# On the Quality of Optimal Assignment for Data Association

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**Abstract.** In this paper, we present a method based on belief functions to evaluate the quality of the optimal assignment solution of a classical association problem encountered in multiple target tracking applications. The purpose of this work is not to provide a new algorithm for solving the assignment problem, but a solution to estimate the quality of the individual associations (pairings) given in the optimal assignment solution. To the knowledge of authors, this problem has not been addressed so far in the literature and its solution may have practical aspects for improving the performances of multisensor-multitarget tracking systems.

**Keywords:** Data association; PCR6 rule; Belief function.

## 1 Introduction

Efficient algorithms for modern multisensor-multitarget tracking (MS-MTT) systems [1, 2] require to estimate and predict the states (position, velocity, etc) of the targets evolving in the surveillance area covered by the sensors. The estimations and the predictions are based on sensors measurements and dynamical models assumptions. In the monosensor context, MTT requires to solve the data association (DA) problem to associate the available measurements at a given time with the predicted states of the targets to update their tracks using filtering techniques (Kalman filter, Particle filter, etc). In the multisensor MTT context, we need to solve more difficult multi-dimensional assignment problems under constraints. Fortunately, efficient algorithms have been developed in operational research and tracking communities for formalizing and solving these optimal assignments problems. Several approaches based on different models can be used to establish rewards matrix, either based on the probabilistic framework [1, 3], or on the belief function (BF) framework [4–7]. In this paper, we do not focus on the construction of the rewards matrix[1], and our purpose is to provide a method to evaluate the quality (interpreted as a confidence score) of each association (pairing) provided in the optimal solution based on its consistency (stability) with respect to all the second best solutions.

[1] We assume that the rewards matrix is known and has been obtained by a method chosen by the user, either in the probabilistic or in the BF framework.

The simple DA problem under concern can be formulated as follows. We have $m > 1$ targets $T_i$ $(i = 1, \ldots, m)$, and $n > 1$ measurements[2] $z_j$ $(j = 1, \ldots, n)$ at a given time $k$, and a $m \times n$ rewards (gain/payoff) matrix $\boldsymbol{\Omega} = [\omega(i, j)]$ whose elements $\omega(i, j) \geq 0$ represent the payoff (usually homogeneous to the likelihood) of the association of target $T_i$ with measurement $z_j$, denoted $(T_i, z_j)$. The data association problem consists in finding the global optimal assignment of the targets to some measurements by maximizing[3] the overall gain in such a way that no more than one target is assigned to a measurement, and reciprocally.

Without loss of generality, we can assume $\omega(i, j) \geq 0$ because if some elements $\omega(i, j)$ of $\boldsymbol{\Omega}$ were negative, we can always add the same maximal negative value to all elements of $\boldsymbol{\Omega}$ to work with a new payoff matrix $\boldsymbol{\Omega}' = [\omega'(i, j)]$ having all elements $\omega'(i, j) \geq 0$, and we get the same optimal assignment solution with $\boldsymbol{\Omega}$ and with $\boldsymbol{\Omega}'$. Moreover, we can also assume, without loss of generality $m \leq n$, because otherwise we can always swap the roles of targets and measurements in the mathematical problem definition by working directly with $\boldsymbol{\Omega}^t$ instead, where the superscript $t$ denotes the transposition of the matrix. The optimal assignment problem consists of finding the $m \times n$ binary association matrix $\mathbf{A} = [a(i, j)]$ which maximize the global rewards $R(\boldsymbol{\Omega}, \mathbf{A})$ given by

$$R(\boldsymbol{\Omega}, \mathbf{A}) \triangleq \sum_{i=1}^{m} \sum_{j=1}^{n} \omega(i, j) a(i, j) \tag{1}$$

$$\text{Subject to} \quad \begin{cases} \sum_{j=1}^{n} a(i, j) = 1 \quad (i = 1, \ldots, m) \\ \sum_{i=1}^{m} a(i, j) \leq 1 \quad (j = 1, \ldots, n) \\ a(i, j) \in \{0, 1\} \quad (i = 1, \ldots, m \text{ and } j = 1, \ldots, n) \end{cases} \tag{2}$$

The association indicator value $a(i, j) = 1$ means that the corresponding target $T_i$ and measurement $z_j$ are associated, and $a(i, j) = 0$ means that they are not associated $(i = 1, \ldots, m$ and $j = 1, \ldots, n)$.

The solution of the optimal assignment problem stated in (1)–(2) is well reported in the literature and several efficient methods have been developed in the operational research and tracking communities to solve it. The most well-known algorithms are Kuhn-Munkres (a.k.as Hungarian) algorithm [8, 9] and its extension to rectangular matrices proposed by Bourgeois and Lassalle in [10], Jonker-Volgenant method [11], and Auction [12]. More sophisticated methods using Murty's method [13], and some variants [3, 14–19], are also able to provide not only the best assignment, but also the $m$-best assignments. We will not present in details all these classical methods because they have been already well reported in the literature [20, 21], and they are quite easily accessible on the

[2] In a multi-sensor context targets can be replaced by tracks provided by a given tracker associated with a type of sensor, and measurements can be replaced by another tracks set. In different contexts, possible equivalents are assigning personnel to jobs or assigning delivery trucks to locations.

[3] In some problems, $\boldsymbol{\Omega} = [\omega(i, j)]$ represents a cost matrix whose elements are the negative log-likelihood of association hypotheses. In this case, the data association problems consists in finding the best assignment that minimizes the overall cost.

web. In this paper, we want to provide a confidence level (i.e. a quality indicator) in the optimal data association solution. More precisely, we are searching an answer to the question: how to measure the quality of the pairings $a(i,j)=1$ provided in the optimal assignment solution $\mathbf{A}$? The necessity to establish a quality indicator is motivated by the following three main reasons:

1. In some practical tracking environment with the presence of clutter, some association decisions ($a(i,j)=1$) are doubtful. For these unreliable associations, it is better to wait for new information (measurements) instead of applying the hard data association decision, and making potentially serious association mistakes.
2. In some multisensor systems, it can be also important to save energy consumption for preserving a high autonomy capacities of the system. For this goal, only the most trustful specific associations provided in the optimal assignment have to be selected and used instead of all of them.
3. The best optimal assignment solution is not necessarily unique. In such situation, the establishment of quality indicators may help in selecting one particular optimal assignment solution among multiple possible choices.

Before presenting our solution in Section 2, one must recall that the best, as well as the 2nd-best, optimal assignment solutions are unfortunately not necessarily unique. Therefore, we must also take into account the possible multiplicity of assignments in the analysis of the problem. The multiplicity index of the best optimal assignment solution is denoted $\beta_1 \geq 1$, and the multiplicity index of the 2nd-best optimal assignment solution is denoted $\beta_2 \geq 1$, and we will denote the sets of corresponding assignment matrices by $\mathcal{A}_1 = \{\mathbf{A}_1^{(k_1)}, k_1 = 1\ldots,\beta_1\}$ and by $\mathcal{A}_2 = \{\mathbf{A}_2^{(k_2)}, k_2 = 1\ldots,\beta_2\}$. The next simple example illustrates a case with multiplicity of 2nd-best assignment solutions for the reward matrix $\boldsymbol{\Omega}_1$.

**Example**: $\beta_1 = 1$ and $\beta_2 = 4$ (i.e. no multiplicity of $\mathbf{A}_1$ and multiplicity of $\mathbf{A}_2$)

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} 1 & 11 & 45 & 30 \\ 17 & 8 & 38 & 27 \\ 10 & 14 & 35 & 20 \end{bmatrix}$$

This reward matrix provides a unique best assignment $\mathbf{A}_1$ providing $R_1(\boldsymbol{\Omega}_1, \mathbf{A}_1) = 86$, and $\beta_2 = 4$ second-best assignment solutions providing $R_2(\boldsymbol{\Omega}_1, \mathbf{A}_2^{k_2}) = 82$ ($k_2 = 1,2,3,4$) given by

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}_2^{k_2=1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ \mathbf{A}_2^{k_2=2} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ \mathbf{A}_2^{k_2=3} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \ \mathbf{A}_2^{k_2=4} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

## 2 Quality of the Associations of the Optimal Assignment

To establish the quality of the specific associations (pairings) $(i, j)$ satisfying $a_1(i, j) = 1$ belonging to the optimal assignment matrix $\mathbf{A}_1$, we propose to use both $\mathbf{A}_1$ and 2nd-best assignment solution $\mathbf{A}_2$. The basic idea is to compare the values $a_1(i, j)$ with $a_2(i, j)$ obtained in the best and in the 2nd-best assignments to identify the change (if any) of the optimal pairing $(i, j)$. Our quality indicator will depend on both the stability of the pairing and its relative impact in the global reward. The proposed method works also when the 2nd-best assignment solution $\mathbf{A}_2$ is not unique (as in our example). The proposed method will also help to select the best (most trustful) optimal assignment in case of multiplicity of $\mathbf{A}_1$ matrices.

### 2.1 A Simplistic Method (Method I)

Before presenting our sophisticate method based on belief functions, let's first present a simplistic intuitive method (called Method I). For this, let's assume at first that $\mathbf{A}_1$ and $\mathbf{A}_2$ are unique (no multiplicity occurs). The simplistic method uses only the ratio of global rewards $\rho \triangleq R_2(\boldsymbol{\Omega}, \mathbf{A}_2)/R_1(\boldsymbol{\Omega}, \mathbf{A}_1)$ to measure the level of uncertainty in the change (if any) of pairing $(i, j)$ provided in $\mathbf{A}_1$ and $\mathbf{A}_2$. More precisely, the quality (trustfulness) of pairings in an optimal assignment solution $\mathbf{A}_1$, denoted[4] $q_I(i, j)$, is simply defined as follows for $i = 1, \dots, m$ and $j = 2, \dots, n$:

$$q_I(i,j) \triangleq \begin{cases} 1, & \text{if } a_1(i,j) + a_2(i,j) = 0 \\ 1-\rho & \text{if } a_1(i,j) + a_2(i,j) = 1 \\ 1, & \text{if } a_1(i,j) + a_2(i,j) = 2 \end{cases} \tag{3}$$

By adopting such definition, one commits the full confidence to the components $(i, j)$ of $\mathbf{A}_1$ and $\mathbf{A}_2$ that perfectly match, and a lower confidence value (a lower quality) of $1 - \rho$ to those that do not match. To take into account the eventual multiplicities (when $\beta_2 > 1$) of the 2nd-best assignment solutions $\mathbf{A}_2^{k_2}$, $k_2 = 1, 2, \dots, \beta_2$, we need to combine the $\mathbf{Q}_I(\mathbf{A}_1, \mathbf{A}_2^{k_2})$ values. Several methods can be used for this, in particular we can use either:

- **A weighted averaging approach**: The quality indicator component $q_I(i, j)$ is then obtained by averaging the qualities obtained from each comparison of $\mathbf{A}_1$ with $\mathbf{A}_2^{k_2}$. More precisely, one will take:

$$q_I(i,j) \triangleq \sum_{k_2=1}^{\beta_2} w(\mathbf{A}_2^{k_2}) q_I^{k_2}(i,j) \tag{4}$$

  where $q_I^{k_2}(i, j)$ is defined as in (3) (with $a_2(i, j)$ replaced by $a_2^{k_2}(i, j)$ in the formula), and where $w(\mathbf{A}_2^{k_2})$ is a weighting factor in $[0, 1]$, such that $\sum_{k_2=1}^{\beta_2} w(\mathbf{A}_2^{k_2}) = 1$. Since all assignments $\mathbf{A}_2^{k_2}$ have the same global reward value $R_2$, then we suggest to take $w(\mathbf{A}_2^{k_2}) = 1/\beta_2$. A more elaborate method

[4] The subscript I in $q_I(i, j)$ notation refers to Method I.

would consist to use the quality indicator of $\mathbf{A}_2^{k_2}$ based on the 3rd-best solution, which can be itself computed from the quality of the 3rd assignment solution based on the 4th-best solution, and so on by a similar mechanism. We however don't give more details on this due to space constraints.

- **A belief-based approach** (see [22] for basics on belief functions): A second method would express the quality by a belief interval $[q_I^{\min}(i,j), q_I^{\max}(i,j)]$ in $[0,1]$ instead of single real number $q_I(i,j)$ in $[0,1]$. More precisely, one can compute the belief and plausibility bounds of the quality by taking $q_I^{\min}(i,j) \equiv Bel(a_1(i,j)) = \min_{k_2} q_I^{k_2}(i,j)$ and $q_I^{\max}(i,j) \equiv Pl(a_1(i,j)) = \max_{k_2} q_I^{k_2}(i,j)$, with $q_I^{k_2}(i,j)$ given by (3) and $a_2(i,j)$ replaced by $a_2^{k_2}(i,j)$ in the formula. Hence for each association $a_1(i,j)$, one can define a basic belief assignment (BBA) $m_{ij}(.)$ on the frame of discernment $\Theta \triangleq \{T = \text{trustful}, \neg T = \text{not trustful}\}$, which will characterize the quality of the pairing $(i,j)$ in the optimal assignment solution $\mathbf{A}_1$, as follows:

$$\begin{cases} m_{ij}(T) = q_I^{\min}(i,j) \\ m_{ij}(\neg T) = 1 - q_I^{\max}(i,j) \\ m_{ij}(T \cup \neg T) = q_I^{\max}(i,j) - q_I^{\min}(i,j) \end{cases} \tag{5}$$

**Remark**: In practice, only the pairings[5] $(i,j)$ such that $a_1(i,j) = 1$ are useful in tracking algorithms to update the tracks. Therefore, we don't need to pay attention (compute and store) the qualities of components $(i,j)$ such that $a_1(i,j) = 0$.

### 2.2 A More Sophisticate and Efficient Method (Method II)

The previous method can be easily applied in practice but it does not work very well because the quality indicator depends only on the $\rho$ factor, which means that all mismatches between the best assignment $\mathbf{A}_1$ and the 2nd-best assignment solution $\mathbf{A}_2$ have their quality impacted in the same manner (they are all taken as $1-\rho$). As a simple example, if we consider the rewards matrix $\boldsymbol{\Omega}_1$ given in our example, we will have $\rho = R_2(\boldsymbol{\Omega}_1, \mathbf{A}_2^{k_2})/R_1(\boldsymbol{\Omega}_1, \mathbf{A}_1) = 82/86 \approx 0.95$, and we will get using method I with the weighting averaging approach (using same $w(\mathbf{A}_2^{k_2}) = 1/\beta_2 = 0.25$ for $k_2 = 1,2,3,4$) the following quality indicator matrix:

$$\mathbf{Q}_I(\mathbf{A}_1, \mathcal{A}_2) = \frac{1}{\beta_2} \sum_{k_2=1}^{\beta_2} \mathbf{Q}_I(\mathbf{A}_1, \mathbf{A}_2^{k_2}) = \begin{bmatrix} 1.0000 & 1.0000 & \mathbf{0.5233} & 0.5233 \\ 0.5233 & 1.0000 & 0.7616 & \mathbf{0.2849} \\ 0.7616 & \mathbf{0.2849} & 0.7616 & 0.7616 \end{bmatrix} \tag{6}$$

We observe that optimal pairings (2,4) and (3,2) get the same quality value 0.2849 with the method I (based on averaging), even if these pairings have different impacts in the global reward value, which is abnormal. If we use the method I with the belief interval measure based on (5), the situation is worst because the three optimal pairings (1,3), (2,4) and (3,2) will get exactly same belief interval values [0.0465,1]. To take into account, and in a better way, the

[5] given in the optimal solution found for example with Murty's algorithm.

reward values of each specific association given in the best assignment $\mathbf{A}_1$ and in the 2nd-best assignment $\mathbf{A}_2^{k_2}$, we propose to use the following construction of quality indicators depending on the type of matching (called Method II):

- When $a_1(i,j) = a_2^{k_2}(i,j) = 0$, one has full agreement on "non-association" $(T_i, z_j)$ in $\mathbf{A}_1$ and in $\mathbf{A}_2^{k_2}$ and this non-association $(T_i, z_j)$ has no impact on the global rewards values $R_1(\boldsymbol{\Omega}, \mathbf{A}_1)$ and $R_2(\boldsymbol{\Omega}, \mathbf{A}_2^{k_2})$, and it will be useless. Therefore, we can set its quality arbitrarily to $q_{II}^{k_2}(i,j) = 1$.
- When $a_1(i,j) = a_2^{k_2}(i,j) = 1$, one has a full agreement on the association $(T_i, z_j)$ in $\mathbf{A}_1$ and in $\mathbf{A}_2^{k_2}$ and this association $(T_i, z_j)$ has different impacts in the global rewards values $R_1(\boldsymbol{\Omega}, \mathbf{A}_1)$ and $R_2(\boldsymbol{\Omega}, \mathbf{A}_2^{k_2})$. To qualify the quality of this matching association $(T_i, z_j)$, we define the two BBA's on $X \triangleq (T_i, z_j)$ and $X \cup \neg X$ (the ignorance), for $s = 1, 2$:

$$\begin{cases} m_s(X) = a_s(i,j) \cdot \omega(i,j)/R_s(\boldsymbol{\Omega}, \mathbf{A}_s) \\ m_s(X \cup \neg X) = 1 - m_s(X) \end{cases} \tag{7}$$

  Applying the conjunctive rule of fusion, we get

$$\begin{cases} m(X) = m_1(X)m_2(X) + m_1(X)m_2(X \cup \neg X) + m_1(X \cup \neg X)m_2(X) \\ m(X \cup \neg X) = m_1(X \cup \neg X)m_2(X \cup \neg X) \end{cases} \tag{8}$$

  Applying the pignistic transformation[6] [24], we get finally $BetP(X) = m(X) + \frac{1}{2} \cdot m(X \cup \neg X)$ and $BetP(\neg X) = \frac{1}{2} \cdot m(X \cup \neg X)$. Therefore, we choose the quality indicator as $q_{II}^{k_2}(i,j) = BetP(X)$.
- When $a_1(i,j) = 1$ and $a_2^{k_2}(i,j) = 0$, one has a disagreement (conflict) on the association $(T_i, z_j)$ in $\mathbf{A}_1$ and in $(T_i, z_{j_2})$ in $\mathbf{A}_2^{k_2}$, where $j_2$ is the measurement index such that $a_2(i, j_2) = 1$. To qualify the quality of this non-matching association $(T_i, z_j)$, we define the two following basic belief assignments (BBA's) of the propositions $X \triangleq (T_i, z_j)$ and $Y \triangleq (T_i, z_{j_2})$

$$\begin{cases} m_1(X) = a_1(i,j) \cdot \frac{\omega(i,j)}{R_1(\boldsymbol{\Omega}, \mathbf{A}_1)} \\ m_1(X \cup Y) = 1 - m_1(X) \end{cases} \quad \text{and} \quad \begin{cases} m_2(Y) = a_2(i,j_2) \cdot \frac{\omega(i,j_2)}{R_2(\boldsymbol{\Omega}, \mathbf{A}_2^{k_2})} \\ m_2(X \cup Y) = 1 - m_2(Y) \end{cases} \tag{9}$$

  Applying the conjunctive rule, we get $m(X \cap Y = \emptyset) = m_1(X)m_2(Y)$ and

$$\begin{cases} m(X) = m_1(X)m_2(X \cup Y) \\ m(Y) = m_1(X \cup Y)m_2(Y) \\ m(X \cup Y) = m_1(X \cup Y)m_2(X \cup Y) \end{cases} \tag{10}$$

  Because we need to work with a normalized combined BBA, we can choose different rules of combination (Dempster-Shafer's, Dubois-Prade's,Yager's

[6] We have chosen here BetP for its simplicity and because it is widely known, but DSmP could be used instead for expecting better performances [23].

rule [23], etc). In this work, we recommend the Proportional Conflict Redistribution rule no. 6 (PCR6), proposed originally in DSmT framework [23], because it has been proved very efficient in practice. So, we get with PCR6:

$$\begin{cases} m(X) = m_1(X)m_2(X \cup Y) + m_1(X) \cdot \frac{m_1(X)m_2(Y)}{m_1(X)+m_2(Y)} \\ m(Y) = m_1(X \cup Y)m_2(Y) + m_2(X) \cdot \frac{m_1(X)m_2(Y)}{m_1(X)+m_2(Y)} \\ m(X \cup Y) = m_1(X \cup Y)m_2(X \cup Y) \end{cases} \tag{11}$$

Applying the pignistic transformation, we get finally $BetP(X) = m(X) + \frac{1}{2} \cdot m(X \cup Y)$ and $BetP(Y) = m(Y) + \frac{1}{2} \cdot m(X \cup Y)$. Therefore, we choose the quality indicators as follows: $q_{II}^{k_2}(i, j) = BetP(X)$, and $q_{II}^{k_2}(i, j_2) = BetP(Y)$.

The absolute quality factor $Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2})$ of the optimal assignment given in $\mathbf{A}_1$ conditioned by $\mathbf{A}_2^{k_2}$, for any $k_2 \in \{1, 2, \dots, \beta_2\}$ is defined as

$$Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2}) \triangleq \sum_{i=1}^{m} \sum_{j=1}^{n} a_1(i, j) q_{II}^{k_2}(i, j) \tag{12}$$

**Example** (continued): If we apply the Method II (using PCR6 fusion rule) to the rewards matrix $\boldsymbol{\Omega}_1$, then we will get the following quality matrix (using weighted averaging approach)

$$\mathbf{Q}_{II}(\mathbf{A}_1, \mathcal{A}_2) = \frac{1}{\beta_2} \sum_{k_2=1}^{\beta_2} \mathbf{Q}_{II}(\mathbf{A}_1, \mathbf{A}_2^{k_2}) = \begin{bmatrix} 1.0000 & 1.0000 & \mathbf{0.7440} & 0.7022 \\ 0.7200 & 1.0000 & 0.8972 & \mathbf{0.5753} \\ 0.8695 & \mathbf{0.4957} & 0.9119 & 0.8861 \end{bmatrix}$$

with the absolute quality factors $Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2=1}) \approx 1.66$, $Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2=2}) \approx 1.91$, $Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2=3}) \approx 2.19$, $Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2=4}) \approx 1.51$. Naturally, we get

$$Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2=3}) > Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2=2}) > Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2=1}) > Q_{abs}(\mathbf{A}_1, \mathbf{A}_2^{k_2=4})$$

because $\mathbf{A}_1$ has more matching pairings with $\mathbf{A}_2^{k_2=3}$ than with other 2nd-best assignment $\mathbf{A}_2^{k_2}$ ($k_2 \neq 3$), and those pairings have also the strongest impacts in the global reward value. One sees that the quality matrix $\mathbf{Q}_{II}$ differentiates the qualities of each pairing in the optimal assignment $\mathbf{A}_1$ as expected (contrariwise to Method I). Clearly, with Method I we obtain the same quality indicator value 0.2849 for the specific associations (2,4) and (3,2) which seems intuitively not very reasonable because the specific rewards of these associations impact differently the global rewards result. If the method II based on the belief interval measure computed from (5) is preferred[7], we will get respectively for the three optimal pairings (1,3), (2,4) and (3,2) the three distinct belief interval [0.5956,0.8924], [0.4113,0.7699] and [0.3524,0.6529]. These belief intervals show that the ordering of quality of optimal pairings (based either on the lower bound, or on the upper bound of belief interval) is consistent with the ordering of quality of optimal pairings in $\mathbf{Q}_{II}(\mathbf{A}_1, \mathcal{A}_2)$ computed with the averaging approach. Method II provides a better effective and comprehensive solution to estimate the quality of each specific association provided in the optimal assignment solution $\mathbf{A}_1$.

[7] just in case of multiplicity of second best assignments.

## 3 Conclusion

In this paper we have proposed a method based on belief functions for establishing the quality of pairings belonging to the optimal data association (or assignment) solution provided by a chosen algorithm. Our method is independent of the choice of the algorithm used in finding the optimal assignment solution, and, in case of multiple optimal solutions, it provides also a way to select the best optimal assignment solution (the one having the highest absolute quality factor). The method developed in this paper is general in the sense that it can be applied to different types of association problems corresponding to different sets of constraints. This method can be extended to SD-assignment problems. The application of this approach in a realistic multi-target tracking context is under investigations and will be reported in a forthcoming publication if possible.

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