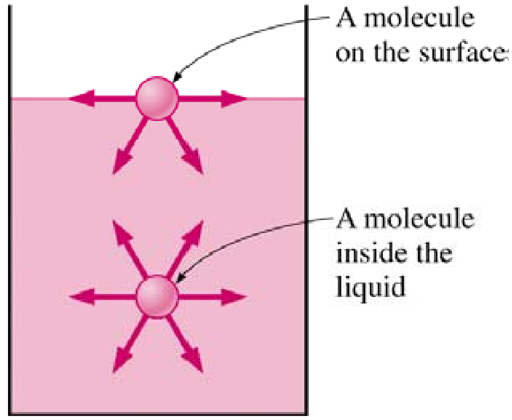
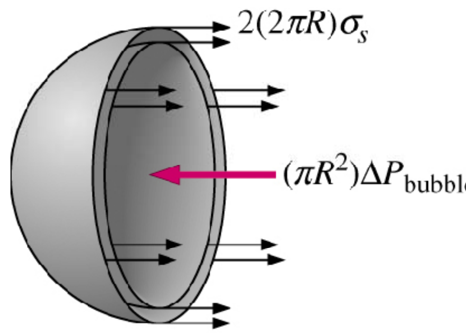
Surface Tension and Capillarity

Surface tension is a property of liquids which is felt at the interface between the liquid and another fluid (typically a gas). Surface tension has dimensions of force per unit length, and always acts parallel to the interface. Surface molecules are subject to an attractive force from nearby surface molecules so that the surface is in a state of tension. A soap bubble is a good example to illustrate the effects of surface tension. How does a soap bubble remain spherical in shape? The answer is that there is a higher pressure inside the bubble than outside, much like a balloon. In fact, surface tension in the soap film acts much the same as the tension in the skin of a balloon.



Consider a soap bubble of radius R with internal pressure p_{in} and external (atmospheric) pressure p_{out}. The excess pressure bubbleinout $\Delta p = p_{in} - p_{out}$ can be found by considering the free-body diagram of half a bubble. Note that surface tension acts along the circumference (resulting from cutting across the two interfaces) and the pressure acts on the area of the half-bubble. By statics (to be explained later), the net force due to the pressure is equal to the pressure times the projected area. Hence, balancing the forces due to surface tension and pressure difference:



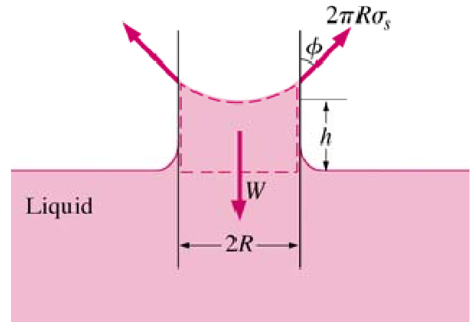
(*b*) Half a bubble

$$\boldsymbol{2(2\pi R)\,\sigma_s = (\pi R^2)\,\Delta p_{bubble}}$$

$$\boldsymbol{\Delta p_{bubble} = 4\,\sigma_s / R}$$

where σ_s is the surface tension of the fluid in air.

You may repeat this exercise for a droplet, and show that $\boldsymbol{\Delta p_{droplet} = 4\,\sigma_s / R}$



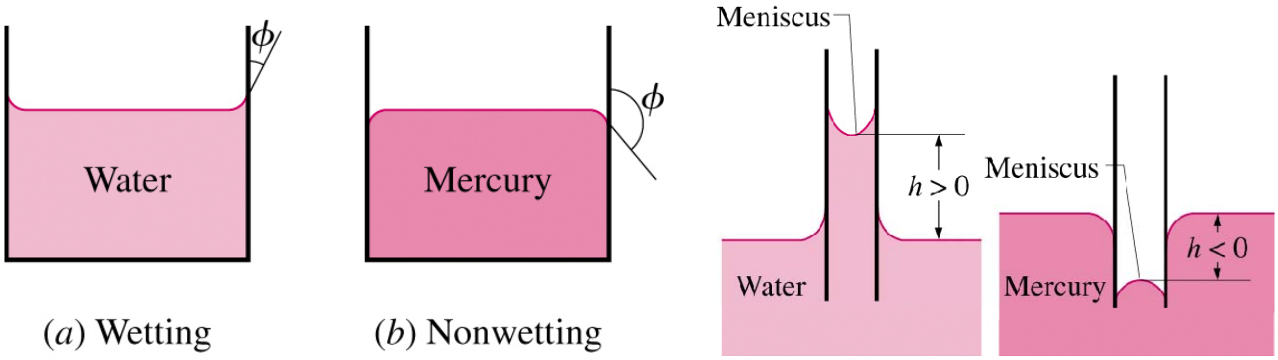
Surface tension is also important at the interface between a liquid, a gas, and a solid. For example, a meniscus occurs when the surface of a liquid touches a solid wall, as most readily noticed when a capillary tube is placed in a liquid. Consider a glass capillary tube inserted into a liquid, such as water. The water will rise up the tube to a height *h*, because surface tension pulls the surface of the water towards the glass, as shown. The meniscus is the curved surface at the top of the water column. The height of the water column can be found by summing all forces acting on the water column as a free body diagram. (This is a statics problem since there is no acceleration.) The downward force is due to gravity, i.e. the weight of the water column. The only upward force available to balance the weight is that caused by surface tension (pressure forces all cancel out, as will be explained in a later lecture). Column height *h* can be determined as follows:

weight of fluid column = surface tension pulling force

$$\rho g\,(\pi R^2 h) = 2\pi R\,\sigma_s\,\cos\phi$$

$$h = \frac{2\,\sigma s\,\cos\phi}{\rho g R}$$

The contact angle is defined as the angle between the liquid and solid surface, as shown in the sketch. Contact angle depends on both the liquid and the solid. If ϕ is less than 90^o, the liquid is said to "wet" the solid. However, if ϕ is greater than 90^o, the liquid is repelled by the solid, and tries not to "wet" it. For example, water wets glass, but not wax. Mercury on the other hand does not wet glass.



(*a*) Wetting fluid (*b*) Nonwetting fluid

Vapor Pressure

Vapor pressure is defined as the pressure at which a liquid will boil (vaporize). Vapor pressure rises as temperature rises. For example, suppose you are camping on a high mountain (10,000 ft. or roughly 3,000 m in altitude). the atmospheric pressure at this elevation is about 70 kPa., the vapor pressure of water is also around 70 kPa. From this it can be stated that at 10,000 ft. of elevation, water boils at around 90^{o}C, rather than the common 100^{o}C at standard sea level pressure. This has consequences for cooking. For example, eggs have to be cooked longer at elevation to become hard-boiled since they cook at a lower temperature. A pressure cooker has the opposite effect. Namely, the tight lid on a pressure cooker causes the pressure to increase above the normal atmospheric value. This causes water to boil at a temperature even greater than 100oC; eggs can be cooked a lot faster in a pressure cooker!

Vapor pressure is important to fluid flows because, in general, pressure in a flow decreases as velocity increases. This can lead to *cavitation*, which is generally destructive and undesirable. In particular, at high speeds the local pressure of a liquid sometimes drops below the vapor pressure of the liquid. In such a case, *cavitation* occurs. In other words, a "cavity" or bubble of vapor appears because the liquid vaporizes or boils at the location where the pressure dips below the local vapor pressure. Cavitation is not desirable for several reasons. First, it causes noise (as the cavitation bubbles collapse when they migrate into regions of higher pressure). Second, it can lead to inefficiencies and reduction of heat transfer in pumps and turbines (turbomachines). Finally, the collapse of these cavitation bubbles causes pitting and corrosion of blades and other surfaces nearby.

Compressibility

All fluids are compressible under the application of external forces. The compressibility of a fluid is expressed by its **bulk modulus of elasticity** *E*, which is the ratio of the change in unit pressure to the corresponding volume change per unit volume.

$$E = \frac{\Delta p}{-\Delta V/V} = \frac{\Delta p}{-\Delta \rho/\rho}$$

Note that the bulk modulus of elasticity has the same dimensions as pressure: [*E*] = [ML-1T-2].

For water at room temperature, *E* is approximately 2.2×10^9 N/m^2, while for air at atmospheric pressure the isentropic bulk modulus of elasticity is approximately 1.4×10^5 N/m^2. That is, air is typically four orders of magnitude more compressible than water.

For most practical purposes liquids may be regarded as incompressible. However, there are certain cases, such as unsteady flow in pipes (e.g., water hammer), where the compressibility should be taken into account. Gases may also be treated as incompressible if the change in density is very small (typically less than 3%).

An ideal fluid is an incompressible fluid.

Pressure disturbances imposed on a fluid move in waves. These pressure waves move at a velocity equal to that of sound through the fluid. The velocity, or celerity, *c*, is given by

$$c = \sqrt{E/\rho}$$

Problems - Properties

a) If 6 m of oil weighs 47 kN, find its specific weight, density, and relative density.

(Ans. 7.833 kN/m^3, 798 kg/m^3, 0.800)

b) At a certain depth in the ocean, the pressure is 80 MPa. Assume that the specific weight at the surface is 10 kN/m and the average bulk modulus is 2.340 GPa.

Find:

a) the change in specific volume between the surface and the large depth;

b) the specific volume at the depth, and;

c) the specific weight at the depth.

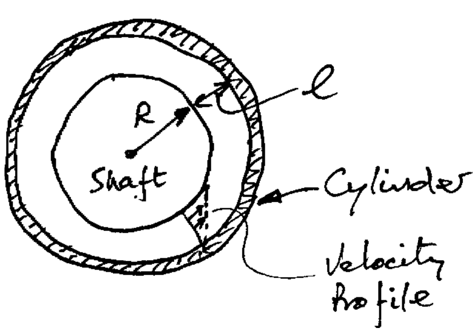
(Ans. -0.335x10^{-4} m^3/kg, 9.475x10^{-4} m^3/kg, 10.35 kN/m^3)

c) A 100 mm deep stream of water is flowing over a boundary. It is considered to have zero velocity at the boundary and 1.5 m/s at the free surface. Assuming a linear velocity profile, what is the shear stress in the water?

(Ans. 0.0195N/m^2)

d) The viscosity of a fluid is to be measured using a viscometer constructed of two 750 mm long concentric cylinders. The outer diameter of the inner cylinder is 150 mm and the gap between the two cylinders is 1.2 mm. The inner cylinder is rotated at 200 rpm and the torque is measured to be 10 Nm.

a) Derive a generals expression for the viscosity of a fluid using this type of viscom eter, and;



b) Determine the viscosity of the fluid for the experiment above.