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MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE ASSESSMENT INTERNATIONAL EDUCATION
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER S

INDEX NUMBER

ADDITIONAL MATHEMATICS

4047/01

Paper 1

October/November 2020

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

This document consists of **19** printed pages and **1** blank page.

Singapore Examinations and Assessment Board

Cambridge Assessment International Education

DC (NF/CB) 183969/3

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The roots of the quadratic equation $2x^2 - 5x + 8 = 0$ are α and β. Find a quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [5]

Sum of roots:

$\alpha + \beta = \frac{-5}{2}$ —— 1

Pdt of roots:

$\alpha\beta = \frac{8}{2} = 4$ —— 2

Sum of $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$: $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(\frac{-5}{2})^2 - 2(4)}{4}$

$= -\frac{7}{16}$

Pdt of $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$: $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

∴ Quadratic equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$:

$x^2 - (\frac{-7}{16})x + 1 = 0$

$16x^2 + 7x + 16 = 0$

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2 (a) Given that $\left[\left(\frac{50}{3}\right)^{-2} \times \sqrt{3^3}\right] \div \frac{5}{2} = 2^a\, 3^b\, 5^c$, find the value of each of a, b and c. [3]

Given $\left(\frac{3}{50}\right)^2 \times 3^{\frac{3}{2}} \times \frac{2}{5} = 2^a 3^b 5^c$

Hence, $\frac{3^2}{(2\times 5^2)^2} \times 3^{\frac{3}{2}} \times \frac{2}{5} = \frac{3^{\frac{7}{2}} \times 2}{2^2 \times 5^4 \times 5}$

$= 3^{\frac{7}{2}} \times 2^{-1} \times 5^{-5}$

$= 2^{-1} 3^{\frac{7}{2}} 5^{-5}$

$\therefore a = -1, \; b = \frac{7}{2}, \; c = -5$

(b) The value of a painting increases by 7% each year. Given that its value at the beginning of 2014 was \$1 000 000, find its value, to 2 significant figures, at the beginning of 2020. [2]

$V = P\left(1 + \frac{r}{100}\right)^n$

$= 1\,000\,000\left(1 + \frac{7}{100}\right)^6$

$= 1500730.352$

$= \$1\,500\,000$

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3 Express $\dfrac{4x^2-7x+9}{2x^2-x-3}$ in partial fractions. [5]

Step 1:

$$\begin{array}{r|l} & \quad 2 \\ 2x^2-x-3 & 4x^2-7x+9 \\ & -(4x^2-2x-6) \\ \hline & \quad -5x+15 \end{array}$$

Step 2: $\dfrac{4x^2-7x+9}{(2x-3)(x+1)} = 2 + \dfrac{-5x+15}{(2x-3)(x+1)}$

$= 2 + \dfrac{A}{(2x-3)} + \dfrac{B}{(x+1)}$

Step 3: $-5x+15 = A(x+1) + B(2x-3)$

Sub $x=-1$: $20 = B(-5)$

$\therefore B = -4$

Sub $x = \frac{3}{2}$: $-5\left(\frac{3}{2}\right)+15 = \frac{5}{2}A$

$\therefore A = 3$

hence, $\dfrac{4x^2-7x+9}{(2x-3)(x+1)} = \underline{2 + \dfrac{3}{2x-3} - \dfrac{4}{x+1}}$

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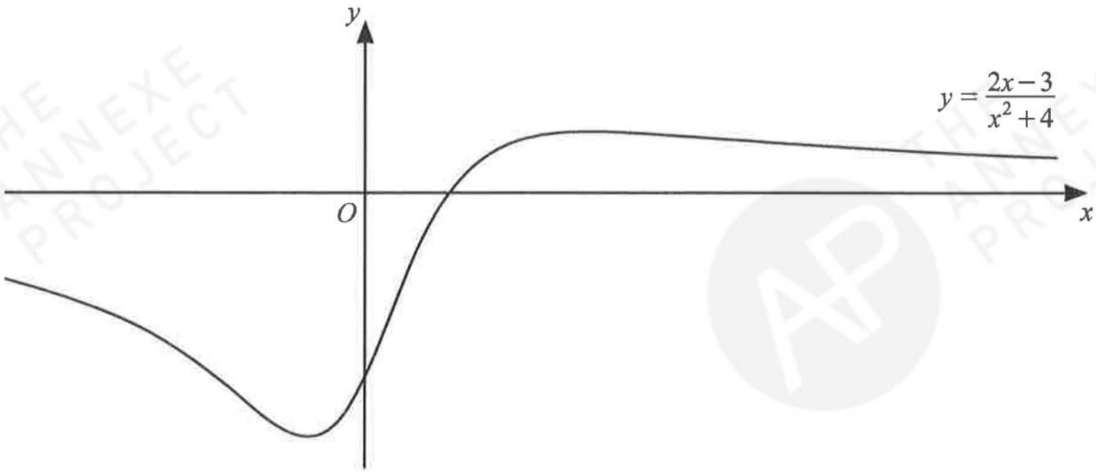


The diagram shows part of the graph of $y = \dfrac{2x-3}{x^2+4}$. Find the values of x for which y is increasing. [4]

$$y = \frac{2x-3}{x^2+4}$$

$$\frac{dy}{dx} = \frac{(x^2+4)(2) - (2x-3)(2x)}{(x^2+4)^2}$$

$$= \frac{2x^2+8-4x^2+6x}{(x^2+4)^2}$$

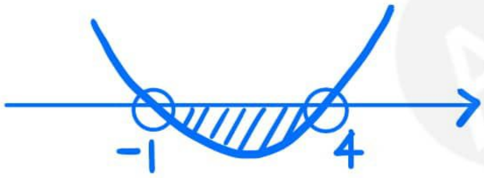
$$= \frac{-2x^2+6x+8}{(x^2+4)^2}$$

$$= \frac{-2(x^2-3x-4)}{(x^2+4)^2}$$

$$= \frac{-2(x-4)(x+1)}{(x^2+4)^2}$$

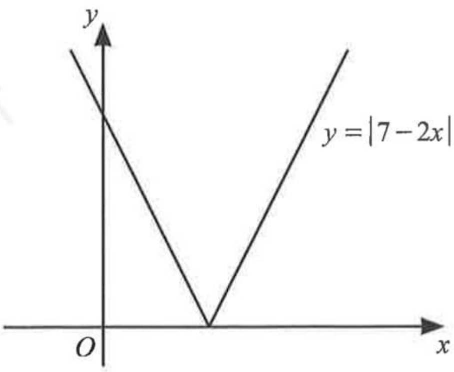
for y to be increasing, $\frac{dy}{dx} > 0$

hence, $(x-4)(x+1) < 0$



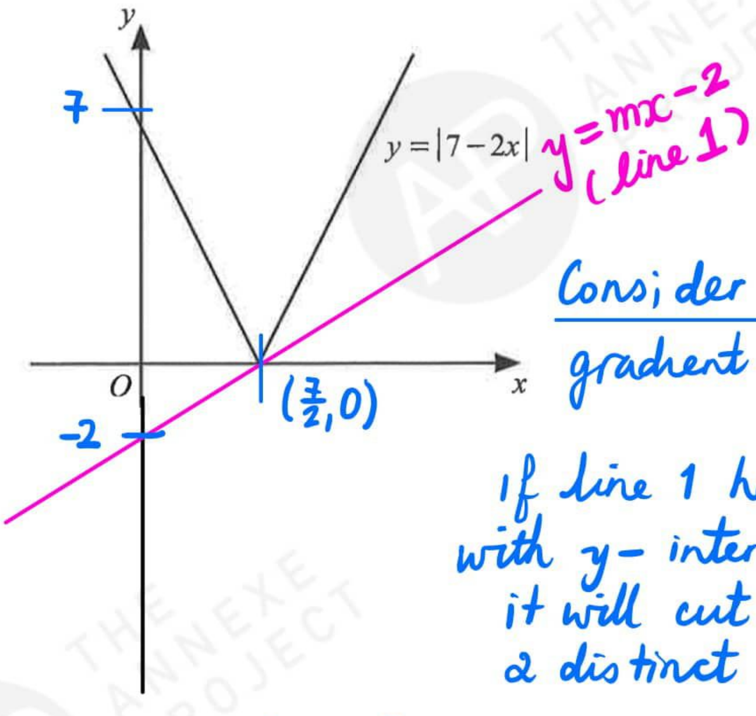
$\therefore$ $-1 < x < 4$

5



The diagram shows part of the graph of $y = |7-2x|$. The line $y = mx - 2$, where m is a constant, cuts $y = |7-2x|$ at two distinct points. Explain why $\frac{4}{7} < m < 2$. [4]

The left arm of $y = |7-2x|$ has a gradient of -2.
The right arm of $y = |7-2x|$ has a gradient of 2.



Consider line 1:

gradient $= \dfrac{0-(-2)}{\frac{7}{2}-0} = \dfrac{4}{7}$

If line 1 has gradient $> \frac{4}{7}$, with y-intercept -2, it will cut $y = |7-2x|$ at 2 distinct points.

However, when line 1 is parallel to the right arm of $y = |7-2x|$, i.e. gradient $= 2$, it will only cut the left arm of $y = |7-2x|$ once.
Hence, for $y = mx - 2$ to cut $y = |7-2x|$ at 2 distinct points, $\frac{4}{7} < m < 2$.

6 Find the coordinates of the stationary points of the curve $y = x^2 + \frac{4}{x^2}$ and determine the nature of each stationary point. [6]

$y = x^2 + 4x^{-2}$

$\frac{dy}{dx} = 2x - 8x^{-3} = 2x - \frac{8}{x^3}$

$= \frac{2x^4 - 8}{x^3}$

Let $\frac{dy}{dx} = 0$

$\therefore 2x^4 - 8 = 0$

$x^4 - 4 = 0$

$(x^2)^2 - 2^2 = 0$

$(x^2 - 2)(x^2 + 2) = 0$

$x^2 - 2 = 0$ $\quad$ $x^2 + 2 = 0$

$x = \pm\sqrt{2}$ $\quad$ (No Solution)

When $x = \sqrt{2}$, $y = 2 + \frac{4}{2} = 4$; When $x = -\sqrt{2}$, $y = 2 + \frac{4}{2} = 4$

For $(\sqrt{2}, 4)$:

x	$\sqrt{2}^-$	$\sqrt{2}$	$\sqrt{2}^+$
$\frac{dy}{dx}$	\	—	/

$(\sqrt{2}, 4)$ is a minimum pt.

For $(-\sqrt{2}, 4)$

x	$-\sqrt{2}^-$	$-\sqrt{2}$	$-\sqrt{2}^+$
$\frac{dy}{dx}$	\	—	/

$(-\sqrt{2}, 4)$ is a minimum pt.

The suggested solutions are prepared by Mr Alvin Yeo. Mr Yeo will hold no liability for any errors.

7 Solve the equation $3\cos A = \sec A - 5\tan A$ for $0° < A < 360°$. [7]

$$3\cos A = \frac{1}{\cos A} - \frac{5\sin A}{\cos A}$$

$$3\cos^2 A = 1 - 5\sin A$$

$$3(1-\sin^2 A) = 1 - 5\sin A$$

$$3\sin^2 A - 5\sin A - 2 = 0$$

$$(3\sin A + 1)(\sin A - 2) = 0$$

$\therefore \sin A = -\frac{1}{3}$ or $\sin A = 2$ (No solution)

Basic angle:

$A = 19.471°$

but A lies in 3rd or 4th quad.

$\therefore A = 180° + 19.471°$ or $360° - 19.471°$

$= 199.5°$ or $340.5°$

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8 (a) The remainder when x^3+ax, where a is a constant, is divided by $x-2$ is the same as the remainder when it is divided by $x+1$. Find the value of a. [3]

Let $f(x) = x^3 + ax$

$f(2) = 8 + 2a$

$f(-1) = -1 - a$

$8 + 2a = -1 - a$

$3a = -9$

$\therefore \underline{a = -3}$

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(b) Solve the equation $x^3 - 2x^2 - 4x + 3 = 0$, expressing non-integer roots in surd form. [4]

Step 1: Let $f(x) = x^3 - 2x^2 - 4x + 3$

$f(3) = 27 - 18 - 12 + 3$

$= 0$

$\therefore (x-3)$ is a factor.

Step 2:

$$
\begin{array}{r|l}
 & x^2 + x - 1 \\
x-3 & x^3 - 2x^2 - 4x + 3 \\
 & -(x^3 - 3x^2) \\
\hline
 & x^2 - 4x + 3 \\
 & -(x^2 - 3x) \\
\hline
 & -x + 3 \\
 & -(-x + 3) \\
\hline
 & 0
\end{array}
$$

$\therefore f(x) = (x-3)(x^2 + x - 1)$

Step 3: To find the roots of $x^2 + x - 1$:

$$x = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

Hence, solving $x^3 - 2x^2 - 4x + 3 = 0$

$x = \frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}$ or 3

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9 The equation of a circle is $x^2+y^2+4x-6y-12=0$.

(i) Find the radius of the circle and the coordinates of its centre. [4]

$x^2+4x+y^2-6y-12=0$

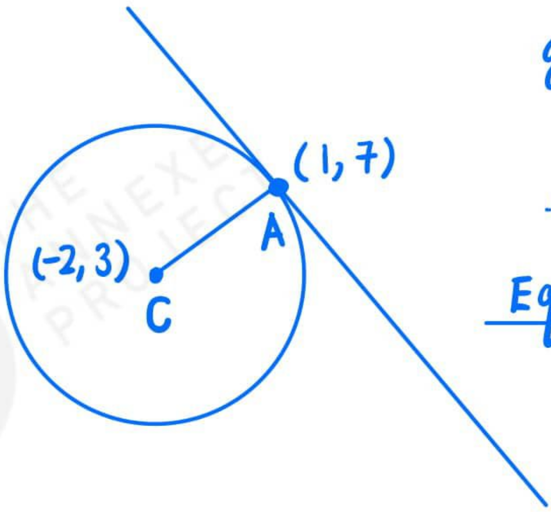
$(x+2)^2-2^2+(y-3)^2-3^2-12=0$

$(x+2)^2+(y-3)^2=5^2$

$\therefore$ radius $= 5$,

centre $= (-2, 3)$

(ii) Find the equation of the tangent to the circle at the point (1, 7). [3]



gradient of $CA = \frac{7-3}{1-(-2)}$

$= \frac{4}{3}$

$\therefore$ gradient of tangent $= -\frac{3}{4}$

Equation of tangent:

$y-7 = -\frac{3}{4}(x-1)$

$y = -\frac{3}{4}x + \frac{31}{4}$

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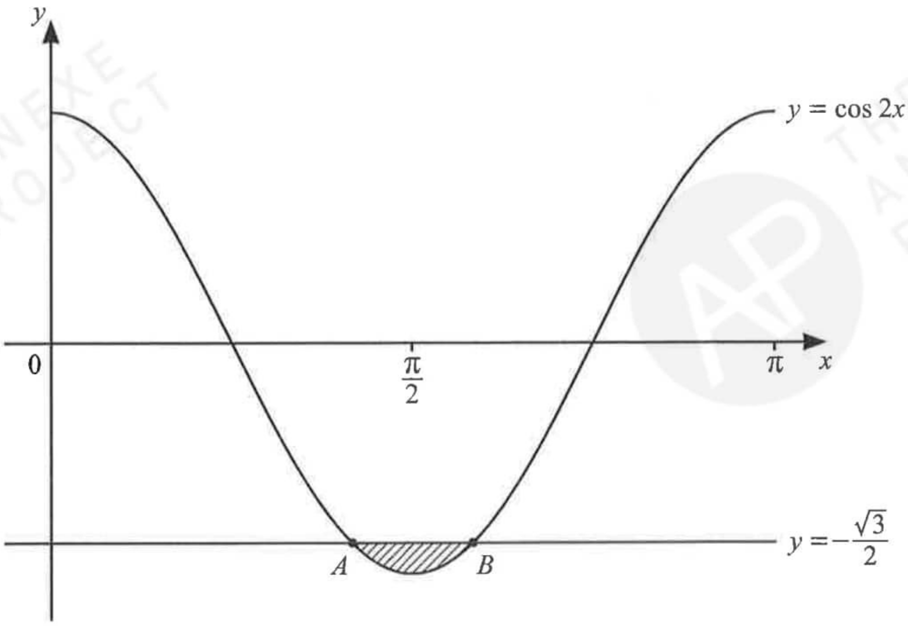
Continuation of working space for question **9(ii)**.

10



The diagram shows, for $0 \leqslant x \leqslant \pi$, the curve $y = \cos 2x$ and the line $y = -\frac{\sqrt{3}}{2}$. The curve and the line intersect at the points A and B.

(i) Find the x-coordinate of A and of B. [3]

Let $\cos 2x = -\frac{\sqrt{3}}{2}$

Basic angle $2x = \frac{\pi}{6}$

$2x$ lies in 2^{nd} or 3^{rd} quad.,

$\therefore 2x = \frac{5\pi}{6}$ or $\frac{7\pi}{6}$

$x = \frac{5\pi}{12}$ or $\frac{7\pi}{12}$

hence, x-coordinate of $A = \frac{5\pi}{12}$

x-coordinate of $B = \frac{7\pi}{12}$

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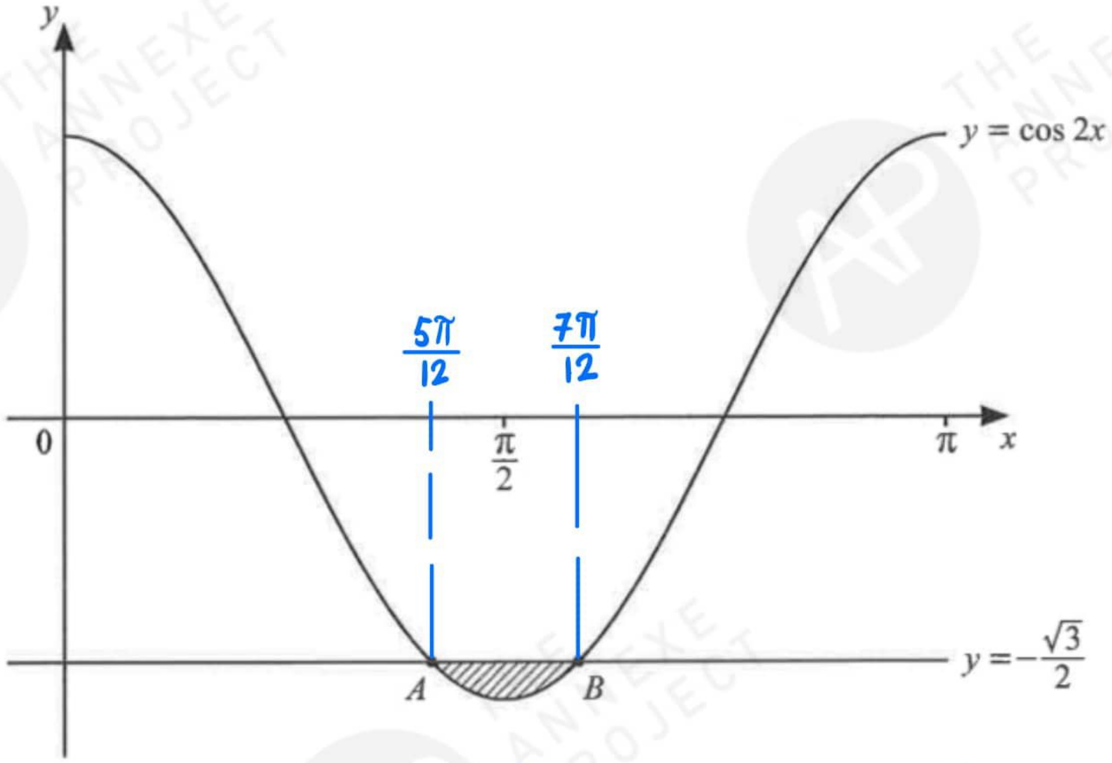
(ii) Showing all your working, find the exact area of the shaded region. [6]



$$\text{Shaded Area} = -\left[\int_{\frac{5\pi}{12}}^{\frac{7\pi}{12}} \cos 2x \, dx\right] - \left[\left(\frac{7\pi}{12} - \frac{5\pi}{12}\right) \cdot \frac{\sqrt{3}}{2}\right]$$

$$= -\left[\frac{\sin 2x}{2}\right]_{\frac{5\pi}{12}}^{\frac{7\pi}{12}} - \left[\frac{\pi}{6} \cdot \frac{\sqrt{3}}{2}\right]$$

$$= -\left[\frac{\sin \frac{7\pi}{6}}{2} - \frac{\sin \frac{5\pi}{6}}{2}\right] - \frac{\pi\sqrt{3}}{12}$$

$$= -\left[-\frac{1}{4} - \frac{1}{4}\right] - \frac{\pi\sqrt{3}}{12}$$

$$= \frac{1}{2} - \frac{\pi\sqrt{3}}{12}$$

$$= \frac{6 - \pi\sqrt{3}}{12} \text{ sq. units}$$

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11 A motorcyclist, travelling along a straight road, passes a point A and, 10 seconds later, passes a point B. The motorcyclist's speed at B is twice his speed at A. For the journey from A to B, the motorcyclist's speed, v m/s, t seconds after passing A, is given by

$$v = 15\left(\frac{t}{20} + e^{kt}\right), \quad 0 \leqslant t \leqslant 10,$$

where k is a constant.

(i) Find the speed of the motorcyclist at B. [1]

When $t = 0$ s, $v = 15(0 + e^0)$
$= \underline{15 \text{ m/s}}$ (speed at A)

$\therefore$ speed at $B = 2 \times 15$
$= \underline{30 \text{ m/s}}$

(ii) Find the distance between A and B. [6]

When $t = 10$s, $v = 30$ m/s

$\therefore \quad 30 = 15\left(\frac{10}{20} + e^{10K}\right)$

$2 = \frac{1}{2} + e^{10K}$

$\frac{3}{2} = e^{10K}$

$\ln\frac{3}{2} = 10K$

$\therefore K = \underline{\frac{1}{10}\ln\frac{3}{2}}$ or 0.040547

$v = 15\left(\frac{t}{20} + e^{0.040547t}\right)$

$= \frac{3}{4}t + 15e^{0.040547t}$

$s = \int \frac{3}{4}t + 15e^{0.040547t}\,dt$

$= \frac{3}{4}\left(\frac{t^2}{2}\right) + \frac{15e^{0.040547t}}{0.040547} + C$

$= \frac{3}{8}t^2 + 369.9455e^{0.040547t} + C$

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When $t = 0$, $s = 0$

$\therefore 0 = 369.9455 e^{0} + c$

$c = -369.9455$

$\Rightarrow s = \frac{3}{8}t^2 + 369.9455 e^{0.040547t} - 369.9455$

When $t = 10$ s,

$s = \frac{3}{8}(10)^2 + 369.9455 e^{0.040547(10)} - 369.9455$

$= 222.46$ m

$= \underline{222 \text{ m } (3 \text{ s.f.})}$

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(iii) Find the acceleration of the motorcyclist when $t = 2$. [3]

$$v = 15\left(\frac{t}{20} + e^{0.040547t}\right)$$
$$= \frac{3}{4}t + 15e^{0.040547t}$$

$$\therefore a = \frac{dv}{dt} = \frac{3}{4} + 15(0.040547)e^{0.040547t}$$
$$= \frac{3}{4} + 0.608205e^{0.040547t}$$

When $t = 2s$, $a = \frac{3}{4} + 0.608205e^{0.040547(2)}$
$$= 1.4096$$
$$= \underline{1.41 \text{ m/s}^2}$$

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12 Do not use a calculator in this question.

(i) Show that $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = \tan\alpha$. [2]

$$\text{LHS} = \frac{\sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta} + \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta}}{\cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} + \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta}}$$

$$= \frac{\cancel{2}\sin\alpha\,\cancel{\cos\beta}}{\cancel{2}\cos\alpha\,\cancel{\cos\beta}}$$

$$= \tan\alpha$$

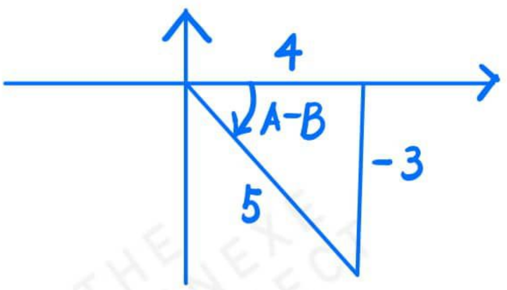
$$= \text{RHS (shown).}$$

Angles A and B are both acute and $\sin(A-B) = -\frac{3}{5}$.

(ii) What can be deduced about the size of B relative to A? [1]

$\angle B > \angle A$

(iii) Find the value of $\cos(A-B)$. [2]



$$\cos(A-B) = \frac{4}{5}$$

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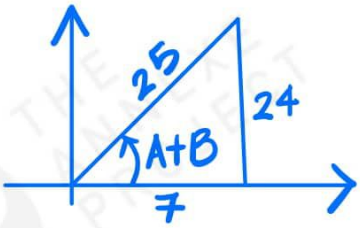
(iv) Given that $\cos(A+B)=\frac{7}{25}$ and $\tan(A+B)=\frac{24}{7}$, state the value of $\sin(A+B)$. [1]



$\sin(A+B) = \frac{24}{25}$

(v) Using the identity from **part (i)**, show that $\tan A = \frac{1}{3}$. [2]

Using $\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\cos(\alpha+\beta)+\cos(\alpha-\beta)} = \tan\alpha$.

$\therefore \frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)} = \tan A$

$\frac{\frac{24}{25}+\left(-\frac{3}{5}\right)}{\frac{7}{25}+\frac{4}{5}} = \tan A$

$\therefore \tan A = \frac{\frac{9}{25}}{\frac{27}{25}} = \frac{9}{27} = \frac{1}{3}$ (shown).

(vi) Hence find the exact value of $\tan B$. [3]

Given $\tan(A+B) = \frac{24}{7}$

$\frac{\tan A + \tan B}{1-\tan A\tan B} = \frac{24}{7}$

$\frac{\frac{1}{3}+\tan B}{1-\frac{1}{3}\tan B} = \frac{24}{7}$

$\frac{1}{3} + \tan B = \frac{24}{7} - \frac{8}{7}\tan B$

$\frac{15}{7}\tan B = \frac{65}{21}$

$\therefore \tan B = \frac{13}{9}$

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