Figure 6.** Movement of orbits.

The orbits' movement analysis showed two attraction points: $(p_1 = 1,\ p_2 = 0; p_3 = 0)$ and $(p_1 = 0,\ p_2 = 0; p_3 = 1)$.

Applying (9), the total AEC is computed for both points. For the $(p_1 = 1,\ p_2 = 0; p_3 = 0)$, it is 1.354; whereas for the $(p_1 = 0,\ p_2 = 0; p_3 = 1)$, it is 2.5. As anticipated, the model demonstrated that out of these two stable points, a population composed solely of *Enthusiast* is preferable. Specifically, the model where only the *Enthusiast* received a reward resulted in the lowest total AEC compared to all other scenarios.

Given that the starting structure of the employee population may be unknown, it is critical to segment the area of attraction based on two distinct potential long-term scenarios, each with its own total AECs. The issue is whether additional rewards can enhance the area of attraction to align with the preferred state $(p_1 = 1,\ p_2 = 0; p_3 = 0)$, despite leading to a higher total AEC. By increasing the reward, $a$, the principal can enlarge this area. Illustrated in Figure 7, the adjustment of the isocline (17) to cross the horizontal axis at the point $(p_1 = 0,\ p_2 = 0; p_3 = 1)$ indicates desire state and the feasible enlargement of the attraction area for the point $(p_1 = 1,\ p_2 = 0; p_3 = 0)$.

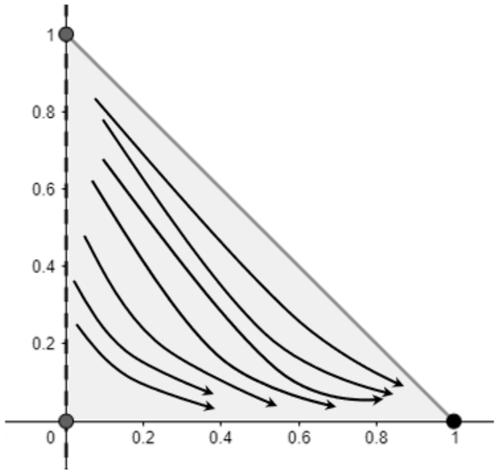


**Figure 7.** *Enthusiast* reward increased case.

With (10) and (12), alongside the necessity to shift the *Enthusiast* isocline, the replicator dynamic equation will be derived to achieve the target state $(p_2 = 0)$.

$$(E_1 - E_{TOTAL})(p_2 = 0) = \frac{1}{4}\left(-2 + 4p_1 - 2p_1{}^2\right) + a(1 - p_1)$$
$$= -\frac{1}{2} + p_1 - \frac{1}{2}p_1{}^2 + a - ap_1 = -\frac{1}{2}p_1{}^2 + (1-a)p_1 + a - \frac{1}{2} = 0.$$

The calculated result is given by:

$$(p_1)_{1,2} = 1 - a \pm a \ \Rightarrow\ p_1 = 1 \text{ or } p_1 = 1 - 2a.$$

Given the requirement for the isocline to intersect at point (0,0), then $1 - 2a = 0 \Rightarrow a = \frac{1}{2}$.

For the identical value of $a$, it is imperative to observe where the isocline intersects the vertical axis to confirm that it lies beyond the shaded triangle.

$$(E_1 - E_{TOTAL})(p_1 = 0,\ a = \tfrac{1}{2}) = \frac{1}{4}\left(-2 + \left(6 - \sqrt{2}\right)p_2 - 2p_2{}^2\right) + \frac{1}{2}$$
$$= 2p_2\left(3 - \frac{\sqrt{2}}{2} - p_2\right) = 0 \ \Rightarrow p_2 = 0 \text{ or } p_2 = 3 - \frac{\sqrt{2}}{2}$$

As $3 - \frac{\sqrt{2}}{2} > 1$, it is evident that the isoclines for $a = \frac{1}{2}$ cross the shaded triangle solely at the point (0,0). In this scenario, total AEC is 1.5 and is greater than 1.354 (total AEC for $a = \frac{\sqrt{2}}{4}$).

By raising the reward, the principal can widen the area of attraction in the direction of the preferred point, where only the *Enthusiast* group is present. To achieve this, the principal must incur a cost to reduce the risk and shrink the area of attraction, leading to a point solely populated by the *Parasite* group. This will cost 0.146 more in expenses. Through the strategic adjustment of rewards, the principal can minimise the risk of ending up with an undesirable population composition.

In this scenario, discrimination can be an issue (particularly against the *Worker* group). This may prompt some *Worker* agents to adopt *Parasite*-like behaviours, and this has the potential to alter the trajectory of the orbit, consequently impacting stability and costs to a considerable extent. Additionally, the approach of penalising *Parasite* types for their actions could be considered as well. These considerations represent potential future enhancements that will not be presented in this paper. Equally critical is the management of a population exclusively made up of *Enthusiasts*, considering their high motivational requirements and how much will cost the organisation to fulfil those requirements.

### 4.3. Scenario with No Incentives for All and with Reward Parameter for Enthusiast

The next scenario analysis proceeds under conditions where there are no incentives ($s$) for all agents and with a reward ($a$) provided exclusively for the *Enthusiast*. Using (4) and Table 1, Table 5 shows the payments in this situation.

**Table 5.** Scenario of payment matrix with reward ($a$) only for the *Enthusiast* and no incentive ($s$) for all.

| | $p_1$ | $p_2$ | $p_3 = 1 - p_1 - p_2$ |
|---|---|---|---|
| **Employee Type** | ***E*** | ***W*** | ***P*** |
| $E$ | $1+a$ | $1+a$ | $1+a$ |
| $W$ | 1 | 1 | 1 |
| $P$ | 1 | 1 | 1 |

The expected payments for each agent are calculated.

$$E_1 = p_1(1+a) + p_2(1+a) + (1 - p_1 - p_1)(1+a) = 1+a$$

$$E_2 = p_1 * 1 + p_2 * 1 + (1 - p_1 - p_1) * 1 = 1$$

$$E_3 = p_1 * 1 + p_2 * 1 + (1 - p_1 - p_1) * 1 = 1$$

For dynamic $E_{TOTAL}$ is needed:

$$E_{TOTAL} = p_1 E_1 + p_2 E_2 + (1 - p_1 - p_2) E_1 = p_1 a + 1$$

The replicator dynamic equation gives:

$$\dot{p_1} = p_1(E_1 - E_{TOTAL}) = p_1(1 + a - (p_1 a + 1)) = a p_1 (1 - p_1) \quad (23)$$

$$\dot{p_2} = p_2(E_2 - E_{TOTAL}) = p_2(1 - (p_1 a + 1)) = -a p_1 p_2 \quad (24)$$

From (23) and (24):

- From (23), isoclines for *Enthusiast* are $p_1 = 0$ and $p_1 = 1$;
- From (24), isoclines for *Worker* are $p_1 = 0$ and $p_2 = 0$.

In Figure 8, the isoclines (represented by dashed lines) are depicted, with a focus on the shaded triangle; and potential equilibrium points are identified as A and B.

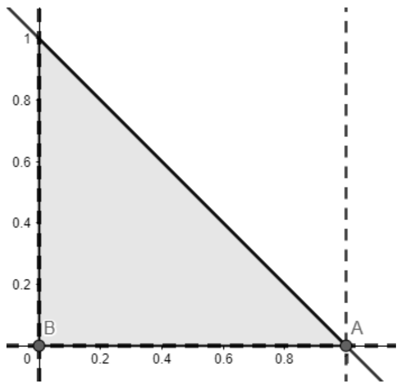


**Figure 8.** Isoclines: scenario with no incentive and only reward for *Enthusiast*.

Within the region bounded by the isoclines (the shaded triangle) $E_1 - E_{TOTAL} > 0$ and $E_2 - E_{TOTAL} < 0$. The vector field is formed (see Figure 9).

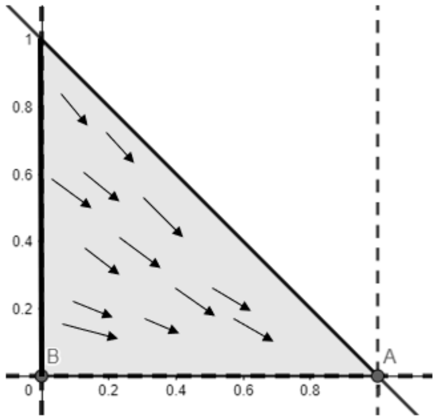


**Figure 9.** Vector field: scenario with no incentive and only reward for *Enthusiast*.

The rationale behind this is that every orbit passes through the point (1,0). The equation line with a slope of k, passing through the point (1,0), can be expressed as:

$$p_2 - 0 = k(p_1 - 1) \Rightarrow p_2 = k(p_1 - 1) \tag{25}$$

Putting (25) into (24):
$\dot{p_2} = -ap_1k(p_1 - 1) = k * ap_1(1 - p_1) = k\dot{p_1} \Rightarrow$ orbits represent lines that intersect at the point $(1, 0)$.

The orbits' movements are depicted (see Figure 10), and they tend towards the point $A$, $(p_1 = 1,\ p_2 = 0,\ p_3 = 0)$. From Table 5 and the effort of *Enthusiast* $e_1 = 1$, it can be calculated that total expected cost is $2 + 2a$ and total effective work is 2. This leads finally to a total average expected cost of $1 + a$.

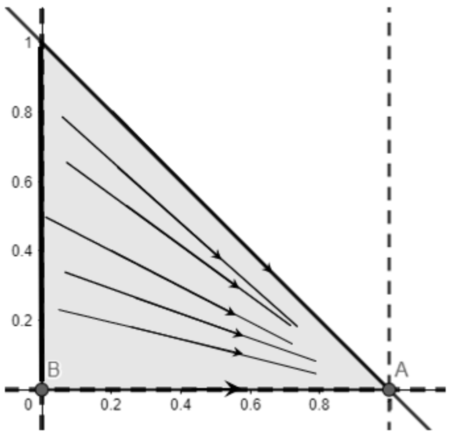


**Figure 10.** Movement of orbits in "no incentive" scenario.

According [32] (p. 123), "extreme case shows that the cost per unit of efficient work can be further reduced by eliminating the incentive and very small rewards. However, in the short run, the company would not have enough of the efficient workforce necessary for business (because there is not enough *Enthusiast* type). In the long run, the number of *Enthusiast* types would increase to 100%. In this extreme case, it is confirmed that the incentive is a good short-run motivational tool, while the reward is a good long-run motivational tool. By combining these two tools, the organisation minimises (manages) the total AEC and makes sure to have enough workforce in the short run. However, again, Enthusiast's characteristics should be considered so that the company can manage them effectively (for future research). The general conclusion is that work is stimulated with incentive, while a reward motivates behaviour change".

In this extreme scenario, it is demonstrated that the cost per unit of efficient work can be further reduced by eliminating incentives and providing minimal rewards. However, in the short term, the company may face a shortage of the efficient workforce required for operations (due to insufficient *Enthusiast* types). Over the long term, the proportion of *Enthusiast* type would grow to 100%. This extreme case confirms that incentives are effective as short-term motivational tools, while rewards are effective in the long term. By integrating these two approaches, the organisation minimises the total AEC and ensures an adequate workforce in the short term. Nonetheless, it is essential to consider the specific characteristics of *Enthusiast* for effective management. In summary, incentives stimulate work, whereas rewards motivate behavioural change.

## 5. Findings and Discussion

This study extends the theoretical research established by [1], illustrating the application of traditional and evolutionary game theory along with replicator dynamics in researching the dynamics between leaders and their teams. Two significant novelties have been introduced into the existing model. The first novelty introduces a motivating reward parameter aimed at a specific employee group, while the second embeds evolutionary game theory within the modelling approach to evaluate the emergence of ESS.

When rewards were allocated solely to the *Enthusiast* category, the potential for optimising costs emerged; evidenced by a reduced cost of 1.354 compared to 2.296 without the *Enthusiast*'s reward, as calculated by [1]. A total population of *Enthusiast* emerges as the equilibrium, which is theoretically stable but practically challenging. The *Enthusiast* category, by nature demanding, may face decreased motivation when tasked with work typically assigned to *Worker* category; leading to potential disengagement and attrition. This could leave the organisation short-staffed and, at its most critical, shift the population to an equilibrium consisting solely of *Parasites*. Thus, the extreme scenario suggests that leaders should also consider rewards for the *Worker* category and disincentives for *Parasites* to achieve a balanced workforce structure, potentially refining the model in future iterations.

Another observed scenario is the further reduction in costs with the removal of incentives for all categories, offering minimal reward to the *Enthusiast*. Here, the organisation faces the short-term challenge of an inefficient workforce. These scenarios reveal that incentives may act as a short-term motivational tool, while rewards serve as a long-term strategy tool to induce favourable behavioural changes. There may be circumstances where increasing rewards for the *Enthusiast* to 1.5 from 1.354, although raising costs slightly, strategically decreases the risk of a *Parasite*-dominated workforce with significantly higher costs. This increase can be seen as paying a premium for risk insurance in various contexts.

While these represent extreme scenarios, they demonstrate the potential of theoretical models in guiding leadership to apply rewards and incentives for structuring their teams to achieve optimal outcomes. The paper's scientific contribution lies not only in modelling the interplay between a leader and a diverse team but also in applying these findings to driving leadership strategies and organisational structure definition, and better definitions of employees' roles. It suggests scenarios that enable future refinement of the model.

The paper suggests the ideal evolutionary structure based on a theorem, outlining the criteria for strategies to become evolutionarily stable. For identified equilibrium points, the ESS status is verified through the theorem, and the optimal ratios are determined.

The results obtained reveal several key findings that align with, enhance, or diverge from the existing literature. The traditional game theory, as applied in the study, emphasises the principal–agent relationship and strategic interactions among different employee types (*Enthusiast*, *Worker*, *Parasite*). This approach aligns with the perspectives offered by Linder and Foss [27], and O'Donnell and Sanders [28], who discuss the principal–agent framework in the context of leadership and organisational dynamics.

The use of traditional game theory to model leadership interactions and employee behaviours confirms the foundational insights provided by these authors, enhancing the understanding of strategic decision-making processes within organisations. The extension of the model to include evolutionary game theory provides insights into the stability of employee-type structures and the dynamics of strategy evolution. This integration supports the findings of Szolnoki and Perc [10], who emphasise the importance of non-conformist strategies in achieving competitive advantage. The study's results, showing that a balanced workforce structure can be maintained through strategic adjustments, echo the significance of ESS in fostering cooperative and competitive behaviours. ESS extension within the model is an additional upgrade of the model developed by [1], where evolutionary game theory was not observed.

The modelling results demonstrate that targeted rewards for the *Enthusiast* category can optimise organisational costs and improve overall performance. This finding aligns with the research by Zhang et al. [13], who highlight the importance of equity-based incentives in promoting digital transformation and cooperative behaviour. The study's empirical evidence that rewards can significantly influence employee motivation and engagement corroborates the theoretical assertions made by these authors. This outcome also supports the work of Li et al. [17], who noted that enhanced prosocial behaviours driven by rewards can significantly influence group dynamics.

A comprehensive review of the relationship between game theory and AI indicated that the use of AI to optimise workforce management strategies is a novel aspect of this study. This resonates with the work of Hazra and Anjaria [43], who discuss the intersection of game theory and deep learning. The study's idea of leveraging AI for dynamic strategy adjustments and decision-making enhances the practical applicability of theoretical models, supporting the notion that AI can significantly improve organisational outcomes by enabling more informed and adaptive leadership strategies. This part extends this field for future research.

The empirical results indicate that a mix of short-term incentives and long-term rewards is necessary to maintain an efficient and motivated workforce. This conclusion is in line with the findings of Amalia and Prayekti [24], who argue that transformational leadership and well-designed incentive programmes can enhance employee morale and performance. The study's insights into the balance of incentives and rewards provide a deeper understanding of how leadership can strategically influence employee behaviour to achieve optimal performance.

The integration of evolutionary game theory and replicator dynamics into the workforce management model offers a more nuanced understanding of how employee types interact and evolve. This enhancement addresses the gap identified by previous studies that primarily focused on traditional game theory without considering the evolutionary aspects of strategy dynamics and more or less only one agent type, such as those by Bierman and Fernandez (1993) [6], or Stankova and Olsder (2006) [30].

The research reveals how the integrated three-theory model, TER—comprising traditional game theory (T), evolutionary game theory (E), and replicator dynamics (R)—can assess the impact of principal–agent relations on organisational performance. Collectively, these theories forge a robust tool for determining stable, cost-effective workforce ratios that aid leaders in decision-making.

This integrative approach marks a significant advance in scientific inquiry, positing a critical framework not just for leadership studies but for broader research applications as well. The methodological framework is depicted in Figure 11.

The TER model—considering a comprehensive perspective as displayed in Figure 11—fuses three distinct theories with their respective concepts, methodologies, and tools. The other aspect represents the system's state, whether static or dynamic. These perspectives integrate into a cohesive methodological structure that combines static analysis with dynamic processes, evaluating both the structure and stability of populations. Its applications extend beyond the scenarios presented in the manuscript.

| | STATIC | | DYNAMIC |
|---|---|---|---|
| THEORY | Basic game theory | Evolutionary game theory | Replicator Dynamic |
| CONCEPT | Nash Equilibrium | Evolutionary Stable Strategies (ESS) | Replicator Dynamic Equation |
| METHODS | Best Response (pure strategies), Expected Payment (mix strategies) | ESS chek and relation among Nash Equilibrium and ESS | Stability theorem and Replicator Dynamic Equation |
| TOOLS | Linear Algebra | Linear Algebra and Probability | Phase Portrait |

**Figure 11.** TER methodological framework.

The theoretical framework provides a robust concept for understanding and managing workforce diversity, offering practical insights for leaders to optimise organisational performance through the strategic use of rewards and incentives. The alignment with existing findings and the introduction of new perspectives through AI integration and evolutionary game theory represent significant contributions to the field of strategic workforce management.

While the theoretical models provide valuable insights, the practical application of these models may face challenges; particularly in managing a population of *Enthusiast* employees. The high motivational requirements and potential for disengagement among *Enthusiast* types when performing routine tasks highlight the need for further refinement of the model to incorporate practical considerations and real-world constraints.

Like any scientific work, this one also has room for enhancement and opens the way for new research areas; thereby contributing further to the field. The theoretical and methodological framework outlined here offers the groundwork for future studies that may build upon or refine the results of this research or even give way for entirely new applications of the developed methodological framework.

The categorisation of employee groups in the model is informed by extensive literature and the author's profound experience in leadership. Current literature and the author's knowledge do not provide a validated theoretical base for such classifications. Although this limitation does not significantly impact the model's outcomes, validation would give the credibility of the defined categories and enhance the rationale behind their selection, suggesting a potential direction for future research to define these classifications better.

The model deals with a specific structure of employee groups but does not detail the criteria for assigning individuals to these categories. Identifying the group for a new hire is not addressed in this paper, but it represents another direction for enhancing the model. Concepts such as moral hazard and adverse selection from game theory could inform the development of methods for classifying individual employees.

Furthermore, the model explores the dynamics of employee transition between groups, without delving into the psychological and sociological drivers of such shifts; indicating an area for in-depth analysis of these transformative behaviours.

A novel aspect of the model is its observation of the interaction between leaders and diverse employees. Future enhancements could consider scenarios with multiple leaders or hierarchical structures where agents manage other agents, introducing complexities to the calculations due to more intricate variable structures.

While the model initially supposes three employee groups, expanding this to four, five, or more is a possibility for subsequent research. Although not recommended beyond five for visual representation purposes, the use of replicator dynamics and ESS remains viable with more complex computations.

The presumption that the *Enthusiast* always gives maximum effort is questioned, suggesting the integration of a coefficient to vary this effort, allowing for diverse simulations and potential model refinements.

Incorporating social network analysis could provide additional value, examining the influence within and between groups, identifying key employees, and enabling leaders to focus their efforts where most impactful strategically. The innovative SNA approach of Damij et al. [57] could serve as an initial point for further investigation.

The impact of varying organisational cultures on the model's efficacy is another aspect not accounted for, presenting an opportunity to assess how different cultural contexts might alter the results. Validation across diverse organisations and subsequent model adjustments constitute promising directions for the model's evolution, acknowledging the challenge of creating a universally applicable framework.

One significant contribution of such models in the future should be their integration into AI-driven processes and the digitalisation of systems. This is in line with the results of Tagscherer and Carbon [22], who observed digital leadership and pointed out that digitalisation requires leaders with new skills to navigate digital transformation effectively.

Today's AI/ML principles operate in a manner that provides the system with a large amount of data (both structured and unstructured), and the system learns from the data according to the goal set for it. All these systems attempt to learn autonomously without, or with less, human involvement.

However, it raises the question of whether these learning processes could still be improved with the human factor. The models from this and similar papers were created through the conceptual thinking of experts in the field of game theory. A goal was set, an approach was presented, necessary functions were defined, and results were obtained by utilising certain areas of game theory and replicator dynamics.

In integrating human reasoning and artificial intelligence, it is possible to achieve a significant synergistic effect between humans and technology. The entire process of conceptual thinking, defining necessary functions, including elements of game theory and replicator dynamics, can be given by humans as input in the AI modelling process as additional context in reasoning. Furthermore, with quality human guidance, the AI system performs modelling and further develops the model.

The fact that the AI system is not only given data from which it learns, which often represents a "black box" to humans, but is also provided with a theoretical foundation that has clearly defined theorems and postulates, makes such a system explainable. In today's AI world, making the results of AI systems explainable is a significant challenge.

Given that the calculations in this work were performed on a group of three types of employees with sophisticated artificial intelligence systems, introducing even more different types of employees would not be a problem. In such a case, more complex calculations could be transferred from humans to technology. Robust AI systems would also not have a problem with calculating stationary points, drawing vector fields and orbits, and checking for ESS and optimal equilibria. This, indeed, represents a huge potential for further development. Moreover, with good human guidance, AI can also help initially define the utility function and behaviour function of each type of employee.

Based on this, the software could be developed with such a model at its core, which would allow for fine-tuning of results by simply changing different parameters to achieve optimal outcomes. The ultimate benefit would be a model adapted to the culture and structure of each organisation. Finally, if—in collaboration with the HR team—employees of the organisation were grouped into defined types through a specific survey, such software would enable leaders to manage each type of employee strategically, monitor the structure by type, and make informed decisions in achieving the optimal structure of their employees.

This would then be a real "cockpit" based on AI and game theory expertise, through which a leader could simulate and evaluate the impact of different structures on the organisation's results and thereby plan the hiring of new employees depending on the type of employee who would bring optimal results. Besides new hires, such a system would assist in managing existing employees and their motivation.

In summary, this manuscript not only progresses the model designed [1] through parameter adjustments and the introduction of new variables but also showcases its practical application as a theoretical guide to the dynamics between leaders and their teams. The integration of evolutionary game theory offers fresh insights into the ongoing leader–follower narrative, addressing the research query posited at the outset and fulfilling the study's objectives.

## 6. Conclusions

In conclusion, the modelling approach in this paper is an innovative perspective that significantly contributes to the literature on workforce management by integrating traditional game theory, evolutionary game theory, and replicator dynamics into a cohesive framework. The primary aim was to optimise employee engagement and organisational performance through strategic management of workforce diversity, particularly by identifying optimal proportions of different employee types—*Enthusiast*, *Worker*, and *Parasite*—using mathematical modelling.

This research extends the theoretical foundation laid by Talajić, Vrankić, and Kopal [1] by introducing two novel elements: a motivating reward parameter and the incorporation of evolutionary game theory to identify ESS. The integration of these theoretical approaches provides a robust framework for understanding the complex dynamics of workforce management. Specifically, the study demonstrates how strategic manipulation of rewards and incentives can lead to more stable and efficient employee populations, thus contributing new insights into the application of game theory in human resource management.

The modelling approach in this paper is an innovative perspective on leadership dynamics, charting new possibilities for leaders globally. The added value of the modelling is the incorporation of evolutionary game theory techniques; a fresh addition to the relationship between leadership and team dynamics. Analysing these dynamics through the lens of population shifts—tracking the temporal distribution of different employee types—marks a progressive step in the study and understanding of leadership's impact on team structure. The model predicts the trajectory of various employee types within the workforce and allows leaders to balance the current structure to an evolutionary stable state where cost efficiency is maximised.

The model specifically addresses the rewarding of the *Enthusiast* type. This represents one of the more radical scenarios, yielding a homogeneous *Enthusiast-based* workforce as the stable state. An alternate scenario explored is the absence of initial incentives for any employee category, with only the *Enthusiast* receiving additional rewards, which similarly results in a stable *Enthusiast-centric* population. Though such extreme workforce compositions may seem implausible, they underscore the model's comprehensive scope and its potential for future research enhancements. Adjustments to the model's variables can lead to revised outcomes and more informed decision-making. Considerations might include varied incentives for the *Worker* type or sanctions for the *Parasite* type, as well as the introduction of different leader types, among other variables.

The practical implications of this study are manifold. By demonstrating that rewards can effectively motivate *Enthusiast* employees and that a combination of incentives and rewards can optimise workforce structure in both the short and long term, this research offers actionable strategies for organisational leaders. The findings suggest that while incentives are effective as short-term motivators, rewards play a crucial role in long-term behavioural changes. This dual approach can help organisations maintain a balanced and motivated workforce, thereby enhancing overall performance and reducing operational costs.

The novelty of this study lies in its comprehensive approach to workforce management, combining multiple theoretical frameworks to provide a detailed analysis of employee dynamics. This integrative model not only offers a theoretical contribution by filling a notable gap in the literature but also provides practical guidance for leaders in structuring their teams. Future research could build on these findings by exploring the impact of varying reward structures on different types of employees and further refining the model to include additional parameters, such as penalties for undesirable behaviours.

The theoretical framework opens an expansive field of possibilities for monitoring the long-term stability of employee populations, laying the groundwork for future research initiatives and broader application of this framework. It is particularly well-suited for larger organisations with sizable workforces, where it demonstrates its practical value by calculating the ideal proportion of each employee type and thereby influencing the organisation's cost structure.

By providing such models (frameworks) as additional context to AI systems, it is possible to create a strong synergy between humans and technology to achieve even better and more explainable results. This synergy offers great potential for the development of AI-based systems in this field in the future.

This study offers a significant contribution to both the theory and practice of workforce management. Combining traditional game theory, evolutionary game theory, and replicator dynamics, it provides a nuanced understanding of how strategic management of employee diversity can enhance organisational performance. The findings underscore the importance of tailored motivational strategies and offer a roadmap for leaders seeking to leverage these insights in the context of digital transformation and competitive advantage in the modern knowledge economy. This research not only advances theoretical models but also provides actionable guidance for managers, investors, and policymakers navigating the complexities of workforce management in an increasingly digital and competitive landscape. The idea of integrating this theoretical framework with AI models serves as an impulse for further research and the development of robust, explainable AI models driven together by humans and technology, which could represent a significant practical and scientific contribution in the future.

**Author Contributions:** Conceptualisation, M.T. and I.V.; methodology, M.T. and I.V.; software, M.T.; validation, M.P.B., M.T. and I.V.; formal analysis, M.T.; investigation, M.T.; data curation, M.T.; writing—original draft preparation, M.T.; writing—review and editing, M.P.B.; visualisation, M.T.; supervision, M.T.; project administration, M.T.; funding acquisition, M.P.B. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research received no external funding.

**Institutional Review Board Statement:** Not applicable.

**Informed Consent Statement:** Informed consent was obtained from all subjects involved in the study.

**Data Availability Statement:** The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding author.

**Conflicts of Interest:** The authors declare no conflicts of interest.

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