

# Interactive Visual Analysis of Multi-Parameter Families of Function Graphs

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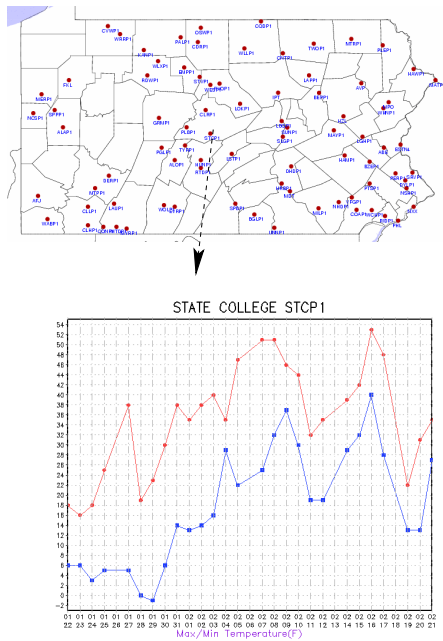


Figure 1. Map with temperature curves at some locations

## Abstract

The paper describes a method developed for interactive data visualization and exploration with application in automotive industry. Input data set contains a large number of function graphs. Each of the graphs is characterized by a set of basic attributes. The technique that is used for visualization includes two linked views: a map view (or attribute space view), where all function graphs are represented as a point or an icon on the map, and a linked function graph view. The map view provides additional visualization possibilities and allows user interaction. Additional features like brushing in both views, graph management, and related issues like interpolation of the graphs are described.

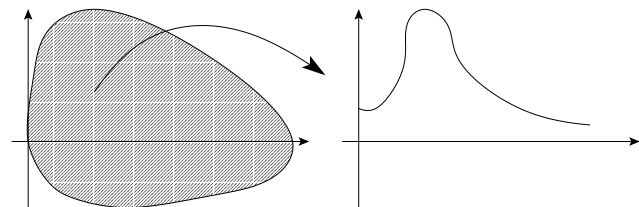


Figure 2. Example of an engine characteristic diagram

**Keywords**— interactive visualization, linked views, brushing, time series data, interactive exploration, Kennfeld diagram

## 1. Introduction

The amount of information we are being confronted with nowadays increases constantly. Effectively exploring and analyzing huge amounts of data is a challenging problem, perhaps even more than collecting or processing those data.

Plain data tables or simple 2D charts are not very useful when it comes to large data sets. Furthermore, people often look for various correlations and want to compare specific data. Information visualization tries to make it easier for the user to analyze and explore large data sets by cleverly and interactively displaying information.

Information to be explored can come from various disciplines and application areas and there is no universal solution that can fit all of those areas. In this paper we describe a method which can be used to analyze and explore data sets which contain families of function graphs, where each of them corresponds to a point in a 2D domain. The data set looks like this: we have a 2D domain and for each point in this domain (or for some of the points) there is a corresponding graph describing an attribute in other 2 dimensions.

One example for such data set is a diagram of temperature over time measured on a number of points within a certain geographical area. The geographical map represents the 2D domain, and the temperature vs time data at specific points on the map represent additional two dimensions of the data set. Figure 1 illustrates the example.

Another example of similar data set comes from automotive engine design. An *engine characteristic diagram* is a 2D diagram that shows engine speed (in rpm) on X axis and torque at crankshaft on Y axis. At every engine speed there is a maximum torque that the engine can produce when the the accelerator is fully depressed. There is also a minimum torque (with the accelerator not depressed at all) which is negative and can be used to decelerate the car. While driving a car the working point of the engine follows a path within the engine characteristic diagram.

An example of the engine characteristic diagram is shown in Figure 2.

For each point of this diagram there are several interesting properties to be investigated, for instance gas pressure in cylinders, main bearing forces, speed irregularity and torsional vibrations as functions of crank angle.

## 2. Motivation

When users work with large sets of function graphs associated with points in an attribute space they often want to explore the data first. They would try to find locations in attribute space that generate function graphs which have some interesting or unusual property, or conversely, ones that exhibit some regularity or follow a pattern. They often have little a priori knowledge about the data set. They do not exactly know what pattern or property they are looking for, or where the feature is likely to appear in the attribute space.

This calls for a visualization method that allows quick and easy, yet accurate exploration of the data. Two simultaneous views of the data set should be provided: (1) a good overall view of all function graphs to support navigation in the data set as well as (2) a detailed view which offers more information about specific graphs. Picking graphs from the attribute space must be made as effortless as possible and should require minimum user interface action.

After some interesting locations have been identified in the attribute space, another typical task is the detailed comparison of graphs associated with these areas. In order to perform this efficiently one must be able to observe the specific graphs at the same time and still not loose the context where the graphs belong. [3]

Working with static 2D charts is obviously not practical if the attribute space is large. For example, numerical simulations where a large number of parameterized cases are calculated produce large sets of result graphs. These graphs must be examined in search of optimal boundary conditions

and domains in the attribute space. It is clearly impossible to explore a set of thousand function graphs without the possibility of navigating the data set interactively. More complex queries like "show me points in the attribute space whose function graphs fulfill this-and-this criteria" are obviously even less feasible without interactive visualization methods.

Although displaying the data set as a 3D animated sequence where the time represents one of the coordinates is an obvious approach, it can only be useful in some cases. It is suitable to offer an overview of the data set but it is less practical for interactive navigation. It is even less suitable to show detailed and accurate information about graphs of specific points in the attribute space.

Therefore we suggest a different approach. Naturally, one must note that there is no perfect solution which could aid all visualization tasks even for one specific type of data. Existing methods that show animated views for better overview in time domain lack numeric accuracy. Sheets of numeric data and charts are accurate but lack overview. We assume that the method suggested in this paper could complement these existing methods and offer overview and detailed information simultaneously.

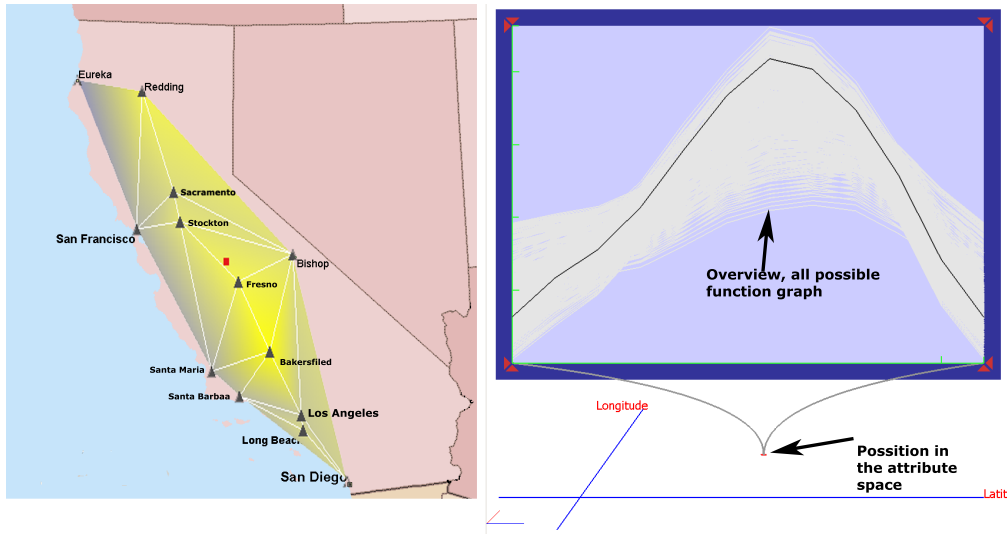
This is a way to explore the data in particular when many variants have to be examined, which is a typical scenario in optimization, for example. Interaction opens new ways of data exploration.

## 3. Technology content

The main idea is to have two linked views: the attribute view and the function graph view. The attribute view displays the domain (map, engine characteristic diagram,...) while the function graph view displays the data available for each point from the domain (temperature over time, cylinder pressure over crank angle,...) The attribute view can contain additional information such as glyphs for points where actual data is available, or a color-map of some interesting data characteristic (e.g. the maximum, the average, or mean value of the data) The system is interactive and allows the user to navigate and select points both in the attribute view and in the function graph view. The function graph view can display more graphs simultaneously in 2D or, optionally, in 3D.

Furthermore, the function graph view contains an overview part (showing the actual position of the selected point in the attribute space) and the graph corresponding to that point. Besides multiple selection in the attribute view, a collecting principle is described as well. The user can collect various graphs during the exploration, and compare them afterwards.

Finally, we describe an application which is implemented based on above mentioned principles. The application is used to visualize engine characteristic dia-



**Figure 3.** Attribute view and function graph view

gram, and it has been developed together with AVL [2], one of the leading companies in the field of powertrain simulation, measurement and design.

### 3.1. Basic idea

We describe a general framework for interactive analysis and exploration of data sets containing a family of function graphs each of them belonging to a point in 2D domain. The main idea is to have an interactive visualization method, which will make it possible to easily select a point in the attribute space and get the corresponding graph. Furthermore, the inverse process, selecting a function graph with a certain characteristic in order to locate similar graphs in the domain should also be possible.

### 3.2. Function graphs and interpolation

The data set we are concentrating on consists of a number of function graphs:

$$f_P(x)$$

Each of the function graphs is characterized by a set of parameters (or attributes)  $P = \{p_1, p_2, \dots, p_N\}$ . In a general case this gives  $N$ -dimensional attribute space. The function graphs from the input data set are represented as points in this  $N$ -dimensional space and they are scattered non-uniformly across this space. The complete input data set can be formalized as:

$$\mathcal{F} = \{f_{P_i}\}$$

In this paper we concentrate on a case where 2 out of  $N$  parameters are visualized in the attribute view. This gives

us a 2D space which can be thought of as a map. The linked view displays interactively selected function graphs.

In most cases the function graphs are not defined for each point in the map view. If the defined points are dense enough, it might be sufficient to choose the nearest defined point. However, there might be some applications where only few points in the 2D domain are available. These points can be scattered in the parameter domain on an unstructured grid and just picking the nearest defined graph when performing a selection in the map view is not always satisfactory.

The natural solution is to provide interpolated curves at all points within the map view for which the data is not available [9]. Of course, the interpolation can only be done if the nature of the data allows it. The first requirement is that the data set is continuous in nature.

Let us introduce the interpolation operator  $\Omega$  :

$$\begin{aligned} \hat{f}_P(x) &= \Omega(\mathcal{F}, P) \\ &= \Omega_P \mathcal{F} \end{aligned}$$

The unknown function graph  $\hat{f}_P(x)$  can be interpolated from the graphs of the neighboring data points with a method that best fits the data.

To enable efficient interpolation, the Delaunay triangulation is performed as a first step on all the defined points within the 2D map .

If a graph  $\hat{f}_P$  at point  $\hat{P}$  needs to be interpolated the following steps are performed:

1. The triangle  $\triangle(P_1, P_2, P_3)$  containing the point  $\hat{P}$  is located.
2. The homogeneous barycentric coordinates  $(t_1, t_2, t_3)$  of the point  $\hat{P}$  are computed. Homogeneous barycentric coordinates are barycentric coordinates normalized such that they become the actual areas

of the subtriangles  $\triangle(\hat{P}, P_2, P_3)$ ,  $\triangle(P_1, \hat{P}, P_3)$ ,  $\triangle(P_1, P_2, \hat{P})$  normalized by the area of the original triangle  $\triangle(P_1, P_2, P_3)$ . These coordinates are also called *areal coordinates* [4].

3. The resulting function graph is computed as the weighted sum of the three related function graphs using homogeneous barycentric coordinates as weighting factors.

$$\hat{f}_P(x) = t_1 f_{P_1}(x) + t_2 f_{P_2}(x) + t_3 f_{P_3}(x)$$

Using linear interpolation for interpolating temperature data on a map is likely to be a valid approach, assuming the data is not sampled too sparse. In this case trilinear interpolation of the graphs of the three nearest points on the map is an acceptable way to fill in the missing data.

Interpolating gas pressure curves in an engine characteristic diagram is an interesting example where a more sophisticated interpolation method is required. Using a simple linear interpolation may lead to incorrect results due to the specific shape of the curve (Figure 4), and possible phase shifts between the cylinder pressure curves at various engine speeds.

One possible solution to the problem is to align all involved cylinder pressures by cyclic shifting so that the  $x$  - coordinate of the peak value for all involved curves matches. The phase shift of the resulting graph is interpolated between the phase shifts that were used for the initial alignment using the same weighting factors ( $t_1, t_2, t_3$ ) (Figure 5)

Another solution to this problem is to use interpolation in frequency domain instead of direct interpolation. The function graphs are transformed to the frequency domain first, then the weighted sum using homogeneous barycentric coordinates is calculated for phase and amplitude data before converting them back to the original domain. Both methods are specific and usable for cylinder pressure diagrams because the nature of the diagram allows and requires such interpolation.

### 3.3. Visual metaphor

Since there is an attribute space and a family of graphs we propose to use two views for visualization [7]. The attribute space view is used to depict the domain. Points where function graphs exist are of main interest in the attribute space. If there are just a few points, all of them will be depicted. If there are a lot of points, maybe even more points per pixel, only a subset will be shown. Furthermore, a characteristic of the corresponding function graph, such as maximum value, minimum value, average,... can be shown in the attribute space as well. Trilinear color interpolation will be used to show this characteristic. If the points are too

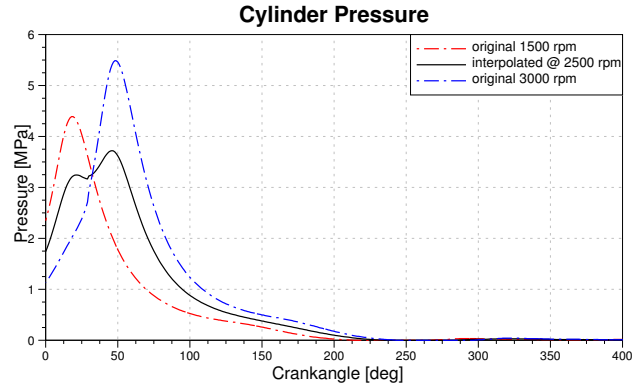


Figure 4. An erroneous interpolation of cylinder pressure

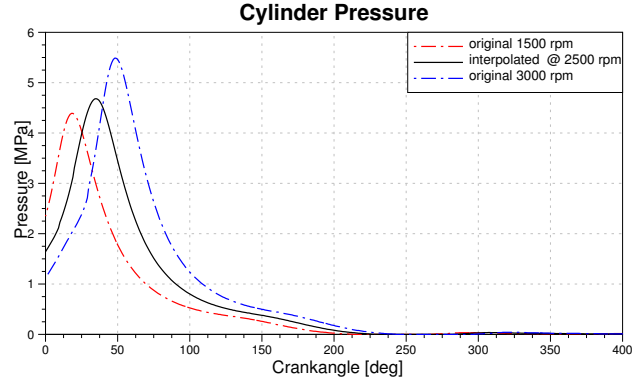


Figure 5. The correct interpolation of cylinder pressure

sparse, additional points have to be inserted into the triangulation in order to avoid the visual artifacts of the trilinear interpolation. To create these additional nodes the interpolation described in Section 3.2 is used—same as when interpolating the data for the function graph view.

The second view is used to show function graphs. It shows function graphs in a plane. Besides the current function graph, all possible function graphs are shown in light gray in the same plane as an overview. Furthermore the 2D attribute space is depicted perpendicular to the plane, and origin point of the highlighted graph is shown as well. This additional overview information helps the user in seeing a particular graph in an overall context. Note that graph's source is highlighted in attribute space itself as well. This dual highlighting helps the user to keep concentrated on the graph view, and still have an overview of the graph's location in the attribute space. Without overview feature, the user would have to constantly change focus from graph view to the attribute view. Figure 3 shows the attribute view, with maximum value depicted using a color scale, and function graph view with overview and one curve highlighted. A real map image has been used as background for attribute view.

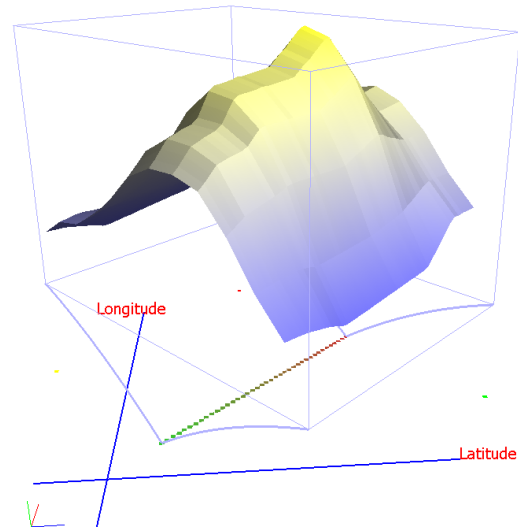
### 3.4. Workflow and interaction

The usual workflow while exploring a data set consisting of family of function graphs involves two main actions:

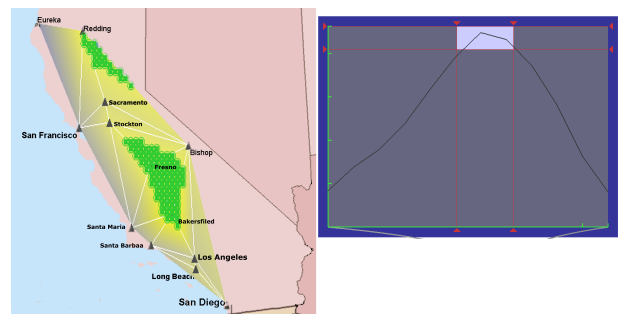
- attribute based curve exploration, and
- locating graphs in attribute space based on their properties

This means that user usually browses through the attribute space and observes the function graphs, and later-on, selects some characteristics in the function graphs and finds corresponding positions in the attribute space. The straightforward interaction for the first action is mouse-over, which depicts a function graph corresponding to the point which is currently under the mouse pointer. This can be either a provided function graph, or interpolated one, depending on mouse position. This information should also be displayed to the user. Seeing one graph at a time makes it difficult for the user to compare the curves. A method for selection in attribute space is required. There are numerous possibilities to select multiple points in attribute space. A line selection method which allows the user to select a line in the attribute space is the simplest. If we want to have separable, independent axis in the attribute selection, a rectangle selection with the axis aligned with the attribute space is needed. With both line and rectangle selection, the selection can be colored using a color gradient to provide additional information about sources of the function graphs. When the user selects multiple attributes, more function graphs are displayed simultaneously. Each graph is drawn in the color of its selection point. Figure 6 illustrates simple line selection, using the temperature data set. Note that the colors of the function graphs are the same as that of the corresponding points in the selection in attribute space. Having multiple function graphs corresponding to a line in the attribute space, the resulting curves can be extruded to create a 3D surface. 3D surfaces can offer new insights into the data. Figure 7 shows another line selection depicted as 3D surface. Note that extrusion to 3D is possible only for line selections in the attribute view.

It is possible to combine more selections using Boolean algebra, which adds additional possibilities to the data exploration and analysis. Selections are very powerful for exploration, but they have a limited lifetime. When the user makes a new selection the old one is lost. That is why a collection principle is introduced. Collections are actually containers that hold a number of graphs. As the user browses the attribute space and finds a graph of interest he can add it to a collection and continue browsing. Graphs can be added to existing collections at any later point. Several collections may be defined and used simultaneously, much like having multiple clipboards in a text editor. It is also possible to create new collections via Boolean operations on other collec-



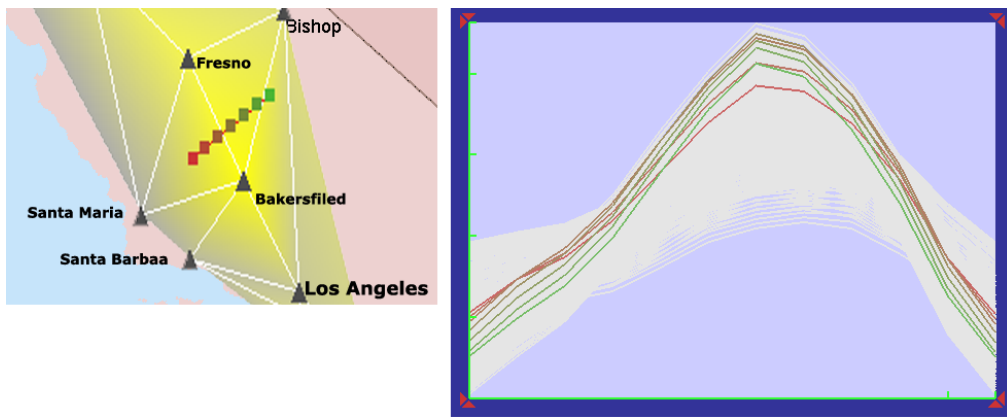
**Figure 7.** 3D display of function graphs belonging to a line in the attribute space



**Figure 8.** Brushing example. Areas that with a very high July temperature are found in the attribute space

tions. After the user had created collections he can simply display and explore them.

Once a selection is made or a collection is displayed, the user can start working with the function graph view. The user often wants to locate graphs in the attribute space having certain characteristics. The user can use a rectangular brush [1] [5], for example, to select a certain area in the function graph view. The points corresponding to the graphs passing through the selected rectangle are highlighted in the attribute space. For instance, brushing makes it possible to find which areas on the map had average temperatures in July over some threshold. Figure 8 shows such an example.



**Figure 6.** 2D display of function graphs belonging to a line in the attribute space. Note the overview depicted as context, and colors used to distinguish between seven function graphs

#### 4. Application

We have applied the above described principles in the *KennfeldView* tool developed together with AVL. The application is implemented in C++ using OpenGL [8] and FOX Toolkit [6]. The tool is available on several platforms, including MS Windows and Linux. Primary application of the tool is the exploration and analysis of engine characteristic diagrams. After the data is gathered either by simulation or measurement on real engines, the user loads the data, and starts exploration. The attribute view can display maximum, minimum, average and mean value of the corresponding graphs as underlying color map. Due to our experience the maximum value was the most important feature, as engineers are interested in high, rather than in low pressure (or temperature) areas. The *KennfeldView* supports three types of interpolation: linear, modified linear and interpolation in Fourier space. A proper interpolation has to be selected based on the input data. In order to perform the interpolation the input points in the attribute view are tessellated. The user can choose to display tessellation of the attribute view, or the icons on the points where graph data is available, or both.

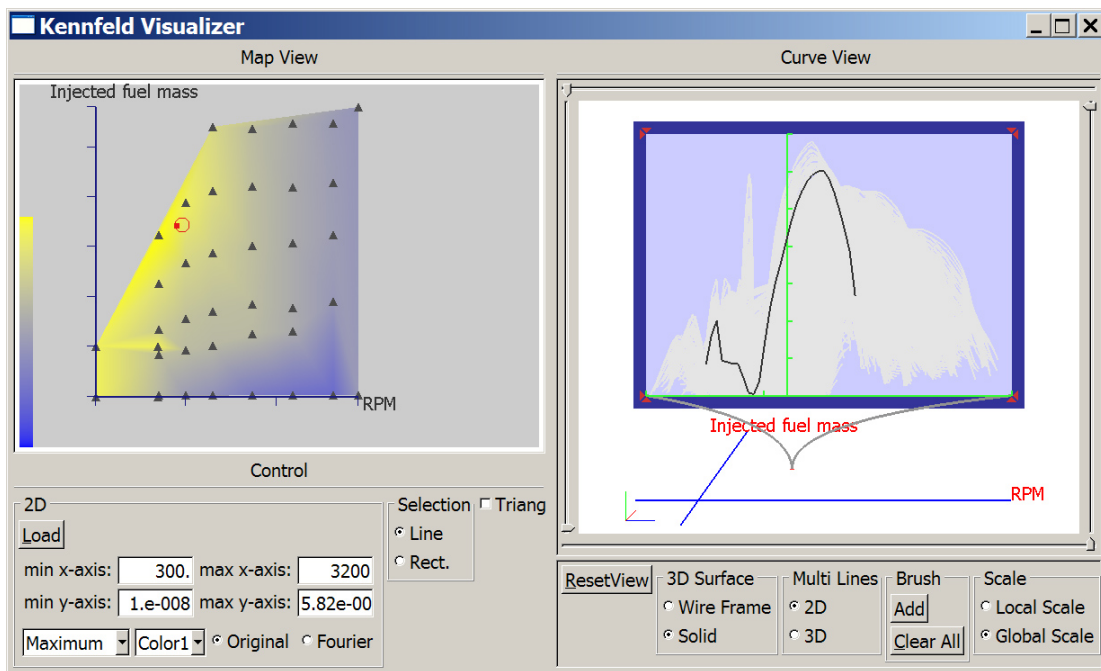
Once the data is loaded, interpolation type, color scale and underlying map is chosen the user can start data exploration. Simply moving over the attribute view will display corresponding graphs in the graph view. Single or multiple selection, as described in Section 3.4 are supported. If the user chooses to display multiple selections as a 3D surface (possible only for line selection) the surface can be displayed either as a surface or a wire-frame. The camera can be freely moved in this mode. 2D display of multiple selections does not allow camera movement, but allows brushing in the graph view, which is a very important feature for engineers.

Let us show how an actual data exploration might look like. We have two data sets, the first describing the rate of

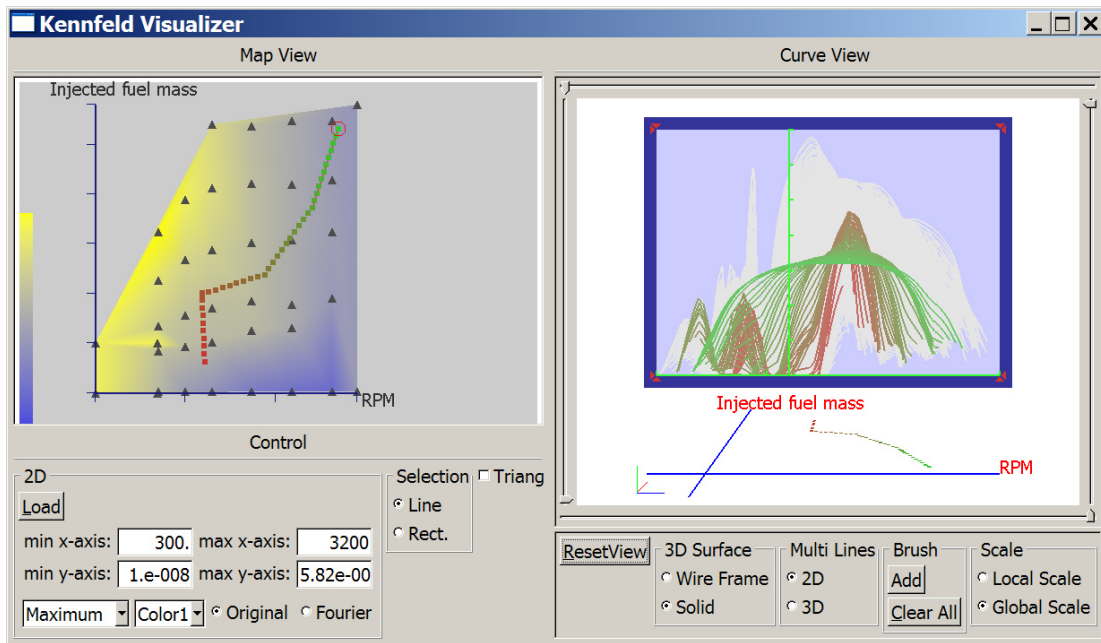
injection as a function of a crankangle. Rate of injection indicates how fuel mixture is injected in the cylinder. Figure 9 shows a characteristic injection curve. Two peaks are clearly visible, the first one so called *pre-injection*, and the second one the main injection. Note that this form of injection curve is desired and engine designers try to achieve such engine characteristics. On the left hand side in the figure 9 the corresponding attribute space is shown, which depicts injected fuel mass and engine speed in rpm. The depicted attribute space represents the working range of an engine. The engine's working point can be only in the depicted area in the attribute view. The engineers can use the tool to explore injection curves at various working points and to quickly see if there are some areas where the curve has only one peak (undesired curve shape) or has too low pre-injection ratio. Figure 10 shows an example of a multi-line selection in attribute space. Note how the color changing from red to green aids the user in identifying areas that have an undesired (single) peak curve shape in the selection.

An inverse kind of exploration is shown in Figure 11. Here we have selected an area with a pre-injection amount greater than certain value. We can clearly see where in the attribute space function graphs with such a characteristics are located. Finally, Figure 4 shows us the cylinder pressure in the function graph view and the working area of the engine is described with the load signal and engine speed in rpm. The load signal is directly proportional to the pressure on the accelerator. The attribute space depicts the domain where the engine's operating point can be while running. A rectangle selection is shown which is a usual way in exploring this kind of data.

The *KennfeldView* tool has been recently demonstrated to a group of mechanical engineers at AVL List GmbH. They had good initial impression about the application and are interested in integrating it into one of AVL's software products.



**Figure 9.** Typical shape of injection curve. Two peaks, smaller pre-injection, and main injection are clearly visible

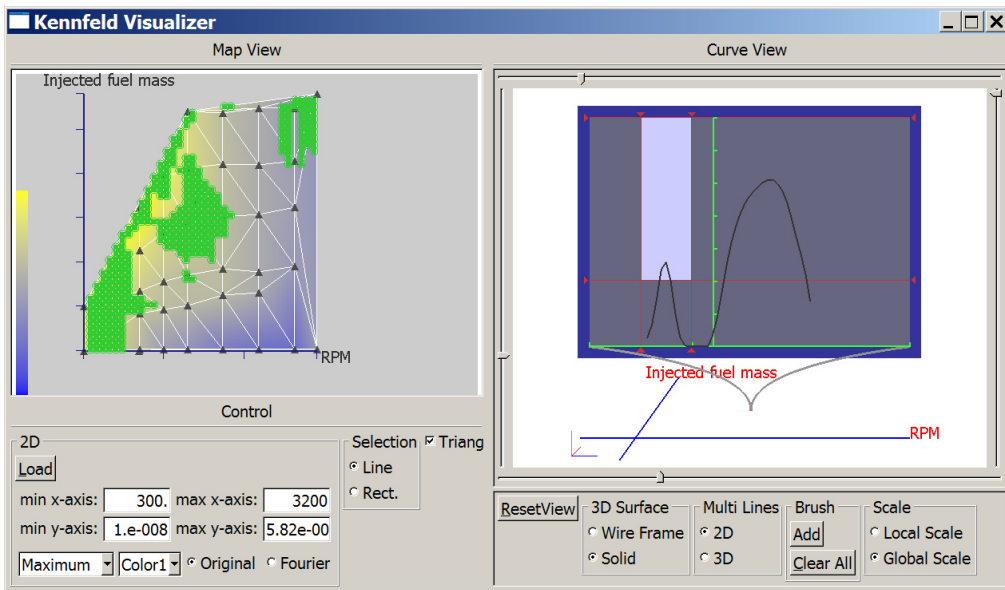


**Figure 10.** Multi line selection in the attribute space with overview and color coded curves displayed in 2D

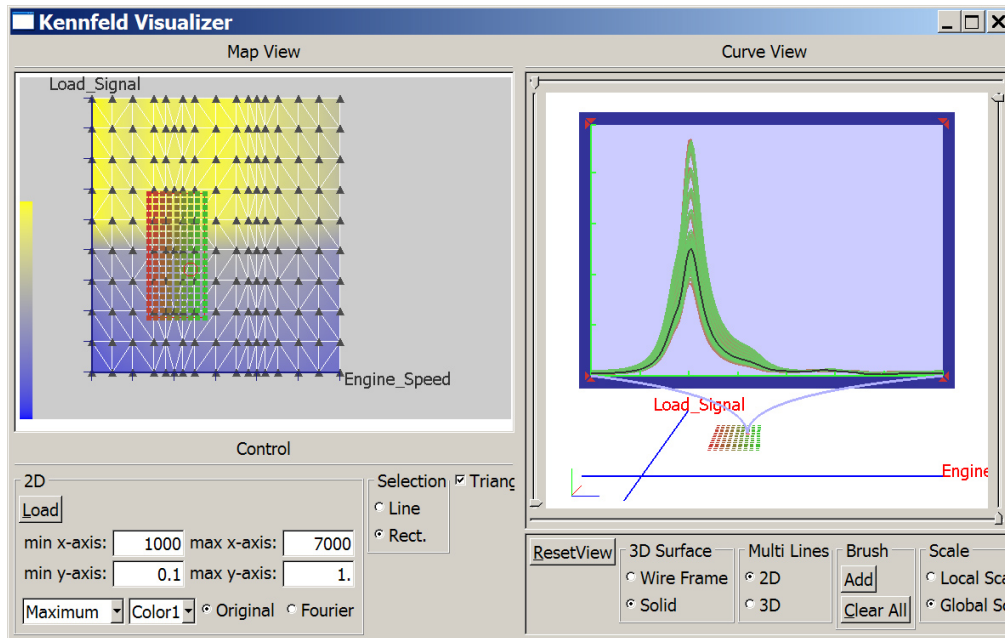
## 5. Summary and conclusion

We have presented a method to visualize a family of function graphs where each of them is associated with point in a 2D domain. Such data sets are ubiquitous, from all kinds of data bound to the map of an area (meteorological data, population, pollution, etc.), to special cases in engineering, for example the working domain of an engine and

various attributes over crank angle at each point. We have suggested an interactive, dual linked view solution consisting of an attribute view and a graph view. Users can browse through the data by simply moving of the pointer over the attribute view and explore data in the graph view. Brushing in the graph view adds additional dimension to the exploration by showing the locations of graphs with certain characteristics on the 2D map. The engineers at AVL deal-



**Figure 11.** Brushing example, the user tries to find areas with pre-injection ratio above certain amount



**Figure 12.** Rectangle selection in attribute space. The curves in this example depict pressure in cylinders over crankshaft angle. The attribute space is defined with load signal (which is proportional to the pressure on the gas pedal) and engine speed in rpm

ing with engine design are pleased with the new interactive proposal which has many advantages over the previously used static 2D charts. *KennfeldView* may evolve to be component of AVL's software suite.

## 6. Acknowledgements

We express special thanks to Andreas Münzer of AVL for supplying the engine characteristic diagram data set. Parts of this work have been carried out in the scope of applied and basic research at the VRVis Research Center which is funded by an Austrian governmental research program called K plus ([www.kplus.at](http://www.kplus.at)).



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