

# **RUSH: Real-time Burst Subgraph Detection in Dynamic Graphs**

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# ABSTRACT

Graph analytics have been effective in the data science pipeline of fraud detections. In the ever-evolving landscape of e-commerce platforms like Grab or transaction networks such as cryptos, we have witnessed the phenomenon of 'burst subgraphs,' characterized by rapid increases in subgraph density within short timeframes-as a common pattern for fraud detections on dynamic graphs. However, existing graph processing frameworks struggle to efficiently manage these due to their inability to handle sudden surges in data. In this paper, we propose RUSH (Real-time bUrst SubgrapH detection framework), a pioneering framework tailored for real-time fraud detection within dynamic graphs. By focusing on both the density and the rate of change of subgraphs, RUSH identifies crucial indicators of fraud. Utilizing a sophisticated incremental update mechanism, RUSH processes burst subgraphs on large-scale graphs with high efficiency. Furthermore, RUSH is designed with user-friendly APIs that simplify the customization and integration of specific fraud detection metrics. In the deployment within Grab's operations, detecting burst subgraphs can be achieved with approximately ten lines of code. Through extensive evaluations on real-world datasets, we show RUSH's effectiveness in fraud detection and its robust scalability across various data sizes. In case studies, we illustrate how RUSH can detect fraud communities within various Grab business scenarios, such as customer-merchant collusion and promotion abuse, and identify wash trading in crypto networks.

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#### **PVLDB Artifact Availability:**

The source code, data, and/or other artifacts have been made available at https://github.com/Xtra-Computing/RUSH.

#### **INTRODUCTION** 1

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Figure 1: Fraud Detection Pipeline of Grab



#### Figure 2: An example of burst subgraph detection

Graph analytics have become increasingly prevalent in data science pipelines of various applications, e.g., fraud detection on ecommerce platforms. These platforms often represent temporal data in the form of graphs [21, 22], constructing various networks such as transaction networks [8, 10, 17], user-device networks [7, 15], and finance networks [35, 40]. In dynamic graph environments, the identification of burst subgraphs [11, 29]-defined by sharp density increases in short durations-has become a pivotal pattern for fraud detection. This observation highlights the importance of scalable data science techniques for effective fraud mitigation in complex networked contexts. For example:

(1) Anomalous Surge in Transaction Activity. Consider the example of a transaction network. A high frequency of transactions occurring within a short time span indicates potential collusion, fraudulent activities such as brushing, or exploitation of promotions. Efficient detection of these patterns is essential for maintaining the integrity of the network and safeguarding the interests of genuine participants. Swiftly processing incoming transactions and detecting emerging fraud in real time are vital for minimizing losses, underscoring the necessity for a scalable fraud detection pipeline.

(2) Identifying Fraud via Device Activity. Another example is the userdevice graph, where each node represents a user or a device, and each edge denotes the usage of a device by a user. Recent studies [1, 38] report that 21.4% of traffic to e-commerce portals was malicious bots in 2018. In instances of fraudulent behavior on ecommerce platforms, it is often observable that both fraudsters and their numerous associated devices become active within a short time span. This pattern of burst behavior can be systematically utilized by platform moderators as a heuristic to detect and investigate fraudulent communities. Moreover, the swift surge in such fraudulent activities places a significant strain on detection efforts.



Figure 3: Statistics of Burst Subgraph Detected by RUSH: (a) indicates the percentage of nodes in burst subgraphs, (b) indicates average percentage of fraud nodes in burst subgraphs Fraud Detection Pipeline in Grab (Figure 1). (1) The existing transactions form a weighted graph, where the weight of an edge might represent the volume of funds transferred, and the weight of a vertex could indicate the associated suspiciousness of a user.

(2) Grab recursively peels the nodes from the transaction graph, generating a peeling sequence. (3) As new transactions occur, Grab integrates them into the existing graph and progressively reduces edge and vertex weights to emphasize recent activities. This approach enhances the detection of emerging fraudulent behaviors marked by bursts. (4) To identify fraudulent activities, algorithms specialized in detecting dense subgraphs (e.g., peeling algorithm) are utilized. (5) Upon detection, moderators take immediate action to prevent further financial damage.

*Example 1.1.* Figure 2 illustrates a burst subgraph detection example within a transaction graph, where nodes represent users and edges represent transactions. From  $T_0$  to  $T_1$ , a brief time period, a fraudulent community conducts four transactions. Traditional dense subgraph detection systems often fail to identify such communities because loyal users engaged in regular transactions can also form dense subgraphs, thereby obscuring the fraudulent activity. In contrast, the burst subgraph detection system decays the weights of outdated transactions. As a result, the fraudulent community becomes more distinguishable due to the high volume of transactions conducted in a short period.

Figure 3 shows the percentage of nodes in burst subgraphs within NFT transaction and industrial graphs from Grab. It indicates that between 3.1% and 33.7% of nodes partake in burst subgraphs, whereas 64.4% to 93.4% of identified fraudulent nodes demonstrate burst behavior. This phenomenon stems from fraudulent communities aiming for quick profits through promotion exploitation or brushing schemes. Within the NFT graph ba, a single day saw a volume of fraudulent transactions amounting to 813.2 ETH. The lack of efficient detection systems for such burst activities can result in significant financial losses for both users and platforms. Thus, the urgency for real-time detection of burst subgraphs is highlighted.

Numerous systems target the mining of burst subgraphs but tend to specialize in identifying specific patterns like burst core or burst cohesiveness subgraphs. Even state-of-the-art systems such as MBC [29] and DBS [11] reconstruct these subgraphs from scratch, compromising real-time detection efficiency. Systems aimed at finding the densest subgraphs [18, 23, 28] may overlook rapid weight changes in fraudulent communities. They typically detect burst fraud communities only when they become the densest, leading to high detection latency. While systems such as Spade [25] can be modified for burst subgraph detection, they lack efficient methods for adjusting to weight changes, hindering timely fraud detection.

A key strategy to improve the detection of burst activities involves applying a time decay function to graph weights, thereby accentuating the significance of recent transactions. However, this approach faces three primary challenges. Firstly, there is an imperative need for rapid response to new transactions, especially critical in the financial sector, where threats must be neutralized within 100 milliseconds [25, 27]. Achieving incremental updates of burst subgraphs within such stringent time frames poses a considerable challenge. Secondly, the application of a time decay function to weights with each new transaction presents its own set of difficulties. Recomputing the burst subgraph from scratch for each new transaction incurs untenably high latency, necessitating a method for incrementally updating weight decay. Thirdly, deployment is challenging for engineers. Implementing detection for burst subgraphs necessitates establishing a mechanism to manage the decay of weights throughout the entire graph and designing incremental algorithms for burst subgraphs based on their density metrics.

In response to these challenges, we introduce RUSH, a real-time framework designed for burst subgraph detection. To address the first challenge, RUSH enhances the process of incremental edge weight updates using innovative optimization techniques to facilitate real-time detection. To address the second challenge, RUSH employs batch update and batch selection strategies to accelerate the weight decay process. To address the third challenge, RUSH supports a suite of friendly APIs that allow users to specify dense metric functions or tailor detection system parameters. In summary, this paper makes the following contributions:

- RUSH utilizes an innovative metric with a half-life decay function embedded in transaction networks, emphasizing temporal dynamics for enhanced detection precision.
- RUSH implements several optimization techniques, including batch updates, graph compression, and efficient selection strategies, to boost real-time detection efficiency.
- RUSH provides extensive APIs for easy customization, enabling quick deployment with about ten lines of code for fraud detection in diverse applications, including Grab's business scenarios.
- Extensive experiments on industry and NFT datasets not only show that RUSH delivers results four orders of magnitude faster than the state-of-the-art methods for burst subgraph detection but also highlight its robust scalability, maintaining sublinear growth in update times and latency across graphs scaling.
- The case studies highlight RUSH's impact on fraud detection pipelines, significantly lowering detection time for fraudulent communities and preventing up to 97.4% of frauds.

# 2 BACKGROUND

# 2.1 Preliminaries

In this paper, we focus on the (un)directed graph G = (V, E) where V is a set of vertices and  $E \subseteq V \times V$  is a set of edges. For each vertex  $v_i$  or edge  $(v_i, v_j)$ , they have a non-negative weight for them, denoted as  $a_i$  or  $c_{ij}$ . Given a vertex  $v \in V$ , N(v) is the neighbor set of v, i.e.,  $\{v' | e(v, v') \in E\}$  where e(v, v') denotes the edge between v and v'. d(v) is the degree of v, i.e., d(v) = |N(v)|.

**Induced Subgraph.** Given  $S \subseteq V$ , we denote the induced subgraph by G[S] = (S, E[S]), where  $E[S] = \{(u, v) | (u, v) \in E \land u, v \in S\}$ .

**Density Metrics** *g*. We adapt the class of density metrics *g* in previous studies [23, 25],  $g(S) = \frac{f(S)}{|S|}$ , where *f* is the total weight of *G*[*S*], *i.e.*, the sum of the weight of *S* and *E*[*S*]:

$$f(S) = \sum_{u_i \in S} a_i + \sum_{u_i, u_j \in S \land (u_i, u_j) \in E} c_{ij}$$
(1)

The weight of a vertex  $u_i$ , denoted by  $a_i$  where  $a_i \ge 0$ , quantifies the suspiciousness associated with vertex  $u_i$ . Similarly, the weight of an edge  $(u_i, u_j)$  reflects the suspiciousness of the transaction between  $u_i$  and  $u_j$ , denoted by  $c_{ij}$ . The function g(S) represents the density of the induced subgraph G[S]. However, existing works, such as [23, 25], assume that the weights of the edges remain constant over time. This assumption overlooks that the influence of transactions progressively weakens as time passes.

**Weight Decay via Half-Life Principle.** We deviate from relying solely on the original edge weight  $c_{ij}$  and instead employ a half-life function [5, 20, 33] to progressively reduce the influence of an edge on the graph over time, denoted by  $\frac{c_{ij}}{2^{\lfloor \frac{l_{ij}}{\delta} \rfloor}}$ , where  $t_{ij}$  is the time elapsed since the edge  $(u_i, u_j)$  was established and  $\delta$  is the half-life duration. We use a similar decay function  $\frac{a_i}{2^{\lfloor \frac{l_i}{\delta} \rfloor}}$  for the node weight, where  $t_i$  is the time elapsed since the node weight was assigned. Hence, the density function can be redefined as follows:

$$f(S) = \sum_{u_i \in S} \frac{a_i}{2^{\lfloor \frac{t_i}{\delta} \rfloor}} + \sum_{u_i, u_j \in S \ \bigwedge (u_i, u_j) \in E} \frac{c_{ij}}{2^{\lfloor \frac{t_{ij}}{\delta} \rfloor}} \tag{2}$$

Using a half-life decay mechanism in our analysis is crucial for highlighting emerging fraudulent patterns. It diminishes the weight of outdated transactions, directing attention to recent activities essential in identifying emerging fraudulent communities. Thus, we use G(l) to denote the graph at time l, where all weights have been reduced using the half-life decay function.

**Graph Updates**  $\Delta G$ . We denote the set of updates to *G* by  $\Delta G = (\Delta V, \Delta E)$ . We denote the graph obtained by updating  $\Delta G$  to *G* as  $G \oplus \Delta G$ . Therefore,  $G \oplus \Delta G = (V \cup \Delta V, E \cup \Delta E)$ . Similarly, we also denote the graph deletion operation as  $G \oplus \Delta G$ , which is deleting all the edges and vertices in  $\Delta G$  from *G*.

**Problem Definition.** Given a graph G(l), a dense subgraph metric g(S), a specified half-life duration  $\delta$ , and graph updates  $\Delta G_{l+1}$ , our objective is to efficiently detect the densest subgraph within the updated graph G(l + 1). The graph G(l + 1) is created by updating  $\Delta G_{l+1}$  to G(l) and then applying decay to all weights that reach the half-life at time l + 1.

# 2.2 Peeling Algorithm and Reordering

**Peeling Weight.** The peeling weight function  $w_{u_i}(S)$  quantifies a node's contribution to the dense subgraph and is contingent upon the density metric employed. If we use the density metrics in Section 2.1, the weight function will be:

$$w_{u_i}(S) = \frac{a_i}{2^{\lfloor \frac{t_i}{\delta} \rfloor}} + \sum_{u_i, u_j \in S \ \land (u_i, u_j) \in E} \frac{c_{ij}}{2^{\lfloor \frac{t_{ij}}{\delta} \rfloor}} + \sum_{u_j, u_i \in S \ \land (u_j, u_i) \in E} \frac{c_{ji}}{2^{\lfloor \frac{t_{ji}}{\delta} \rfloor}}$$

Prior research [23, 25] has employed a peeling strategy to identify the densest subgraph within a given graph. This approach conducts the methodology delineated in Algorithm 1 to ascertain the peeling sequence. Subsequently, the data graph is peeled according to this

#### Algorithm 1: Execution of Peeling Algorithms

**Input:** A graph G = (V, E)**Output:** The peeling sequence order *O* 

 $S_0 = V$ 

2 for i = 1, ..., |V| do

select the vertex  $u \in S_{i-1}$  with the minimum peeling weight  $w_u(S_{i-1})$ 

 $S_i = S_{i-1} \setminus \{u\}; O.add(u)$ 

5 return O

4



Figure 4: Example of Half-life Decay (w.r.t.  $\delta = 2$ ).

sequence. The densest subgraph as revealed through this peeling procedure provides a 2-approximation to the true densest subgraph.

*Example 2.1.* Figure 4 illustrates an example of the research problem. The system takes a data graph, a specified half-life duration, and edge updates as inputs. With each update, it appends the incoming edges to the graph and applies a decay function to adjust the weights. The system then identifies the subgraph with the highest density, which is highlighted in grey. The half-life duration is set to 2 units. Consequently, the weights of all incoming edges at time  $t_1$  (highlighted in red) are halved by time  $t_3$  (depicted in blue). Consider the graph  $G(t_4)$  depicted in Figure 4(d). The application of the weight function to nodes  $v_1$  to  $v_5$  yields peeling weight values of 3.5, 3, 3, 7.5, and 1 respectively. Hence, the first node to be peeled is  $v_5$ . This process is replicated sequentially until all nodes are removed, resulting in the peeling sequence  $[v_5, v_3, v_2, v_1, v_4]$ .

**Peeling Sequence Reordering.** The core of the peeling algorithm involves maintaining the peeling sequence (*O*) after graph updates. In this paper, we integrate Spade [25], which takes the data graph *G*, an existing peeling sequence *O*, and graph updates  $\Delta G^+$  and  $\Delta G^-$  as inputs. It outputs the updated peeling sequence *O'*. We use  $O' = \text{SPADE}(G, O, \Delta G^-, \Delta G^+)$  to denote the update algorithm.

The key steps of Spade include: (1) Initialization of a priority queue that maintains the affected peeling subsequence, ordering vertices in ascending order of their peeling weight. (2) Insertion of the updated vertices into the priority queue and marking the neighbors of these vertices as affected. (3) Recursive insertion of affected vertices in the peeling sequence into the priority queue. (4) Peeling of vertices from the priority queue. Spade reduces the update cost from  $O(|E| \log |V|)$  to  $O(|E_{AFF}| \log |V_{AFF}|)$  [25], where  $|E_{AFF}|$  and  $|V_{AFF}|$  denote the number of edges and vertices in the affected area, respectively.

#### **3 FRAMEWORK OF RUSH**

# 3.1 The RUSH Architecture

Figure 5 shows the architecture for RUSH. RUSH imports the transaction logs and constructs the graph (Step 1). Utilizing the userspecified dense metric function, RUSH then applies a peeling algorithm to generate the initial peeling sequence (Section 2.2). As



Figure 5: The Architecture of RUSH

1	class RUSH{
2	public:
3	<pre>void LoadGraph(string path);</pre>
4	<pre>// Set the duration for half-life decay</pre>
5	<pre>void SetHalfLifeDuration(int time_length);</pre>
6	<pre>void SetBatchSize(int batch_size);</pre>
7	<pre>void SetSelectionMethod(Type selection_method);</pre>
8	<pre>set<vertex> InsertEdge(Edge &amp;E);</vertex></pre>
9	<pre>// Set the edge/node weight function</pre>
10	<pre>Void EdgeFunction(Function<double(edge &e)=""> eFunc);</double(edge></pre>
11	<pre>Void NodeFunction(Function<double(node &v)=""> vFunc);</double(node></pre>
12	private:
13	<pre>vector<vertex> CalculatePeelingSequence();</vertex></pre>
14	};

Listing 1: RUSH's API

new transactions arrive, RUSH incorporates each incoming edge and incrementally updates the existing peeling sequence (Step 2.1). Concurrently, RUSH monitors edges reaching their half-life thresholds, halving their weights accordingly (Step 2.2). With the batch updating algorithm, RUSH utilizes the user-selected or adaptive approach (Step 3) to incrementally update edges in batches (Step 4). Finally, RUSH reports burst communities to the moderators, who can then enact preventive measures (Step 5).

# 3.2 APIs of RUSH

**Density Metrics.** RUSH accommodates a wide range of density functions, conforming to the formula  $g = \frac{f(S)}{|S|}$ , where f(S) adheres to Equation 2 and  $a_i$  and  $c_{ij}$  are non-negative. By varying  $a_i$  and  $c_{ij}$ , RUSH can seamlessly adapt to half-life versions of the popular peeling algorithms such as DG [6], DW [19], and FD [23].

**APIs of** RUSH **(Listing 1).** RUSH empowers users to customize their peeling algorithms for their specific applications. Developers can define their density metrics by implementing custom weight functions. Unlike Spade, RUSH provides APIs such as SETHALFLIFE-DURATION and SETBATCHSIZE, which allow users to set decay mechanisms without manually performing decay operations across the entire graph. In the architecture of RUSH, implementing the time-decayed DW density can be achieved with just 14 lines of code. In contrast, accomplishing the same functionality within Spade requires approximately 40 lines of code. Detailed implementation for DW, DG and FD are provided in Appendix B of [9].

# **4 IMPLEMENTATION OF RUSH**

# 4.1 Baseline Approaches in Half-Life Dynamics

Given a graph G = (V, E), and let  $\Delta G = (\Delta V, \Delta E)$  denote a subset of *G*'s elements (both edges and vertices) subject to decay. The decay of the graph is denoted by  $\frac{\Delta G}{2^n}$ , where *n* represents the number of half-life periods, and  $\frac{\Delta G}{2^n}$  symbolizes the division of the weights

Algorithm 2: Adapting Spade for Half-Life Dynamics									
<b>Input:</b> $G = (V, E)$ , existing peeling sequence $O, \Delta G$									
Output: Updated peeling sequence order O'									
1 Initialize $\Delta G^-, \Delta G^+ \leftarrow \emptyset$									
2 for $x \in V \cup E$ do									
3 if x is pending halving then									
$4 \qquad \qquad$									
5 $\Delta G^+ = \frac{\Delta G^-}{2} \cup \Delta G$ ; /* Halve weights of $\Delta G^-$ and new insertions */									
6 $O' = \text{Spade}(G, O, \Delta G^-, G^+)$ ; /* Peeling sequence reordering */									
7 return O'									

of all elements in  $\Delta G$  by a factor of  $2^n$ . This results in the progressive reduction of weights, reflecting the diminishing influence or significance of these elements in the graph. More specifically,

- Vertex Weights: For each vertex u<sub>i</sub> ∈ V, if u<sub>i</sub> ∈ ΔV, its weight or attribute after decay is a'<sub>i</sub> = <sup>a<sub>i</sub></sup>/<sub>2<sup>n</sup></sub>.
- Edge Weights: Similarly, For each edge  $e = (u_i, u_j) \in E$ , if  $e \in \Delta E$ , its weight after decay is  $w'_{ij} = \frac{w_{ij}}{2^n}$ .

Adapting Spade for Half-Life Dynamics (Algorithm 2). In a system where both edges and vertices are subject to time decay, whenever a new edge  $(u_i, u_j)$  is introduced at time  $T_n$ , the framework requires a reevaluation of the weight for existing edges and nodes. Each edge  $(u_i, u_j)$  and each node  $u_i$  is characterized by its creation time,  $T_{ij}$  for edges and  $T_i$  for nodes, alongside its respective weight,  $c_{ij}$  for edges and  $a_i$  for vertices. If an edge or node meets the decay criteria based on its timestamp and current system time, it is added to the pending update set, denoted as  $\Delta G^-$  (Line 2-4). After assembling all elements requiring updates, the system removes  $\Delta G^-$  and replaces them with their decayed counterparts  $\Delta G^+ = \frac{\Delta G^-}{2}$  (Line 5). Once the edges and vertices requiring halving are identified, RUSH invokes Spade's incremental method to maintain the peeling sequence by removing  $\Delta G^-$  and incorporating  $\Delta G^+$ , *i.e.*,  $O' = \text{SPADE}(G, O, \Delta G^-, \Delta G^+)$  (Line 6).

Traditionally, updating the weights and attributes of edges and vertices based on their temporal dynamics has been computationally intensive. It involves scanning each element to assess its decay requirement. This approach, while comprehensive, is resourceintensive and impractical in scenarios characterized by frequent updates due to the excessive computational overhead.

**Time complexity.** Given an edge insertion, the process necessitates an O(|V| + |E|) traversal to determine which edges and nodes require their weights to be halved. Additionally, given the  $\Delta G^+$  and  $\Delta G^-$ , the time required for reordering the peeling sequence is bounded by  $O(|E| \log |V|)$  [25]. Consequently, the worst-case cost for processing a newly inserted edge is  $O(|E| \log |V|)$ .

# 4.2 Batch Update

Notably, Algorithm 2 requires traversing the entire graph for each graph update, determining which edges and nodes will decay. On the other hand, some users exhibit regular patterns where, even after the peeling weight has halved, new transactions might increase the peeling weight again, resulting in minimal overall change in peeling weight. Based on this observation, we recognize that maintaining individual edge insertions can lead to considerable overhead. To enhance efficiency, RUSH employs a batch processing strategy. **Batch Update Scenario.** As illustrated in Figure 7, edges in the dynamic graph are grouped and processed in batches. Each batch



**Figure 7: Batch Compression Example** 

encompasses a time interval defined as  $((m-1) \cdot t, m \cdot t]$ , where m is an integer that indexes each interval and t is the length of the interval. If the half-life duration is k times the duration of the interval, then the half-life period can be expressed as  $\delta = k \cdot t$ . This period defines the frequency at which edge weights decay in accordance with batch processing. For the remainder of this section, since all times are multiples of the unit time t, we will use l to represent the actual time  $l \cdot t$  for clarity and ease of understanding.

Edge and Vertex Set Decay Within an Interval. For a given batch, the set of edges updated within the time interval (m - 1, m]is denoted as  $\Delta G_m$ . The time *m* marks the completion of the batch processing for  $\Delta G_m$ . After this, the weights of edges and vertices in  $\Delta G_m$  are subject to decay, following the half-life principle over time. Specifically, after *n* half-life periods  $\delta$  at time  $m + n \cdot k$ , the weights of the edges and nodes in  $\Delta G_m$  decay to  $\frac{\Delta G_m}{2^n}$ .

**Graph Representation Over Time.** At any given time  $n \cdot k + m$ , the graph can be represented by appending the batches as follows:

$$G(n \cdot k + m) = \sum_{l=1}^{n \cdot k + m} \left( \frac{\Delta G_l}{2^{\lfloor \frac{n \cdot k + m - l}{k} \rfloor}} \right)$$
(3)

Example 4.1. Figure 6 illustrates the efficiency of batch update over the simple edge updating strategy. In the case of simple edge updating, each modification, such as halving the weight of the edge between node  $v_2$  and  $v_3$  at time  $t_1$ , requires individual processing. However, batch updating merges these operations over the duration of the batch (2 units in this example). Consequently, at time  $t_2$ , it is unnecessary to recompute the peeling sequence since the net effect of reducing and subsequently adding an edge of equivalent halved weight negates any change. Therefore, the peeling sequence at time  $t_2$  remains identical to the initial sequence. Similarly, the transition from graph  $G(t_2)$  to graph  $G(t_4)$  can be perceived as simply adding a new edge with weight 1 between  $v_1$  and  $v_2$ .

We observe that both the decay and reordering operations are positively correlated with the size of the graph. Hence, the compression of different batches can improve the efficiency.

Definition 4.2 (Batch Compression). Given two batches  $\Delta G_{m_1}$  =  $(V_{m_1}, E_{m_1})$  and  $\Delta G_{m_2} = (V_{m_2}, E_{m_2})$  satisfying  $m_2 > m_1$  and  $m_1 \equiv$  $m_2 \pmod{k}$ , they are compressed into  $\Delta G_m = (V_m, E_m)$  where:

- $V_m = V_{m_1} \cup V_{m_2}$  and  $E_m = E_{m_1} \cup E_{m_2}$ .
- $\forall u_i \in V_m$ , its weight  $a_i$  is calculated as  $a_{m_1}/2 \frac{m_2 m_1}{k} + a_{m_2}$ , where  $a_{m_1}$  and  $a_{m_2}$  are vertex weights in  $\Delta G_{m_1}$  and  $\Delta G_{m_2}$ . •  $\forall (u_i, u_j) \in E_m$ , its weight  $c_{ij}$  is calculated as  $c_{m_1}/2^{\frac{m_2-m_1}{k}}$
- where  $c_{m_1}$  and  $c_{m_2}$  are edge weights in  $\Delta G_{m_1}$  and  $\Delta G_{m_2}$ .

The batch compression technique can be extended to recursively compress multiple batches whenever any two of these batch updates,  $\Delta G_{m_i}$  and  $\Delta G_{m_i}$ , satisfy the condition  $m_i \pmod{k} = m_i$ (mod k) for  $m_i \neq m_j$  and  $m_i, m_j \in \mathbb{N}^+$ . For example, if k = 5, then batches  $\Delta G_2$  and  $\Delta G_7$  will decay simultaneously, as 2 (mod 5) = 7 (mod 5). When l = 7,  $\Delta G_2$  will be compressed with  $\Delta G_7$ , since they share the same decay period and will decay at subsequent times such as l = 12, 17, ... Similarly, when  $l = 12, \Delta G_{12}$  will be compressed. We formally define the batch congruence class below.

Definition 4.3 (Batch Congruence Class). Let G be a set of batch updates, each denoted as  $\Delta G_m$ . A congruence class within  $\mathcal{G}$  is defined as a subset of  $\mathcal{G}$  where two batches  $\Delta G_{m_1}$  and  $\Delta G_{m_2}$  are considered equivalent if  $m_1 \pmod{k} = m_2 \pmod{k}$  for a given modulus k. Formally, the congruence class for  $\Delta G_m$  is defined as:

$$[\Delta G_m]_k = \{\Delta G_n \in \mathcal{G} \mid m \pmod{k} = n \pmod{k}\}$$
(4)

Batch Compression via Congruence Classes. We observe that batches within the same congruence class share identical half-life decay points. Hence, we compress batches in each congruence class. After applying batch compression to each of the k congruence classes, we obtain k compressed batches:  $\Delta G_1^*, \Delta G_2^*, \dots, \Delta G_k^*$ . Each compressed batch  $\Delta G_m^*$  is the result of compressing all batches within the congruence class  $[\Delta G_m]_k$ .

Intuitively, batch compression benefits the system by reducing the graph size through compression within congruence classes. Specifically, the scale of the original graph G(l) is significantly reduced after compression, from  $\sum_{i=1}^{l} |\Delta G_i|$  to  $\sum_{m=1}^{k} |\Delta G_m^*|$ , where  $|\Delta G_i|$  denotes the size of each individual batch before compression, and  $|\Delta G_m^*|$  represents the size of compressed batch. It also decrease update times by reducing the number of edges that require decay.

Example 4.4. Figure 7 illustrates an example of graph compression. When the half-life duration is k times the batch duration, batches such as  $\Delta G_0$ ,  $\Delta G_k$ , and others belong to the same congruence class  $[\Delta G_0]_k$ . RUSH aggregates all batches within this class to create a compressed batch update  $\Delta G_0^*$ . Regardless of the original number of batches, it is only necessary to maintain k compressed batches since there are at most k batch congruence classes.

**Time Complexity.** While the time complexity remains  $O(|E| \log |V|)$ , the efficiency is improved due to the reduced scale of |E| and |V|.

LEMMA 4.5. Utilizing the batch update strategy within RUSH, the resulting densest subgraph achieves a 4-approximation.

PROOF. The edge update strategy yields a 2-approximate result for the densest subgraph, as demonstrated in [23]. Our batch update strategy achieves a 2-approximation relative to the edge update strategy, thereby concluding the proof. Due to space limitations, the complete proof is provided in Appendix C of [9]. 

Lemma 4.5 demonstrates the accuracy guarantee of RUSH.

# 4.3 Batch Selection Strategy

Due to the half-life setting, the detection of dense subgraphs at time  $G(n \cdot k + m)$  can be incrementally maintained either from the dense subgraphs of  $G((n-1) \cdot k + m)$  along with the intermediate batches, or from  $G(n \cdot k + m - 1)$  in addition to the most recent batch. We introduce two distinct selection methods in this subsection.

LEMMA 4.6. Let G be a graph with a peeling sequence O, and associated peeling weights  $\Delta_i$  for each  $u_i$  in O. Consider the graph  $G' = \frac{G}{n}$ , obtained by dividing all weights in G by a constant n > 0. Then, the peeling sequence O' in G' satisfies O' = O, and for each  $u_i$ in O, the corresponding peeling weight  $\Delta'_i$  in  $\frac{G}{n}$  is given by  $\Delta'_i = \frac{\Delta_i}{n}$ .

**PROOF.** Lemma 4.6 is verified by first demonstrating that the initial vertex peeled in *G* is identically the first peeled in  $\frac{G}{n}$  due to uniform peeling weight adjustments. This initial step is then extended through induction to confirm that the sequence holds for the subsequent (k + 1)th nodes after the initial *k* nodes are peeled. The complete proof can be found in Appendix C of [9].

Given the graph  $G((n - 1) \cdot k + m)$ , we can derive the peeling sequence and peeling weight for the same graph at time  $n \cdot k + m$  by retaining the original peeling sequence and halving the previous peeling weights, as indicated by Lemma 4.6. This method reduces the computational cost from  $O(|E| \log |V|)$  to O(V).

**Option 1: Incremental Maintenance Post-Half-Life Updates** (RUSH-PHL). We can get the peeling sequence of  $G(n \cdot k + m)$  by incorporating the recent k batches to the decayed subgraph  $G((n - 1) \cdot k + m)$ . Formally, we define  $O' = \text{SPADE}(G((n - 1) \cdot k + m)/2, O, \emptyset, \Delta G^+)$ , where O represents the peeling sequence of  $G((n - 1) \cdot k + m)/2$  and  $\Delta G^+ = \sum_{i=(n-1)\cdot k+m+1}^{n\cdot k+m} \Delta G_i$ .

**Option 2: Incremental Maintenance Post-Batch Update (**RUSH-PBU). Formally, we define  $O' = \text{SPADE}(G(n \cdot k + m - 1), O, \Delta G^-, \Delta G^+)$ , where O represents the peeling sequence computed at the end of the last batch.  $\Delta G^-$  denotes the set of edges and vertices subject to halving, while  $\Delta G^+$  includes the edges and vertices to be inserted post-halving. Specifically,  $\Delta G^- = \Delta G^*_m$  and  $\Delta G^+ = \frac{\Delta G^*_m}{2} \oplus \Delta G_{n \cdot k + m}$ .

**Selection by the Number of Updates (**RUSH-S): The efficiency is directly proportional to the number of updates. Therefore, we introduce the selection strategy aimed at choosing the updating option that requires the fewest number of updates. In Option 1, there are a total of  $x_1 = \sum_{i=(n-1)\cdot k+m+1}^{n\cdot k+m} |\Delta G_i|$  updates, while in Option 2, there are  $x_2 = 2|\Delta G_m^*| + |\Delta G_{n\cdot k+m}|$  updates. When  $x_1 \le x_2$ , RUSH selects Option 1; otherwise, RUSH selects Option 2. As the number of batches increases, the value of *k* rises accordingly, leading to an increase in  $x_1$  and a decrease in  $x_2$ . This leads to longer update times for RUSH-PHL and shorter ones for RUSH-PBU.

#### 5 EXPERIMENT

# 5.1 Experiment Setup

The codebase<sup>1</sup> was compiled using g++ (version 8.3.1) with the -03 optimization. The experiments were executed on a Linux-based system with two Intel Xeon Gold 6246R CPUs and 384 GB of RAM. **Datasets.** Experiments were conducted on seven distinct datasets, as elaborated in Table 1. Among these datasets, two originate from

**Table 1: Experiment Datasets** 

Datasets	Name	V	E	Avg. Degree	$ \Delta E $	Time Interval (day)
dotdotdot	dot	11.4K	19.0K	3.4	1.9K	191.8
Terraforms	tf	16.3K	35.6K	4.4	3.6K	100.7
BAYC	ba	21.4K	65.6K	6.1	6.5K	103.5
Meebits	mb	38.3K	69.7K	3.6	7.0K	85.0
ArtBlock	ab	250.3K	452.5K	3.6	45.3K	85.9
Grab1	gb1	6.7M	75.2M	22.1	75K	1.41
Grab2	gb2	14.5M	32.0M	4.4	32K	0.78

industrial sources at Grab, denoted as gb1 and gb2. Additionally, five datasets were utilized from the NFT repository [2]. The construction of the graph *G* commences with an initial setup comprising the vertex set *V* and 90% of the edges *E*, reserving the remaining 10% of *E* for incremental testing.

**Competitors.** We conducted a comparative analysis between RUSH and four state-of-the-art systems specialized in detecting burst subgraphs and dense subgraphs. The chosen benchmarks for this comparison include MBC [29], DBS [11], Fraudar [23], and Spade [25]. We compare RUSH with SCAN [37] and DOMINANT [13], which represent the commonly used clustering and graph neural network (GNN)-based strategies for detecting graph outliers, respectively.

**Default Setting.** Within RUSH, the default configuration set the half-life duration to 60 seconds for industrial datasets, such as those from Grab, and to 3,600 seconds for all other datasets. By default, the batch duration for processing was determined as one quarter of the respective half-life durations across all datasets. For the comparison frameworks, MBC and DBS, we defined the time window to be four times the half-life duration specified for RUSH. We use the default parameters for MBC, SCAN and DOMINANT.

**Density Function.** Following [25], we employed three density functions (DG, DW, and FD) to assess their influence on various systems. By default, our approach employs DW. Additional experiments on DG and FD are detailed in the supplementary material [9]. **Metric.** We employ *updating time* and *latency* to measure the efficiency. Updating time refers to the average duration a system takes to incorporate a new edge insertion within the updated graph. Latency is defined as the interval from the moment a fraud node makes its first transaction to when it is detected by the system.

**Updating Time.** Figure 9(a) showcases the updating times for systems employing the DW density function. RUSH significantly outperforms SCAN, DOMINANT, MBC, DBS, and Fraudar, achieving speedups by factors of  $5.4 \times 10^2$ ,  $3.9 \times 10^2$ , 17.1,  $1.6 \times 10^4$ , and  $2.9 \times 10^3$ , respectively. This remarkable efficiency gain can be attributed to RUSH's incremental update methodology, which avoids recalculating from scratch for each update. RUSH surpasses Spade by being 18.8 times faster, a feat achieved through the implementation of the graph compression technique and the selection strategic use of batch updates. These innovations considerably enhance the efficiency of the updating process, allowing RUSH to swiftly respond to the explosive transactions exhibited by fraudulent communities. RUSH achieves an improvement in throughput by approximately two orders of magnitude compared to other systems. The detailed comparison can be found in Appendix A of [9].

#### 5.2 Efficiency

**Latency.** Figure 9(b) contrasts the latency of RUSH with other systems. On average, RUSH achieves speedups of 821.9, 774.5, 332.2, 801.9, and 645.9 times compared to SCAN, DOMINANT, MBC,

<sup>&</sup>lt;sup>1</sup>https://github.com/Xtra-Computing/RUSH





DBS, and Fraudar, respectively. This improvement in latency is due to RUSH's design, optimized for dynamic updates and explosive insertions. In contrast to MBC, DBS, and Fraudar's reliance on intensive recalculations, RUSH proposes incremental algorithms that quickly adapt to graph updates. Despite being built for dynamic graphs, Spade overlooks the link between time and weight, allowing fraudsters to act longer before detection and thus increasing Spade's latency, which is 254.4 times higher than that of RUSH.

**Impact of Batch Update.** Figure 10 compares the batch update strategy with the edge updating method. RUSH-E signifies the edge updating approach, in contrast to the other labels, which represent different selection methods within the batch update framework. The method employed by RUSH-E involves scanning the entire graph and reordering the peeling sequence after each edge update, as detailed in Section 4.1. This approach is significantly more time-consuming, taking 12.3X longer than RUSH-S.

**Impact of Batch Selection.** Figure 10 compares the performance of three different batch selection methods. As the number of batches increases, the number of batches that need updating increases for RUSH-PHL and decreases for RUSH-PBU as analyzed in Section 4.3. Meanwhile, RUSH-S evaluates the number of edges requiring updates and selects the most cost-effective strategy. As a result, RUSH-S consistently achieves a speedup of 1.2 times on average compared to both RUSH-PBU and RUSH-PHL.

**Impact of Batch Compression.** Figure 12 evaluates the batch compression technique. On average, it achieves a speedup of 1.3*X*. This improvement is attributed to the method's ability to merge batches in the same congruence class, thereby reducing the number of edges that need to be updated during the batch update phase.

**Evaluation of Scalability.** Scalability plays a crucial role in the fraud detection pipelines, as it determines the ability to handle increasing volumes of data without compromising performance. Figure 11 showcases RUSH's scalability across varying scales of the Grab graph. As the graph scales from 1 million to 64 million edges, the updating time and latency increase by only 2.3 and 1.6 times, respectively, demonstrating a sublinear growth pattern. This efficiency is attributed to the graph compression technique, which becomes increasingly effective at larger scales, significantly reducing the graph's size. Concurrently, the time decay function enhances RUSH's capability to rapidly identify burst behavior, ensuring swift detection despite the graph's expansive size. Hence, both updating time and latency maintain a sublinear trajectory, underscoring RUSH's robust performance to large-scale environments.

**Metric.** We employ metrics such as *precision, recall,* and *F1 score* to evaluate the effectiveness of the systems. Precision quantifies the proportion of transactions that the system accurately identifies as fraudulent. Recall measures the percentage of fraud nodes successfully detected by the system. F1 score serves as a balanced metric, representing the harmonic mean of precision and recall.

Overall Effectiveness. Figure 8 showcases the superior effectiveness of RUSH when utilizing the DW density function, with an F1 score that surpasses MBC, DBS, Fraudar, and Spade by factors of 1.5, 9.7, 2.7, and 1.7, respectively. Unlike MBC, which focuses on detecting burst k-core, RUSH is tailored to identify burst subgraphs, a strategy that more accurately captures the characteristics of fraudulent behavior, thus significantly enhancing precision by 1.31. In contrast to both DBS and Fraudar, which target subgraphs within a specific timespan, RUSH takes a holistic approach by considering the entire temporal span of the graph. This methodology enables the detection of a broader range of fraudsters, elevating RUSH's recall above these systems by 9.51 and 1.64, respectively. While Spade achieves a precision comparable to RUSH, its recall is comparatively lower across several datasets. This discrepancy is attributed to Spade's equal treatment of outdated and new data, making it challenging to detect emerging fraudulent communities. On average, RUSH achieves higher F1 value than SCAN and DOMI-NANT by factors of 3.1X and 2.5X. SCAN underperforms primarily because fraudulent nodes often exhibit similar behaviors (e.g., high volume of transactions in a short period) but do not necessarily belong to the same cluster. DOMINANT is less effective because the relatively small number of fraudulent nodes (fewer than 1000) challenges the system's ability to learn from the data effectively.

**Parameter Tuning.** As batch size increases, the system's recall slightly decreases due to the increasing of the granularity of time between updates. This leads to less timely adjustments in the weights of nodes and edges. Details on how parameters affect detection effectiveness are provided in Appendix A.3 of [9].

# 5.4 Case Study within Grab

Spade has been deployed within Grab to identify fraudulent activities. Consequently, in our case studies, we conduct a comparative analysis between RUSH and Spade to evaluate their performance. **Customer-Merchant Collusion.** Figure 13(a) illustrates a case of customer-merchant collusion, where customers and merchants participate in sham transactions on promotional offers for financial benefits. In this scenario, a normal user may engage in numerous transactions with legitimate merchants, forming a dense but genuine community. Without decay function, systems like Spade takes more time (8,732s) to find the fraudulent community, while RUSH takes less time (673s) and prevent more fraudulent transactions.



Figure 13: Case Study Example: For (a) and (b), the loss is measured in the number of transactions, rather than amount of fund, due to confidentiality issues. The loss in (c) is quantified in terms of the number of ETH involved in fraudulent transactions.

**Promotion Abuse.** Promotion abuse happens when a single user creates multiple accounts on one device to repeatedly use a voucher intended for single-use per individual. In scenarios of promotion abuse, fraudsters exploit limited device access to frequently log in, thereby creating an extremely dense subgraph within a short timeframe. Traditional systems like Spade may fail to distinguish between legitimate and malicious densities, as seen in Figure 13(b), where the benign community appears denser (density of 79.3) than the fraudulent one (density of 59.7) without time-based decay mechanisms. Conversely, RUSH reduces the benign community's density to 43.4, belowing the fraudulent's 59.7.

# 5.5 Case Study on Crypto Wash Trading

**Wash Trading.** Figure 13(c) showcases a wash trading example, illustrating how fraudulent users manipulate a token's price via coordinated buy-sell transactions. Spade [25] fails to recognize fraudulent behaviors until they evolve into the densest subgraph. During such delays, 42 transactions have been executed by fraudsters, amounting to over 1,000 ETH. In contrast, RUSH detects the fraudulent community after just five transactions within the group, reducing the loss to approximately 200 ETH.

# 6 RELATED WORK

**Dense Subgraph Detection**. Numerous studies have employed dense subgraph mining techniques to detect fraudulent activities, identify spam, or uncover communities within social and review networks [23, 30, 31]. While these approaches are adept at handling static graph structures, their adaptation to dynamic graphs, though explored [3, 16, 24, 25], often faces inherent limitations. Specifically, methodologies, such as [32], assume that the weights on edges and vertices remain constant over time. Moreover, the throughput of these methods is not sufficient for scenarios characterized by burst patterns. In contrast, RUSH integrates a time decay mechanism, facilitating the rapid identification of burst subgraphs. This innovative approach reduces latency in detecting fraudulent communities, thereby enhancing the system's efficiency and effectiveness.

**Burst Subgraph Detection.** Several systems [11, 29, 39] focus on the mining of burst subgraphs. However, their scope is generally limited to identifying particular types, such as burst core or burst cohesiveness subgraphs. In contrast, RUSH offers an extensive range of APIs, allowing users to employ or develop various density metrics to identify different types of burst subgraphs.

**Fraud Detection Using Graph Techniques.** In the landscape of graph-based fraud detection, strategies such as COPYCATCH [4] and GETTHESCOOP [26] have been instrumental, utilizing local search heuristics to unearth dense subgraphs in bipartite graphs. The method of label propagation, as discussed in [36], emerges as an effective technique for community detection within this context. The exploration of link analysis for fraud identification further enriches the domain, as seen in [12, 37]. Graph Neural Networks (GNNs) have significantly advanced graph-based fraud detection, as demonstrated in studies like [13, 14, 34].

# 7 CONCLUSION

Burst subgraphs are effective common patterns for fraud detections in many data science pipelines of on-line applications such as e-commerce and crypto transactions. In this paper, we present RUSH, a novel system for efficiently detecting burst subgraphs in dynamic graphs using a half-life time decay approach. RUSH introduces graph compression and batch updating methods to speed up graph updates. It also offers user-friendly APIs for customizable fraud detection across different scenarios. Our experiments show RUSH greatly outperforms existing systems in update speed and latency reduction. Case studies confirm its effectiveness in combating frauds in Grab and crypto networks, demonstrating its real-world utility in real-time fraud detection data science pipelines.

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# REFERENCES

- [1] [n.d.]. Distil Networks: The 2019 Bad Bot Report. https://www.bluecubesecurity. com/wp-content/uploads/bad-bot-report-2019LR.pdf.
- [2] 2023. Live Graph Lab. https://livegraphlab.github.io/. Accessed: 2023-12-10.
- [3] Bahman Bahmani, Ravi Kumar, and Sergei Vassilvitskii. 2012. Densest Subgraph in Streaming and MapReduce. Proceedings of the VLDB Endowment 5, 5 (2012).
- [4] Alex Beutel, Wanhong Xu, Venkatesan Guruswami, Christopher Palow, and Christos Faloutsos. 2013. Copycatch: stopping group attacks by spotting lockstep behavior in social networks. In Proceedings of the 22nd international conference on World Wide Web. 119–130.
- [5] Robert E Burton and Richard W Kebler. 1960. The "half-life" of some scientific and technical literatures. American documentation 11, 1 (1960), 18–22.
- [6] Moses Charikar. 2000. Greedy approximation algorithms for finding dense components in a graph. In International Workshop on Approximation Algorithms for Combinatorial Optimization. Springer, 84–95.
- [7] Nidhika Chauhan and Prikshit Tekta. 2020. Fraud detection and verification system for online transactions: a brief overview. *International Journal of Electronic Banking* 2, 4 (2020), 267–274.
- [8] Tianyi Chen and Charalampos Tsourakakis. 2022. Antibenford subgraphs: Unsupervised anomaly detection in financial networks. In Proceedings of the 28th ACM SIGKDD Conference on Knowledge Discovery and Data Mining. 2762–2770.
- [9] Yuhang Chen, Jiaxin Jiang, Shixuan Sun, Bingsheng He, and Min Chen. 2024. RUSH: Real-time Burst Subgraph Discovery in Dynamic Graphs (Complete Version). https://github.com/Xtra-Computing/RUSH/blob/main/RUSH\_paper. pdf
- [10] Dawei Cheng, Xiaoyang Wang, Ying Zhang, and Liqing Zhang. 2020. Graph neural network for fraud detection via spatial-temporal attention. *IEEE Transactions* on Knowledge and Data Engineering 34, 8 (2020), 3800–3813.
- [11] Lingyang Chu, Yanyan Zhang, Yu Yang, Lanjun Wang, and Jian Pei. 2019. Online density bursting subgraph detection from temporal graphs. *Proceedings of the VLDB Endowment* 12, 13 (2019), 2353–2365.
- [12] Corinna Cortes, Daryl Pregibon, and Chris Volinsky. 2003. Computational methods for dynamic graphs. *Journal of Computational and Graphical Statistics* 12, 4 (2003), 950–970.
- [13] Kaize Ding, Jundong Li, Rohit Bhanushali, and Huan Liu. 2019. Deep anomaly detection on attributed networks. In *Proceedings of the 2019 SIAM International Conference on Data Mining*. SIAM, 594–602.
  [14] Yingtong Dou, Zhiwei Liu, Li Sun, Yutong Deng, Hao Peng, and Philip S Yu. 2020.
- [14] Yingtong Dou, Zhiwei Liu, Li Sun, Yutong Deng, Hao Peng, and Philip S Yu. 2020. Enhancing Graph Neural Network-based Fraud Detectors against Camouflaged Fraudsters. In Proceedings of the 29th ACM International Conference on Information and Knowledge Management (CIKM'20).
- [15] HangYuan Du, Duo Li, and WenJian Wang. 2022. Abnormal User Detection via Multiview Graph Clustering in the Mobile e-Commerce Network. Wireless Communications and Mobile Computing 2022 (2022).
- [16] Alessandro Epasto, Silvio Lattanzi, and Mauro Sozio. 2015. Efficient densest subgraph computation in evolving graphs. In Proceedings of the 24th international conference on world wide web. 300–310.
- [17] Michael Fleder, Michael S Kester, and Sudeep Pillai. 2015. Bitcoin transaction graph analysis. arXiv preprint arXiv:1502.01657 (2015).
- [18] Aristides Gionis and Charalampos E Tsourakakis. 2015. Dense subgraph discovery: Kdd 2015 tutorial. In Proceedings of the 21th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. 2313–2314.
- [19] Naga VC Gudapati, Enrico Malaguti, and Michele Monaci. 2021. In search of dense subgraphs: How good is greedy peeling? *Networks* 77, 4 (2021), 572–586.
- [20] Jericho Hallare and Valerie Gerriets. 2020. Half life. (2020).
   [21] Wentao Han, Youshan Miao, Kaiwei Li, Ming Wu, Fan Yang, Lidon
- [21] Wentao Han, Youshan Miao, Kaiwei Li, Ming Wu, Fan Yang, Lidong Zhou, Vijayan Prabhakaran, Wenguang Chen, and Enhong Chen. 2014. Chronos: a graph engine for temporal graph analysis. In Proceedings of the Ninth European Conference on Computer Systems. 1–14.
- [22] Frank Harary and Gopal Gupta. 1997. Dynamic graph models. Mathematical and Computer Modelling 25, 7 (1997), 79–87.

- [23] Bryan Hooi, Hyun Ah Song, Alex Beutel, Neil Shah, Kijung Shin, and Christos Faloutsos. 2016. Fraudar: Bounding graph fraud in the face of camouflage. In Proceedings of the 22nd ACM SIGKDD international conference on knowledge discovery and data mining. 895–904.
- [24] Jiaxin Jiang, Yuhang Chen, Bingsheng He, Min Chen, and Jia Chen. 2024. Spade+: A Generic Real-Time Fraud Detection Framework on Dynamic Graphs. *IEEE Transactions on Knowledge and Data Engineering* (2024).
- [25] Jiaxin Jiang, Yuan Li, Bingsheng He, Bryan Hooi, Jia Chen, and Johan Kok Zhi Kang. 2022. Spade: A Real-Time Fraud Detection Framework on Evolving Graphs. Proc. VLDB Endow. 16, 3 (nov 2022), 461–469. https://doi.org/10.14778/3570690. 3570696
- [26] Meng Jiang, Peng Cui, Alex Beutel, Christos Faloutsos, and Shiqiang Yang. 2014. Inferring strange behavior from connectivity pattern in social networks. In *Pacific-Asia conference on knowledge discovery and data mining*. Springer, 126– 138.
- [27] Mingxuan Lu, Zhichao Han, Susie Xi Rao, Zitao Zhang, Yang Zhao, Yinan Shan, Ramesh Raghunathan, Ce Zhang, and Jiawei Jiang. 2022. BRIGHT-Graph Neural Networks in Real-Time Fraud Detection. In Proceedings of the 31st ACM International Conference on Information & Knowledge Management. 3342–3351.
- [28] Atsushi Miyauchi and Akiko Takeda. 2018. Robust densest subgraph discovery. In 2018 IEEE International Conference on Data Mining (ICDM). IEEE, 1188–1193.
- [29] Hongchao Qin, Rong-Hua Li, Ye Yuan, Guoren Wang, Lu Qin, and Zhiwei Zhang. 2022. Mining Bursting Core in Large Temporal Graphs. Proceedings of the VLDB Endowment (2022).
- [30] Yuxiang Ren, Hao Zhu, Jiawei Zhang, Peng Dai, and Liefeng Bo. 2021. Ensemfdet: An ensemble approach to fraud detection based on bipartite graph. In 2021 IEEE 37th International Conference on Data Engineering (ICDE). IEEE, 2039–2044.
- [31] Kijung Shin, Tina Eliassi-Rad, and Christos Faloutsos. 2016. Corescope: Graph mining using k-core analysis-patterns, anomalies and algorithms. In 2016 IEEE 16th international conference on data mining (ICDM). IEEE, 469-478.
- [32] Kijung Shin, Bryan Hooi, Jisu Kim, and Christos Faloutsos. 2017. Densealert: Incremental dense-subtensor detection in tensor streams. In Proceedings of the 23rd ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. 1057–1066.
- [33] Sanya B Taneja, Tiffany J Callahan, Mary F Paine, Sandra L Kane-Gill, Halil Kilicoglu, Marcin P Joachimiak, and Richard D Boyce. 2023. Developing a knowledge graph for pharmacokinetic natural product-drug interactions. *Journal of Biomedical Informatics* 140 (2023), 104341.
- [34] Chen Wang, Yingtong Dou, Min Chen, Jia Chen, Zhiwei Liu, and S Yu Philip. 2021. Deep Fraud Detection on Non-attributed Graph. In 2021 IEEE International Conference on Big Data (Big Data). IEEE, 5470–5473.
- [35] Daixin Wang, Jianbin Lin, Peng Cui, Quanhui Jia, Zhen Wang, Yanming Fang, Quan Yu, Jun Zhou, Shuang Yang, and Yuan Qi. 2019. A semi-supervised graph attentive network for financial fraud detection. In 2019 IEEE International Conference on Data Mining (ICDM). IEEE, 598–607.
- [36] Meng Wang, Chaokun Wang, Jeffrey Xu Yu, and Jun Zhang. 2015. Community detection in social networks: an in-depth benchmarking study with a procedureoriented framework. *Proceedings of the VLDB Endowment* 8, 10 (2015), 998–1009.
- [37] Xiaowei Xu, Nurcan Yuruk, Zhidan Feng, and Thomas AJ Schweiger. 2007. Scan: a structural clustering algorithm for networks. In Proceedings of the 13th ACM SIGKDD international conference on Knowledge discovery and data mining. 824– 833.
- [38] Chang Ye, Yuchen Li, Bingsheng He, Zhao Li, and Jianling Sun. 2021. GPU-Accelerated Graph Label Propagation for Real-Time Fraud Detection. In Proceedings of the 2021 International Conference on Management of Data. 2348–2356.
- [39] Ming Zhong, Junyong Yang, Yuanyuan Zhu, Tieyun Qian, Mengchi Liu, and Jeffrey Xu Yu. 2024. A Unified and Scalable Algorithm Framework for User-Defined Temporal (k, X)-Core Query. IEEE Transactions on Knowledge and Data Engineering (2024).
- [40] Hangjun Zhou, Guang Sun, Sha Fu, Linli Wang, Juan Hu, and Ying Gao. 2021. Internet financial fraud detection based on a distributed big data approach with node2vec. *IEEE Access* 9 (2021), 43378–43386.