THE BALLISTIC COEFFICIENT

William T. McDonald
Ted C. Almgren
December, 2008

The term “ballistic coefficient” is familiar to most shooters today. They know that the ballistic coefficient of a bullet is a measure of how well it retains velocity as it travels downrange and how well it “bucks” the wind. However, few shooters really know exactly what a ballistic coefficient is. These authors wrote about the ballistic coefficient in the first Sierra Reloading Manual in 1971. At that time there was very little published information about ballistic coefficients. Since then, much has been written by many authors, but there is still a general misunderstanding among shooters of what the ballistic coefficient is and why it is important.

The purpose of this article is to try to explain the ballistic coefficient in language easily understood by all shooters. The article describes where the ballistic coefficient came from, how it relates to aerodynamic drag (retardation force) on a bullet, and how it is used to calculate bullet trajectories.

Where the Ballistic Coefficient Came From

In 1742 Benjamin Robins, an English mathematician, invented the ballistic pendulum. This device made direct measurements of bullet velocities possible. Up until then bullet velocities in flight had been calculated using an analysis of projectile trajectories published by Galileo in 1636. Galileo had thought that air drag was very small compared to gravity, so that a bullet traveled as if it were in a vacuum. This was because Galileo’s experiments were with early firearms and the muzzle velocities of those arms were much less than the speed of sound in the air. The ballistic pendulum immediately led to two momentous discoveries, (1) muzzle velocities of bullets were much higher than previously thought, and (2) bullets slowed down in flight much faster than had been predicted by Galileo’s work. Both discoveries resulted from the fact that muzzle velocities at that time had increased to above the speed of sound. These discoveries showed that air drag greatly affected bullet trajectories and could not be neglected.

It was to be another century and a half before air drag was understood well enough to calculate bullet trajectories that were even approximately accurate. During that time countless experiments on air drag were made, together with a great deal of theoretical developments in mathematics and physics. But the situation around 1850 was that air drag on a projectile had to be measured; it could not be calculated from scientific knowledge in that day. [This situation is still largely true today for bullets.]

Up until about 1850, aiming a shoulder arm or handgun was little more than pointing the barrel at the target and pulling the trigger. Almost all arms were smoothbores and not very accurate. Targets were encountered at short ranges. Game animals were plentiful and were taken by hunters at very short ranges (typically 20 to 40 yards). In warfare armies marched up to within point blank range of their muskets before firing ("Don't shoot until you see the whites of their eyes." -- Colonial Colonel William Prescott, Battle of Bunker Hill, June 17, 1775).

Around 1850 rifled barrels were becoming popular. Rifled bores greatly improved the accuracy of firearms, both rifles and artillery pieces. This extended the effective range of fire well beyond 100 yards for rifles and several hundred yards for artillery. Weapon and ammunition technology advanced rapidly in the mid-1800s. Rifled barrels, elongated, pointed bullets, and improvements in the quality of black powder further improved muzzle velocities and the effective range of fire. This in turn created a need for precise aiming. To aim a weapon precisely it was necessary to know where the bullet would hit when the shot was made at a target, and then adjust the weapon sights or the hold so that the bullet would strike the target. This means that the trajectory of the bullet had to be calculated accurately to form firing tables (ballistics charts). Thus, aerodynamic drag on a bullet had to be measured.

Many, many measurements of aerodynamic drag on a wide variety of projectiles were made from about 1850 until well after 1900. The early measurements consisted of a whole lot of data and many mysteries from the interpretation of those data. This was especially true because instrumentation was crude and the effects of different atmospheric conditions were not understood.

There was a leap forward about 1870. The Reverend Francis Bashforth in England, who also was a mathematician and experimenter, had invented an early type of chronograph. Between 1865 and 1870 he made drag measurements on a variety of cylindrical artillery projectiles with different point shapes. From his observations he conceived the idea of a "standard bullet." This was a cylindrical flat-based bullet of a certain diameter, weight, length, and point shape. The idea was that drag could be measured on the "standard bullet" and then simply scaled in magnitude for practical bullets of the same basic shape but different weight and diameter. The basic shape that Bashforth chose was in fact pretty "standard" for all projectiles of that era, both rifle and artillery. Thus, there was to be a "standard bullet" with a measured "standard drag." Then each actual bullet would have a scale factor (a number) associated with it which would multiply the "standard drag" values to adjust them to be the actual drag on the actual bullet. The advantage of this approach was that drag measurements on each and every actual bullet could be avoided, provided that a way of determining the drag scale factor could be found.

Theoretical work at that time provided a way to determine the drag scale factor, at least initially. By applying Newton's Laws to an actual bullet in flight it was shown theoretically that:

$$\begin{aligned}\text{Drag scale factor} &= [\text{drag deceleration of the actual bullet}]\ \mathbf{/}\ [\text{drag deceleration of the standard bullet}] \\ &= [d_{\text{act bullet}} \wedge 2\ \mathbf{/}\ w_{\text{act bullet}}]\ \mathbf{/}\ [d_{\text{std bullet}} \wedge 2\ \mathbf{/}\ w_{\text{std bullet}}]\end{aligned}$$

Where $d_{act\ bullet}$ = diameter (inches) of the actual bullet
$d_{std\ bullet}$ = diameter (inches) of the standard bullet
$w_{act\ bullet}$ = weight (lbs) of the actual bullet
$w_{std\ bullet}$ = weight (lbs) of the standard bullet
Drag deceleration of the bullet was also called the retardation of the bullet

This was wonderful!!! The drag scale factor could be calculated from the diameter and weight of the actual bullet, compared to the diameter and weight of the standard bullet.

This wonderful idea was adopted, and this is how the ballistic coefficient began. At any value of bullet velocity, the standard drag measured for that velocity could be multiplied by the drag scale factor to calculate the actual drag on the real bullet. This not only avoided the necessity to measure the drag on every type of bullet at all bullet velocities, but it also led to great theoretical simplifications in calculating bullet trajectories. This in turn was wonderful, because the age of computers was still a century away.

Formal Definition of the Ballistic Coefficient

Bashforth selected the diameter of the standard bullet to be one inch and the weight to be one pound. This meant that the denominator in the drag scale factor equation above was identically equal to 1.0:

$[d_{std\ bullet} \wedge 2 / w_{std\ bullet}] = 1.0$

In turn this meant that, as long as the diameter of an actual bullet was measured in inches and weight was in pounds, the drag scale factor could be calculated as

Drag scale factor = $[d_{act\ byllet} \wedge 2 / w_{act\ bullet}]$

From this equation it appears that the drag scale factor has physical units of (sq in **/** lb), but this is not true. It must be remembered that the right hand side of this equation is divided by 1.0 (sq in **/** lb). The drag scale factor is a pure number; it has no physical units whatever.

But, alas, a problem developed. It was discovered experimentally around 1880 that the drag scale factor computed in this manner did not give the correct answer for the drag on the actual bullet. The reason was not understood then, but today we know that even small changes in the size or shape of an actual bullet cause a different air flow around the bullet, and therefore change the drag on the actual bullet. To handle this situation a “form factor” (or ‘shape factor”) was added to the equation above. This “form factor” is usually denoted by the letter “i”, and the equation is then written:

Drag scale factor = $[\ i * d_{act\ bullet} \wedge 2 / w_{act\ bullet}]$

The form factor “i” was to be another constant, to be used to adjust the value of the drag scale factor so that it accurately predicted the drag on the real bullet. At first (late 1800s) it was believed that this would work well at all bullet velocities.

That the form factor was necessary was very unfortunate, because the value of the form factor for any particular bullet had to be determined by firing test measurements. So, the analytical convenience of the drag scale factor was lost, at least partially. Again, experimental measurements had to be made for every bullet. However, it turned out that measuring the drag scale factor was far easier than measuring the drag deceleration of a bullet in flight. It was for this reason that the idea of a drag scale factor survived, and the analytical convenience of a scale factor was partially regained.

Actually, it was convenient to define the **ballistic coefficient** of a bullet to be the **inverse** of the **drag scale factor**:

$$\text{Ballistic coefficient} \equiv [1.0 \,\boldsymbol{/}\, \text{drag scale factor}] = [w_{\text{act bullet}} \,\boldsymbol{/}\, (\, i * d_{\text{act bullet}} \text{ ^ } 2)]$$

A reason for this definition is that “bigger is better.” When comparing two bullets, the one with the larger drag scale factor has a larger drag, while the one with the larger ballistic coefficient has smaller drag. It just feels better that as ballistic coefficient grows larger, ballistic performance of a bullet gets better. Again, remember that bullet weight is in pounds (1.0 pound = 7000 grains), and bullet diameter is in inches. Remember also that the ballistic coefficient, although it appears to have physical units of (lbs**/**sq in), actually has no physical units at all. It is a pure number.

The ratio of bullet weight to the square of its diameter in the equation above is recognized today as the **sectional density** of the bullet. The ballistic coefficient can then be remembered as sectional density divided by form factor.

Since the ballistic coefficient is the inverse of the drag scale factor, at any bullet velocity number, the standard drag deceleration of the standard bullet is **divided** by the ballistic coefficient to obtain the actual drag deceleration of the real bullet.

Why Ballistic Coefficients are not Constant, but Change with Bullet Velocity

It was explained above that the form factor (or shape factor) had to be used in the drag scale factor equation because the actual drag on a real bullet did not always scale as expected with bullet weight and diameter. It also was found experimentally that the form factor was not a constant, but in fact it changed value in different bullet velocity ranges (see, for example, **Exterior Ballistic Charts**, Wallace H. Coxe and Edgar Beuglesss, E. I. du Pont de Nemours & Company, Wilmington, DE, 1936). These authors have been measuring bullet ballistic coefficients since about 1970. We also have found that ballistic coefficients of almost all bullets change with bullet velocity. What does this mean?

It means that **the standard drag on the standard bullet does not faithfully represent the actual drag on an actual bullet at all velocities. Therefore, a single ballistic coefficient value cannot produce the actual drag values at all bullet velocities.** So, if scaling the standard drag idea is to be made to work, then we must either (1) allow the ballistic coefficient to change values in different velocity regions in order to have a value in each region which scales the standard drag sufficiently well for accurate trajectory computations, or (2) find a new standard bullet which is a more appropriate model for the actual bullet.

Both approaches have been used through the years. In our own work we have found that the first approach works very well, mainly because it is easy to measure ballistic coefficients of bullets in flight with simple laboratory equipment (chronographs and tape measures). The second approach has had some success in that a standard drag model can be found for a certain type of bullet shape, such that ballistic coefficients of actual bullets of similar shape change more slowly over the useful range of bullet velocities.

Recall that a ballistic coefficient always is measured with respect to a specific standard drag model. In the first approach the ballistic coefficient of a real bullet is measured at a number of velocity levels (typically 30 or more) within the useful velocity range of the bullet. A sequence of velocity regions (typically 3 to 5) can be established that span the useful velocity range of the bullet from muzzle velocity down to very low subsonic velocity levels. In each velocity region an average value of ballistic coefficient can be used to scale the standard drag so that the bullet trajectory can be accurately calculated for bullet velocities within that region. Allowing the ballistic coefficient of the bullet to change in value from one region to the next then allows the complete trajectory of the bullet to be accurately calculated by the scaling technique.

Sierra has long used this technique for all bullets in the line. Sierra uses G1 as the standard drag model for all bullets and allows up to five velocity regions to span the full velocity range for each bullet, although for most Sierra bullets three velocity regions are sufficient.

The second approach, i.e., multiple standard bullets, is described in the next paragraphs.

What are Drag Functions (G-Functions)?

The idea of a “standard bullet” with measured “standard drag” as a reference for computing actual drag on a real bullet of the same shape by simply scaling the standard drag has continued from the late 1800’s to the present day. This was true especially in military establishments up until the time of WWII. The Bashforth standard bullet was used as a reference for bullets of flat based, cylindrical shape with rather blunt points, which was the basic bullet shape in the late 1800’s.

With the appearance of smokeless powder in 1886 and jacketed bullets shortly thereafter, muzzle velocities began to increase markedly. Then, it was realized that drag on a bullet could be reduced dramatically if the point were sharpened. This was a basic change to the Bashforth standard bullet shape. Other shape changes (lubrication grooves, rotating bands, boat tails, etc.) were soon to follow for a variety of purposes. In the late 1800's and early 1900's instrumentation improved dramatically, and it was easier to measure drag deceleration with fair accuracy. These circumstances made it possible to have different standard bullets and standard projectiles, and many appeared especially in the military forces in Europe, England, and America.

For purposes of trajectory calculation each "standard drag" was a table of numbers. Each pair of entries in the table was the drag deceleration (retardation) measured for the standard bullet or projectile and the value of the projectile's velocity at which the measurement was made. Now, drag deceleration changes very sharply with bullet velocity. For computational purposes it was more convenient to divide each value of drag deceleration by the velocity at which it was measured, producing a new table of "drag function" values versus the velocities at which they were measured. The standard drag function for each standard bullet was unique for the shape of that standard bullet. What this meant was that **at sea level standard atmospheric conditions** the standard drag deceleration of the standard bullet was the product of velocity of the bullet multiplied by the value of the standard drag function at that velocity. At higher altitudes or nonstandard atmospheric conditions the standard drag function had to be corrected for the air density at the true temperature, pressure, and relative humidity at the location of the bullet in flight, and corrected also for the speed of sound at the air temperature at the location of the bullet in flight. Furthermore, for an actual bullet the standard drag deceleration at each value of bullet velocity had to be divided by the ballistic coefficient.

All this sounds very complex, and it is, but this approach led to important simplifications in the calculation of bullet trajectories. Military forces throughout the western world adopted the drag function method of describing drag effects and computing bullet trajectories. These methods were widely used through WWII. Many measurements of drag functions were made on standard bullet and projectile shapes used by military services. Each drag function was given a name composed of the letter G followed by a number, such as G1, G5, G6, G7, etc. The letter G was chosen in honor of the Gavre Commission in France, which existed from 1873 to 1898 and performed firing tests and conducted analyses of firing test data from other countries.

Today, two drag functions, G1 and G7, are popular for commercial bullets. G1 is the drag function for a slightly modified Bashforth standard bullet shape, and G7 is the drag function for long, slender bullets with long ogival points and boat tails, the so-called very low drag bullets. G1 is used widely by most bullet manufacturers, while makers of very low drag type of bullets are adopting G7. It is important to remember that the ballistic coefficient of any bullet is measured with reference to a particular G-function. **In other words, a ballistic coefficient measured with reference to G1 cannot be used with G7, and vice versa.**

In Summary

- The ballistic coefficient idea emerged from about 200 years of efforts by ballisticians to understand drag and to be able to calculate the effect of drag on a bullet trajectory in flight.
- The ballistic coefficient appears to have physical units of sectional density (lbs/sq in), but actually it is a **pure number** for the reason explained above..
- The ballistic coefficient of a real bullet always is measured with respect to a specific standard drag function (G1, G7, etc.).
- **The ballistic coefficient of a bullet is a scale factor (a number) which divides the standard drag to predict the actual drag on the real bullet.**
- A standard drag function (G1, G7, etc.) is a table of numbers. Each pair of numbers in that table are (1) a specific speed of the bullet in the air, and (2) the drag deceleration of the standard bullet at that bullet speed, divided by that bullet speed.
- Ballistic coefficients are relatively easy to measure in a shooting laboratory. The technique is to measure initial velocity and final velocity of each fired round (using chronographs) over a measured range distance between the chronographs. Then a software analysis program is used to compute the ballistic coefficient value which would cause the standard bullet starting at the initial velocity to have a computed final velocity equal to the measured final velocity.
- **Because a ballistic coefficient always relates to a specific standard drag model, say G1, that ballistic coefficient cannot be used with any other drag model, say G7. This is always true.**